Robust supply chain coordination modeling: A revenue management perspective

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Abstract

The revenue management concept and techniques are applied to model the coordination of supply chain elements. The fundamental premise of this approach is synchronization of a group of business entities consisting of a manufacturer and multiple suppliers to achieve an optimal supply chain capacity plans. The output of the supply chain can be various products and thus it is measured in terms of capacity. In our paper, the demand is stochastic. As a result, the chain faces uncertainty when it comes to determine the volume of contract between manufacturer and suppliers. The model developed in this paper provides the basis for long-term contracts between manufacturer and its supply network for coordinated and non-coordinated supply relationship. It also provides decision rules to increase flexibility in responding to consumer demand shifts without cost overlays in resource utilization, while increasing the overall capacity utilization and market share. The collaboration framework versus independency of supply chain members is introduced to investigate the rules for competition through optimal demand management and capacity allocation. An important result is that the models are robust, as they are independent of demand distribution function. We also study the effect of a supplier who supplies two competitive chains on capacity and price basis. Our analysis of supply-chain shows that at the presence of uncertainty of demand collaborative chains are more robust to capacity reservation plan comparing to independent identities as far as capacity is concerned. The robustness of a supply network to support the chain leader (manufacturer) is also measured and the trade off for sustainable network is proposed.

Keywords: Supply chain modeling; Revenue management; Supply chain contract; Capacity option pricing; Robust planning

1. Introduction

Although the domain of applicability of revenue management is mainly service industry, we extend it to be applied for coordination modeling of supply chain. A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of the finished products to customers. Traditionally, marketing, distribution, planning, manufacturing, and the purchasing departments along the supply chain operate independently. These departments have their own objectives, which are often conflicting. Therefore, many manufacturing operations are designed to maximize revenue and lower costs with little consideration for the impact on service levels and market share. This concept is usually applied through optimization models of capacity utilization.

Furthermore, due to globalization trend, as the producer market is changing to customer market, the concept of increasing service level by inventory is changing to make-to-order (MTO) production policies as well as JIT production system. Such environments for production system transform the concept of product sales to capacity sales by reserving a specific unit of capacity for each customer, which is addressed as capacity planning optimization models and revenue management. Another trend in business environment is the need for production flexibility, which has a counterpart in man-

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ufacturing policy as lean paradigm and supply chain management. Therefore, coordination between the various players in the chain is the key in its effective management.

In this paper, we develop a supply network model. The aim is addressing the new challenges of customer centric market with supply network planning, in favor of network internal consistency and utilization. The result is more capability for adoption to market changes and robust behavior of network. We show a supply network partnership, can be the best basis of long-term collaborative competitive network. This is true especially where there is not a pure single owned supply chain and all individual partners have their own degree of freedom and interest.

Our model combines the capacity reservation with demand uncertainty and focus on a long-term coordination mechanism. The objective is to increase the utilization of network capacity by reservation. This approach can deal with uncertainty of demand and pull up the market demand to planned capacity.

The term robust supply chain in our paper introduces a new concept. It is beneficiary for both manufacturer and supplier. The model combines flexible pricing and capacity utilization through a dynamic interaction with demand patterns.

What makes our paper distinguished from the previous ones is the unique approach and demand dynamism. This is done by introducing a demand independent model that reflects robustness in modeling as well as capacity-price trade off decisions. It reflects robustness to demand changes over time or dynamic pricing in relation to market behavior.

The scope of the paper is related to purchasing and supply side of supply chain management. Physical distribution problem and related planning cases is not considered in our model.

This paper organized as follows. In the next section, we review the related literature. The model is described in Section 3. The main part of the analysis is also introduced in this section. In Sections 4, 5 and 6 different coordination scenarios in supply chain analyzed. Section 7 summarizes the results on network design and competitive analysis.

2. Literature review

The effects of decision-making in supply chains have been investigated in several papers and from different perspectives in the recent years. The researches mainly consider a firm that owns a fixed capacity of a resource that is consumed in the production or delivery of multiple products through maximization of its total expected revenues over a finite horizon. The company may choose a dynamic pricing strategy for each product or, if prices are fixed, by selecting a dynamic rule that controls the allocation of capacity to requests for the different products to its customers [25]. From supply side, in another paper, Zhou, Fan, and Cho [16] consider the setup and focus on the optimal purchasing strategy.

Further researches analyze the single strategic customer Shen [34] as a base for supply chain relationship. In particular, a number of these papers study supply chains with random demand in a single-period setting based on generalizations of the newsvendor framework. Review papers by Tsay, Nahmias and Agrawal [37], Lariviere [23], Cachon [10] and Netessine [30] provide comprehensive pointers to this literature. The papers that investigate decentralized supply chains employing stochastic models in an infinite horizon setting are relatively fewer. Lee and Whang [24] and Chen [14] focus on coordination mechanisms that use nonlinear pricing schemes. Cachon and Zipkin [12] study the two-stage decentralized supply chain in detail and look into coordination issues through linear transfer payments. Cachon [11] extends this analysis to the single supplier and multi-retailer system. The above-mentioned optimization approach to supply chain contracts mainly focus on an inside out view, which is reactive to demand. We restrict our review to three areas, a) supply coordination and capacity planning, b) pricing and demand planning and c) coordination and robust supply network.

2.1. Supply coordination and capacity planning

There are several studies related to contractual agreements between the supply chain members for capacity reservation. Cachon [10] provides an excellent literature review of the supply chain contract. However, contract mechanisms have been proposed in the past. These include Quantity Flexibility [36], Deductible reservation [21], back-up agreements [20], Buy back [23], Pay-to-delay [8], and Take-or-pay [21]. Other studies employed modifications of the above contracting mechanisms [33] and Donohue, [18]. A more general review of the supplier contract literature is reported in an excellent survey by Tsay, Nahmias and Agrawal [37].

One of researches more related to ours is [8, 9] who study two-stage flexible supply contracts for advance reservation of capacity or advance procurement of supply. The demand uncertainty is analyzed to a two period problem with options by Barnes-Schuster, Bassok and Anupindi [5]. Araman, Kleinknecht and Akella [2] do extension of demand to more than one market. They consider the optimal procurement strategy using a mix of the long-term contracts and the spot market supply and provide a necessary and sufficient condition for the contracts to achieve channel coordination. A dynamic capacity allocation procedure introduced by Braut and Sridharan [6] for design purposes is a new approach to evaluate supply chain parameters.

Recently we have seen an increasing interest in auction research within the operations management community (see Vulcano, Van Ryzin and Maglaras [39]; Chu and Shen [16]). It shows that consideration on bidders' behaviors in this line of research is important. For instance, Vulcano, van Ryzin and Maglaras [39] analyze a dynamic auction in which a seller with C units to sell faces a sequence of buyers separated into T time periods. They assume the buyers' valuations for a single unit are private and independent.

