Optimum target value of multivariate processes with unequal non-conforming costs

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Abstract

In quality control charts, the problem of determining the optimum process mean arises when the deviation of a quality characteristic in one direction is more harmful than in the opposite direction. The failure mode in these two directions is usually different. A great majority of researches in this area have considered asymmetric cost function for processes with single quality characteristics. In this paper, we consider processes in which there are more than one quality characteristics to monitor. The quality characteristics themselves may or may not be independent. Based upon the specification limits and the costs associated with the deviations we derive a formula to determine the optimum process mean. To illustrate the proposed formula and to estimate the costs associated with the optimum process mean we present four numerical examples by simulation. The results of the simulation studies show that considerable amount of savings can be obtained by applying the proposed process means.

1. Introduction and literature review

The selection of the optimal process target has become an important research area in which the focus is to decrease costs. Production processes are usually designed in a way that their product satisfies a set of specifications. In quality control of the product, one of the important parameters that must be determined is the process mean. Determining the optimal mean of a manufacturing process involves a complex and financially important decision. If deviations in the two directions of a quality characteristic have equal cost associated with, then the middle point of the tolerance limits specifies the mean of the process. However, when the deviation of a quality characteristic in one direction is more harmful than in the opposite direction, the optimal mean of the process is not the middle point of the tolerance limits. This problem is identified with the so-called "optimal filling problem," where the product is typically a pocket of food or a bottle of liquid [38].

Several researchers have studied this problem so far. Springer [32] and Bettes [4] considered a filling univariate process, where the upper and the lower specification limits are given. They obtained the optimal mean of the process such that the total costs of the reprocessing and material for overfilled and under filled items are minimized.

Hunter and Kartha [16] and Nelson [30] found the best target value for the mean of a univariate process so that producer's profit is maximized. Also, under asymmetric price/cost condition Bisgaard et al. [5] developed a method to find the optimal univariate process mean. Carlsson [8] presented a method to determine the most profitable process level. Golhar [14] determined the best mean value of a canning

process. Boucher and Jafari [6] and Al-sultan [1] discussed situations in which the items are subjected to lot-by-lot acceptance sampling rather than complete inspection plan. Leon and Wu [18] extended the discussion of Taguchi [33,34] problem of parameter design to considerations of asymmetric costs. Elsayed and Chen [12] determined optimum levels of process parameters for products with multiple characteristics. Bai and Lee [3] and Tang and Lo [35] presented an economic model to determine the process mean and the cut off value of the quality characteristic when items are sold or scrapped. Arcelus and Rahim [2] developed a model for simultaneously selecting optimum target in situations when both variable-type and attribute-type quality characteristics were present. Chen and Chung [10] considered an economic model to determine the most profitable target value and the optimum inspection precision level for a production process.

Cain and Janssen [7] proposed target selection in process control under asymmetric costs. Li [19,20,21, 22,23], Li et al. [24,25,26], Li and Wu [27], Maghsoodloo and Li [28], Philips and Cho [31] and Wu and Tang [39] found the target value of the process mean in an unbalanced tolerance design with asymmetric linear and quadratic quality loss function. Cho and Leonard [11] showed that the piecewise linear quality loss function for product is roughly proportional to the deviation of the quality characteristic from its specification limits.

Misiorek and Barnett [29] presented the sense concerned with optimal targeting of the mean (and standard deviation) of a quality characteristic. Golhar and Pollock [13] studied the linear loss function applied in the filling-canning problem for determining the optimum-manufacturing target. Hong et al. [15] studied optimum mean value and screening limits with multi-class screening. Lee et al. [17] considered the problem of determining the optimum target value of the quality characteristic of interest and screening limits for a correlated variable under single and two-stage screenings.

Chandra [9] adopted step loss function and selected the optimum process mean based on balancing the cost of not meeting the upper and the lower specification limits. However, he neglected to consider the quality loss for a product which is within specifications in his model. Teeravaraprug and Cho [37] studied a multivariate quality loss function and determined the most economical process target levels for multiple quality characteristics; the characteristics being the customer's viewpoints. Veevers and Sparks [38] presented a method to set the optimal target

value of the process mean and deviation. Teeravaraprug [36] examined a situation where a product was classified into two grades with respect to market specification. He determined the optimal process mean that gave a maximum profit to the manufacturer.

