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# Lot sizing with rework and different inspection costs

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**Abstract**

This paper deals with the single machine multi-product lot scheduling problem in which defective items are produced in any production run of each product. We have adopted the common cycle scheduling policy and assumed that the setup times for production of each product can be non-zero. Further, we have assumed that defective items will be reworked and the inspection costs during the normal production and rework processing times are different. For this system we obtained the optimal batch sizes for each product such that the total cost per unit time is minimized.

**Keywords:** Rework; Lot scheduling; Production control

## 1. Introduction

Consider the problem of obtaining a low cost schedule for a production system in which a number of products are manufactured on a single facility in a fixed sequence repeated from cycle to cycle. This problem is known as economic lot scheduling problem (ELSP). For any given problem, optimum manufacturing frequency for individual products and cycle time can be easily determined but the problem arises when we try to obtain a feasible schedule. If it is possible to obtain a feasible schedule without altering the optimum manufacturing frequencies or cycle time for individual products then it is the optimum production schedule. In practice such a happy coincidence of events rarely occurs. Unless there is plenty of idle time, the independent lot sizing and scheduling of products running the facility will likely lead to interferences among different products. That is, facility will be required to produce more than one item at the same time, which is physically impossible (Hax and Canada [12], Johnson and Montgomery [15]). There is, at the present time, no algorithm available which solves the problem optimally, and several different types of approaches have been presented in the literature ( see Elmaghraby [3] for a comprehensive literature review through 1978, for recent contribution see Cook [1], Dobson [2], Gallego and Shaw [4], Glass [5], Graves [7], Gunter [8], Haessler [9], Haji and Mansuri [10], Park and Yun [20], Roundy [21] and Zipkin [22] among others). One approach, the common cycle approach proposed by Hanssman [11], is to schedule exactly one lot of each product in a time interval called common cycle (CC) or *T*. The CC approach always finds a feasible schedule and consists of a very simple procedure. The CC approach also requires much less computational effort than the other approaches. Jones and Inmann [16] have shown that the CC approach produces optimal and near optimal schedules in many realistic situations. They also gave upper bounds for the maximum percentage deviation of the common cycle's schedule from optimality.

In the ELSP, it is assumed that a perfectly reliable facility produces items at a fixed production rate and the products produced are all non-defective. But in practice there are many situations in which a certain amount of defective products results due to various reasons including poor production quality and material defects, and subsequently a portion of them may be scrapped as well. Depending on the proportion of defectives, the amount of optimal batch sizes also varies depending on several cost factors such as setup, processing and inventory carrying costs. In a production system where there is no repair facility,

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defective items are wasted as scraps, and as a result, they lose a big share of profit margin. Researchers that consider rework option are meager Jamal et al. [14].

A study by Goyal and Gunashekharan [6] showed the effect of process control while they ignored the situation of producing defects. The issue of imperfect production and quality control in a lot-sizing problem had been addressed in the literature by Hayek and Salameh [13], Lee [17] and Lee and Rosenblatt [19]. Lee et al. [18] developed a model of batch quantity in a multi-stage production system considering various proportions of defective items produced in every stage, but they did not consider the rework option of the defective items. In a recent paper, Jamal et al. [14] considering the reworking of defective items for the case of a single product, developed two models for obtaining the economic batch quantity for the single product.

In this paper we address the reworking of defective items for the case of a multi-product single machine system. Adopting the common cycle time approach for all products, allowing non-zero setup times for each product, and assuming different inspection costs for the normal production and the rework periods, we obtained the optimal common cycle time, hence the optimal batch sizes for all products, which minimize the sum of inventory holding, setup, production process and inspection costs.

## 2. Assumptions

In this paper, all the standard assumptions of the general ELSP hold true ( see for example, Johnson and Montgomery [15]). The most relevant assumptions used in this paper are as follows:

- There are $n$ products, all of which must be produced on a single machine, which can make only one product at a time.
- Demand rates for all products are constant, known, and finite.
- Production rates for all products are constant, known, and finite.
- All demands must be filled immediately, so no shortages are permitted.
- Due to common cycle scheduling approach, each product is produced only once in each cycle *T*.

