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مجلّه‌ی پژوهش‌های آماری ایران

J. Statist. Res. Iran **5** (2008): 221–233

دوره‌ی ۵، شماره‌ی ۲، پاییز و زمستان ۱۳۸۷، صص ۲۲۱–۲۳۳

Point and Interval Estimation for the Burr Type III Distribution

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Abstract. In this paper, we study the estimation problems for the Burr type III distribution based on a complete sample. The maximum likelihood method is used to derive the point estimators of the parameters. An exact confidence interval and an exact joint confidence region for the parameters are constructed. Two numerical examples with real data set and simulated data, are presented to illustrate the methods proposed here.

Keywords. Burr type III distribution; confidence interval; joint confidence region; maximum likelihood estimation.

1 Introduction

Burr (1942) introduced 12 different forms of cumulative distribution functions for modeling lifetime data or survival data. Two members of the family, the Burr types III and XII are important because they are inherently more flexible than the Weibull. Both the Burr types III and XII cover a much larger area of the skewness kurtosis plane than the Weibull (Rodriguez, 1977; Tadikamalla, 1980; Fry, 1993; Lindsay et al., 1996), with the type III being the most flexible of the three. In this paper, we therefore restrict attention to the type III Burr. The Burr type III distribution has been introduced to forestry by Lindsay et al. (1996). This distribution has been used in other fields and goes by various synonyms. For example, in economics it is known as the Dagum distribution (Dagum, 1977), and McDonald (1984) noted that it was related to the generalized beta of the second kind. Mielke (1973) and

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Mielke and Johnson (1974) have used this distribution in meteorology and water resources applications and shown it to be a special case of their Kappa distribution (Mielke and Johnson, 1973; Tadikamalla, 1980).

The probability density function (pdf) and cumulative distribution function (cdf) of the two-parameter Burr type III distribution are given, respectively, by

$$F(x;\theta,c) = (1+x^{-c})^{-\theta}, \qquad x>0,\ \theta>0,\ c>0, \tag{1}$$

and

$$f(x;\theta,c) = \theta c x^{-(c+1)}(1+x^{-c})^{-(\theta+1)} \qquad x>0,\ \theta>0,\ c>0. \tag{2}$$

The joint confidence regions based on complete and censored samples have discussed for a wide array of distributions by many authors, including Chen (1996, 1998), Kus and Kaya (2007), Wu et al. (2007), and Wu (2008). The main purpose of this paper is to construct the exact confidence interval and exact joint confidence region for the two parameters c and θ of the Burr type III distribution. We obtain the maximum likelihood estimators of the parameters in section 2. In section 3, we derive an exact confidence interval for the parameter c and an exact joint confidence region for the parameters c and θ. Two numerical examples are presented to illustrate the methods proposed in section 4.

2 Point Estimation

In this section, the maximum likelihood estimators (MLEs) of the parameters of the Burr type III distribution are obtained.

Suppose that $X_1,\ldots,X_n$ be a sample of size n from the Burr type III distribution in (1). The likelihood function is given by

$$L(\theta,c) = \prod_{i=1}^{n} f(x_i;\theta,c) = (\theta c)^n \prod_{i=1}^{n} x_i^{-(c+1)}(1+x_i^{-c})^{-(\theta+1)}. \tag{3}$$

The log-likelihood function can be written as

$$\ln L(\theta,c) = n\ln\theta + n\ln c - (c+1)\sum_{i=1}^{n}\ln x_i - (\theta+1)\sum_{i=1}^{n}\ln(1+x_i^{-c}), \tag{4}$$

and hence we derive the likelihood equations for θ and c as

$$\frac{\partial \ln L(\theta, c)}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^{n} \ln(1 + x_i^{-c}) = 0, \tag{5}$$

and

$$\frac{\partial \ln L(\theta, c)}{\partial c} = \frac{n}{c} - \sum_{i=1}^{n} \ln x_i + (\theta + 1) \sum_{i=1}^{n} \frac{x_i^{-c} \ln x_i}{1 + x_i^{-c}} = 0. \tag{6}$$

