Optimal lot size of EPQ model considering imperfect and defective products

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Abstract

The economic production quantity (EPQ) is a commonly used inventory model. An assumption in the EPQ model is that all units produced are perfect. Some researchers have studied the effects after relaxing this assumption on the inventory models. The objective of this paper is to determine the economic production quantity with reduced pricing, rework and reject situations in a single-stage system in which rework takes place in each cycle after processing to minimize total system costs. The assumption entertained in this paper is that processing leads to different products classified in the four groups of perfect products, imperfect products, defective but reworkable products, and, finally, non-reworkable defective products. The percentage of each type is assumed to be constant and deterministic. A mathematical model is developed and numerical examples are presented to illustrate the usefulness of this model compared to previous ones.

Keywords: Economic production quantity; Reduced price; Rework; Reject

1. Introduction

The classical EPQ model has been in use for a long time. It is a well-established and widely used technique in inventory management (Bedworth and Bailey, [1]). The EPQ model can be considered as an extension of the well-known economic order quantity, EOQ, model introduced by Harris [10] to minimize total inventory cost for a single-stage production system. A usual unrealistic assumption in EPQ is that all units produced are of good quality (Warets, [21]). The classical EPQ model shows that the optimal lot size will generate minimum manufacturing cost, thus producing minimum total setup and inventory costs. However, this is only true if all manufactured products are of perfect quality. In reality this is not the case; therefore, it is necessary to allow cost for handling imperfect products as this cost can influence the decision for selecting the economic lot size (Chan et al., [4]). Hence, in recent decades, researchers tried to determine the optimal batch quantity of imperfect production system considering different operating conditions. A brief discussion of their work is given as follows:

Gupta and Chakraborty [9] considered the reworking option of rejected items. They considered recycling from the last stage to the first stage and obtained an economic batch quantity model. Porteus formulated the relationship between process quality improvement and setup cost reduction and illustrated that the annual cost can be further reduced when a joint investment in both process quality improvement and setup reduction is optimally made [15]. Cheng [5] validates Porteus's model by including the learning effects on setup frequency and process quality. Rosenblatt and Lee [16] assumed that the time from the beginning of the production run until the process goes out of control is exponential and that defective

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items can be reworked instantaneously at a cost and kept in stock. Tapiero et al. [19] have presented a theoretical framework to examine the tradeoffs between pricing, reliability, design and quality control issues in manufacturing operations.

Schwaller presented a procedure that extends EOQ model by adding the assumptions that a known proportion of defectives existed in arriving lots and that fixed and variable inspection costs were required in seeking and eliminating the defectives [18]. Zhang and Gerchak [22] considered a joint lot sizing and inspection policy in order to develop the EOQ model where the number of defective items in each lot is random and defective units cannot be used and, thus, they must be replaced with non-defective ones. Cheng [6] addressed an EOQ model with demanddependent unit cost and imperfect production processes and formulated the optimization problem as a geometric program to obtain a closed-form optimal solution. Lee et al. [13] developed a model of batch quantity in a multi-stage production system considering various proportions of defective items produced in every stage while they ignored the rework situation. Salameh and Jaber [17] surveyed an EOQ model where each lot contains a certain percentage of defective items with a continuous random variable. They also considered that imperfect items could be sold as a single batch at a reduced price by the end of 100% inspection but they did not address the impact of the reject and the rework and ignored the factor of when to sell. However, they made an error in their final formulation later corrected by Cardenas [3]. In their paper, Salameh and Jaber did not declare what point in the cycle would be appropriate for selling the imperfect products. This was the point taken up by Papachristos and Konstantaras [14] for clarification and elucidation. They also look at the sufficient conditions given in the Chan et al. [4] and Salameh and Jaber [17] papers, which are related to the issue of non-shortages and pointed out that the proposed conditions cannot prevent shortages from happening. Because the portion of non-perfect items in our model in this paper is assumed constant, there is not this problem in our model. Goyal and Cardenas-Barron [8] presented a simple approach for determining the economic production quantity for an item with imperfect quality and suggested that this simple approach was comparable to the optimal method of Salameh and Jaber. Hayek and Salameh assumed that all of the defective items produced were repairable and obtained an optimal point for EPQ model under the effect of reworking of imperfect quality items [11]. Teunter and van der Laan [20] tried to find the

