

Recurrence Relations for Single and Product Moments of Generalized Order Statistics from p th Order Exponential Distribution and its Characterization

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Abstract. In this paper, we establish some recurrence relations for single and product moments of generalized order statistics from p th order exponential distribution. Further the results are deduced for the recurrence relations of record values and ordinary order statistics and using a recurrence relation for single moments we obtain characterization of p th order exponential distribution.

Keywords. Generalized order statistics; order statistics; record values; single moment; product moment; recurrence relations; p th order exponential distribution; characterization.

MSC 2010: 62E10, 62E15.

1 Introduction

A random variable X is said to have an p th order exponential distribution if its probability density function (*pdf*) is given by

$$f(x) = (a_0 + a_1x + a_2x^2 + \cdots + a_px^p) e^{-\left(a_0x + a_1\frac{x^2}{2} + a_2\frac{x^3}{3} + \cdots + a_p\frac{x^{p+1}}{p+1}\right)}$$

$$x \geq 0, \quad a_i > 0, \quad i = 1, 2, \dots, p \quad (1)$$

and the cumulative distribution function (*cdf*) is

$$F(x) = 1 - e^{-\left(a_0x + a_1\frac{x^2}{2} + a_2\frac{x^3}{3} + \cdots + a_p\frac{x^{p+1}}{p+1}\right)}, \quad (2)$$

where p is some positive integer. For more details on this distribution and its application one may refer to Kamal (2010).

Kamps (1995) introduced the concept of generalized order statistics (*gos*) as follows: Let X_1, X_2, \dots be a sequence of independent and identically distributed (*iid*) random variables ($r\nu$) with absolutely continuous *cdf* $F(x)$ and *pdf* $f(x)$, $x \in (\alpha, \beta)$. Let $n \in \mathbb{N}$, $n \geq 2$, $\tilde{m} = (m_1, m_2, \dots, m_{n-1}) \in \mathbb{R}^{n-1}$, $k > 0$, be the parameters such that

$$\gamma_r = k + (n - r) + M_r > 0 \quad \text{for all } r \in \{1, 2, \dots, n - 1\}$$

where $M_r = \sum_{j=r}^{n-1} m_j$. Then $X(r, n, \tilde{m}, k)$, $r = 1, 2, \dots, n$ are called if their joint *pdf* is given by

$$k \left(\prod_{j=1}^{n-1} \gamma_j \right) \left[\prod_{i=1}^{n-1} \{1 - F(x_i)\}^{m_i} f(x_i) \right] \{1 - F(x_n)\}^{k-1} f(x_n) \quad (3)$$

on the cone $F^{-1}(0) < x_1 \leq \dots \leq x_n < F^{-1}(1)$.

The model of *gos* contains as special cases, order statistics, record values, sequential order statistics, Stigler's order statistics.

Here we assume two cases:

Case I: $m_1 = m_2 = \dots = m_{n-1} = m$;

Case II: $m_i \neq m_j$, $\gamma_i \neq \gamma_j$, $i, j = 1, 2, \dots, n - 1$.

For Case I, *gos* will be denoted as $X(r, n, m, k)$ with its *pdf* (Kamps, 1995)

$$f_r(x) = \frac{C_{r-1}}{(r-1)!} \{\bar{F}(x)\}^{\gamma_{r-1}} f(x) [g_m \{F(x)\}]^{r-1} \quad (4)$$

and the joint *pdf* of $X(r, n, m, k)$ and $X(s, n, m, k)$, $1 \leq r < s \leq n$, is

$$f_{rs}(x, y) = \frac{C_{s-1}}{(r-1)!(s-r-1)!} \{\bar{F}(x)\}^m [g_m \{F(x)\}]^{r-1} \\ \times [h_m \{F(y)\} - h_m \{F(x)\}]^{s-r-1} \{\bar{F}(y)\}^{\gamma_{s-1}} f(x)f(y), \quad x < y \quad (5)$$

where

$$\bar{F}(x) = 1 - F(x), \quad C_{r-1} = \prod_{i=1}^r \gamma_i,$$

$$h_m(x) = \begin{cases} -\frac{1}{m+1}(1-x)^{m+1}, & m \neq -1 \\ -\ln(1-x), & m = -1 \end{cases}$$

and

$$g_m(x) = h_m(x) - h_m(0), \quad x \in [0, 1).$$

For Case II, the *pdf* of $X(r, n, \tilde{m}, k)$ is (Kamps and Cramer, 2001)

$$f_{X(r, n, \tilde{m}, k)}(x) = C_{r-1} f(x) \sum_{i=1}^r a_i(r) \{\bar{F}(x)\}^{\gamma_i-1} \quad (6)$$