From (5), we obtain the MLE of θ as a function of c, say $\hat{\theta}(c)$, as

$$\hat{\theta}(c) = \frac{n}{\sum_{i=1}^{n} \ln(1 + x_i^{-c})}. \tag{7}$$

Substituting $\hat{\theta}(c)$ in (4), we obtain the profile log-likelihood of c as

$$\begin{aligned} g(c) &= \ln L\{\hat{\theta}(c), c\} \\ &= K + n \ln c - (c+1) \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \ln(1 + x_i^{-c}) \\ &\quad - n \ln \left[\sum_{i=1}^{n} \ln(1 + x_i^{-c}) \right], \end{aligned} \tag{8}$$

where K is a constant. Therefore, the MLE of c, say $\widehat{c}_{MLE}$, can be obtained by maximizing (8) with respect to c. It can be shown that the maximum of (8) can be obtained as a fixed point solution of the following equation

$$h(c) = c, \tag{9}$$

where, $h(c)$ is given by

$$h(c) = \left[-\frac{1}{n} \sum_{i=1}^{n} \frac{\ln x_i}{1 + x_i^{-c}} - \frac{\sum_{i=1}^{n} \frac{x_i^{-c} \ln x_i}{1 + x_i^{-c}}}{\sum_{i=1}^{n} \ln(1 + x_i^{-c})} \right]^{-1}. \tag{10}$$

We apply iterative procedure to find the solution of (9). Once $\hat{c}_{MLE}$ is obtained, the MLE of θ, say $\hat{\theta}_{MLE}$, can be obtained from (7) as $\hat{\theta}_{MLE} = \hat{\theta}(\hat{c}_{MLE})$.

3 Interval Estimation

In this section, an exact confidence interval for c and an exact joint confidence region for c and θ are constructed.

3.1 Confidence Interval for c

Let $X_1, X_2, \ldots, X_n$ be a sample of size n from a two-parameter Burr type III distribution, and $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the corresponding order statistics. Further, let $Y_{(i)} = -\ln F[X_{(i)}] = \theta \ln[1 + X_{(i)}^{-c}]$, $i = 1, \ldots, n$. It can be seen that $Y_{(n)} < Y_{(n-1)} < \cdots < Y_{(1)}$ is the order statistics from the standard exponential distribution. Now let us consider the following transformation:

$$\begin{cases} S_1 = nY_{(n)} \\ S_2 = (n-1)(Y_{(n-1)} - Y_{(n)}) \\ S_3 = (n-2)(Y_{(n-2)} - Y_{(n-1)}) \\ \vdots \\ S_n = Y_{(1)} - Y_{(2)}. \end{cases} \tag{11}$$

It is easy to show that the spacing $S_1, S_2, \ldots, S_n$, as defined in (11), are independent and identically distributed as a standard exponential distribution. Hence,

$$V = 2S_1 = 2nY_{(n)},$$

has a chi-square distribution with 2 degrees of freedom and

$$U = 2\sum_{i=2}^{n} S_i = 2\left(\sum_{i=1}^{n} Y_{(i)} - nY_{(n)}\right),$$

has a chi-square distribution with $2n-2$ degrees of freedom. It is also clear that U and V are independent random variables. Let

$$T_1 = \frac{U/(2n-2)}{V/2} = \frac{U}{(n-1)V} = \frac{\sum_{i=1}^{n} Y_{(i)} - nY_{(n)}}{n(n-1)Y_{(n)}}, \tag{12}$$

and

$$T_2 = U + V = 2\sum_{i=1}^{n} Y_{(i)}. \tag{13}$$

It is easy to show that T_1 has an F distribution with $2n-2$ and 2 degrees of freedom and T_2 has a chi-square distribution with $2n$ degrees of freedom. Furthermore, T_1 and T_2 are independent, see Johnson et al. (1994, P. 350).

To derive the exact confidence interval for c and the exact joint confidence region for c and θ, we need the following lemma.

Lemma 4 *Based on the observed order statistics $x_{(1)} < \cdots < x_{(n)}$, suppose that*

$$T_1(c) = \frac{\sum_{i=1}^{n} \ln\left\{1 + x_{(i)}^{-c}\right\} - n\ln\left\{1 + x_{(n)}^{-c}\right\}}{n(n-1)\ln(1 + x_{(n)}^{-c})}.$$

Then

(1) $T_1(c)$ is strictly increasing in c for any $c > 0$.

(2) For $x_{(n)} \geqslant 1$ and any $t > 0$, the equation $T_1(c) = t$ has a unique solution for any $c > 0$.