solution for the non-optimal condition in an inventory model with remanufacturing. Chiu [7] considered a finite production model with random defective rate, scrap, the reworking of repairable defective items and backlogging to derive an optimal operating policy including lot size and backordering levels that minimized overall inventory costs. Chan et al. [4] provided a framework to integrate lower pricing, rework and reject situations into a single EPQ model. They found that the time factor of when to sell the imperfect items is critical, as this decision would affect the inventory cost and the batch quantities. They also assumed that defective items could be reworked instantaneously at a cost and kept in stock. Jamal et al. considered a single production system with rework options incorporating two cases of rework process to minimize the total system cost. In the first case, they considered that the rework executed within the same cycle and the same stage of production. In the second case, the defective items are accumulated up to N cycles to be then reworked in the next cycle. Jamal et al. assumed that all defective products could be reworked [12]. Recently, Ben-Daya et al. [2] developed integrated inventory inspection models with and without replacement of nonconforming items discovered during inspection. Inspection policies include no inspection, sampling inspection, and 100% inspection. They proposed a solution procedure for determining the operating policies for inventory and inspection consisting of order quantity, sample size, and acceptance number.

This paper extends the work by Jamal et al. [12] and case II of Chan et al. [4] and studies the optimal run time problem of EPQ model with imperfect products, reworking of the reparable defective products and rejecting of non-reworkable defective items. Neglecting the production of imperfect products and scraps, Jamal et al. assumed that all defective products could be reworked, but in some real situations, it is observed that some non-perfect products cannot be reworked and they should be either sold at a lower price or rejected altogether. In this paper, the different scenarios for imperfect quality products are investigated. While Chan et al. assumed that defective items are reworked instantaneously with processing at no additional time, our assumption is that these products should be reworked at the end of the processing period in a kind of reprocessing stage. In other words, reworking an imperfect item takes time and money as does the processing of a product [12].

This paper is organized along the following lines: Problem definition, notations and assumptions used throughout this study are presented in Section 2. In Section 3, the mathematical models are derived in order to minimize the total cost per unit time. The various costs of the inventory system considered here include setup, production, inspection, rejection, and inventory holding costs. In this section, the optimal solution to the problem is also introduced. Numerical examples are provided in Section 4 to demonstrate the applicability of the proposed model. In Section 5, we present some conclusions and recommendations for possible future work.

2. Problem definition

Consider the classical EPO model and assume that a process produces a single product in a batch size of Q. Producing these items takes place at a finite production rate, P units per unit time. A 100% inspection, which has a fixed cost for each unit, is performed in order to identify the quality of each product. Demand for perfect product is continuous with Dunits per unit time. Each lot produced contains p_1 percent of imperfect quality items (See Figure 1). The perfect and imperfect products are kept in stock when identified. The imperfect product is sold at the end of processing period, i.e., end of T_p in Figure (2), as a single batch at a reduced price per unit. The lot also contains a percentage of defectives, p_2 , so that these defective products can be reprocessed, or reworked, after the processing period and kept in stock. These products are assumed to be of good quality after reprocessing. Thus, the reworked products will need no inspection. Each lot produced also contains a percentage of defectives, p_3 , so that these units are rejected with an associated cost when identified. In other words, a defective product that cannot be reworked is rejected immediately after its work operation completes with an associated cost. The main objective of the present study is to minimize the total system cost of the inventory system. Below are the notations used and assumptions made:

2.1. Notations

- *A* Setup cost for each lot.
- *C* Production cost per unit.
- *I* Inspection cost per unit.
- J Reject cost per unit.
- *H* Inventory holding cost per unit per unit time.