and the joint *pdf* of $X(r, n, \tilde{m}, k)$ and $X(s, n, \tilde{m}, k)$, $1 \leq r < s \leq n$, is

$$f_{X(r, n, \tilde{m}, k), X(s, n, \tilde{m}, k)}(x, y) = C_{s-1} \sum_{i=r+1}^s a_i^{(r)}(s) \left\{ \frac{\bar{F}(y)}{\bar{F}(x)} \right\}^{\gamma_i}$$

$$\times \left[\sum_{i=1}^r a_i(r) \{\bar{F}(x)\}^{\gamma_i} \right] \frac{f(x)}{\bar{F}(x)} \cdot \frac{f(y)}{\bar{F}(y)}, \quad x < y \quad (7)$$

where

$$a_i(r) = \prod_{\substack{j=1 \\ j \neq i}}^r \frac{1}{(\gamma_j - \gamma_i)}, \quad \gamma_j \neq \gamma_i, \quad 1 \leq i \leq r \leq n,$$

$$a_i^{(r)}(s) = \prod_{\substack{j=r+1 \\ j \neq i}}^s \frac{1}{(\gamma_j - \gamma_i)}, \quad \gamma_j \neq \gamma_i, \quad r+1 \leq i \leq s \leq n.$$

Now since $\lim_{m \rightarrow -1} h_m(x) = \log\left(\frac{1}{1-x}\right)$, therefore, we will consider only the case $h_m(x) = -\frac{1}{m+1}(1-x)^{m+1}$ for all m , unless needed otherwise.

Several authors utilized the *gos* in their work, such as Kamps and Gather (1997), Keseling (1999), Cramer and Kamps (2000), Ahsanullah (2000), Pawlas and Szynal (2001), Ahmed and Fawzy (2003), Ahmed (2007), Khan, et al. (2007) among others. Kamps (1998) investigated the importance of

recurrence relations of order statistics in characterization. Recurrence relations for moments of k -records were investigated, among others, by Grudzien and Szynal (1997), Pawlas and Szynal (1998, 1999).

In this paper, we establish recurrence relations for single and product moments of gos from p th order exponential distribution. Result for order statistics and record values can be deduced as special cases from gos and characterization of this distribution has been obtained on using a recurrence relation for single moments.

2 Recurrence Relations for Single Moments

Note that for p th order exponential distribution defined in (1)

$$f(x) = \left(\sum_{u=0}^p a_u x^u \right) \bar{F}(x). \quad (8)$$

Recurrence relations for single moments of gos from df (2) can be derived in the following theorem.

Case I: $m_i = m_g = m$, $i, j = 1, 2, \dots, n-1$.

Theorem 1. For the distribution given in (2) and $n \in \mathbb{N}$, $m \in \mathfrak{R}$, $r = 1$

$$E \{X^j(1, n, m, k)\} = \sum_{u=0}^p \frac{\gamma_1 a_u}{(j+u+1)} E \{X^{j+u+1}(1, n, m, k)\} \quad (9)$$

and for $2 \leq r \leq n$

$$E \{X^j(r, n, m, k)\} = \sum_{u=0}^p \frac{\gamma_r a_u}{(j+u+1)} [E \{X^{j+u+1}(r, n, m, k)\} - E \{X^{j+u+1}(r-1, n, m, k)\}] \quad (10)$$

Proof. From (4) and (8), we have

$$E \{X^j(r, n, m, k)\} = \sum_{u=0}^p a_u \frac{C_{r-1}}{(r-1)!} I_r(x), \quad (11)$$

where

$$I_r(x) = \int_0^\infty x^{j+u} \{\bar{F}(x)\}^{\gamma_r} g_m^{r-1} \{F(x)\} dx.$$

Integrating by parts treating x^{j+u} for integration and the rest of the integrand for differentiation, we get

$$I_r(x) = \frac{\gamma_r}{(j+u+1)} \int_0^\infty x^{j+u+1} \{\bar{F}(x)\}^{\gamma_r-1} f(x) g_m^{r-1} \{F(x)\} dx \\ - \frac{(r-1)}{(j+u+1)} \int_0^\infty x^{j+u+1} \{\bar{F}(x)\}^{\gamma_r+m} f(x) g_m^{r-2} \{F(x)\} dx.$$

Now substituting for $I_1(x)$ and $I_r(x)$ in equation (11), we derive the relations in (9) and (10). \square

Remark 1. Setting $m = 0$, $k = 1$ in Theorem 1, we obtain recurrence relations for single moments of order statistics of the p th order exponential distribution in the form

$$E(X_{1:n}^j) = n \sum_{u=0}^p \frac{a_u}{(j+u+1)} E(X_{1:n}^{j+u+1}) \quad (12)$$

and

$$E(X_{r:n}^j) = (n-r+1) \sum_{u=0}^p \frac{a_u}{(j+u+1)} \left\{ E(X_{r:n}^{j+u+1}) - E(X_{r-1:n}^{j+u+1}) \right\}. \quad (13)$$

These result was obtained by Kamal (2010).

