Journal of Industrial Engineering International *Islamic Azad University, South Tehran Branch*
July 2008, Vol. 4, No. 7, 33-37

A multi-objective geometric programming model for optimal production and marketing planning

S. J. Sadjadi*

Associate Professor, Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

Sh. Arabzadeh

Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

Abstract

This paper presents a multi-objective geometric programming model which determines the product's selling price in two markets. We assume demand is a function of price and marketing expenditure in two markets. The cost of production is also assumed to be a function of demands in both markets. Our model is a posynomial function which is solved using Geometric Programming (GP). In our GP implementation, we use a transformed dual problem to change the model into an optimization of an unconstraint problem with a single variable solved using a simple line search. In order to study the behavior of the model we analyze the solution in different cases and a numerical example is used to demonstrate the implementation for each case.

Keywords: Geometric programming; Production and operation management; Economics

1. Introduction

One of the most important issues on having a fair price discrimination strategy is to choose a right model.

Many traditional discrimination models assume production as a function of price in a form of linear or quadratic. In this paper the production and cost function are considered to be an exponential form of price and marketing, respectively. These types of modeling have been widely used in the literature (4, 6, 7, 8 and 9). They consider production as a function of price and marketing expenditure and assume that when demand increases, production will be less costly. Sadjadi et. al. [10] study the effects of integrated production and marketing decisions in a profit maximizing firm. Their model formulation is to determine price, marketing expenditure, demand or production volume, and lot size for a single product with stable demand when economies of scale are given. Lee [7] considers the same demand function for determining order quantity and selling price. In their implementation, they use a previous model formulation [9], with an adaptation of Geometric Programming (GP), to determine the global solution of model.

The primary assumption of this paper is to determine price and marketing strategy in two markets. We assume that we have competition in two markets. Therefore it is necessary to have advertisement on selling goods in two markets. In the other word, pricing strategy is the only way to promote market. The objective function of our modeling is to maximize the profit in two markets. The proposed model of this paper considers the first market as a primary objective and optimizes it first. Then, we optimize the profit for the second market while we keep the first optimal solution. We use an arbitrary value in order to compromise between the profits for two markets. The resulted problems for two stage of algorithm are in posynomial GP problem [3]. We use GP method to find the global maximize of the resultant model. In order to analyze the behavior of the proposed method under different conditions postoptimaly analysis is presented.

This paper is organized as follows: First, we present problem statement in stage one. Next, GP method is used to find the optimal solution of the problem formulation and then find the optimal solution of the problem statement in the second market with a constraint, which means that profit in second

* *Corresponding author. E-mail:sjsadjadi@iust.*

market must be bigger than the first market's profit. Throughout the paper, we use some numerical examples in order to show the implementation of the algorithm and analyze the behavior of the parameters of our model.

2. Problem statement

Consider a single product where its demand is affected by selling price and consider the following notation:

- P_i Selling price per unit,
- α_i The price elasticity to demand,
- M Marketing expenditure per unit,
- γ_i Marketing expenditure elasticity to demand in the market i=1,2,
- k_i Represent other related factors,
- c_i The production cost per unit for market i=1,2,
- D_i The production lot size (unit) for two markets, i=1,2,
- u_i The scaling constants for unit production cost in two markets, i=1,2,
- β_i Lot size elasticity of production unite cost, i=1,2
- t An arbitrary number.

For both markets we assume,

$$D_i = k_i P_i^{-\alpha i} M^{\gamma i} \qquad i = 1,2, \tag{1}$$

where production (D_i ,i=1,2) are defined as a function of price per unit (Pi) and marketing expenditure (M) with α_i>1 0< γ_i<1 , i=1,2.

The scaling constants k_i represent other related factors and the assumption α_i>1, i=1,2 implies that D_1 , D_2 increase at a diminishing rate as P_1 and P_2 decrease. This type of relationship is widely used in the literature [6-10]. Besides, (1) can be easily estimated by applying linear regression to the logarithm of the function.

We assume that the unit production cost (c_1) and (c_2) can be discounted with β_1 and β_2, respectively. Therefore we have:

$$c_1 = u_1 D_1^{-\beta_1} \quad , \quad c_2 = u_2 D_2^{-\beta_2}, \tag{2}$$

where D_1 and D_2 are production lot size (units), u_1 and u_2 are the scaling constants for unit production cost in market one and two, respectively. The exponent β_1 and β_2 represent lot size elasticity of production unite cost with 0<β_1 , β_2<1 which are almost the same as price elasticity α and we suggest a small value for it, say β_1 , β_2 =0.01. We will also explain that the algorithm we use imposes some other limitations for all the parameters in our model.