- *D* Demand rate in units per unit time.
- *P* Production rate in units per unit time.
- C_S Setup cost per unit time.
- C_M Production cost per unit time.
- C_I Inspection cost per unit time.
- C_J Reject cost per unit time.
- Q Lot size in number of units per cycle.
- S_P Unit selling price for good quality products.
- *S_I* Unit selling price for imperfect quality products.
- p_1 Percentage of imperfect quality products.
- *p*₂ Percentage of rework products.
- p_3 Percentage of reject products.
- *T* Cycle time.
- T_p Processing time in each cycle.
- T_r Reprocessing, reworking, time in each cycle.
- T_m Total production run time, sum of the processing and reprocessing times in each cycle.
- T_d Time in each cycle when there is no production.
- I_1 Total quantity of inventory in stock during the processing period.
- *I*₂ Total quantity of inventory in stock during the reprocessing period.
- I_3 Total quantity of inventory in stock when there is no production.

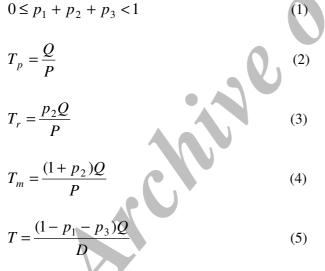
2.2. Assumptions

- No Shortage is allowed.
- The demand rate for the good product is deterministic and constant.
- The demand for the imperfect product with reduced price always exits.
- Proportions of imperfect, reworked and rejected products are constant in each cycle.

- No imperfect or defective product is produced during the rework.
- The processing and reprocessing are accomplished using the same resources at the same speed.
- No stop is allowed during the manufacturing operations of one lot.
- All parameters including production and demand rates, setup time, etc. are constant and deterministic.

3. Modeling

Figure 2 presents the behavior of the inventory level in stock during one cycle. The purpose is to minimize total cost (TC) per unit time. The total relevant cost per unit time comprises such costs as setup, production (processing and reprocessing), inspection, rejection and inventory holding. According to Figure 2, it can be easily shown that:



To ensure that inventory level will not run into shortages, we must have:

$$P(1 - p_2 - p_3) \ge D \tag{6}$$

The various costs per unit time derived with respect to Equation 1 to 5 are as follows:

3.1. Setup cost

In our notations, the setup cost for the production system during a cycle is designated as A. Using Equation 5, therefore, the setup cost per unit time is equal to:

$$C_{S} = \frac{A}{T} = \frac{D}{Q(1 - p_{1} - p_{3})}A$$
(7)

3.2. Production cost

Production cost in each cycle consists of two parts: Processing cost at time T_p , and reprocessing or reworking cost at time T_r . The quantity of products reworked during each cycle is p_2Q . Therefore, according to the notation used and from Eq. 5, production cost per unit time will be:

$$C_M = \frac{CQ + C(p_2Q)}{T} = \frac{CD}{(1 - p_1 - p_3)} (1 + p_2)$$
(8)

3.3. Inspection cost

According to the problem definition, inspection only takes place during the processing time. Therefore, inspection cost per unit time will be:

$$C_{I} = \frac{IQ}{T} = \frac{D}{(1 - p_{1} - p_{3})}I$$
(9)

3.4. Rejection cost

The number of rejected items during each cycle is Qp_3 . So rejection cost per unit time is given by:

$$C_J = \frac{JQp_3}{T} = \frac{Dp_3}{(1 - p_1 - p_3)}J$$
(10)

3.5. Inventory holding cost

The inventory holding cost per cycle is obtained as the average inventory times holding cost per product per cycle. Following Jamal et al., we don't consider any inventory holding costs for the defective items by the machine waiting for rework. The reason for this is the low percentage of the items as well as the low level of such costs as storage, etc. The average inventory level can be evaluated as:

$$\bar{\mathbf{I}} = \frac{\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3}{T}$$
(11)