Our research is different, as we have extended the contract mechanism to multi supplier on different coordination mechanism including both independent and dependent relationships.

2.2. Supply coordination and pricing

Existing studies in the literature focus mostly on deriving the conditions on pricing for channel coordination. The issue of pricing the supply flexibility and its role in supply contract negotiation has yet to be addressed in detail in the literature. A review on contracts from information perspective is available in Corbert and Tang [17], and the quantity commitment under stochastic demand is reviewed by Anupindi and Bassok [4] to relate the capacity offers to market.

Wu, Kleindorfer and Zhang [40] provide a related model that addresses the option pricing issue in a slightly different setting, where they consider a long-term supply contract between a seller and a buyer with a capacity limit specified in the contract. There is a reservation cost per unit of capacity that the buyer needs to pay in advance, as well as an execution cost per unit of output when the capacity is actually used. The paper by Wu *et al.* derives the seller's optimal bidding and buyer's optimal con-

tracting strategies. Albeniz and Simchi-Levi [1] studies a purchasing process between a buyer and many suppliers for option contracts in a single period supply environment. The papers by Elmghraby and Keskinocak [19], Bitran and Caldentey [7], and McGill and van Ryzin [28], and the book by Talluri and van Ryzin [35] provide comprehensive overviews of the areas of dynamic pricing and revenue management.

An important difference between their model and ours is that there is no supply chain for a multi item product in the model of Wu *et al.*, while in our model the buyer has to manage the orders to a group of suppliers. The manufacturer/buyer can buy capacity options to have the right to maximize its market share with a capacity adjustment of supply and reacting to demand through price adjustment.

2.3. Demand planning and robust supply network

One of the central assumptions of classical optimization problems is those problems that at some time after performing an action the resulting state variables can be predicted. Often, a combination of parameter values is used to obtain a result, which is followed by a sensitivity analysis to determine the set of other combinations that would lead to the same result [27, 3], or to determine the "nearest" alternative result [31]. Referring to a system that holds up well under exceptional conditions, robust is an adjective commonly applied in marketing literature to products in several ways. It derives from the Latin *robustus*, meaning "strength."

Roy expresses robustness as a measure of adaptation to change or "good enough for every possible realization of the chance node". March and Shapira [26] explain robustness for actions decided by decision makers. In their point of view, "The robust action is that which avoids dramatic loss whatever the events happen to be". Zäpfel [41] describes robustness from solution space point of view. He defines "A robust solution accommodates finding a feasible solution of the detailed planning for each possible realization of demand."

Roy [32] defines robustness analysis as a procedure to identify robust conclusions, given sets of combinations for the parameter values. Vincke [38] refers to robust solution as one that is always near the solution initially found by a method, for any acceptable combination of parameter values. Kouvelis and Yu [22] define robust solution in an optimization problem as the one, which has the best performance under in its worst case (max-min rule).

They reviewed a more comprehensive modeling in their book titled robust discrete optimization.

Competitiveness as a proactive response to make a supply chain robust to its environment is considered by Cachon [13]. A robust model for logistic problems is being introduced which introduces the robustness in modeling by Chian and Lin [15]. Implication of robustness in supply network is being considered in our research in modeling the demand and capacity allocation as well as the supply network adaptability to be proactive in a competitive environment.

3. Model description and analysis

In this section, we elaborate the problem, formalize the costs and decisions involved and develop the basic model. Then, in subsequent sections, the model is analyzed for a variety of cases.

3.1. The problem

Consider a supply chain with a manufacturing firm and several suppliers. The output of this chain can be some different products (or services). However, all outputs are measured in terms of the manufacturing capacity. To assemble the final output, the manufacturer procures parts, components or services from some qualified suppliers. The final output of the chain is assembled which includes n types of inputs delivered by the suppliers to the manufacturer. The output of the suppliers (input to the manufacture) is also measured in terms of the capacity of the manufacturing capacity.

The buyer signs the contract with n suppliers through a reverse auction program. In fact, the buyer pre-qualifies a number of suppliers for each type of input (n types) to participate in some bidding process by considering the reverse auction rules such as price, technical capabilities and financial records. The buyer obtains quote from the participating suppliers in the form of price – capacity curve, as we will discuss later. Then, for each type of input one supplier (or in some case more) is selected to sign a long-term contract.

The demand for the output is stochastic. Although the buyer (manufacturer) faces uncertain demand for the capacity (final output), he has to reserve some deterministic amount of capacity through an advanced purchasing contract entered with the suppliers. The planning horizon is divided into T planning periods. The decision to reserve

capacity is made at the beginning of each planning period, when only available information is the demand distribution function. From the buyer point of view, there are two important decisions to make.

How much of capacity to be procured through capacity reservation? How much to pay to each supplier for the reserved capacity? In other words, what would be the contract price of each input? Therefore, the objective of the basic model is to determine the optimal expected capacity and its corresponding price for each period. In the next sections, we analyze the relation between the price and capacity under different conditions.

3.2. Supply chain coordination problem

The final product is assembled by putting together n inputs. As mentioned before, this product and all inputs are measured in terms of the manufacturing capacity.

Definition. By one capacity unit of Sth supplier, it means the total required input from this supplier in order to manufacture one capacity unit of the final product. The relation between one capacity unit of each supplier and one capacity unit of manufacture can be determined by the bill of material. Since all inputs are required to be included in the final products, the capacity of the final output is equal to the minimum capacity of all inputs, called coordinated capacity. In other words, the production volume of q, or what we call it the "coordinated capacity", is the required output of each supplier to produce q units of final product. Obviously, the output of all suppliers should be equal in order to have a balanced relation in the supply chain. Without loss of generality, we assume the assembling capacity is not limited.

The demand (D_t) for the supply chain output is a random variable, which depends on both price (P_t) and planning period (t), denoted by function of $D_t = X_t P_t^{-b}$ with a stochastic pattern of $X_1, X_2, X_3, ..., X_T$. The elasticity coefficient of demand with respect to price is constant and is equal to b. Let C be the total coordinated capacity of the supply chain for the duration of planning horizon, where it is divided into T planning periods.

The problem is to find the optimal capacity size to be contracted in advance and to determine the price of the supply chain output for different periods. Since the demand for the output capacity depends on the price offered by the manufacturer, a

tradeoff between the price and the amount of capacity offered to end user customers have to be made.

3.3. Notation

From this point on and for simplicity, we set the manufacturer index as *s=0* while indices s=1,2,…,n refer to the suppliers. We also define the following decision variables as well as parameters of the model.

