

## Econometric Estimation of the Demand in Iran (A Systems Approach with the CBS Model)

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### **Abstract:**

*In this study, we estimate a complete demand system for Iran that emphasizes main groups demand, using the CBS differential demand system specification. The results of this study, indicated that the expenditure elasticity of “Furniture and upholstery” and “Transportation and communication” groups are greater than one, and expenditure elasticity of “Housing group”; “Hygiene and medical care ” and “Clothing and Footwear ” lesser than one. Also, the lowest compensated own-price elasticity and the lowest uncompensated own-price elasticity are found for Food and for “Clothing and Footwear”, respectively. From the compensated cross-price elasticities view point, some groups are Allen-Hicks complements, although the values of elasticities are small (For example Hygiene and medical care and Food).*

**JEL classification:** D1, C39

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## 1. Introduction

In the last several decades, consumer demand analyses have moved in the direction of the system-wide approach. There are now numerous algebraic specifications of demand system, including the linear and quadratic expenditure systems, the Rotterdam model, the CBS<sup>1</sup> model, Translog models, The Almost Ideal demand systems, et ct. (Brown et al, 1994)

However, the assumptions used to parameterize these models have different implications. For example, the marginal expenditure share and the Slutsky terms are assumed constant in the Rotterdam model, while they are assumed functions of budget shares in the AIDS.

Modern consumer theory is valuable in indicating plausible assumptions for making estimation of demand parameters in a statistically tractable framework. In particular, the theory offers conditions under which own- and cross-price and income elasticities of demand can be estimated with an economy of parameter and with systematic behavioural interrelations.

## 2. Literature Review

Demonstrated demand analysis in a probabilistic manner. The paper discussed the usefulness of information theory for demand system.

Barten (1969) utilized maximum likelihood estimation for a complete system of demand equations. The objective of his study was to estimate a system of demand equations under various constraints imposed upon the coefficient of demand functions.

Lee and Seale (1992) investigated demand relationships among fresh fruits in Canada using the differential approach for the time series data. The Rotterdam and CBS model were used with the usual theory restrictions.

Alston and Chalfont (1993) discussed and compared the Almost Ideal and Rotterdam models with the statistical measure.

Barten (1993) illustrated choice of functional form for consumer income allocation models to satisfy theoretical

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<sup>1</sup> "Central Bureau Voor de Statistiek", the Dutch name of Statistics Netherlands.

properties. (In this paper models such as AIDS, CBS, NBR AND Rotterdam have been used to describe consumer behavior).

Neves (1994), discussed the theoretical performance of differential demand systems (The AIDS, CBS, NBR AND Rotterdam). The restrictions that they imposed on the evaluation of demand elasticities over time were illustrated and compared theoretically.

Brown and Lee (2000) utilized a uniform substitute's demand model with varying coefficients to specify demand systems. The synthetic modeling approach has been extended to increase the flexibility of the model.

Fousekis and Revell (2000) employed differential demand system to analyze demand in the United Kingdom. The Rotterdam, CBS, AIDS, NBR and Synthetic model (with imposing homogeneity and symmetry restrictions) were estimated using Seemingly Unrelated Regression (SUR) method.

Laajimi et al (2003) used a differential system approach in Tunisia. they found that in comparison of several models, the CBS was the best one.

Regarding the literature on consumer behavior, the differential demand system models could be categorized into four groups: (the Rotterdam model, the CBS model, the NBR<sup>2</sup> model, the Almost Ideal Demand System (AIDS) model. (See table 1.)

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<sup>2</sup> the model is named after the Netherlands Central Bureau of Statistics and the National Bureau of research, where Neves worked when the model was developed.

**Table 1:** numerous specifications of differential demand systems

Model	Functional Form	description	Dependent Variable
Rotterdam	$w_i d \log q_i = q_i d \log Q + \sum_j p_{ij} d \log p_j$ $d \ln Q = d \ln m - \sum_j w_j d \ln p_j$	$w_i$ Presents the average budget of commodity $i$ , $q_i$ and $p_i$ are quantity and price of good $i$ respectively, $b_i = w_i \frac{\partial \ln q_i}{\partial \ln m}$ , $q_i = g_i b_i$ and $m$ is total expenditure.	$w_i d \log q_i$
CBS	$w_i (d \log q_i / Q) = b_i d \log Q + \sum_j p_{ij} d \log p_j$ $d \ln Q = d \ln m - \sum_j w_j d \ln p_j$	In this case, the dependent variable is the ( $w_i$ ) deviation of the log change in the quantities of all $n$ goods.	$w_i (d \log q_i / Q)$
AIDS	$dw_i = b'_i d \log Q + \sum_j g_{ij} d \log p_j$ $d \ln Q = d \ln m - \sum_j w_j d \ln p_j$	This model is very similar on the right-hand side to the Rotterdam model, although the dependent variable is different.	$dw_i$
NBR	$dw_i + w_i d \log Q = q_i d \log Q + \sum_j g_{ij} d \log p_j$ $d \ln Q = d \ln m - \sum_j w_j d \ln p_j$	This is another hybrid system because it has the Rotterdam income coefficient and the AIDS price coefficient.	$dw_i + w_i d \log Q$