3. The proposed model

In this section we present our proposed production lot sizing and marketing model (π_i ,i=1,2) based on the explained assumptions. As we explained the proposed method of this paper has two stages. In the first stage we are interested in maximizing the profit $\pi_1(P_1,M)$ simultaneously in order to determine the prices and marketing expenditure for the planning horizon as follows:

Max $\pi_1(P_1 ,M)$ = Revenue in Market 1 - Production cost in Market 1 - Marketing expenditure in Market 1 = $P_1D_1 - C_1D_1 - MD_1$. (3)

In the second stage, we optimize the profit for the second market keeping the profitability for the first market. Therefore we have:

$$\text{Max } \pi_2 = P_2D_2 - C_2D_2 - MD_2$$
Subject to:
$$\pi_1 = P_1D_1 - C_1D_1 - MD_1 > t\, \pi_1^*, \tag{4}$$

where $0 \le t \le 1$. Obviously, when t = 0, we prefer the profitability of the second market to the first one. As *t* increases, we are more interested in keeping the profitability of the first market as we optimize π_2.

In order to solve (4), we need to have the optimal solution π_1*. The optimal solution for π_1 is obtained as follows:

$$\text{Max } \pi_1 = P_1D_1 - C_1D_1 - MD_1 . \tag{5}$$

Problem (5) is in Geometric Programming which can be easily formulated in posynomial form. Since there are two variables and three terms associated with (5) the degree of difficulty is equal to 3-(2+1) =0 [3].Therefore we have:

Max π_1 or Min π_1^{-1}
Subject to: $P_1D_1 - C_1D_1 - MD_1 > \pi_1$, (6)

or

$\text{Min } \pi_1^{-1}$,

Subject to:

$$k_1 P_1^{1-\alpha 1} M^{\gamma 1} - u_1 k_1^{1-\beta 1} P_1^{\alpha 1(\beta 1-1)} M^{\gamma 1(1-\beta 1)}$$

$$-k_1 P_1^{-\alpha 1} M^{\gamma 1+1} > \pi_1. \quad (7)$$

Therefore we have:

$\text{Min } \pi_1^{-1}$,

Subject to:

$$u_1 k_1^{-\beta 1} P_1^{\alpha 1 \beta 1-1} M^{-\beta 1 \gamma 1} + P_1^{-1} M$$

$$+ k_1^{-1} P_1^{\alpha 1-1} M^{-\gamma 1} \pi_1 \leq 1. \quad (8)$$

Problem (8) is in posynomial form and can be solved using its dual problem formulation as follows:

$$d(\pi_1) = \text{Max } f(w) = \left[\frac{1}{w_0}\right]^{w_0} \left[\frac{u_1 k_1^{-\beta_1} \lambda}{w_1}\right]^{w_1} \left[\frac{\lambda}{w_2}\right]^{w_2} \left[\frac{k_1^{-1} \lambda}{w_3}\right]^{w_3}$$

Subject to:

$w_0 = 1$,

$-w_0 + w_3 = 0$,

$(\alpha_1 \beta_1 - 1) w_1 - w_2 + (\alpha_1 - 1) w_3 = 0$,

$-\beta_1 \gamma_1 w_1 + w_2 - \gamma_1 w_3 = 0, \quad (9)$

Thus,

$w_1 = (\gamma_1 + 1 - \alpha_1)/(\alpha_1 \beta_1 - \beta_1 \gamma_1 - 1)$,

$w_2 = (\beta_1 \gamma_1 - \gamma_1)/(\alpha_1 \beta_1 - \beta_1 \gamma_1 - 1)$,

$w_3 = 1$,

$\lambda = (\alpha_1 \beta_1 - \alpha_1)/(\alpha_1 \beta_1 - \beta_1 \gamma_1 - 1), \quad (10)$

Using w_i $i = 0,\ldots,3$ from (10), one can determine the optimal solution π_1^* from (9) and solve (4) as follows:

$\text{Min } \pi_2^{-1}$

Subject to:

$P_1 D_1 - C_1 D_1 - MD_1 \geq t\, \pi_1^*, \quad (11)$

$P_2 D_2 - C_2 D_2 - MD_2 \geq \pi_2,$

or

$\text{Min } \pi_2^{-1}$,

Subject to: (12)

$$u_1 k_1^{-\beta 1} P_1^{\alpha 1 \beta 1-1} M^{-\beta 1 \gamma 1} + P_1^{-1} M + t\, \pi_1^* k_1^{-1} P_1^{\alpha 1-1} M^{-\gamma 1} \leq 1,$$

$$u_2 k_2^{-\beta 2} P_2^{\alpha 2 \beta 2-1} M^{-\beta 2 \gamma 2} + P_2^{-1} M + k_2^{-1} P_2^{\alpha 2-1} M^{-\gamma 2} \pi_2 \leq 1.$$