D_t The number of output capacities demanded in period *t*;

P_t Price of a unit output capacity sold to the end user customers in period *t*;

p_s Price of a unit input capacity paid to supplier s, $s = 1, 2, ..., n$;

C Coordinated capacity within supply chain for the total duration of planning horizon;

C_s Input capacity offered by supplier s, $s = 1, 2, ..., n$;

v_s Variable cost of a unit capacity for entity s, ($s = 0$ for manufacturer and $s = 1, 2, ..., n$ for suppliers);

F_s Fixed cost of a unit capacity for entity, *s* ($s = 0$ for manufacturer and $s = 1, 2, ..., n$ for suppliers);

q_t Coordinated capacity within supply chain for the duration of remaining periods of $[t, t+1, .., T]$;

$\phi_{st}(q)$ Total expected sale revenue of entity s for the duration of remaining periods of $[t, t+1, .., T]$, if the coordinated capacity for the same periods is q, ($s = 0$ for manufacturer and $s = 1, 2, ..., n$ for suppliers);

$\pi_{s,t}(q)$ Total expected net profit of entity s during the remaining periods of $[t, t+1, .., T]$, if the coordinated capacity for the same periods is q, ($s = 0$ for manufacturer and $s = 1, 2, ..., n$ for suppliers);

π_s Total expected net profit of entity *s* (or manufacturer) during the planning horizon.

3.4. Dynamic programming model

The problem is formulated by applying dynamic programming approach. The state of system is defined as q_t, the coordinated capacity or the total output to be produced in the remaining periods of $[t, t+1, .., T]$. The decision variable in each period is the price to be determined.

The total revenue during period *t* is $D_t P_t$. As mentioned before, since the demand is stochastic and denoted by $D_t = X_t P_t^{-1}$ in period *t*. Then, the following recursive equation represents the total income of manufacturer during the remaining periods of $[t, t+1, .., T]$.

$$\phi_{0,t}(q) = \max_{P_t} \{ P_t E(X_t P_t^{-b}) + \phi_{0,t-1}(q - E(X_t P_t^{-b})) \} \quad (1)$$

The profit of the manufacturer for the same periods is as follows:

$$\pi_{0,t}(q) = \phi_{0,t}(q) - v_0 q - q \sum_{s=1}^{n} p_s - F_0 \quad (2)$$

Similarly, for supplier $s, s = 1, ...n$.

$$\pi_{s,t}(q) = \phi_{s,t}(q_s) - v_s q_s - F_s \quad (3)$$

Although for numerical cases the recursive Equation (1) can be solved, deriving a general relation between capacity and price is not possible, directly. Therefore, an algorithm is developed to achieve this objective. First we introduce a new variable of $z_t, t = 1, ..., T$, as follow:

$$z_t = \frac{q_t}{P_t^{-b}} \quad (4)$$

3.5. Algorithm of solving recursive equations

The algorithm consists of two stags. In the first stage, determine $z_t, t = 1, ..., T$, which optimizes the following set of equations:

$$r_t = \frac{z_t - E[z_t - X_t] + r_{t-1}^* E[(z_t - X_t)^m]}{z_t^m} \quad (5)$$

where $m=1-1/b$. In this stage, first calculate z_1^* and then $z_1^*, z_1^*, ..., z_T^*$, and also its corresponding optimal value of r_t^*. In the second stage, set:

$$\phi_{0,t}(q_t) = r_t^*(q_t)^m \tag{6}$$

where $r_0 = 0$, and $C = C_t$ see Modarres and Nazemi (2006).

Note: The important point about obtaining the optimal value of z_t is that the set of Equations (5) are independent of capacity C_t.

The following theorem is applied to verify the validity of the algorithm:

Theorem 1. Following the above-mentioned algorithm results in an optimal solution of recursive Equations of (2).

Proof. The proof is by induction and we follow an approach similar to the one introduced by Monahan, *et al.* [3]. From (4) and also by definition of m, it is implied that:

$$P_t E[X_t P_t^{-b}] = \frac{E[X_t]}{z_t^m}(q_t)^m = \frac{z_t - E[z_t - X_t]}{z_t^m}(q_t)^m \tag{7}$$

We first prove (5) holds for $t=1$. By (1) and (7),

$$\phi_{0,1}(q_1) = \underset{P_1}{Max}[P_1 E(X_1 P_1^{-b})] = \max_{z_1}\{\frac{z_1 - E[z_1 - X]}{z_1^m}(q_1)^m\}$$

Thus, $\phi_{0,1}(q_1) = r_1^*(q_1)^m$. Now, let (5) holds for (t-1), i.e. $\phi_{0,t-1}(q_{t-1}) = r_{t-1}^*(q_{t-1})^m$. From (1) and (7) it is implied that:

$$q_t - E(X_1 P_1^{-b}) = \frac{q_t}{z_t} E[z_1 - X]$$

Then, from (5):

$$\phi_{0,t-1}(q_t - E(X_1 P_1^{-b})) = r_{t-1}(\frac{q_t E[z_1 - X]}{z_t})^m$$

Thus,

$$\phi_{0,t}(q_t) = \max_{z_t}\{\frac{z_t - E[z_t - X_t]}{z_t^m} + r_{t-1}(\frac{E[z_t - X_t]}{z_t})^m\}(q_t)^m$$

As a result, $\phi_{0,t}(q_t) = r_t^*(q_t)^m$.

4. Collaborative versus independent suppliers

In this section, we investigate the effect of supplier independence on the optimal capacity of the network. The suppliers are either independent entities or they are collaborative and consolidated with the manufacturer. In the first case, each supplier charges the manufacturer the price, which is optimal for its business while in the second case the optimality of the total chain is sought.

First, we study the cases where there exists a strong relationship of supplier-manufacturer and the profits of all partners are consolidated in the same corporation. Due to such relationship, the cost structures of all partners are transparent. In this case, the total profit of the total chain is as follows:

$$\pi = r_T C^m - CV_0 - C\sum_{s=1}^{n}v_s - \sum_{s=0}^{n}F_s \tag{8}$$

Lemma 1. In case of collaborative identities with transparent relationship and central planning where maximization of the total profit of the system is followed, the optimal network capacity has the following relationship.

$$C^* = \left(\frac{mr_T^*}{(v_0 + \sum_{s=0}^{n}v_s)}\right)^b \tag{9}$$

Proof. It is implied from (8) that π is a concave function of C. Thus, to gain the maximal profit of the chain, the optimal network capacity of C^* is calculated by obtaining the root of $\partial\pi/C = 0$.

4.1. Coordinated supply network with independent entities

Now consider the situation where the suppliers are independent entities. Each supplier charges the manufacturer a price, which is optimal for its business. In this case, the total profit of the manufacturer is as follows:

$$\pi_0 = r_T C^m - v_0 C - C\sum_{s=1}^{n}p_s - F_0 \tag{10}$$

Lemma 2. In case of independent suppliers, the optimal network capacity is as follows:

$$C^* = \left(\frac{mr_T^*}{v_0 + \overline{V_s}} \right)^b \tag{11}$$

and

$$\overline{V_s} = \frac{z_t^{1-m}}{mr_T} \sum_{s=1}^{n} v_s \tag{12}$$

where $\overline{V_s}$ is the total amount paid to the suppliers for one unit of capacity.