Source: current research

### 3. The CBS Model

In previous empirical studies, different models of demand systems have been applied to estimate price and income

elasticities of consumer demand. In this paper, using CBS model, which was developed by Keller and Van Driel(1985), the income and price elasticities of Iranian consumer will be estimated.

The CBS model based on differential system, based on differential equation for the budget shares of consumer goods.

it is also based on Rotterdam model. The absolute-price version of the Rotterdam model, developed by Theil (1965), is expressed as:

$$w_i d \ln q_i = g_i b_i d \ln Q + \sum_j p_{ij} d \ln p_j \quad (1)$$

Where  $w_i = \frac{p_i q_i}{m}$  represents the average budget share of commodity  $i$ ;  $q_i$  and  $p_j$  are quantity and price of good  $i$ , respectively,  $g_i$  is treated as change in consumer's behaviour in good  $i$ ,  $b_i = w_i \frac{\partial \ln q_i}{\partial \ln m}$  is the marginal propensity to consume,  $m$  is total expenditure; and  $d \ln Q$  is the Divisia volume index which can be expressed as:

$$d \ln Q = d \ln m - \sum_j w_j d \ln p_j = \sum_j w_j d \ln q_j \quad (2)$$

The marginal shares  $b_i$  and Slutsky coefficients  $p_{ij}$  were treated as constants. These equations (1) satisfy adding-up condition if  $\sum_i g_i = 0$ ; and Engle and Slutsky aggregation if  $\sum_i b_i = 1$  and  $\sum_i p_{ij} = 0$ . The homogeneity condition in the Rotterdam system requires  $\sum_j p_j = 0$ , while the Slutsky symmetry condition implies  $p_{ij} = p_{ji}$ .

The system defined in (1) has an important limitation. It assumes that marginal budget shares are constant. However, there is no strong *a priori* basis for this conclusion. Various researchers conclude that this assumption is a severe handicap that may limit

the validity of the model (Gao and Spreen, 1994, Lee and et al, 1994, Gao et al, 1995). To escape this dilemma, an alternative parameterization is on the Working's model (Working, 1945).

$$w_i = a_i + b_i \ln m \quad (3)$$

As the sum of budget shares is unity, it follows from (3) that  $\sum_i a_i = 1$  and  $\sum_i b_i = 1$ .

To drive the marginal shares implied by Working's model, we multiply (3) by  $m$  and then differentiate with respect to  $m$ , which results in:

$$\frac{\partial(p_i q_i)}{\partial m} = a_i + b_i(1 + \ln m)$$

Or,  $b_i = w_i + b_i$

Hence, under Working's model the  $i$ th marginal share differs from the corresponding budget share by  $b_i$  as the budget share is not constant with respect to income, neither is the associate budget share (Laajimi and et al, 2003).

By replacing  $b_i$  in (1) with (3) and rearranging terms, we obtain the CBS model:

$$w_i(d \ln q_i - d \ln Q) = g_i b_i d \ln Q + \sum_j p_{ji} d \ln p_j$$

$$w_i d \ln \left( \frac{q_i}{Q} \right) = g_i b_i d \ln Q + \sum_j p_{ji} d \ln p_j \quad (4)$$

Neoclassical consumer theory imposes some well-known restrictions on the parameters of these equations:

i) Adding-up:  $\sum_i g_i b_i = 0$  and  $\sum_i p_{ij} = 0$

ii) Homogeneity :  $\sum_j p_{ij} = 0$

iii) Utility maximization: the matrix  $\Pi$  is symmetric and negative semi definite of rank  $k-1$ .

#### 4. Data and Estimation Method

Data necessary to estimate the parameters required for the CBS model are retail prices, per capita consumption of main groups, and per capita total expenditure. To estimate The CBS demand system, it also requires the income flexibility parameters.