Problem (12) is a minimization of a nonlinear posynomial objective function subject to two posynomial constraints. Since there are eight terms and five variables, the degree of difficulty is 8-(5+1) =2. Therefore we have:

$$d(\pi_2) = \text{Max } f(W) = \left[\frac{1}{w_{01}}\right]^{w_{01}} \left[\frac{u_1 k_1^{-\beta_1} \lambda_1}{w_{11}}\right]^{w_{11}} \left[\frac{\lambda_1}{w_{12}}\right]^{w_{12}} \left[\frac{t \pi_1^* k_1^{-1} \lambda_1}{w_{13}}\right]^{w_{13}}$$

$$\times \left[\frac{u2 k_2^{-\beta_2} \lambda_2}{w_{21}}\right]^{w_{21}} \left[\frac{\lambda_2}{w_{22}}\right]^{w_{22}} \left[\frac{k_2^{-1} \lambda_2}{w_{23}}\right]^{w_{23}}$$

Subject to:

$w_{01} = 1$,

$- w_{01} + w_{23} = 0$,

$(\alpha_1 \beta_1 - 1) w_{11} - w_{12} + (\alpha_1 - 1) w_{13} = 0$,

$- \beta_1 \gamma_1 w_{11} + w_{12} - \gamma_1 w_{13} - \beta_2 \gamma_2 w_{21} + w_{22} - \gamma_2 w_{23} = 0$,

$(\alpha_2 \beta_2 - 1) w_{21} - w_{22} + (\alpha_2 - 1) w_{23} = 0$,

$\lambda_1 = w_{11} + w_{12} + w_{13}$,

$\lambda_2 = w_{21} + w_{22} + w_{23}, \quad (13)$

We rewrite the linear equation of (13) in terms of two variables, w_{21} and w_{13}. Therefore we have:

$w_{01} = 1$,

$w_{23} = 1$,

$w_{12} = w_{21} (\alpha_2 \beta_2 - 1) + \alpha_2 - 1, \quad (14)$

$w_{22} = -[(-\alpha_1 + \gamma_1 + 1)\, w_{13} + (\beta_2\, \gamma_2 + 1 - \alpha_2\beta_2)\, w_{21} + \gamma_2 - \alpha_2 + 1] / (\beta_1\gamma_1 - \alpha_1\beta_1 + 1),$

$w_{11} = -[(\alpha_1\beta_1\beta_2\gamma_2 + \alpha_1\beta_1 - \alpha_1\beta_1\alpha_2\beta_2 - \beta_2\gamma_2 - 1 + \alpha_2\beta_2)\, w_{21} + w_{13}(-\gamma_1 + \gamma_1\beta_1) + [-\alpha_1\beta_1\alpha_2 + \alpha_1\beta_1 + \beta_1\alpha_1\gamma_2 + \alpha_2 - 1 - \gamma_2]] / (\beta_1\gamma_1 - \alpha_1\beta_1 + 1).$

As we can observe, the linear constraints in (13) can be converted into (14) where there are only two unknowns. Therefore, we may use a simple grid search to find the optimal solution. Note that in order to have a feasible solution in (13) the following must hold:

$t_1 = (1 - \alpha_2) / (-1 + \alpha_2\, \beta_2),$

$t_2 = [[(-\alpha_1 + \gamma_1 + 1)\, (-\alpha_1\beta_1\alpha_2 + \alpha_1\beta_1 + \alpha_1\beta_1\gamma_2 + \alpha_2 - 1 - \gamma_2)] / [(\beta_1\gamma_1 - \gamma_1)\, (\beta_2\gamma_2 + 1 - \alpha_2\beta_2)]] - [(\gamma_2 - \alpha_2 + 1) / (\beta_2\gamma_2 + 1 - \alpha_2\beta_2)],$

$t_3 = 1 / [1 - [(-\alpha_1 + \gamma_1 + 1)\, (\alpha_1\beta_1\beta_2\gamma_2 + \alpha_1\beta_1 - \alpha_1\beta_1\alpha_2\beta_2 - \gamma_2\beta_2 - 1 + \alpha_2\beta_2) / (\beta_1\gamma_1 - \gamma_1) / (\beta_2\gamma_2 + 1 - \alpha_2\beta_2)]],$