Proof. From the supplier point of view, the optimal capacity allocation should lead to its maximum profit, regardless of the profit of the other elements of supply chain. The profit of supplier s with respect to its coordinated capacity C_s is as follows:

$$\Omega_s = r_T C_s^m - v_s C_s - F_s \qquad s = 1, ..., n \tag{13}$$

The maximal profit of the supplier is obtained by setting the derivative of π_s with respect to C_s equal to zero and by noting that π_s in (13) is a concave function of C_s. As a result:

$$C_s^* = \left(\frac{mr_T^*}{v_s} \right)^b$$

From (4):

$$p_s = \left(\frac{z_T^*(v_s)^b}{(mr_T)^b} \right)^{1-m} = \frac{v_s}{mr_T} (z_T^*)^{1-m}$$

Then, by substituting p_s in (10) and setting the derivative of π_0 with respect to C equal to zero, the optimal value of capacity of (11) is derived.

Note. The optimal price for supplier s is p_s, as calculated above. However, the optimality is violated if the coordinated capacity is not C_s^*. On the other hand, since the coordinated capacity is determined as the minimum capacity offered by all suppliers,

then the optimal capacity C^*, as determined in this lemma, provides the optimality of the manufacturer and not the supplier, although each supplier quotes the manufacturer a price based on its system optimality.

4.2. Comparison of independent and collaborative suppliers

Let define:

$$\gamma = \frac{z_t^{1-m}}{mr_T} = \frac{z_t^{\frac{1}{b}}}{(1 - \frac{1}{b})r_T}$$

as market characteristic ratio. In fact, γ indicates the ratio of market price to the total variable cost of suppliers.

Note. This definition indicates that γ is also independent of the coordinated capacity, because z_t and r_T are the function of demand, only.

In Figure 1, the contours of graph for the effect of r_T and z_t with a given b are depicted.

From (12):

$$\overline{V_s} = \gamma \sum_{s=1}^{n} v_s \tag{14}$$

As mentioned before, $\overline{V_s}$ is the total amount paid to the suppliers for one unit of capacity, which depends on γ, the market characteristic ratio. Therefore, by relationship (14) three cases may happen.

For $\gamma = 1$ the optimal coordinated capacity of independent entity scenario has the same behavior as dependence-coordinated scenario.

For $\gamma \succ 1$ the network capacity of a central coordinated scenario is higher than that of the other scenario. Consequently, the coordinated independent entity scenario has lower utilization of capacity in comparison with dependence-coordinated scenario.

For $\gamma \prec 1$, the coordinated independent entity scenario has the most capacity utilization and makes the lowest margin (or makes loss). This case only applies when a supplier tries to push out its competitors from the market.

In real practice, γ is the boundary limit for the network to analyze its market position in comparison with its competitors or with substitute products.

Figure 2 shows that infeasible area (loss area) for a supplier increases where the demand elasticity factor (b) gets higher. Also any increase in z_t and decrease in r_T reduces the infeasible area for the supplier capacity plan.

5. Coordinated Capacity plan under Different criteria

In the previous section, the optimal capacity was calculated under two extreme conditions regarding supplier independence. In this section we study the cases where suppliers offer their available capacity in a reasonable price, under different criteria.

5.1. Capacity plan, minimizing the loss of supplier

As shown in previous section the total capacity offer can be different for the suppliers with different cost structure. As each supplier has no idea on the price offer of other suppliers in the network, the cost structure of manufacturer is a function of suppliers offer. It means the effect of supplier pricing offer does not affect only on the profit and capacity utilization of manufacturer and the supplier itself but also to the whole supply network. Let C^* and C_s^* be the optimal capacity from manufacturer and suppliers point of view, respectively. Since for $\gamma \geq 1$ (feasible), $v_0 + \overline{V_s} > v_S$, then, $C^* \leq C_s^*$. As a result:

$$\left(\frac{m r_T^*}{v_0 + \overline{V_s}} \right)^b \leq \left(\frac{m r_T^*}{v_s} \right)^b , \quad s = 1, 2, \dots n \quad (15)$$

Figure 3 shows the optimal capacity for manufacturer and supplier s. The range of capacity that causes loss is beyond feasible manufacturer capacity plan. We define any reduction from the highest expected profit as loss of supplier s. In other words, loss in our terminology is the lost profit. Let π_S^* and π_s^{**} be the profit of supplier s, if capacity of C^* or C_s^* is ordered by the manufacturer, respectively. Then:

$$\pi_S^{**} = r_T C_s^m - v_s C_s^* - F_s C_s^*$$
$$\pi_s^* = r_T (C^*)^m - v_s C^* - F_s C^*$$
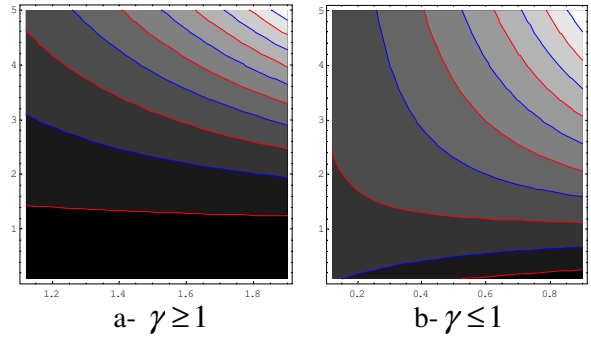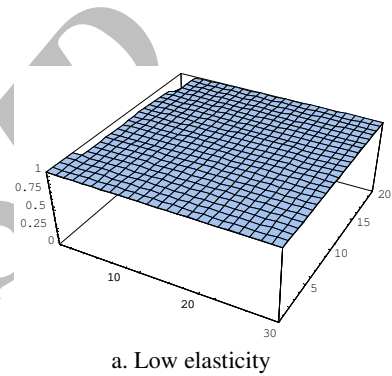

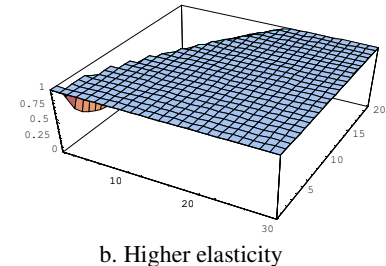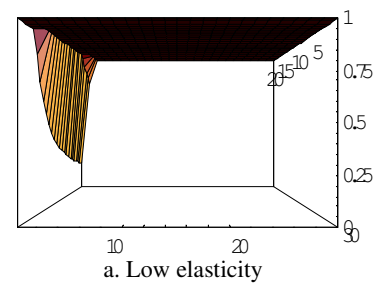a- $\gamma \geq 1$ b- $\gamma \leq 1$

Figure 1. Contour graph for the effect of r_T & Z_t with a given b.
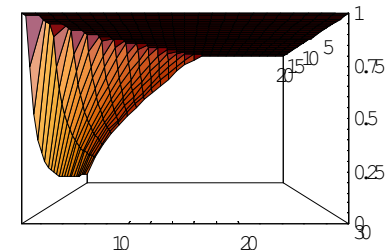

a. Low elasticity

b. Higher elasticity

a. Low elasticity

b. Higher elasticity

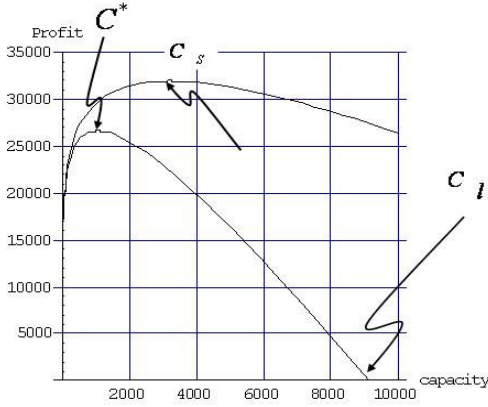**Figure 2.** Infeasible area for supplier.