(3) For $x_{(n)} < 1$ and any $0 < t < \frac{1}{n(n-1)}\sum_{i=1}^{n-1}\frac{\ln x_{(i)}}{\ln x_{(n)}} - \frac{1}{n}$, the equation $T_1(c) = t$ has a unique solution for some $c > 0$.

Proof. (1) The function $T_1(c)$ can be written as

$$T_1(c) = \frac{1}{n(n-1)}\sum_{i=1}^{n-1}\frac{\ln\left\{1 + x_{(i)}^{-c}\right\}}{\ln\left\{1 + x_{(n)}^{-c}\right\}} - \frac{1}{n}.$$

By using Lemma 1 from Wu et al. (2007), $\ln\{1 + x_{(i)}^{-c}\}/\ln\{1 + x_{(n)}^{-c}\}$ is a strictly increasing function of c for any $x_{(i)} < x_{(n)}$; $i = 1, 2, \ldots, n-1$, hence $T_1(c)$ is strictly increasing in c.

(2) For $x_{(n)} \geqslant 1$, note that the function $T_1(c)$ is strictly increasing in c that

$$\lim_{c\to 0} T_1(c) = 0,$$

and that

$$\lim_{c\to\infty} T_1(c) = \infty.$$

Thus, if $t > 0$, $T_1(c) = t$ has a unique solution for any $c > 0$.

(3) For $x_{(n)} < 1$, note that the function $T_1(c)$ is strictly increasing in c that

$$\lim_{c\to 0} T_1(c) = 0\ ,$$

and that

$$\lim_{c\to\infty} T_1(c) = \frac{1}{n(n-1)}\sum_{i=1}^{n-1}\frac{\ln x_{(i)}}{\ln x_{(n)}} - \frac{1}{n}.$$

Then, for $0 < t < \frac{1}{n(n-1)}\sum_{i=1}^{n-1}\frac{\ln x_{(i)}}{\ln x_{(n)}} - \frac{1}{n}$, the solution of the equation $T_1(c) = t$ has a unique solution for some $c > 0$.

Let $F_{(\alpha)(\upsilon_1,\upsilon_2)}$ be the percentile of F distribution with right-tail probability α and υ_1 and υ_2 degrees of freedom. An exact confidence interval for the parameter c is given in the following theorem.

Theorem 13 *Suppose that* $X_{(i)},\ i = 1, 2, \ldots, n$, *are order statistics from the Burr type III distribution. Then, for any* $0 < \alpha < 1$,

$$\left(\varphi\left(X_{(1)}, \ldots, X_{(n)}, F_{(1-\frac{\alpha}{2})(2n-2,2)}\right),\ \varphi\left(X_{(1)}, \ldots, X_{(n)}, F_{(\frac{\alpha}{2})(2n-2,2)}\right)\right)$$

is a $100(1-\alpha)\%$ *confidence interval for* c, *where* $\varphi(X_{(1)}, \ldots, X_{(n)}, t)$ *is the solution of* c *for the equation*

$$\frac{\sum_{i=1}^{n}\ln\left\{1 + X_{(i)}^{-c}\right\} - n\ln\left\{1 + X_{(n)}^{-c}\right\}}{n(n-1)\ln\left\{1 + X_{(n)}^{-c}\right\}} = t.$$

Proof. From (12), we know that the pivot

$$\begin{aligned} T_1(c) &= \frac{\sum_{i=1}^{n} Y_{(i)} - nY_{(n)}}{n(n-1)Y_{(n)}} \\ &= \frac{\sum_{i=1}^{n}\ln\left\{1 + X_{(i)}^{-c}\right\} - n\ln\left\{1 + X_{(n)}^{-c}\right\}}{n(n-1)\ln\left\{1 + X_{(n)}^{-c}\right\}}, \end{aligned}$$

has an F distribution with $2n-2$ and 2 degrees of freedom. By Lemma 1, T_1 is strictly increasing in c and hence $T_1(c) = t$ has a unique solution for any $c > 0$. Therefore, for $0 < \alpha < 1$, the event

$$F_{(1-\frac{\alpha}{2})(2n-2,2)} < \frac{\sum_{i=1}^{n}\ln\left\{1 + X_{(i)}^{-c}\right\} - n\ln\left\{1 + X_{(n)}^{-c}\right\}}{n(n-1)\ln\left\{1 + X_{(n)}^{-c}\right\}} < F_{(\frac{\alpha}{2})(2n-2,2)},$$

is equivalent to the event

$$\varphi\left(X_{(1)},\ldots,X_{(n)},F_{(1-\frac{\alpha}{2})(2n-2,2)}\right) < c < \varphi\left(X_{(1)},\ldots,X_{(n)},F_{(\frac{\alpha}{2})(2n-2,2)}\right).$$