In this paper, we focus on determining the target value of the mean of both a single and multiple quality characteristics vector. We emphasize multivariate processes in which there are more than one quality characteristic presented while they do not have equal costs in the two directions of their deviations. In such processes, the optimal process mean determination may result in great savings.

In the rest of the paper, first problem definition and notifications comes. Then, determining the optimum process mean of a process with a single quality characteristic is illustrated and by using the senses of multivariate normal distribution, optimum process mean will be estimated for processes with multiple quality characteristic vectors. In order to demonstrate the importance of the proposed formulae, four numerical examples by adopting simulation will be presented. Then, the obtained results from the simulation study will be compared with those obtained from traditional approaches.

2. Problem definition and notations

Consider a product with several quality characteristics denoted by $X = [X_1, X_2, ..., X_p]$. In a stable process, a suitable model for the variation in X can often be a multivariate normal distribution with mean vector μ and covariance matrix Σ. Moreover, define the upper and the lower specification limits for the ith quality characteristics (X_i) as usl_i and lsl_i, receptively. Also, let us define c_i and c_i' to be the costs of the ith characteristic lying above usl_i and under lsl_i respectively. We assume that for $i=1,2,...,p$ the c_i's and the c_i''s are independent of the distance between usl_i's and lsl_i's, and Σ is constant.

3. Optimum process mean for univariate processes

For a univariate quality characteristic, let the process mean and standard deviation be μ and σ, respec-

tively. Then, the optimal process mean can be found by minimizing Equation (1).

$$z = c'F_x(lsl) + c[1 - F_x(usl)],$$ (1)

where $F_X(.)$ denotes the value of the cumulative probability distribution function of the quality characteristic at point ".". With the normality assumption held, we can write Equation (1) as Equation (2).

$$z = c'\int_{-\infty}^{lsl} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx$$

$$+ c\int_{usl}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx.$$ (2)

Figure 1 shows a univariate normal process with specification limits in two directions of the process mean and the different costs of producing defective items shown.

To minimize z, we take the derivative of z with respect to μ and set it equal to zero. That is:

$$\frac{\partial z}{\partial \mu} = 0.$$ (3)

The derivative of z with respect to μ is:

$$\frac{\partial z}{\partial \mu} = c'\int_{-\infty}^{lsl} (\frac{x-\mu}{\sigma}) \frac{1}{\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx$$

$$+ c\int_{usl}^{\infty} (\frac{x-\mu}{\sigma}) \frac{1}{\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx$$

$$+ c' \frac{-1}{\sigma\sqrt{2\pi}} \frac{(e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2})^2}{2} \Big|_{-\infty}^{lsl}$$

$$= c(\frac{-1}{\sigma\sqrt{2\pi}} \frac{(e^{-\frac{1}{2}(\frac{\infty-\mu}{\sigma})^2})^2}{2}$$

$$+ \frac{1}{\sigma\sqrt{2\pi}} \frac{(e^{-\frac{1}{2}(\frac{usl-\mu}{\sigma})^2})^2}{2}$$

$$+ c'(\frac{-1}{\sigma\sqrt{2\pi}} \frac{(e^{-\frac{1}{2}(\frac{lsl-\mu}{\sigma})^2})^2}{2}$$

$$+ \frac{1}{\sigma\sqrt{2\pi}} \frac{(e^{-\frac{1}{2}(\frac{-\infty-\mu}{\sigma})^2})^2}{2}$$

$$= c\frac{1}{\sigma\sqrt{2\pi}} \frac{(e^{-\frac{1}{2}(\frac{usl-\mu}{\sigma})^2})^2}{2}$$

$$+ c' \frac{-1}{\sigma\sqrt{2\pi}} \frac{(e^{-\frac{1}{2}(\frac{lsl-\mu}{\sigma})^2})^2}{2}.$$

Setting the derivation equal to zero, we will have:

$$c(e^{-\frac{1}{2}(\frac{usl-\mu}{\sigma})^2})^2 = c'(e^{-\frac{1}{2}(\frac{lsl-\mu}{\sigma})^2})^2.$$

Solving for μ, the optimal value of the process mean (μ^*) is obtained by Equation (4).

$$\mu^* = \frac{2\sigma^2(Ln(\sqrt{c'}) - Ln(\sqrt{c})) + usl^2 - lsl^2}{2(usl - lsl)}$$ (4)