Figure 3. Optimal capacity for manufacturer and supplier.

Due to deviation from optimal capacity plan, the loss function for the supplier s is formulated as follows:

$$L_S = \pi_s^{**} - \pi_s^* = r_T (C_s^m - C^m) - v_s (C_s - C) \quad (16)$$

Proposition 4. If the policy in network is to maximize the profit of collaborative identities while the purchasing power for the manufacturer is stronger than supplier, then capacity C_L that prevents the loss of suppliers is as follows:

$$\left(\frac{m r_T^*}{v_0 + \overline{V_s}} \right)^b \le \left(\frac{m r_T^*}{v_s} \right)^b \le \left(\frac{r_T^*}{v_s} \right)^b$$

or

$$C^* \le C_S \le C_L \quad (17)$$

Proof. The capacity limit that causes loss for supplier is determined from the function of supplier gross profit. Then:

$$\pi_s^* = r_T C_l^m - v_s C_l \le 0$$

or

$$C_l \ge \left(\frac{r_T^*}{v_s} \right)^b$$

Also from (9) we have:

$$\left(\frac{m r_T^*}{v_s} \right)^b \le \left(\frac{r_T^*}{v_s} \right)^b \quad \text{as} \quad m^b \le 1.$$

Taking into account the above relation as well as (15), the proof is complete. $\square$

Therefore, for all optimal coordinated capacity plans for manufacturer, it would not make loss for suppliers even though it is not optimum for suppliers.

5.2. Coordinated capacity plan, EQUAL profit for all suppliers

Success of a network is related to long-term win-win business model between suppliers and manufacturer. Therefore, a supply network whose strategic mission is to establish a strong and long-term relationship has to address equal profit sharing for all partners in network.

Proposition 5. The optimal capacity plan for a win-win business model with a transparent cost structure in a network of equal opportunity for all suppliers is as follows:

$$C^* = \left(\frac{m(n+1)(r_T - z_T^{1-m})}{\sum_{s=0}^{n} v_s} \right)^b \quad (18)$$

Proof. Let π be the profit of the chain, including the manufacturer and suppliers. Then from (2) and (3),

$$\pi = \sum_{s=0}^{n} \pi_s = n r_T C^m - C \left(\sum_{s=0}^{n} (v_s + F_s) - nC \left(\frac{Z_T^*}{C} \right)^{1-m} \right)$$

The maximum capacity is found by derivation of this function with respect to capacity. $\square$

As it is clear from the above relationship, the capacity is an inverse function of total costs, which is consistent with the assumption of the problem.

The above conclusion suggests that for establishing an equal policy profit for the suppliers in a supply network, the necessary condition is to divide the job load to equal cost ratio plan.

$$C^* = \left(\frac{m(r_T - z_T^{1-m})}{v_s} \right)^b \quad (19)$$

In fact, in this model all suppliers suffer a similar deviation from the optimal capacity of single business model.

6. Coordinated capacity plan with capacity constraint

There are situations in a market where some suppliers with a limited capacity share their capacity in more than one network. In this section, we investigate the coordination mechanism with capacity constraints.

6.1. Coordinated capacity plan with supplier capacity constraint

Consider there is one supplier with limited capacity that supplies to two supply chain networks. In such an environment, the objective of this supplier is to maximize its profit through a tradeoff between two networks or from demand perspective two different markets.

Let p_1, p_2 be the price paid by networks one and two, respectively. The decision variables are C_1 and C_2, the capacity assigned to the networks. Then:

$$\text{Max} \quad p_1 C_1 + p_2 C_2$$

Subject to:

$$C_1 + C_2 \leq C_S \qquad (20)$$

$$C_i \geq 0, \quad i = 1, 2$$

Substituting C_1 and C_2, the model happen to be a nonlinear knapsack problem, as follows:

$$\text{Max} \quad p_1 \left(\frac{r_1}{v_1 + p_1} \right)^{b1} + p_2 \left(\frac{r_2}{v_2 + p_2} \right)^{b2}$$

Subject to:

$$\left(\frac{r_1}{v_1 + p_1} \right)^{b1} + \left(\frac{r_2}{v_2 + p_2} \right)^{b2} \leq C_S \qquad (21)$$

$$p_i \geq 0, \quad i = 1, 2$$

By solving the problem, the optimal price and capacity, reservation is determined. The solution space for the problem has a specific characteristic (see figure 4) that may help to find competitiveness of network and solution space. t is interesting to note that:

- For two networks with similar characteristics, the network with lager demand elasticity factor (*b*) is supplied at lower price and with the bigger share of supplier capacity.

- For two networks with similar characteristics, a network with larger stochastic revenue parameter (r) is supplied at same price level with the bigger share of supplier capacity.

- For two networks with similar characteristics, a network with lower variable cost, (*v*) is supplied at lower price with the bigger share of supplier capacity.

6.2. Special case

In a competitive market, assume supplier *k* has a limited capacity but all others have enough capacity. Then, the proposed procedure is also applicable, by redefining the variable cost for network excluding the shortage capacity as below:

$$\sum_{s=0, j \neq k}^{n} v_s = v$$

Changing the cost variables for each one of the networks as v_1 and v_2 brings the problem to its standard form as discussed before.

6.3. Non coordinated capacity plan, capacity constraint

Assume that a manufacturer and a supplier share the profit and let the price of this supplier is represented as $p_S = (1 + \alpha) v_s$, where α is a positive parameter.
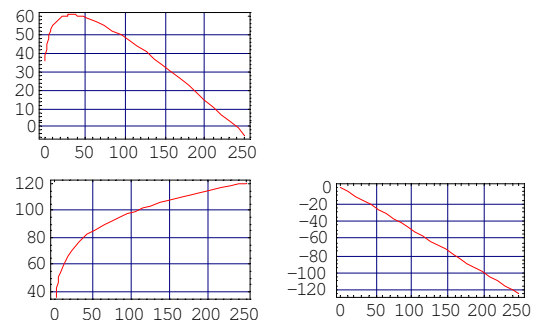

Figure 4. Profit (upper left), revenue (lower left) and cost (lower right).