It should be mentioned here that we can also use $T_1(c)$ to test null hypothesis $H_0: c = c_0$.

3.2 Joint Confidence Region for (c, θ)

Let us now discuss the joint confidence region for the parameters c and θ. Let $\chi^2_{(\alpha,\upsilon)}$ denote the percentile of χ^2 distribution with right-tail probability α and υ degrees of freedom. An exact joint confidence region for (c,θ) is given in the following theorem.

Theorem 14 *Suppose that $X_{(i)},\ i = 1, 2, \ldots, n$, are order statistics from the Burr type III distribution. Then, a $100(1-\alpha)\%$ joint confidence region for c and θ is determined by the following inequalities:*

$$\varphi\left(X_{(1)},\ldots,X_{(n)},F_{\left(\frac{1+\sqrt{1-\alpha}}{2}\right)(2n-2,2)}\right) < c < \varphi\left(X_{(1)},\ldots,X_{(n)},F_{\left(\frac{1-\sqrt{1-\alpha}}{2}\right)(2n-2,2)}\right),$$

$$\frac{\chi^2_{\left(\frac{1+\sqrt{1-\alpha}}{2},2n\right)}}{2\sum_{i=1}^n \ln\left\{1+X_{(i)}^{-c}\right\}} < \theta < \frac{\chi^2_{\left(\frac{1-\sqrt{1-\alpha}}{2},2n\right)}}{2\sum_{i=1}^n \ln\left\{1+X_{(i)}^{-c}\right\}},$$

where $0 < \alpha < 1$, and $\varphi(X_{(1)},\ldots,X_{(n)},t)$ is the solution of c for the equation

$$\frac{\sum_{i=1}^n \ln\left\{1+X_{(i)}^{-c}\right\} - n\ln\left\{1+X_{(n)}^{-c}\right\}}{n(n-1)\ln\left\{1+X_{(n)}^{-c}\right\}} = t.$$

Proof. From (13), we know that

$$T_2 = 2\sum_{i=1}^n Y_i = 2\ \theta\sum_{i=1}^n \ln(1+X_{(i)}^{-c}),$$

has a χ^2 distribution with $2n$ degrees of freedom, and it is independent of T_1. Hence, for $0 < \alpha < 1$, we have

$$P[F_{\left(\frac{1+\sqrt{1-\alpha}}{2}\right)(2n-2,2)} < T_1 < F_{\left(\frac{1-\sqrt{1-\alpha}}{2}\right)(2n-2,2)}] = \sqrt{1-\alpha},$$

and

$$P\left[\chi^2_{\left(\frac{1+\sqrt{1-\alpha}}{2},2n\right)} < T_2 < \chi^2_{\left(\frac{1-\sqrt{1-\alpha}}{2},2n\right)}\right] = \sqrt{1-\alpha}.$$

From these relationships, we conclude that

$$\begin{aligned} P\Bigg[& F_{\left(\frac{1+\sqrt{1-\alpha}}{2}\right)(2n-2,2)} < T_1 < F_{\left(\frac{1-\sqrt{1-\alpha}}{2}\right)(2n-2,2)}, \\ & \chi^2_{\left(\frac{1+\sqrt{1-\alpha}}{2},2n\right)} < T_2 < \chi^2_{\left(\frac{1-\sqrt{1-\alpha}}{2},2n\right)}\Bigg] \\ & = 1-\alpha, \end{aligned}$$

or equivalently

$$\begin{aligned} P\Bigg[& \varphi\left(X_{(1)},\ldots,X_{(n)},F_{\left(\frac{1+\sqrt{1-\alpha}}{2}\right)(2n-2,2)}\right) \\ & < c < \varphi\left(X_{(1)},\ldots,X_{(n)},F_{\left(\frac{1-\sqrt{1-\alpha}}{2}\right)(2n-2,2)}\right), \\ & \frac{\chi^2_{\left(\frac{1+\sqrt{1-\alpha}}{2},2n\right)}}{2\sum_{i=1}^n \ln\left\{1+X_{(i)}^{-c}\right\}} < \theta < \frac{\chi^2_{\left(\frac{1-\sqrt{1-\alpha}}{2},2n\right)}}{2\sum_{i=1}^n \ln\left\{1+X_{(i)}^{-c}\right\}}\Bigg] \\ & = 1-\alpha. \end{aligned}$$