To verify that μ^* is the minimum of Equation (2), $\frac{\partial^2 z}{\partial \mu^2}\Big|_{\mu=\mu^*}$ must be greater than zero. Taking the second derivative of z with respect to μ:

$$\frac{\partial^2 z}{\partial \mu^2} = \frac{1}{\sigma^3\sqrt{2\pi}}[c(usl-\mu)e^{-(\frac{usl-\mu}{\sigma})^2}$$

$$+ c'(\mu-lsl)e^{-(\frac{lsl-\mu}{\sigma})^2},$$

which is a strictly positive function, and in this regard, μ^* is a global minimum to z.

It can be easily seen that if the costs are equal then the optimal process mean is given by Equation (5).

$$\mu^* = \frac{usl + lsl}{2}.$$ (5)

4. Optimum process mean for multivariate processes

Figure 2 shows a bivariate quality characteristics with specification limits in two directions of the process mean vector and the different costs of producing defective items.

Having the previous notations, we define the total cost function in multivariate processes by Equation (6).

$$z = \sum_{i=1}^{p} c_i' F_{x_i}(lsl_i) + \sum_{i=1}^{p} c_i(1 - F_{x_i}(usl_i)), \quad (6)$$

where F_{x_i} is the cumulative probability distribution function of the ith quality characteristic (X_i). Assuming normality, we know that the marginal distribution of the individual quality characteristic is a normal distribution. Then:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{usl_i}^{\infty}...\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{0.5}} e^{-\frac{1}{2}(X-\mu)^T \Sigma^{-1}(X-\mu)}$$

$$= 1 - F_{x_i}(usl_i), \quad (7)$$

and

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{-\infty}^{lsl_i}...\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{0.5}} e^{-\frac{1}{2}(X-\mu)^T \Sigma^{-1}(X-\mu)}$$

$$= F_{x_i}(lsl_i). \quad (8)$$

Then, the cost function becomes:

$$z = \sum_{i=1}^{p} c_i \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{usl_i}^{\infty}...\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{0.5}}$$

$$* e^{-\frac{1}{2}(X-\mu)^T \Sigma^{-1}(X-\mu)} dx_1 dx_2...dx_{i-1}dx_{i+1}...dx_p$$

$$+ \sum_{i=1}^{p} c_i' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}...\int_{-\infty}^{lsl_i}...\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{0.5}} \quad (9)$$

$$* e^{-\frac{1}{2}(X-\mu)^T \Sigma^{-1}(X-\mu)} dx_1 dx_2...dx_{i-1}dx_{i+1}...dx_p.$$

Note that each of the multiple integrals in Equation (9) is a marginal normal probability distribution of the ith characteristic. In this regard, when we take the derivative of z with respect to μ_i, all of the non-μ_i terms become zero. That is for minimizing z we set the derivatives equal to zero as:

$$\frac{\partial z}{\partial \mu_i} = 0 \qquad i=1,2,...,p. \quad (10)$$

In other words, we have p similar equations as for univariate case and we can determine the optimum value of each quality characteristic mean by Equation (11).

$$\mu_i^* = \frac{2\sigma_i^2(Ln(\sqrt{c_i'}) - Ln(\sqrt{c_i})) + usl_i^2 - lsl_i^2}{2(usl_i - lsl_i)}, \quad (11)$$

where $i=1,2,...,p$.

As in the univariate processes, it is obvious that the second derivatives are also strictly positive functions and hence μ_i^*'s are the global minimum points of the z function.

From the derivation process of Equation (11) one may note that when we take the derivative of the cost function with respect to the individual means, for the special case of multi-normal probability distribution the correlation matrix disappears (since the marginal probability distributions become normal themselves). That is why the correlations do not affect the optimal solution.

Equation (11) is an important formula, which determines the optimal process mean vector given the costs, the specifications, and the variances of each quality characteristics. The following numerical examples give a better view of the application and the usefulness of this formula.