Proposition 6. In a network with capacity constraint, at ratio of $\alpha = v_0 / v_s$, the price policy is indifferent for a supplier with independent price policy and a supplier with a partnership policy.

Proof. The total profit is:

$$\pi = rC^m - (v_0 + p_s)C + rC^m - v_s C.$$

If the profit is shared, then:

$$C^* = \left(\frac{2mr}{v_0 + (2+\alpha)v_s} \right)^b. \tag{22}$$

However, if the supplier does not follow the revenue management procedure, then the total profit of the supply chain can be formulated as below:

$$\pi = rC^0 - (v_0 + p_s)C + (p_s - v_s)C$$

or

$$C^* = \left(\frac{mr}{v_0 + v_s} \right)^b \tag{23}$$

Comparing equations (22) and (23) it can be easily shown that indifference capacity reservation value for two collaborative partnership scenarios is at $\alpha = \dfrac{v_0}{v_s}$.

6.4. Coordinated Capacity plan, capacity constraint on a   shared policy approach

Consider a supplier who has some overcapacity and plans to offer some predetermined capacity to the manufacturer by contract and the rest sells it at the spot market. Then, he should determine the size of the capacity assigned to the manufacturer. In such an environment, supplier has the information of demand characteristics and he would like to optimize its revenue through maximization of manufacturer profit.

Proposition 7. In a supply network where there is constraint on supply side, the optimal capacity reservation for each supplier who has the shared capacity plan is as follows:

$$C^* = \left(\frac{mr_T}{v_0 + \sum_{s=0}^{n} \beta_s} \right)^b \tag{24}$$

where

$$\beta_s = r_T^{b(m-1)+1} m^{b(m-1)} v_s^{b(1-m)} \qquad j=1,\dots,n.$$

Proof. With the above assumption, the profit can be written as:

$$\pi_0 = r_T C^m - C\left(v_s - \sum_{s=1}^{n} p_s\right)$$

$$\pi_s = r_T C_s^m - v_s C_s$$

Setting the derivate of the function to zero results in:

$$C_s^* = \left(\frac{r_T}{v_s} \right)^b$$

Then for a unit capacity:

$$\pi_s^u = \frac{\pi_s^*}{C_s^*} = r_T C_s^{m-1} - v_s = r_T \left(\left(\frac{r_T}{v_s} \right)^b \right)^{m-1} - v_s$$

Therefore, the price quoted for the manufacturer is:

$$p_s = \pi_s^u + v_s = r_T^{b(m-1)+1} m^{b(m-1)} V_s^{b(1-m)}$$

On the other hand, for the manufacturer we have:

$$\pi_0 = r_T C^m - C\left(v_m - \sum_{s=1}^{n} r_T^{b(m-1)+1} m^{b(m-1)} v_s^{b(1-m)}\right)$$

By setting the derivative of this function with respect to capacity equal to zero leads to desired result.

Note. It is interesting to note that the optimal revenue for supplier and for the supply chain in decentralized planning is lower than centralized approach and the total market share becomes less in decentralized supply planning. This phenomenon is due to the comparison of relationship between two cases analyzed above. i.e. $\beta_s \geq V_s$).

7. Supply chain network trade off

Up to this point, the coordination capacity reservation planning is based on profit maximization and by considering the internal parameters. Next, we investigate the structure of a network as its competitive advantage in comparison with another network.

Consider a supplier who supplies his capacity to more than one network. The problem in this case of capacity sharing is how to optimize the revenue of this supplier, by assigning the limited capacity to competitive networks. It is clear that an attractive network absorbs more capacity and increases its market share as well as its revenue, in comparison with the competitors. We will show network trade off parameters acts as an index of competitive advantage.

7.1. Network trade off

Consider the market has price elasticity of b over time. There are two networks served by a supplier. Also, consider the following notation:

r_T^1, r_T^2 Revenue multiplier for network 1 and network 2, respectively,

m_1, m_2 Market elasticity factor for network 1 and network 2, respectively,

v_s^1, v_s^2 Variable cost for supplier for network 1 and network 2, respectively.

The total capacity is C. Therefore, capacity reserved for network 2 is $C - C_s$. following this notation, the profit for supplier s is:

$$\pi_s = r_T^1 C_s^{m_1} - V_s^1 C_s + r_T^2 (C - C_s)^{m_2} - V_s^2 (C - C_s)$$

Then, the optimal capacity offered to both networks is as follows:

$$C = C + \left(\frac{m\,r}{m\,r\,C - (v - v)} \right) \quad (25)$$

It can be verified the capacity reserved for the second network increases as its revenue multiplier increases or its competitor's network decreases. Similarly, any increase in cost for the second net-

work increases the capacity reserved for the first one. Such an assignment rule can be established by rearranging the above equation in a general formula for every two competitive network as below:

$$m_1 r_T^1 (C_s)^{m_1-1} - v_s^1 = m_2 r_T^2 (C - C_s)^{m_2-1} - v_s^2 \quad (26)$$

It is interesting to note that in real practice, there are cases where two similar network (equal variable cost and equal price elasticity factor), have different cooperation attractiveness for suppliers. This is due to different market image, which usually is related to brand and marketing distinctive advantage of one network to others (in our model it means different demand probability that means higher, r, and value for superior network).

In such an environment, the above equation changes to a competitive advantage, as the following proposition states.

Proposition 8. There exists a competitiveness factor (χ) for two distinctive supply network which defines the optimal tradeoff for common suppliers of networks.

Proof. Let's define $\chi = \dfrac{m_2 r_T^2}{m_1 r_T^1}$ for general network tradeoff, from (26), we have:

$$(C - C_s)^{\frac{1}{b_2}} = \chi \left[(c)^{\frac{1}{b_1}} \right]$$

Moreover, $\chi = \left(\dfrac{r_T^2}{r_T^1} \right)^b$ for competitor markets. □

7.2. Supply chain network competition

There are situations where a supplier has a long-term business relationship in a certain network. Consider a new network of different product that uses some resources of this existing network. Such a competition enforces price pressure by offering attractive revenue for suppliers in current networks. Then, the expected price may increase because the market leadership of the existing supply network changes. Modeling such an environment helps the market leaders to manage the product policy and product life cycle management. It is expected when such an environment changes, the leader has to revise its plan for introducing new product offers. Let

N_1 represent the existing network. Suppose another network, say N_2 also exists. Then:

$$\pi_{N_2} = r_T^2 (C_N^2)^{m_2} - (v_0 + p_s)C_N^2$$

Thus, the optimal capacity plan for each network is as follows:

$$\hat{C}_N^1 = \left(\frac{m_1 r_1}{v_0 + p_s} \right)^{b1}$$

and

$$\hat{C}_N^2 = \left(\frac{m_2 r_2}{v_0 + p_s} \right)^{b2}$$

As the maximum capacity available for the supplier is equal to C_s, then we have:

$$\hat{C}_N^1 + \hat{C}_N^2 \le C_s$$

or

$$\left(\frac{m_1 r_1}{v_m + p_s} \right)^{b1} + \left(\frac{m_2 r_2}{v_m + p_s} \right)^{b2} \le C_s \qquad (27)$$

Solving for p_S, the optimal capacity is determined.