This completes the proof.

4 Illustrative Example

In this section, we consider the two following examples to illustrate the use of the estimation methods proposed in this paper.

4.1 Example 1. (Real life data)

In this example we consider one real life data set to illustrate the proposed methods of estimation. These data are taken from Badar and Priest (1982), and have been used earlier by Alkasasbeh and Raqab (2009). The data given represent the strength measured in GPA for single carbon fibers of 10 mm in gauge lengths with sample size 63 and they are as follows:

1.901	2.132	2.203	2.228	2.257	2.350	2.361	2.396	2.397	2.445	2.454
2.474	2.518	2.522	2.525	2.532	2.575	2.614	2.616	2.618	2.624	2.659
2.675	2.738	2.740	2.856	2.917	2.928	2.937	2.937	2.977	2.996	3.030
3.125	3.139	3.145	3.220	3.223	3.235	3.243	3.264	3.272	3.294	3.332
3.346	3.377	3.408	3.435	3.493	3.501	3.537	3.554	3.562	3.628	3.852
3.871	3.886	3.971	4.024	4.027	4.225	4.395	5.020.			

Alkasasbeh and Raqab (2009) showed that the generalized logistic distribution (GL) with cdf anf pdf given by

$$G(y;c,\theta)=\left(1+e^{-cy}\right)^{-\theta}, \qquad -\infty<y<\infty,\ c>0,\ \theta>0,$$

$$g(y;c,\theta)=c\,\theta\left(1+e^{-cy}\right)^{-\theta-1}e^{-cy}, \qquad -\infty<y<\infty,\ c>0,\ \theta>0,$$

provides a very good fit to the given data set.

We know that if Y has a GL distribution with parameters c and θ, then $X=e^Y$ has a Burr type III distribution with parameters c and θ. Hence, our method of estimation can be applied to estimate the parameters of c and θ of the GL distribution. Now, we transform the above data to Burr type III form by the transformation $X=e^Y$. Thus, we have the following observations from the Burr type III distribution:

6.693	8.432	9.052	9.281	9.554	10.486	10.602	10.979	10.990
11.531	11.635	11.870	12.404	12.453	12.491	12.579	13.131	13.654
13.681	13.708	13.791	14.282	14.512	15.456	15.487	17.392	18.486
18.690	18.859	18.859	19.629	20.005	20.697	22.760	23.081	23.220
25.028	25.103	25.406	25.610	26.154	26.364	26.950	27.994	28.389
29.283	30.205	31.031	32.884	33.149	34.364	34.953	35.234	37.637
47.087	47.990	48.716	53.038	55.924	56.092	68.374	81.045	151.41.

Using the formula described in section 2, we obtain the MLEs of the parameters c and θ to be $\hat{c}=1.956$ and $\hat{\theta}=225.862$, respectively. To find a 95% confidence interval for c, we need the following percentiles:

$$F_{0.025(124,2)}=39.490, \qquad F_{0.975(124,2)}=0.263.$$

By Theorem 1 and using the S-PLUS package, the 95% confidence interval for c is $(1.320, 3.278)$.

To obtain a 95% joint confidence region for c and θ, we need the following percentiles:

$$F_{0.0127(124,2)} = 78.231, \qquad F_{0.9873(124,2)} = 0.221,$$

and

$$\chi^2_{0.0127(126)} = 164.113, \qquad \chi^2_{0.9873(126)} = 93.208.$$

By Theorem 2 and using the S-PLUS package for solving non-linear equation, the 95% joint confidence region for c and θ is determined by the following inequalities:

$$1.252 < c < 3.534,$$

$$\frac{93.208}{2\sum_{i=1}^{n} \ln\{1 + x_{(i)}^{-c}\}} < \theta < \frac{164.113}{2\sum_{i=1}^{n} \ln\{1 + x_{(i)}^{-c}\}}.$$

Figure 1 shows the 95% joint confidence region for c and θ.