The Annual time series used to estimate the CBS model is for the period 1974 to 2008.

The price data are obtained from Central Bank of Iran (CBI) and year 1997 was considered as the base year. Also household expenditure data are obtained from Statistic Center of Iran (SCI).

In order to estimate equations, it has to be converted to finite changes. We follow the method used by Theil (1976) for the Rotterdam model, which is essentially an application of the

trapezoid rule. Defining  $\bar{w} = \frac{w_{it-1} - w_{it}}{2}$ . And the log difference

operator D is as:  $Dy_t = \log y_t - \log y_{t-1}$

Adding a disturbance term, the finite change will become:

$$\bar{w}_i D\left(\frac{q_i}{Q}\right) = g_i b_i DQ + \sum_j p_{ji} Dp_j + e_{it}$$

Where  $DQ$  is calculated as  $\sum_j \bar{w}_j Dq_{jt}$ , which ensures adding-up (Theil, 1975, p.40), and which differs only in the third order from  $Dm - \sum_j w_j Dp_j = \sum_j w_j Dq_j$ .

To estimate the CBS model, the Full information maximum likelihood (FIML) method was used.

The theoretical restrictions including Adding-up, Homogeneity and Symmetry were imposed in demand equations. The results are shown in table 2.

**Table 2:** Estimates under homogeneity and symmetry, Iran 1974-2008

Variable	Coefficient name	CBS coefficient
Food income coefficient	C(1)	-0.005
Food price coefficient	C(11)	-0.033
Clothing price coefficient	C(12)	0.017
Furniture price coefficient	C(13)	0.003
Hygiene price coefficient	C(14)	-0.016
Housing price coefficient	C(15)	0.030
Transport price coefficient	C(16)	-0.009
Clothing income coefficient	C(2)	-0.014
Clothing price coefficient	C(22)	-0.020
Furniture price coefficient	C(23)	0.004
Hygiene price coefficient	C(24)	-0.008
Housing price coefficient	C(25)	0.016
Transport price coefficient	C(26)	-0.0006
Furniture income coefficient	C(3)	0.014
Furniture price coefficient	C(33)	-0.019
Hygiene price coefficient	C(34)	0.023
Housing price coefficient	C(35)	0.004
Transport price coefficient	C(36)	0.007
Hygiene income coefficient	C(4)	-0.010
Hygiene price coefficient	C(44)	-0.016
Housing price coefficient	C(45)	-0.019
Transport price coefficient	C(46)	0.020
Housing income coefficient	C(5)	-0.084
Housing price coefficient	C(55)	-0.069
Transport price coefficient	C(56)	0.021
Transport income coefficient	C(6)	0.105
Transport price coefficient	C(66)	-0.089

### 5. Elasticity Estimates from the CBS Model

The most interesting economic parameters for policy analysis are elasticities. Using the estimates of price and expenditure coefficient based on CBS model, it would be possible to estimate the prices and expenditure elasticities. Price elasticities could be calculated in two ways. The first is uncompensated elasticity that contains both price and income effects. The second is compensated elasticity that contains only price effects.



The expenditure elasticity of each commodity group ( $h_i$ ), the uncompensated price elasticities ( $E_{ij}$ ) and the compensated price elasticities ( $e_{ij}$ ) for the CBS model are:

$$h_i = \frac{b_i}{w_i} + 1 \quad (5)$$

$$E_{ij} = \frac{p_{ij}}{w_i} - h_i w_j \quad (6)$$

$$e_{ij} = \frac{p_{ij}}{w_i} \quad (7)$$

For calculating (5), (6) and (7), we need income and price coefficient as well as mean of budget share for each group.

The Geometric mean of Budget share in 1974-2008 for Iranian household, reported in Table (3) that “Food “ and “Housing“ groups, have the higher weight in Iranian expenditure in comparison with other groups.

**Table 3:** Geometric mean of Budget share in 1974-2008

Main groups	Transport and communication	Housing	Hygiene and medical care	Furniture and upholstery	Food	Clothing and Footwear	Other consumption
Share of expenditure	0.08	0.30	0.05	0.06	0.33	0.08	0.08

The expenditure elasticities of different groups of goods were estimated through CBS model based on homogeneity and symmetry conditions. The results are shown in Table (4). Regarding the expenditure effects, a commodity is inferior if  $h_i < 0$  or non-inferior if  $h_i > 0$ . In the latter case, it would be a normal good if  $0 \leq h_i \leq 1$  or a luxury if  $h_i > 1$  (Barten, 1993).