Furthermore, it is assumed that for product $j$, $j= 1, \ldots, n$:

- Proportion of defective is constant in each cycle.
- The production rate of non-defectives is greater than the demand rate.
- Scrap is not produced at any cycle.
- No defectives are produced during the rework.
- Production and rework are done using the same resources at the same speed.
- Setup time is allowed to be non-zero.

## 3. Notations

In this paper the following notations are applied for product $j$, $j= 1,\ldots, n$:

$P_j$ Production rate, units/year.

$D_j$ Demand rate, units/year.

$C_j$ Processing cost for each unit of product, \$/unit.

$Lj$ Inspection cost for each unit of product produced in the normal production period, \$/unit.

$m_j$ Inspection cost for each unit of product produced in the rework processing period, \$/unit.

$Q_j$ Batch quantity per cycle, units/batch or units/cycle.

$\beta_j$ Proportion of defectives in each cycle.

$A_j$ Setup cost for product $j$, \$/batch.

$H_j$ Inventory carrying cost, \$/unit/year.

$S_j$ Setup time, year/setup.

## 4. Model

In this model, we have considered a common cycle time $T$ for all products as depicted in Figure 1. Each product is produced only once in each cycle $T$. We assume all the defective items for each product during cycle $T$ are reworked within the same cycle and immediately after the normal processing time of that product. For the feasibility of the problem, not only we should assume $P_j > D_j$ for each $j=1,\ldots,n$, but also we assume that:

$$P_j(1-\beta_j) > D_j. \tag{1}$$

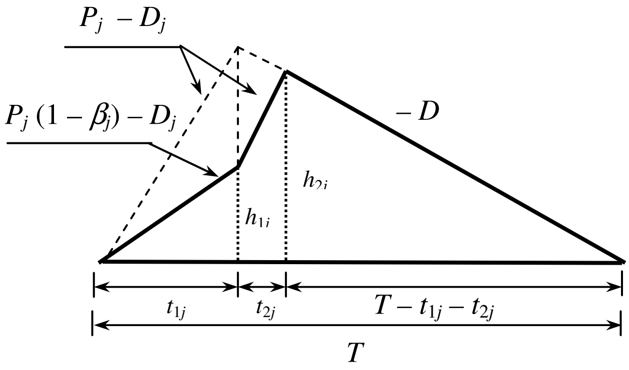


**Figure 1.** On-hand inventory in one cycle.

As shown in Figure1, we see that during the interval $t_{1j}$ the product *j* is produced at rate $P_j$, but the value of non-defectives of this product is produced at rate $P_j(1-\beta_j)$ in the same interval. Hence, the inventory of non-defective of product *j* will increase at rate $[P_j(1-\beta_j)-D_j]$. After $t_{1j}$, immediately rework starts on defective items produced in $t_{1j}$ and continues at rate $P_j$. Therefore, the inventory of non-defective items increases at rate $P_j - D_j$. It is assumed that no defective occurs during the rework process, that is, during $t_{2j}$ because of careful operation or special attention.

We can easily show that in each cycle, the length of the first phase of production of product *j*, $t_{1j}$, and the length of the rework processing time of defectives of the same product, $t_{2j}$, are respectively equal to:

$$t_{1j} = \frac{Q_j}{P_j}, \tag{2}$$

$$t_{2j} = \beta_j\left(\frac{Q_j}{P_j}\right). \tag{3}$$

Since the following relation holds for *j=1,…,n*,

$$Q_j = D_j T, \tag{4}$$

we can write Equations (2) and (3) as follows:

$$t_{1j} = \left(\frac{D_j}{P_j}\right)T, \tag{5}$$

$$t_{2j} = \beta_j\left(\frac{D_j}{P_j}\right)T = \beta_j t_{1j}. \tag{6}$$

Define the on hand inventory of non-defective products at the end of $t_{1j}$ and at the end of $t_{2j}$ respectively by $h_{1j}$ and $h_{2j}$. Now from Figure 1 we can write:

$$\begin{aligned} h_{1j} &= [P_j(1-\beta_j)-D_j]\,t_{1j} \\ &= (P_j - D_j - \beta_j P_j)t_{1j}, \end{aligned} \tag{7}$$