Remark 2. Putting $m = -1$, $k \geq 1$ in (10), we get the recurrence relations for single moments of upper k -records of the p th order exponential distribution in the form

$$E \{X^j(r, n, -1, k)\} = k \sum_{u=0}^p \frac{a_u}{(j+u+1)} [E \{X^{j+u+1}(r, n, -1, k)\} \\ + E \{X^{j+u+1}(r-1, n, -1, k)\}].$$

Case II: $m_i \neq m_j$, $\gamma_i \neq \gamma_j$, $i, j = 1, 2, \dots, n-1$.

Theorem 2. For the distribution given in (2) and $2 \leq r \leq n$, $n \geq 2$ and $k = 1, 2, \dots$

$$E \{X^j(r, n, \tilde{m}, k)\} = \sum_{u=0}^p \frac{\gamma_i a_u}{(j+u+1)} E \{X^{j+u+1}(r, n, \tilde{m}, k)\} \quad (14)$$

Proof. From (6) and (8), we have

$$E \{X^j(r, n, m, k)\} = C_{r-1} \sum_{i=1}^r a_i(r) \sum_{u=0}^p a_u I_r(x), \quad (15)$$

where

$$I_r(x) = \int_0^\infty x^{j+u} \{\bar{F}(x)\}^{\gamma_r} g_m^{r-1} \{F(x)\} dx.$$

Integrating by parts treating x^{j+u} for integration and the rest of the integrand for differentiation, we get

$$I_r(x) = \frac{\gamma_i}{(j+u+1)} \int_0^\infty x^{j+u+1} \{\bar{F}(x)\}^{\gamma_r-1} f(x) dx.$$

Now substituting for $I_r(x)$ in equation (15), we drive the relations in (14). \square

3 Recurrence Relations for Product Moments

Making use of (8), we can drive recurrence relations for product moments of *gos*.

Case I: $m_i = m_j = m, i, j = 1, 2, \dots, n-1$.

Theorem 3. For the given p th order exponential distribution in (2) and $n \geq 2, m \in \mathfrak{R}, 1 \leq r < r+1 \leq n$

$$E \{X^i(r, n, m, k) X^j(r+1, n, m, k)\} = \gamma_{r+1} \sum_{u=0}^p \frac{a_u}{(j+u+1)} [E \{X^i(r, n, m, k) X^{j+u+1}(r+1, n, m, k)\} - E \{X^{i+j+u+1}(r, n, m, k)\}] \quad (16)$$

and for $1 \leq r < s \leq n, s-r \geq 2$, and $i, j \geq 0$

$$E \{X^i(r, n, m, k) X^j(s, n, m, k)\} = \gamma_r \sum_{u=0}^p \frac{a_u}{(j+u+1)} [E \{X^i(r, n, m, k) X^{j+u+1}(s, n, m, k)\} - E \{X^i(r, n, m, k) X^{j+u+1}(s-1, n, m, k)\}] \quad (17)$$

Proof. From (5), we have

$$E \{X^i(r, n, m, k) X^j(s, n, m, k)\} = \frac{C_{s-1}}{(r-1)!(s-r-1)!} \int_0^\infty x^i \{\bar{F}(x)\}^m \times f(x) g_m^{r-1} \{F(x)\} I(x) dx \quad (18)$$

where

$$I(x) = \int_x^\infty y^j [h_m \{F(y)\} - h_m \{F(x)\}]^{s-r-1} \{\bar{F}(y)\}^{\gamma_s-1} f(y) dy \\ = \sum_{u=0}^p a_u \int_x^\infty y^{j+u} [h_m \{F(y)\} - h_m \{F(x)\}]^{s-r-1} \{\bar{F}(y)\}^{\gamma_s} dy$$

upon using the relation in (8). Integrating now by parts treating y^{j+u} for integration and the rest of the integrand for differentiation, we obtain when $s = r + 1$ that