The expenditure elasticities for “Furniture and upholstery”; “Transportation and communication” are greater than one, while for “Housing”; “Hygiene and medical care” and “Clothing and

Footwear”are lesser than one. In other words, these findings indicate that “Furniture and upholstery” and “Transportation and communication “groups are luxury and “Housing”,” Hygiene and medical care “and “Clothing and Footwear” groups are necessary. “Food” group has also unit expenditure elasticity.

**Table 4:** Expenditure elasticities under Homogeneity and Symmetry based on CBS model

Main groups	Food	Housing	Hygiene and medical care	Furniture and upholstery	Transport and communication	Clothing and Footwear
Expenditure coefficient	- 0.005	-0.084	-0.01	0.014	0.105	-0.014
Expenditure elasticity	0.98	0.72	0.79	1.23	2.33	0.83

Considering the figures in tables (5) and (6), we find that Marshallian and Hicksian own-price elasticities for all groups have the expected negative sign, that is, changes in own-prices have inverse impacts on quantities demanded. The resulting demands for all groups (except the transportation) are inelastic.

The lowest compensated own-price elasticity and the lowest uncompensated own-price elasticity are found for “Food” and for “Clothing and Footwear”, respectively.

The relation between goods groups would be determined by the sign of cross-price elasticities. While positive cross-price compensated elasticity indicates to Allen-Hicks substitution, negative cross-price elasticity refers to Allen-Hicks complement.

Considering the compensated cross-price elasticities, the figures indicate that some good groups including” Hygiene and medical care “and “Food” are Allen-Hicks complements.

From the compensated cross-price elasticities view point, some groups are Allen-Hicks complements, although the values of elasticities are small (For example Hygiene and medical care and Food).

**Table 5:** Compensated Price Elasticity ( $e_{ij}$ ) under Homogeneity and Symmetry in CBS model

$e_{ij}$	Food	Clothing and Footwear	Furniture and upholstery	Hygiene and medical care	Housing	Transport and communication
Food	-0.10	0.05	0.01	-0.05	0.09	-0.03
Clothing and Footwear	-	-0.24	0.05	-0.10	0.19	-0.01
Furniture and upholstery	-	-	-0.31	0.38	0.07	0.11
Hygiene and medical care	-	-	-	-0.33	-0.40	0.42
Housing	-	-	-	-	-0.23	0.07
Transport and communication	-	-	-	-	-	-1.13

Uncompensated Cross-price elasticities show substitution or complementary relations among goods. Positive cross-price elasticity indicates substitute goods while negative cross-price elasticity means that goods are complement.

From the uncompensated cross-price elasticities view point, some groups are gross complements, although their elasticities are small (For example Hygiene and medical care and Food).

**Table 6:** Uncompensated Price Elasticity ( $E_{ij}$ ) under Homogeneity and Symmetry in CBS model

$E_{ij}$	Food	Clothing and Footwear	Furniture and upholstery	Hygiene and medical care	Housing	Transport and communication
Food	-0.42	-0.03	-0.05	-0.10	-0.20	-0.10
Clothing and Footwear	-	-0.311	-0.002	-0.137	-0.055	-0.073
Furniture and upholstery	-	-	-0.39	0.32	-0.30	0.02
Hygiene and medical care	-	-	-	-0.37	-0.63	0.35
Housing	-	-	-	-	-0.45	0.01
Transport and communication	-	-	-	-	-	-1.31

## 6. Conclusion

In This paper, the CBS model for evaluating consumer's behaviour in Iran have been used. The finding of the analysis can provide a useful basis for policy-makers, planners, and traders, taking into account the efforts implemented by Iranian government in order to achieve the favourable conditions for tax system and regulated market.

The main results of this paper are:

1) The Geometric mean of Budget shares in 1974-2008 for Iranian household indicated that "Food" and "Housing" groups have the higher weight in Iranian expenditure in comparison with other groups.

2) Expenditure Elasticity under Homogeneity and Symmetry restrictions in the CBS model indicate that "Furniture and Upholstery" and "Transportation and Communication" groups are luxury and "Housing", "Hygiene and Medical Care" and "Clothing and Footwear" groups are necessary. Also, "Food" has unit expenditure elasticity.

3) Marshallian and Hicksian own-price elasticities have the expected negative sign, that is, changes in own-price have inverse

impacts on quantities demanded. The resulting demand for all groups (except the transportation) is inelastic.

4) Compensated cross-price elasticities show some groups are gross complements, though their elasticities are small.

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