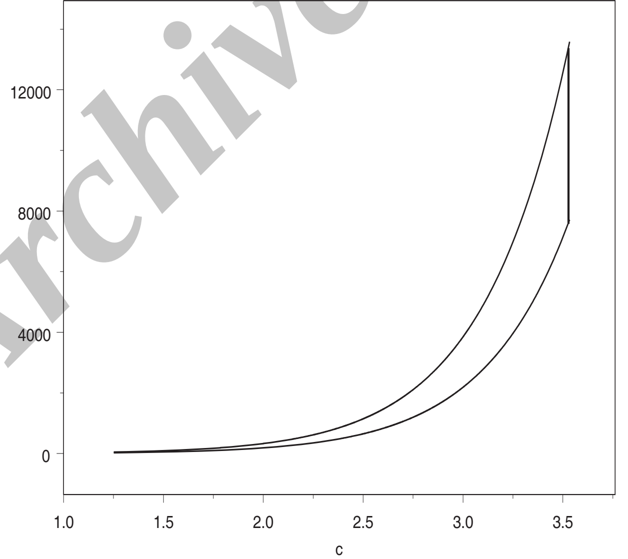

Figure 7. Joint confidence region for c and θ in Example 1.

4.2 Example 2. (Simulated data)

In this example we consider a simulated sample of size $n = 19$ from the Burr type III distribution in (1) with parameters $c = 1.00$ and $\theta = 1.50$. The simulated observations are as follows:

0.0533	0.3975	0.4059	0.5929	0.6592	0.7000	0.8860	0.9465
1.1085	1.2463	1.2975	2.6294	3.0943	5.4194	7.3911	11.2578
15.3966	18.2460	19.1422	25.2919				

Using the formula described in section 2, we obtain the MLEs of the parameters c and θ to be $\hat{c} = 0.944$ and $\hat{\theta} = 1.588$, respectively. To find a 95% confidence interval for c, and a joint confidence region, we need the percentiles:

$$F_{0.025(38,2)} = 39.472, \qquad F_{0.975(38,2)} = 0.247,$$

$$F_{0.0127(38,2)} = 78.213, \qquad F_{0.9873(38,2)} = 0.204,$$

and

$$\chi^2_{0.0127(40)} = 62.591, \qquad \chi^2_{0.9873(40)} = 22.714.$$

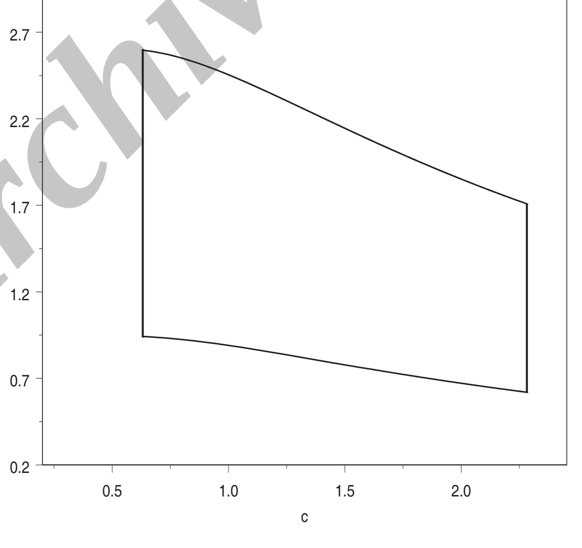

Figure 8. Joint confidence region for c and θ in Example 2.

By Theorem 1 and using the S-PLUS package for solving non-linear equation, the 95% confidence interval for c is $(0.676, 2.093)$. By Theorem 2, the 95% joint confidence region for c and θ is determined by the following inequalities:

$$0.627 < c < 2.288,$$
$$\frac{22.714}{2\sum_{i=1}^{n}\ln\{1+x_{(i)}^{-c}\}} < \theta < \frac{62.591}{2\sum_{i=1}^{n}\ln\{1+x_{(i)}^{-c}\}}.$$

Figure 2 shows the 95% joint confidence region for c and θ.

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