$$I(x) = \sum_{u=0}^p \frac{a_u}{(j+u+1)} \left[-x^{j+u+1} \{\bar{F}(x)\}^{\gamma_{r+1}} + \gamma_{r+1} \times \int_x^\infty y^{j+u+1} \{\bar{F}(y)\}^{\gamma_{r+1}-1} f(y) dy \right] \quad (19)$$

and when $s > r + 1$ that

$$I(x) = \sum_{u=0}^p \frac{a_u}{(j+u+1)} \left\{ \gamma_r \int_x^\infty y^{j+u+1} [h_m \{F(y)\} - h_m \{F(x)\}]^{s-r-1} \{\bar{F}(y)\}^{\gamma_s-1} f(y) dy - \frac{(s-r-1)}{(j+u+1)} \int_x^\infty y^{j+u+1} [h_m \{F(y)\} - h_m \{F(x)\}]^{s-r-2} \{\bar{F}(y)\}^{\gamma_s+m} f(y) dy \right\}. \quad (20)$$

Upon substituting the above expressions for $I(x)$ in (18), we have, after simplifications, the recurrence relations (16) and (17). \square

Remark 3. Setting $m = 0$, $k = 1$ in (16) and (17), we obtain recurrence relations for product moments of order statistics of the p th order exponential distribution in the form

$$E(X_{r,r+1:n}^{(i,j)}) = (n-r) \sum_{u=0}^p \frac{a_u}{(j+u+1)} \left\{ E(X_{r,r+1:n}^{(i,j+u+1)}) - E(X_{r,n}^{(i+j+u+1)}) \right\} \quad (21)$$

and

$$E(X_{r,s;n}^{(i,j)}) = (n - s + 1) \sum_{u=0}^p \frac{a_u}{(j + u + 1)} \left\{ E(X_{r,s;n}^{(i,j+u+1)}) - E(X_{r,s-1;n}^{(i,j+u+1)}) \right\} \quad (22)$$

these result was obtained by Kamal (2010).

Remark 4. Putting $m = -1$, $k \geq 1$ in (17), we get the recurrence relations for product moments of upper k -records of the p th order exponential distribution in the form

$$E \{ X^i(r, n, -1, k) X^j(s, n, -1, k) \} = k \sum_{u=0}^p \frac{a_u}{(j + u + 1)} \left[E \{ X^i(r, n, -1, k) X^{j+u+1}(s, n, -1, k) \} - E \{ X^i(r, n, -1, k) X^{j+u+1}(s-1, n, -1, k) \} \right].$$

Case II: $m_i \neq m_j$, $\gamma_i \neq \gamma_j$, $i, j = 1, 2, \dots, n-1$.

Theorem 4. For the given p th order exponential distribution in (2) and for $1 \leq r < s \leq n-1$, $n \geq 2$ and $k = 1, 2, \dots$

$$E \{ X^i(r, n, \tilde{m}, k) X^j(s, n, \tilde{m}, k) \} = \sum_{u=0}^p \frac{a_u}{(j + u + 1)} \left[\gamma_i E \{ X^i(r, n, \tilde{m}, k) X^{j+u+1}(s, n, \tilde{m}, k) \} - E \{ X^{i+j+u+1}(r, n, \tilde{m}, k) \} \right] \quad (23)$$

Proof. From (7), we have

$$E \{ X^i(r, n, \tilde{m}, k) X^j(s, n, \tilde{m}, k) \} = C_{s-1} \int_0^\infty x^i \sum_{i=r+1}^s a_i^{(r)}(s) \left\{ \frac{1}{\bar{F}(x)} \right\}^{\gamma_i} \left[\sum_{i=1}^r a_i(r) \{ \bar{F}(x) \}^{\gamma_i} \right] \frac{f(x)}{\bar{F}(x)} I(x) dx \quad (24)$$

where

$$\begin{aligned} I(x) &= \int_x^\infty y^j \{ \bar{F}(y) \}^{\gamma_j} \frac{f(y)}{\bar{F}(y)} dy \\ &= \sum_{u=0}^p a_u \int_x^\infty y^{j+u} \{ \bar{F}(y) \}^{\gamma_j} dy \end{aligned}$$

upon using the relation in (8). Integrating now by parts treating y^{j+u} for integration and the rest of the integrand for differentiation

$$I(x) = \sum_{u=0}^p \frac{a_u}{(j+u+1)} [-x^{j+u+1} \{\bar{F}(x)\}^{\gamma_i} + \gamma_i \int_x^\infty y^{j+u+1} \{\bar{F}(y)\}^{\gamma_i-1} f(y) dy]. \quad (25)$$

Upon substituting the above expressions for $I(x)$ in (24), we have, after simplifications, the recurrence relations (23). \square