5. Numerical examples

In order to better understand the importance of the formula in Equation (11) we will present four numerical examples and we will compare the results obtained from the proposed method with those from the traditional method.
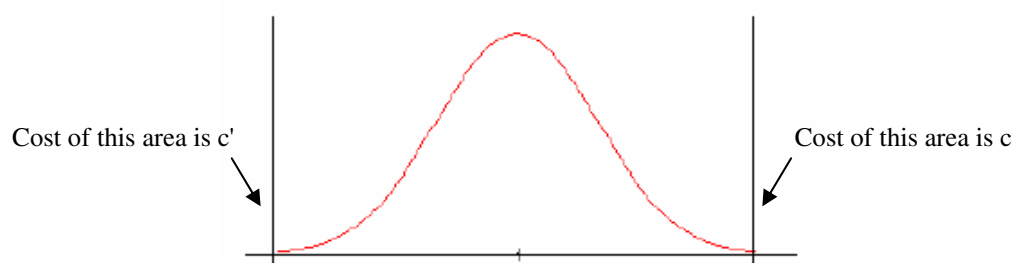
Figure 1. Univariate normal process with specification limits.
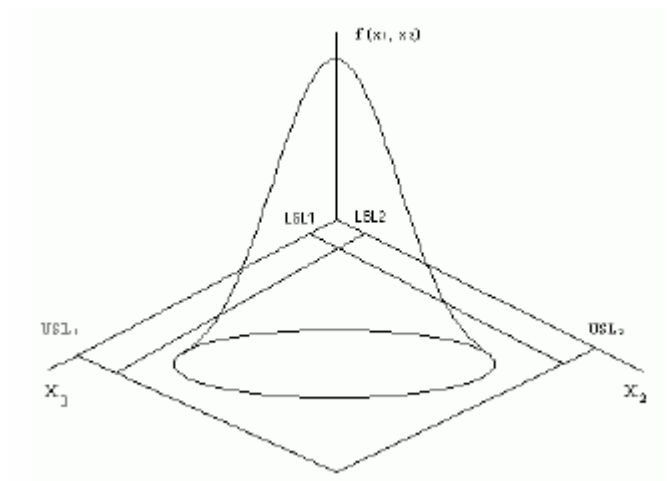
Figure 2. Multivariate normal process with specification limits.

5.1. Example 1

Consider determining target value for process mean of a lumber manufacturing plant when we have only two quality characteristics. The variable x_1 is the stiffness and x_2 is the bending strength in unit of lb inch-2 of a particular grade of lumber. The lower specification limits for x_1 and x_2 are 258 and 460 respectively, and the upper specification limits are 272 and 480 respectively. The costs of lying under lsl for x_1 and x_2 are \$15 and \$20 respectively, while the costs of lying above *usl* are \$10 and \$30.

The standard deviations and the coefficient of correlation between the variables derived from a large amount of past data are $\sigma_{x_1} = 10$, $\sigma_{x_2} = 10$ and $\rho = 0.6$. In traditional method, the target value for the process mean vector is set at the central point of the lower and upper specification limits for x_1 and x_2, that is (265, 470). However, the optimum target value from the process mean by Equation (11) is:

$$\mu_1^* = \frac{2(10)^2(Ln\sqrt{15} - Ln\sqrt{10}) + 272^2 - 258^2}{2(272 - 258)}$$

$$= 266.4481,$$

$$\mu_1^* = \frac{2(11)^2(Ln\sqrt{10} - Ln\sqrt{30}) + 480^2 - 460^2}{2(480 - 460)}$$

$$= 466.6767,$$

$$\mu^* = (266.4481, 466.6767).$$

To determine the effect of using a non-suitable product mean vector, we simulated processes for different sample sizes of production in two cases. In the first case, the process mean vector was the traditional one [$\mu = (265, 470)$] while in the second case it was the proposed one [$\mu = (266.4481, 466.6767)$]. In each case, we generated 100,000 replications by simulation and calculated the corresponding costs. The total costs associated with the first and the second case became \$1329615 and \$1202390, respectively. These costs were \$13285370 and \$12000235, respectively for the cases where there were 1,000,000 replications. Figure 3 shows the total costs of the two cases based on different sample sizes between 1000 and 10000. From Figure 3 it can be seen that when replications become larger the obtained savings by employing the proposed process mean become larger. In other words, in mass production environment, determining the optimal value of the process mean vector can result in a great amount of money savings.