Proposition 9. Price and capacity offered to a network increase as the market parameters increases.

Proof. By relation (27), we have:

$$\hat{C}_N^1 \le C_s - \hat{C}_N^2$$

where,

$$\left(\frac{m_1 r_1}{v_m + p_s} \right)^{b1} = \hat{C}_N^1$$

From equations, by any Increase in r, the capacity (C) for the network increases while keeping other parameters of price unchanged. As the capacity of supplier C_s is fixed, remaining capacity for the rival network decreases respectively.□

It is clearly shown that, the price will change in accordance to network parameter accordingly. Figure (5) shows the proposition behavior.

7.3. Supply chain network competition, extended capacity limited model

Let's consider a general model of competition where the rival network has different characteristics on demand parameters (r) and productivity of manufacturer (v_m). Then,

$$\pi_{N_2} = r_T^2 (C_N^2)^{m_2} - (v_0 + v_s)C_N^2$$

$$\pi_{N_1} = r_T^1 (C_N^1)^{m_1} - (v_0 + v_s)C_N^1$$

Then, the profit sharing for the manufacturer with the cost structure of the supplier in a coordinated planning supply chain is defined as unit revenue per capacity of supplier as follows:

$$\pi_S^1 = \frac{\pi_{N1}}{C_N^1} \cdot \frac{v_S^1}{v_m^1 + v_S^1}$$

$$\pi_S^2 = \frac{\pi_{N2}}{C_N^2} \cdot \frac{v_S^2}{v_m^2 + v_S^2} \qquad (28)$$

Then, the general capacity allocation optimization can be modeled as follows:

$$\text{Max Profit} = \pi_s^1 C_N^1 + \pi_s^2 C_N^2$$

Subject to:

$$C_N^1 \le \left(\frac{m_1 r_1}{v_0 + v_S^1} \right)^{b1} \qquad (29)$$

$$C_N^2 \le \left(\frac{m_2 r_2}{v_0 + v_S^2} \right)^{b2}$$

$$C_N^1 + C_N^2 \le C_s$$

It is implied that the profit function is an increasing, convex function, (see Figure 6). It is apparent that a supplier with a limited capacity reduces the capacity assignment to the network with a lower competitive parameter, (see Figure 7).
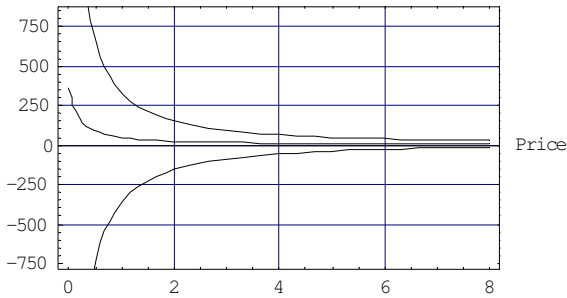
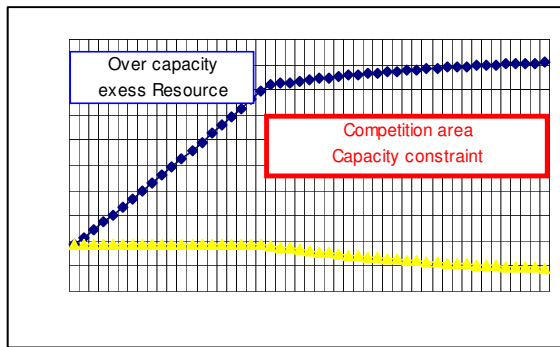**Figure 5.** Capacity plan for networks N1 and N2 and idle capacity versus capacity.

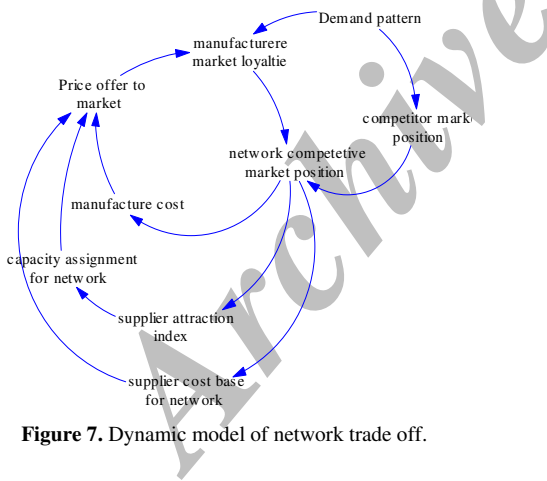**Figure 6.** Network trade off for different market parameter, r.

Figure 7. Dynamic model of network trade off.

The above analysis describes the environment as a dynamic system. Such an effect can be modeled as a viscous circle that has to be addressed by the manufacturer to upgrade the total business competitiveness.

Dynamic model of supply network is arranged into a general dynamic model (see Figure 7) with all parameters discussed in the article. Items in the model are descriptive text for the parameters used in article problem definitions of our article. The new dynamic general simulation model introduces a framework for supply network policy setting for proactive market leaders while using the optimal revenue management model introduced in this paper.

8. Conclusion

The researchers have predominantly studied capacity reservation contracts in a supply chain. However, analysis of capacity contracts in a manufacturing chain & competitive networks has not been done extensively. This paper addresses this issue in detail and analyses alternative strategies that a final assembler (you may call it, manufacturer or buying firm) would like to adopt.

Earlier studies have exogenously assumed the reservation price of the supplier. In contrast, we have explicitly derived the capacity reservation price. It is also, shown that suppliers participating in such contracts are likely to quote lower price at higher levels of utilization which make benefit for themselves as well as the whole supply network.

We have shown the role of capacity reservation in a multiple supplier – one buyer manufacturing setting. In addition to the amount of capacity to be reserved, the number of suppliers with whom the buyer has to engage in a contract and the capacity volume becomes a decision variable. Moreover, the presence of multiple buyer model, which defines more than one supply network for delivery of a group of goods or services, will permit the buyer to set-up a competitive mechanism such as an auction for capacity allocation among the selected suppliers. However, the model also proposes new competitive prices for suppliers with flexibility for providing capacity options to more than one network.

The contract mechanism that we have employed in this paper is an extension to dynamic yield manufacturing mechanism for buyer/manufacturer. We set the higher cost at the network with high demand pattern and lower elasticity to demand and derive a basis for obtaining the capacity reservation cost. In almost all the previous studies in capacity contracts, the capacity reservation price was assumed exogenous and was related to negotiation power of buyer or seller. We however, explicitly model this situation and derive price capacity curves for suppliers.