It is evident from Figure 1 that:

$$\mathbf{I}_{1} = \frac{1}{2}\mathbf{h}_{1}\mathbf{T}_{p} = \frac{Q^{2}}{2P^{2}} \left(P(1 - p_{3} - p_{2}) - D \right) \quad (12)$$

$$I_{2} = \frac{1}{2}(h_{2} + h_{3})T_{r} = \frac{Q^{2}p_{2}}{P^{2}}(P(1 - p_{3} - p_{2} - p_{1}))$$
$$-D + \frac{1}{2}(P - D)p_{2})$$
(13)

$$I_{3} = \frac{1}{2}h_{3}T_{d} = \frac{Q^{2}}{2DP^{2}}(P(1-p_{1}-p_{3}) - D(1+p_{2}))^{2}$$
(14)

As a result and using Equations 11 to 14, the inventory holding cost will be expressed as:

$$C_{H} = H\overline{I} = \frac{HQ}{2P(1-p_{1}-p_{3})} * (P(1-p_{1}-p_{3})^{2} - D(1-2p_{1}+p_{2}(1+p_{2})-p_{3}))$$
(15)

3.6. Total cost per unit time and the optimal solution

As pointed out before, the total cost per unit time can be expressed as:

$$TC = C_S + C_M + C_I + C_J + C_H$$
(16)

Substituting Equations 7 to 10 and 15 in Equation 16 and simplifying, the total cost per unit time will be:

$$TC = \frac{1}{(1 - p_1 - p_3)} * \{A\frac{D}{Q} + \frac{HQ}{2P}(P(1 - p_1 - p_3)^2 - D(1 - 2p_1 + p_2(1 + p_2) - p_3))\} + \frac{D}{(1 - p_1 - p_3)}(C(1 + p_2) + I + p_3J)$$
(17)

Setting the first derivative of *TC* to zero, $\frac{dTC(Q)}{dQ} = 0$, gives the following optimal value of *Q*:

$$Q_{Opt} = \sqrt{\frac{2AD}{H\left[(1-p_1-p_3)^2 - \frac{D}{P}(1-2p_1-p_3+p_2(1+p_2))\right]}}$$
(18)

The second derivate of Equation 17 is negative for all positive values of Q, which implies a unique Q_{opt} that minimizes Equation 17.

If p_2 is zero, the optimal lot size will be the same as the optimal lot size in case II of the Chan et al. model in the event that percentage of defectives is assumed to be constant and deterministic[4]:

$$Q_{Chan} = \sqrt{\frac{2AD}{H(1-p_1-p_3)\left((1-p_1-p_3)-\frac{D}{P}(1-\frac{p_1}{1-p_1-p_3})\right)}}$$
(19)

If p_1 and p_3 are zeros, the optimal lot size will be the same as the policy one in Jamal et al. model [12]:

$$Q_{Jamal} = \sqrt{\frac{2AD}{H\left(1 - \frac{D}{P}(1 + p_2 + p_2^2)\right)}}$$
(20)

If the p_1, p_2 and p_3 are zeros, the optimal lot size in our model will be the same as the optimal lot size in the classical EPQ model:

$$Q' = \sqrt{\frac{2AD}{H\left(1 - \frac{D}{P}\right)}}$$
(21)

4. Numerical examples

To illustrate the usefulness of the model developed in Section 3, consider a production system where the parameters are: A=\$195/cycle; H=\$34/units/year; D=5900units/year; P=9000 units/year; C=\$1/unit; I=\$0.01/unit; J=\$0.01/unit; $p_1=15\%$; $p_2=10\%$ and $p_3=5\%$. From Eqs. 17 and 18, the optimal lot size and minimum total relevant cost are $Q_{Opt}=$ 691units/cycle and $TC(Q_{Opt}) = 12353$ \$/year, respectively.