5.2. Example 2

As an example of a higher dimensional problem, consider a process concerning the test of ballistic missiles. In this example, there are four quality characteristics, $p = 4$, and the cost matrix was given as:

$$\mathbf{c} = \begin{bmatrix} 10 & 6 & 20 & 3 \\ 15 & 20 & 5 & 18 \end{bmatrix}$$

The first row of *c* refers to the costs of lying under the specification limits and the second row shows the costs of lying above the specification limits. The specification limit matrix itself is given by:

$$sl = \begin{bmatrix} 98 & 135 & 80 & 95 \\ 106 & 149 & 88 & 105 \end{bmatrix}$$

where the first and the second row show the lower and the upper specification limits for the four quality characteristics, respectively. The variance-covariance matrix for the quality characteristics from a large amount of past data is:

$$\Sigma = \begin{bmatrix} 64 & 58.576 & 34.5216 & 42.872 \\ 58.576 & 100 & 47.268 & 67.3 \\ 34.5216 & 47.268 & 36 & 45.51 \\ 42.875 & 67.3 & 45.51 & 100 \end{bmatrix}$$

The mean vector in the traditional method is the central points of the specifications as $\mu = (102, 142, 84, 100)$. However, using Equation (11) the optimum value of the process mean is:

$$\mu_1^* = \frac{2(8)^2(Ln\sqrt{15} - Ln\sqrt{10}) + 106^2 - 88^2}{2(106 - 98)}$$

$$= 103.6219,$$

$$\mu_2^* = \frac{2(10)^2(Ln\sqrt{20} - Ln\sqrt{6}) + 149^2 - 135^2}{2(149 - 135)}$$

$$= 146.2999,$$

$$\mu_3^* = \frac{2(6)^2(Ln\sqrt{5} - Ln\sqrt{20}) + 88^2 - 80^2}{2(188 - 80)}$$

$$= 80.88084 ,$$

$$\mu_4^* = \frac{2(10)^2(Ln\sqrt{18} - Ln\sqrt{3}) + 105^2 - 95^2}{2(105 - 95)}$$

$$= 108.9588 .$$

That is:

$$\mu^* = (103.6219, 146.2999, 80.88084, 108.9588) ,$$

is the optimum process mean.

To determine the impact of using the process mean obtained from the traditional method instead of the proposed one, one more time we simulated the process for different sample sizes of production in two cases. In the first case, we applied the traditional mean vector while in the second case, we employed the proposed value of the mean vector. When the sample size is 100,000 replications, the cost associated with the first case is $267,800. This figure in the second case reduced to $203,849. Applying a sample size of 1,000,000 replications, these costs for the case one and two are $2,676,7647 and $20,387,796, respectively. Figure 4 shows that the total costs of the two cases by simulation for different sample sizes from 1000 to 10000 replications.

One more time the figure shows a huge amount of money savings when the proposed process mean is employed especially when the lot size becomes larger.

5.3. Example 3

Consider determining target value for process mean of a process when we have three quality characteristics. The cost matrix is given as:

$$c = \begin{bmatrix} 13 & 20 & 19 \\ 15 & 24 & 20 \end{bmatrix}$$

The first row of c refers to the costs of lying under the specification limits and the second row shows the costs of lying above the specification limits. The specification limit matrix itself is given by:

$$sl = \begin{bmatrix} 120 & 170 & 30 \\ 125 & 178 & 33 \end{bmatrix}$$

where the first and the second row show the lower and the upper specification limits for the four quality characteristics, respectively. The variance-covariance matrix of the quality characteristics from a large amount of past data is:

$$\hat{\Sigma} = \begin{bmatrix} 8 & 1.96 & -2.55 \\ 1.96 & 12 & -2.6 \\ -2.55 & -2.6 & 9 \end{bmatrix}$$

The target value of the process mean vector in the traditional method is the central point of the lower and upper specification limits, that is (122.5, 174, and 31.5). However, the proposed target value is given in Equation (11) as:

$$\mu_1^* = \frac{2(8)^2(Ln\sqrt{13} - Ln\sqrt{15}) + 125^2 - 120^2}{2(125 - 120)}$$

$$= 121.5842 ,$$

$$\mu_2^* = \frac{2(12)^2(Ln\sqrt{20} - Ln\sqrt{24}) + 178^2 - 170^2}{2(178 - 170)}$$