$t_4 = t_2 \times t_3,$

$t_5 = -[-\alpha_1\beta_1\alpha_2 + \alpha_1\beta_1 + \alpha_1\beta_1\gamma_2 + \alpha_2 - 1 - \gamma_2] / (\beta_1\gamma_1 - \gamma_1),$

$t_6 = -(\alpha_1\beta_1\beta_2\gamma_2 + \alpha_1\beta_1 - \alpha_1\beta_1\alpha_2\beta_2 - \beta_2\gamma_2 - 1 + \alpha_2\beta_2) / (\beta_1\gamma_1 - \gamma_1),$

$$0 < w_{21} < \min(t_4, t_1),$$

$$\max(t_6 w_{21} + t_5, 0) < w_{13}. \quad (15)$$

4. Numerical example

In this section we present numerical experience of the implementation of the proposed method. Suppose we have:

$\alpha_1 = 1.5$, $\alpha_2 = 2.0$, $\beta_1 = 0.01$, $\beta_2 = 0.02$,

$\gamma_1 = 0.1$, $\gamma_2 = 0.2$, $t = 0.9218$, $u_1 = 0.2$,

$u_2 = 5$, $k_1 = k_2 = 10^6$.

This example is solved using the procedure explained in section 3. The procedure first finds the optimal solution, π_1^*. In the second stage we find the optimal solution of π_2^* subject to $\pi_1 \geq t\pi_1^*$. The optimal weights are calculated to be:

$w_{13}^* = 4.986$, $w_{01}^* = 1$, $w_{11}^* = 0.492$,

$w_{12}^* = 2.0314$, $w_{21}^* = 0.821$, $w_{22}^* = 0.21184$,

$w_{23}^* = 1$, $\lambda_1^* = 7.509$, $\lambda_2^* = 2.03284$.

And the optimal solution is summarized as follows:

$P_1^* = \$\,16.086$, $P_2^* = \$8.979$, $M^* = \$\,0.936$,

$\pi_1^* = 1.783e{+}005$, $\pi_2^* = 4.775e{+}004$,

$\pi^* = \pi_1^* + \pi_2^* = 2.2509e{+}005$.

5. Conclusion

In this paper, we have presented a new multi objective Geometric Programming model to determine the optimal price discrimination. The proposed model of this paper has considered the production in the first market as a function of price and the production in the second market as a function of price and marketing expenditure. The primary assumption is that the second market is highly competitive. Therefore we need to penetrate the market not only by the competitive price but also by using a sophisticated marketing strategy. The proposed model minimizes the cost of production and marketing subject to maintain minimum acceptable revenue in order to keep the market shares in both markets. We have used geometric programming to determine the optimal solution of the proposed model. Numerical examples have been used to present the implementation of our algorithm. One of the extensions on our model is to maximize the profit as difference between the revenue and the costs. Such a model turns to be a signomial problem and the global solution of the resulted model is not guaranteed. Therefore, we suggest interested researchers to study this problem as future research.

References

[1] Beightler, C. S. and D. T. Phillips, 1976, *Applied Geometric Programming*. NEW YORK: John Wiley & Sons.

[2] Dembo, R. S., 1982, Sensitively analysis in geometric programming. *Journal of Optimization Theory and Applications,* 37, 1-12.

[3] Duffin, R. J., Peterson, E. L. and Zenner, C., 1967, Geometric Programming -Theory and Application. NEW York: John Wiley & Sons.

[4] Freeland, J. R., 2005, Coordination strategies for production and marketing in a functionally decentralized firm. *AIIE Transactions*, 12,126-132.

[5] Kim, D. and Lee, W. J., 1998, Optimal joint pricing and lot sizing with fixed and variable capacity. *European Journal of operation Research*, 109, 212-227.

[6] Frransis Lueng, K., 2005, A generalized geometric programming solution to an economic production quantity model with flexibility and reliability considerations. *European Journal of operation Research,* to appear.

[7] Lee, W. J., 1993, Determining selling price and order quality by geometric programming, optimal solution, bounds and sensitivity. *Decision Sciences*, 24, 76-87.

[8] Lee, W. J. and Kim, D., 1993, Optimal and heuristic decision strategies for integrated production and marketing planning. *Decision Sciences*, 24, 1203-1213.

[9] Lilien, G. L., Kotler, P. and Moorty, K. S., 1992, *Marketing Models*. Englewood Cliffs, NJ; Prentice Hall.

[10] Sadjadi, S. J., Orougee, M. and Aryanezhad, M. B., 2005, Optimal production and marketing planning, *Computational Optimization and Application,* 30(2).

[11] Starr, M. K., 1989, *Managing Production and Operations.* Englewood Cliffs, NJ: Prentice Hall.