Now, using the above example, we will compare the optimal quantity in four models, i.e. Eqs. 18-21, for different values of p_1 , p_2 and p_3 . We assume that $p_3 \in [0, 0.06]$ (See Table 1) $p_2 = 3p_3$ and $p_1 = 4 p_3$. For each set of these values, are shown the optimum value of Q in the four models $(Q_{Opt},$ Q_{Chan} , Q_{Jamal} and Q'), the relations holding between them as $\frac{Q_{Opt}}{Q_{Chan}}$, $\frac{Q_{Opt}}{Q_{Jamal}}$ and $\frac{Q_{Opt}}{Q'}$, the total costs corresponding to the quantity, and the percentage of error in total cost, TC, due to using Q_{Chan} , Q_{Jamal} and Q' values instead of Q_{Opt} . Figure (3) also shows the relation between optimum values for the four models using data from Table (1). It is seen in Figure (3) that more units are to be ordered in each lot when the modified EPQ model is used as compared to previous works, e.g., $Q_{Opt} > Q_{Chan} > Q_{Jamal} > Q'$. Figure (4), plotted using columns 9-12, shows the behavior of TC, i.e. Eq. 17, due to lot size in columns 2 to 5.

As seen in Figure (4) and Column 15 of Table (1), the savings in the total cost per unit time as a result of using the modified EPQ model are more considerable at higher values of p_3 , suggesting that the traditional EPQ formula could be used as an estimate for reasonably very low percentages of imperfect and defective products. Therefore, taking account of product quality for high values of p_1 , p_2 and p_3 is of great importance. For example, if $p_3=6.0\%$ (thereby $p_1=24.0\%$ and $p_2=18.0\%$) and further if the product quality is ignored in lot sizing, that is, if equation 21 is used in determining the lot size, then the error in the total cost per unit time will be around 11.18% compared to the case where the optimal point, Q_{Opt} , is applied. Also, it is observed that the error in the total cost per unit time for Q_{Jamal} is greater than Q_{Chan} .

5. Conclusion

In this paper, the EPQ model was investigated by considering production of various types of nonperfect products. From the results obtained in this study, it is concluded that the optimal lot size is affected by considering the quality of product, with the effect becoming more significant at higher percentages of imperfect and defective items. This study also demonstrated the importance of taking into account the quality of product, especially when percentages of imperfect and defective items are high. Numerical examples were used to show that the error due to ignoring product quality would be high; it was, therefore, concluded that this factor must be taken into account when determining the lot size. The effect of machine breakdown on this model may be recommended for further study. Another important study may investigate the effect of time value of money on optimal lot size.

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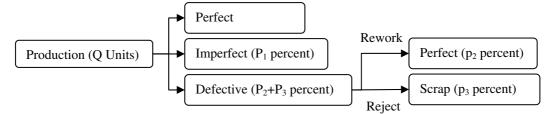
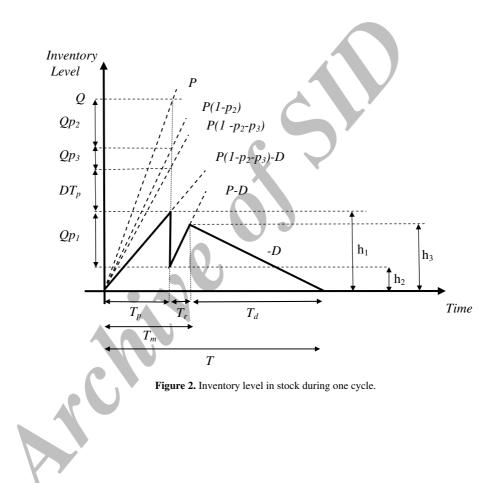


Figure 1. A schematic diagram of our model.