$$= 172.3591 ,$$

$$\mu_3^* = \frac{2(9)^2(Ln\sqrt{19} - Ln\sqrt{20}) + 35^2 - 25^2}{2(35 - 25)}$$

$$= 30.8075 ,$$

$$\mu^* = (121.58, 172.3591, 30.8075) .$$

To determine the effect of using a non-suitable product mean vector, we simulated processes for different sample sizes in two cases. In the first case, the process mean vector is the traditional one as (122.5, 174, 31.5) while in the second case, it is the proposed one as μ^* =(121.58, 172.3591, 30.8075). In each case, we generate 100,000 replications by simulation and calculate the corresponding costs. The total costs associated with the first and the second case become $2385888 and $2372088, respectively. Figure 5 shows the total costs of the two cases based on different sample sizes between 1000 and 10000 replications. From Figure 5, we may conclude that a similar result to the results of Example (2) is obtained.
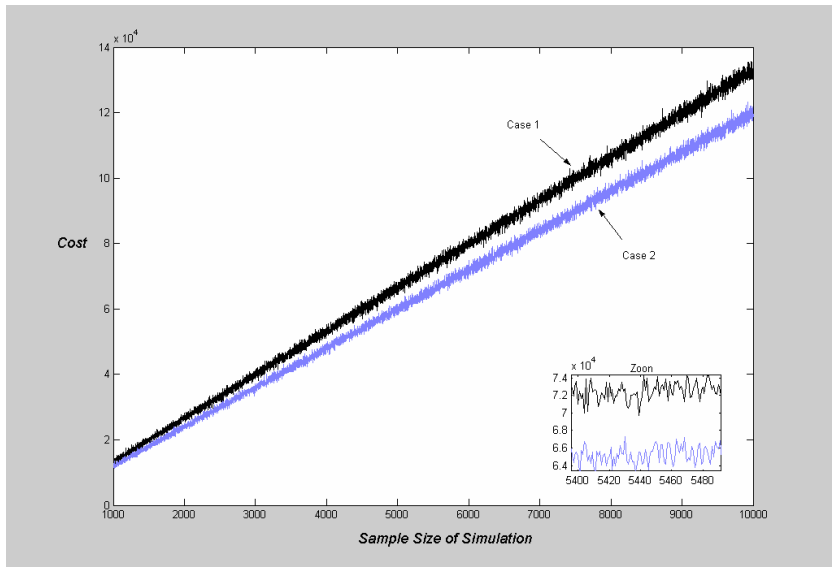
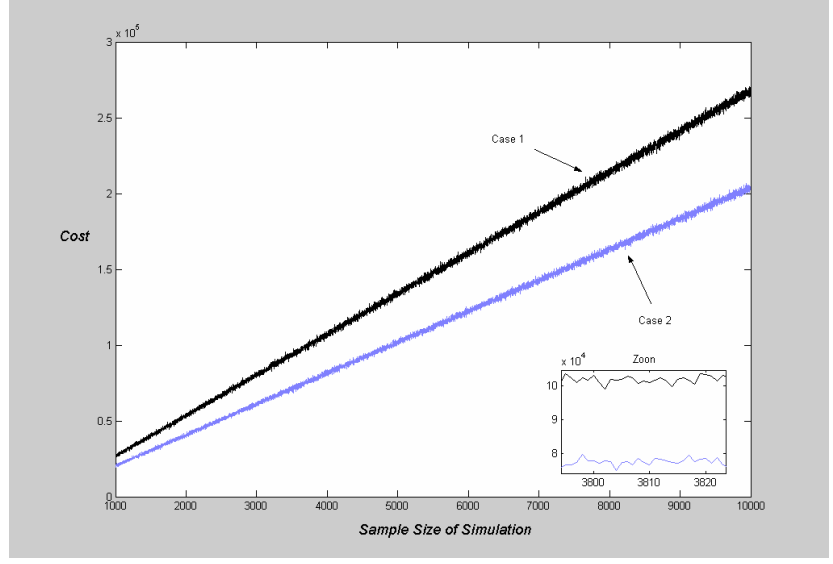**Figure 3.** The costs of the two cases in Example 1.

Figure 4. The costs of the two cases in Example 2.