4 Characterization

Theorem 5. Let X be a non-negative random variable having an absolutely continuous distribution function $F(x)$ with $F(0) = 0$ and $0 < F(x) < 1$ for all $x > 0$, then

$$E \{X^j(r, n, m, k)\} = \sum_{u=0}^p \frac{\gamma_r a_u}{(j+u+1)} E \{X^{j+u+1}(r, n, m, k)\} + \sum_{u=0}^p \frac{\gamma_r a_u}{(j+u+1)} E \{X^{j+u+1}(r-1, n, m, k)\} \quad (26)$$

if and only if

$$F(x) = 1 - e^{-(a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} + \dots + a_p \frac{x^{p+1}}{p+1})}.$$

Proof. The necessary part follows immediately from equation (10). On the other hand if the recurrence relation in equation (26) is satisfied, then on using equation (4), we have

$$\begin{aligned} \frac{C_{r-1}}{(r-1)!} \int_0^\infty x^j \{\bar{F}(x)\}^{\gamma_r-1} f(x) g_m^{r-1} \{F(x)\} dx = \\ \frac{C_{r-1}}{(r-1)!} \sum_{u=0}^p \frac{\gamma_r a_u}{(j+u+1)} \int_0^\infty x^{j+u+1} \{\bar{F}(x)\}^{\gamma_r-1} f(x) g_m^{r-1} \{F(x)\} dx \\ - \frac{(r-1)C_{r-1}}{(r-1)!} \sum_{u=0}^p \frac{\gamma_r a_u}{(j+u+1)} \int_0^\infty x^j \{\bar{F}(x)\}^{\gamma_r+m} f(x) g_m^{r-2} \{F(x)\} dx \end{aligned} \quad (27)$$

Integrating the second integral on the right hand side of equation (27), by parts, we get

$$\begin{aligned} & \frac{C_{r-1}}{(r-1)!} \int_0^\infty x^j \{\bar{F}(x)\}^{\gamma_{r-1}} f(x) g_m^{r-1} \{F(x)\} dx = \\ & \frac{\gamma_r C_{r-1}}{(r-1)!} \sum_{u=0}^p \frac{a_u}{(j+u+1)} \int_0^\infty x^{j+u+1} \{\bar{F}(x)\}^{\gamma_{r-1}} f(x) g_m^{r-1} \{F(x)\} dx \\ & + \frac{C_{r-1}}{(r-1)!} \sum_{u=0}^p \frac{(j+u+1)a_u}{(j+u+1)} \int_0^\infty x^{j+u} \{\bar{F}(x)\}^{\gamma_r} g_m^{r-1} \{F(x)\} dx \\ & - \frac{\gamma_r C_{r-1}}{(r-1)!} \sum_{u=0}^p \frac{a_u}{(j+u+1)} \int_0^\infty x^{j+u+1} \{\bar{F}(x)\}^{\gamma_{r-1}} f(x) g_m^{r-1} \{F(x)\} dx \end{aligned} \quad (28)$$

which reduces to

$$\frac{C_{r-1}}{(r-1)!} \int_0^\infty x^j \{\bar{F}(x)\}^{\gamma_{r-1}} g_m^{r-1} \{F(x)\} dx \left[f(x) - \left(\sum_{u=0}^p a_u x^u \right) \{\bar{F}(x)\} \right] = 0 \quad (29)$$

Now applying a generalization of the Müntz-Szász Theorem (Hwang and Lin, 1984) to equation (28), we get

$$\frac{f(x)}{\bar{F}(x)} = \sum_{u=0}^p a_u x^u$$

which prove that

$$F(x) = 1 - e^{-\left(a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} + \dots + a_p \frac{x^{p+1}}{p+1} \right)}, \quad x \geq 0.$$

□

Remarks

1. Putting $p = 0$, $a_0 = 1$ and $a_u = 0 \quad u \geq 1$ in (13) and (22), we get the results of Joshi (1978, 1982).
2. Setting $p = 1$, $a_0 = 0$, $a_1 = 0$ and $a_u = 0 \quad u \geq 2$ in (13) and (22), we get the results Rayleigh distribution.
3. Setting $p = 1$, $a_0 = \lambda$, $a_1 = \nu$ and $a_u = 0 \quad u \geq 2$ in (13) and (22), we get the results of Balakrishnan and Malik (1986b) and of Mohie El-Din et al. (1997).

5 Conclusion

This paper deals with the generalized order statistics from the p th order exponential distribution. Some recurrence relations between the single and product moments are derived. Characterization of the p th order exponential distribution by using recurrence relation of single moment. Special cases are also deduced.

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