and

$$\begin{aligned} h_{2j} &= h_{1j} + (P_j - D_j)t_{2j} \\ &= [P_j(1-\beta_j)-D_j]\,t_{1j} + (P_j - D_j)\,t_{2j}, \end{aligned}$$

or from Equation (6):

$$\begin{aligned} h_{2j} &= [P_j(1-\beta_j)-D_j]t_{1j} + (P_j - D_j)\beta_j t_{1j} \\ &= (P_j - D_j - \beta_j D_j)t_{1j}. \end{aligned} \tag{8}$$

The total cost of product *j*, denoted by $K_j$, consists of setup cost, processing cost, inspection cost and inventory carrying cost for the product. Let for product *j, j=1,…, n*:

$K_{js}$ Total set up cost per year.

$\bar{I}_j$ Average on hand inventory.

$K_{jH}$ Average inventory carrying cost.

Then, we can write:

$$K_{js} = \left(\frac{D_j}{Q_j}\right)A_j = \frac{A_j}{T}, \tag{9}$$

where $\frac{D_j}{Q_j} = \frac{1}{T}$ stands for the number of cycles per year. We can also write:

$$K_{jH} = H_j \bar{I}_j. \tag{10}$$

From Figure1, the average inventory of product *j* can be written as:

$$\bar{I}_j = \frac{1}{2}[h_{1j}(\frac{t_{1j}}{T}) + (h_{1j} + h_{2j})(\frac{t_{2j}}{T})$$
$$+ h_{2j}(\frac{T - t_{1j} - t_{2j}}{T})]$$
$$= \frac{1}{2T}[(h_{1j} - h_{2j})(t_{1j} + t_{2j}) + h_{2j}(T + t_{2j})] \, . \quad (11)$$

From Equations (5) and (6) we have:

$$t_{ij} + t_{2j} = (1 + \beta_j)t_{1j} \, .$$

And from Equations (7) and (8) we have:

$$h_{2j} - h_{1j} = (P_j - D_j)\beta_j t_{1j} \, .$$

Substituting (8) and these values in Equation (11) we can write:

$$\bar{I}_j = \frac{t_{1j}}{2T}[-\beta_j{}^2 P_j t_{1j} + (P_j - D_j)T - \beta_j D_j T] \, .$$

Now from Equation (5), the above relation can be written as:

$$\bar{I}_j = \frac{1}{2}[P_j - D_j - \beta_j D_j(1 + \beta_j)](\frac{D_j}{P_j})T \, . \quad (12)$$

Thus, from Equations (10) and (12) we have:

$$K_{jH} = (\frac{H_j}{2})[P_j - D_j - \beta_j D_j(1 + \beta_j)](\frac{D_j}{P_j})T \, . \quad (13)$$

Let $K_{jo}$ denote the operation processing cost per year for product *j*. To obtain $K_{jo}$ first we note that in each cycle time *T* the cost of operation process during $t_{1j}$ is equal to $C_j Q_j$ and the cost of operation process during $t_{2j}$, for $\beta_j Q_j$ defective units, is equal to $C_j \beta_j Q_j$. Thus the sum of these two costs in each cycle *T* is $C_j(1 + \beta_j)Q_j$. Therefore, the total production process cost per year is:

$$K_{jo} = \frac{C_j(1 + \beta_j)Q_j}{T} = C_j(1 + \beta_j)D_j \, . \quad (14)$$

Let $K_{ji}$ represent the total inspection cost per year for product *j*. To obtain $K_{ji}$ we first note that in each cycle time the cost of inspection during the normal production time, $t_{ij}$, is equal to $l_j Q_j$ and the cost of inspection during the rework processing time, $t_{2j}$, is equal to $m_j \beta_j Q_j$. Hence, the total inspection cost during each cycle *T* is the sum of these two items, i.e.:

$$l_j Q_j + m_j \beta_j Q_j \, .$$

Therefore, the total inspection cost per year is:

$$K_{ji} = \frac{1}{T}(l_j + m_j \beta_j)Q_j \, ,$$

or from Equation (4):