5.4. Example 4

As an example of a higher dimensional problem, consider a process concerning the test of ballistic missiles. In this example, there were four quality characteristics, *p*=5, and the cost matrix is given as:

$$c = \begin{bmatrix} 12 & 8 & 6 & 15 & 12 \\ 7 & 5 & 10 & 10 & 16 \end{bmatrix}$$

The specification limit matrix itself is given by:

$$sl = \begin{bmatrix} 15 & 42 & 30 & 60 & 25 \\ 20 & 48 & 38 & 70 & 30 \end{bmatrix}$$

The variance-covariance matrix for the four quality characteristics from a large amount of past data is:

$$\hat{\Sigma} = \begin{bmatrix} 4 & 1.5 & 1.8 & 2.3 & 2 \\ 1.5 & 5 & 2.5 & 1 & 2.1 \\ 1.8 & 2.5 & 2 & 1.4 & 2.7 \\ 2.3 & 1 & 1.4 & 7 & 2.9 \\ 1 & 2.1 & 2.7 & 2.9 & 7 \end{bmatrix}$$

In this example, the mean vector in the traditional method is $\mu^* =$(17.5,45,34,65,27.5). However, using Equation (11) the proposed value of the process mean is:

$$\mu_1^* = \frac{2(8)^2(Ln\sqrt{12}-Ln\sqrt{7})+20^2-15^2}{2(20-15)}$$

$$= 18.3624,$$

$$\mu_2^* = \frac{2(10)^2(Ln\sqrt{8}-Ln\sqrt{5})+48^2-42^2}{2(48-42)}$$

$$= 45.9792,$$

$$\mu_3^* = \frac{2(6)^2(Ln\sqrt{6}-Ln\sqrt{10})+38^2-30^2}{2(38-30)}$$

$$= 33.8723,$$

$$\mu_4^* = \frac{2(10)^2(Ln\sqrt{15}-Ln\sqrt{10})+70^2-60^2}{2(70-60)}$$

$$= 65.9934,$$

$$\mu_5^* = \frac{2(10)^2(Ln\sqrt{12}-Ln\sqrt{16})+30^2-25^2}{2(30-25)}$$

$$= 26.0904.$$

That is:

$$\mu^* = (18.3624, 45.9792, 33.8723, 65.9934, 26.0904)$$

is the optimum process mean.

To determine the impact of using the process mean obtained from the traditional method instead of the proposed one, one more time we simulate the process for different replications in two cases. In the first case, we apply the traditional mean vector and in the second case, we employ the proposed value of the mean vector.

For 100,000 replications, the total costs associated with the first and the second case become $931886 and $880585, respectively. These amounts for 1,000,000 replications are $9271179 and $8770072.

Figure 6 shows the total costs of the two cases in simulation of different sample sizes between 1000 to 10000 replications. One more time Figure 6 shows a huge amount of money savings when the proposed process mean is employed, especially when the sample size becomes larger.

6. Conclusions and recommendation for future research

Many processes have several quality characteristics associated with, most of them have different costs in their two directions of specifications. Therefore, the center of the specification limits often may not determine the optimum process mean. In this paper, we derived a formula to determine the optimal process mean of a process with multiple quality characteristics. By four numerical examples, we showed how the proposed formula could be applied in manufacturing environments. In addition, the effectiveness of applying the proposed formula, in terms of money savings, was compared to the one obtained by the traditional method.