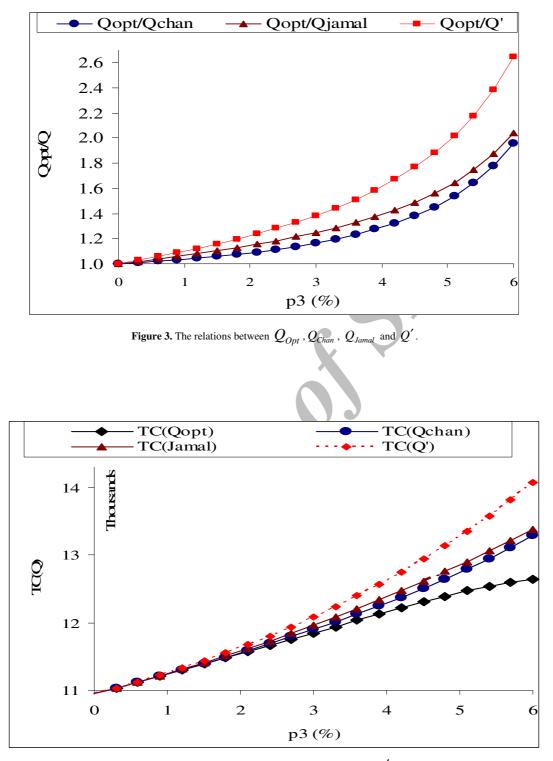


Figure 4. The total cost per unit time for Q_{Opt} , Q_{Chan} , Q_{Jamal} and Q' using Equation 17.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Q_{Opt}	$Q_{\scriptscriptstyle Chan}$	$Q_{\scriptscriptstyle Jamal}$	Qʻ	$rac{Q_{Opt}}{Q_{Chan}}$	$rac{Q_{Opt}}{Q_{Jamal}}$	$rac{Q_{opt}}{Q'}$	ТС			$\frac{TC(Q) - TC(Q_{opt})}{TC(Q)} * 100$			
p_3											$\frac{1}{TC(Q_{opt})} *100$			
(%)								Q_{Opt}	$Q_{\scriptscriptstyle Chan}$	$Q_{\scriptscriptstyle Jamal}$	Q^{\prime}	$Q_{\scriptscriptstyle Chan}$	Q_{Jamal}	Q
0.0	443	443	443	443	1.00	1.00	1.00	11150	11150	11150	11150	0.00	0.00	0.00
0.3	455	451	447	443	1.01	1.02	1.03	11234	11234	11235	11236	0.00	0.01	0.02
0.6	468	459	451	443	1.02	1.04	1.06	11320	11321	11323	11327	0.01	0.03	0.07
0.9	482	468	455	443	1.03	1.06	1.09	11407	11409	11415	11424	0.02	0.07	0.15
1.2	497	476	460	443	1.04	1.08	1.12	11495	11500	11510	11527	0.04	0.13	0.28
1.5	512	484	465	443	1.06	1.10	1.16	11585	11592	11608	11636	0.07	0.20	0.44
1.8	529	493	469	443	1.07	1.13	1.19	11676	11688	11710	11751	0.11	0.30	0.65
2.1	548	501	475	443	1.09	1.15	1.24	11767	11786	11816	11873	0.16	0.41	0.90
2.4	568	510	480	443	1.11	1.18	1.28	11860	11887	11926	12002	0.23	0.55	1.20
2.7	590	518	486	443	1.14	1.21	1.33	11954	11992	12039	12139	0.32	0.72	1.55
3.0	614	527	492	443	1.17	1.25	1.38	12048	12099	12157	12283	0.43	0.91	1.96
3.3	640	535	498	443	1.20	1.29	1.44	12142	12211	12279	12436	0.57	1.13	2.43
3.6	670	543	504	443	1.23	1.33	1.51	12235	12328	12405	12598	0.75	1.39	2.96
3.9	703	551	511	443	1.28	1.37	1.59	12328	12449	12536	12769	0.98	1.69	3.57
4.2	741	559	519	443	1.32	1.43	1.67	12419	12575	12671	12949	1.26	2.03	4.27
4.5	784	567	527	443	1.38	1.49	1.77	12508	12708	12811	13140	1.60	2.43	5.06
4.8	834	574	535	443	1.45	1.56	1.88	12592	12848	12956	13342	2.03	2.89	5.96
5.1	894	581	544	443	1.54	1.64	2.02	12670	12995	13105	13555	2.57	3.43	6.99
5.4	966	588	553	443	1.64	1.75	2.18	12740	13151	13260	13781	3.23	4.08	8.17
5.7	1056	594	563	443	1.78	1.87	2.38	12798	13317	13419	14020	4.06	4.85	9.55
6.0	1173	599	574	443	1.96	2.04	2.65	12838	13494	13583	14273	5.11	5.81	11.18

Table 1. The Q_{Opt} , Q_{Chan} , Q_{Jamal} and Q', the relations between them and the associated costs.