$$K_{ji} = (l_j + m_j \beta_j)D_j \, . \quad (15)$$

Now, $K_j$, the total cost per unit time (year) for product *j*, can be written as:

$$K_j = K_{js} + K_{jH} + K_{jo} + K_{ji} \, . \quad (16)$$

Thus, from Equations (9), (13), (14) and (15):

$$K_j = C_j D_j(1 + \beta_j) + \frac{A_j}{T} + (\frac{H_j}{2})$$
$$*[P_j - D_j - \beta_j D_j(1 + \beta_j)](\frac{D_j T}{P_j})$$
$$+ (l_j + m_j \beta_j)D_j \, . \quad (17)$$

Therefore, the total cost per year for all products, *K*, can be written as:

$$K = \sum_{j=1}^{n} K_j \, ,$$

or

$$K=\sum_{j=1}^{n}D_j[C_j(1+\beta_j)+(l_j+m_j\beta_j)] \tag{18}$$
$$+\sum_{j=1}^{n}\frac{A_j}{T}+\frac{T}{2}\sum_{j=1}^{n}H_j[P_j-D_j-\beta_jD_j(1+\beta_j)](\frac{D_j}{P_j})$$

One can easily show that the second derivative of $K$ with respect to $T$ is positive. Hence $K$ is a convex function. Therefore letting the first derivative of $K$ with respect to $T$ to be equal to zero, we can obtain the optimal value of $T$, denoted by $T_0$, which minimizes $K$.

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$$T_0=\frac{\sqrt{2\sum_{j=1}^{n}A_jP_j}}{\sqrt{\sum_{j=1}^{n}H_jD_j[P_j-D_j-\beta_jD_j(1+\beta_j)]}} \tag{19}$$

$$T=\frac{\sqrt{2\sum_{j=1}^{n}A_j}}{\sqrt{\sum_{j=1}^{n}H_jD_j(1-\frac{D_j}{P_j})}}\,. \tag{20}$$

The value of $T_0$ in Equation (19) is the optimal cycle time if it is a feasible solution. In fact, an arbitrary cycle time with length $T$ is feasible if the sum of the setup times and production times of all products in that cycle is not greater than the length of the cycle. That is, a cycle time $T$ is a feasible solution if:

$$\sum_{j=1}^{n}(S_j+t_{1j}+t_{2j})\le T\,. \tag{21}$$

Now from Equations (5) and (6) we can write:

$$t_{1j}+t_{2j}=(1+\beta_j)(\frac{D_j}{P_j})T\,. \tag{22}$$

Hence, (21) can be written as:

$$T\ge\sum_{j=1}^{n}S_j+T\sum_{j=1}^{n}\frac{D_j}{P_j}(1+\beta_j)\,. \tag{23}$$

Therefore, a cycle time $T$ is feasible if it satisfies the following relation:

$$T\ge\frac{\sum_{j=1}^{n}S_j}{1-\sum_{j=1}^{n}(1+\beta_j)\frac{D_j}{P_j}}\,. \tag{24}$$

Let:

$$T_m=\frac{\sum_{j=1}^{n}S_j}{1-\sum_{j=1}^{n}(1+\beta_j)\frac{D_j}{P_j}}\,.$$

Then, since $K$ is a convex function, we can obtain the optimal value of $T$, denoted by $T^*$, as follows:

$$T^*=T_0 \quad \text{if} \quad T_0\ge T_m$$
$$T^*=T_m \quad \text{if} \quad T<T_m$$

Finally, the optimal batch size for product $j$, $j=1,\dots,n$, is $Q_j^*=D_jT^*$.

## 5. Conclusion

In the general economic lot size scheduling (ELSP), it has been assumed that the items produced are non-defective and do not need any rework. In this paper, we address the rework of defective items in a multi-product single machine system. Adopting the common cycle time approach for all products, allowing non-zero set up times for each product, and assuming different inspection costs for the normal production and the rework periods, we obtain the optimal common cycle time, hence the optimal batch sizes for all products, which minimizes the sum of inventory holding cost, setup cost, production process cost, and inspection cost.

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