One of the future researches in this area is to determine the optimum process mean vector in multivariate processes in which some combinations of quality characteristics lying out of specification limits have different costs and some combinations do not have any cost associated with them.
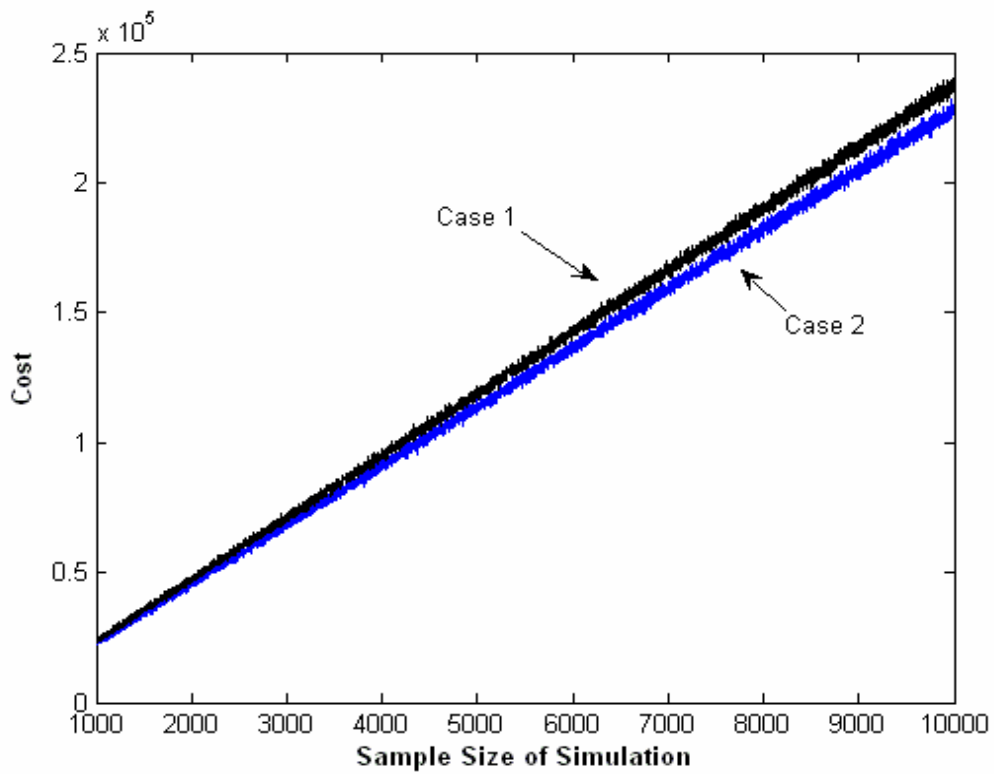
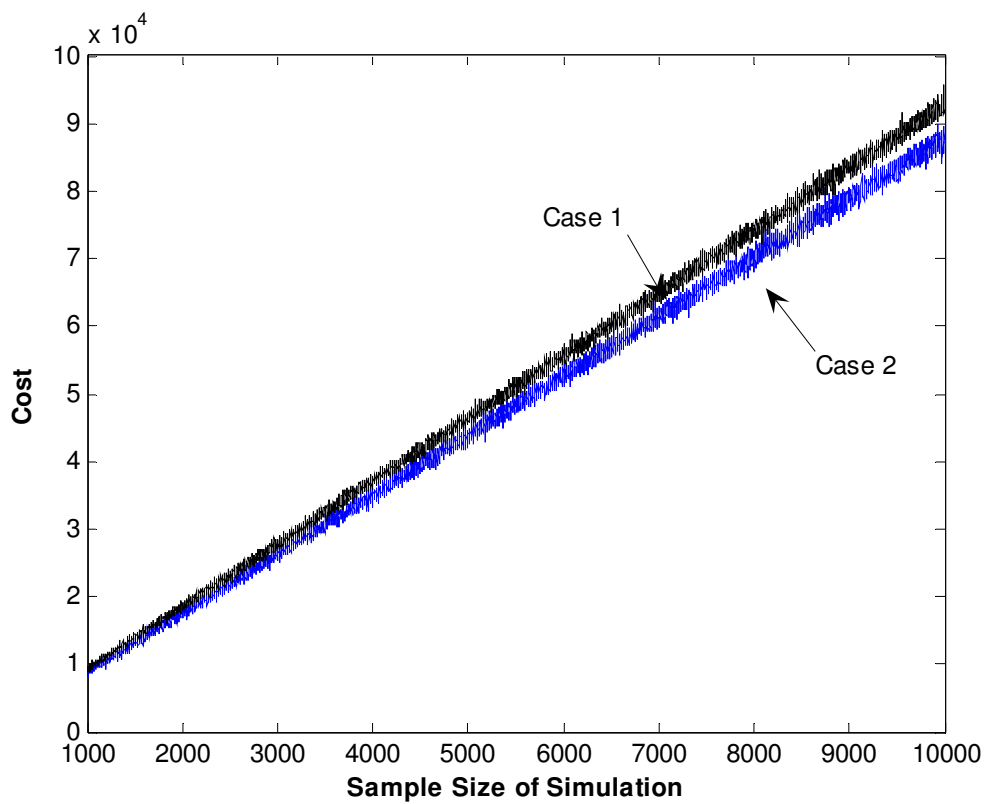**Figure 5.** The costs of the two cases in Example 3.

Figure 6. The costs of the two cases in Example 4.

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