However, variability of demand distribution has significant effect on the optimal capacity to contract, the optimal price & capacity offer of suppliers to manufacturer remains robust to changes in our

model parameters. Even though, earlier studies have addressed the operational perspective of supply contracts, we have shown a robust and sustainable supply network management is related to long-term internal parameters of a supply network instead of suppliers alone. This is clearly in accordance with Taguchi findings that a robust behavior is dependent to internal parameters of a system. We have addressed such internal forces in a network in a closed system and compared rival network effects on supply contracts.

In addition, it is shown that a supply network will behave as a robust system where the competitive market parameters have been utilized to address the competitive pricing of the suppliers in the network. The effect of rival network on pricing of the suppliers has been studied and countermeasures have to be applied to maintain sustainability of the network.

Future work on extending the model includes analyzing for supply-chain policy effect such as tiering and unique supplier policy on the competitive advantage of a chain. Further, modeling transaction costs related to the reselection process and dealing with multiple suppliers in a post-selection setting will sharpen the insights.

References

[1] Albeniz, M., Simchi-levi, D., 2007, Competition in the Supply Option Market, IT working paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1115249.

[2] Araman, V., Kleinknecht, J. and Akella, R., 2001, Supplier and procurement risk management in e-business: Optimal long-term and spot market mix. Working Paper, Department of Management Science and Engineering, *Stanford University*, Stanford, CA 94305-4026, June.

[3] Antunes, C. H. and Clímaco, J. N., 1993, Decision aid for discrete alternative multiple criteria problems: A visual interactive approach, *Information and Decision Technologies*, 19, 185-193.

[4] Anupindi, R. and Bassok, Y., 1998, Supply contracts with quantity commitments and stochastic demand. In Tayur, S., M. Magazine, and R. Ganesan, Editors, Quantitative models for supply chain management (Chapter 7), *Kluwer Academic Publishers*, 197 - 232.

[5] Barnes-Schuster, D., Bassok, Y. and Anupindi., R., 1999, Coordination and flexibility in supply contracts with options. Working paper, Graduate School of Business, University of Chicago.

[6] Barut M. and Sridharan V, 2004, Design and evaluation of a dynamic capacity apportionment procedure. *European Journal of Operational Research* ,155(1), 112-33.

[7] Bitran, G. and Caldentey, R., 2003, An overview of pricing models and revenue management. *M&SOM*, 5, 203-229.

[8] Brown, A. O. and Lee, H. L., 1997, Optimal pay to delay capacity reservation with applications to the semiconductor industry. Working Paper, Stanford University.

[9] Brown, A. and Lee, H., 1998, The win-win nature of options based capacity reservation arrangements. Working Paper, Vanderbilt University, Nashville, TN.

[10] Cachon, G. P., 2002, Supply chain coordination with contracts. The Wharton school of Business, University of Pennsylvania, Philadelphia, PA.

[11] Cachon, G., 2001, Stock wars: Inventory competition in a two echelon supply chain. *Operations Research,* 49, 658-674.

[12] Cachon, G. P. and Zipkin, P., 1999, Competitive and cooperative inventory policies in a two-stage supply chain. *Management Science,* 45, 936-953.

[13] Cachon, G. P., 1998, Competitive supply chain inventory management, Quantitative models for supply chain management (Chapter 5), *Kluwer Academic Publishers*, 111-146.

[14] Chen, F., 1999, Decentralized supply chains subject to information delays. *Management Science,* 45, 1076-1090.

[15] Chian, Son Yu and Han, L., 2000, A robust model for stochastic logistic problems", *International Journal of Production Economics*, 64, 385-397.

[16] Chu, L. Y. and Zuo-Jun Max, Sh., 2006, Agent competition double-auction mechanism. *Management Science*, 52(8), 1215-1222.

[17] Corbett, C. J. and Tang, C. S., 1998, Designing supply contracts: Contract type and information asymmetry, Quantitative models for supply chain management (Chapter 9), *Kluwer Academic Publishers*, 269 - 297.

[18] Donohue, K., 1996, Supply contracts and fashion goods: optimizing channel profit. The Wharton School, *University of Pennsylvania*, Philadelphia, PA.

[19] Elmaghraby, W. and Keskinocak, P., 2003, Dynamic pricing in the presence of inventory considerations: Research overview, current practices and future directions. *Management Science*, 49(10), 1287-1309.

[20] Eppen, G. D. and Iyer , A.V., 1997, Backup agreements in fashion buying - The value of upstream flexibility. *Management Science*, 43, 1469-1484.

[21] Jin, M. and Wu, S. D., 2001, Modeling capacity reservation in high tech manufacturing. Department of Industrial and Systems Engineering, P.C. Rossin College of Engineering, *Lehigh University*, December 2001.

[22] Kouvelis, P. and Yu, G., 1997, Robust Discrete Optimization and Its Applications. *Kluwer Academic Publishers*, Dordrecht.

[23] Lariviere, M. A., 1998, Supply chain contracting and coordination with stochastic demand, Quantitative models for supply chain management (Chapter 8), *Kluwer Academic Publishers*, 233 - 268.

[24] Lee H. and Whang, S., 1999, Decentralized multi-echelon supply chains: Incentives and information. *Management Science,* 45(5), 633-640.

[25] Maglaras, C. and Meissner, J., 2006, Dynamic pricing strategies for multi-product revenue management problems. *Manufacturing & Service Operations Management,* 8(2), 136-148.

[26] March, J. G., Shapira, Z., 1987, Managerial perspectives on risk and risk taking. *Management Science,* 33, 1404-1418.

[27] Mareschal, B., 1988, Weight stability intervals in multi-criteria decision aid. *European Journal of Operational Research*, 33, 54-64.

[28] McGill, J. and Van Ryzin, J. G., 1999, Revenue management: Research overview and prospects. *Transportation Science*, 33, 233-256.

[29] Modarres, M. and Nazemi, J., 2006, Capacity price decisions, a manufacturing yield management perspective. *Journal of Industrial Engineering International*, 2(1), 1-18.

[30] Netessine, S., Shumsky, R. and Simon, W. E., 2002, Introduction to the theory and practice of yield management. *INFORMS Transactions on Education*, 3(1), 34-44.

[31] Rios Insua D. and French, S., 1991, A framework for sensitivity analysis in discrete multi-objective decision-making. *European Journal of Operational Research,* 54, 176-190.

[32] Roy, B., 1998, A missing link in OR-DA: robustness analysis. *Foundations of Computing and Decision Sciences*, 23, 141-160.

[33] Serel, D., Dada M. and Moskowitz, H., 2001, Sourcing decision with capacity reservation contracts. *European Journal of Operational Research*, 131, 635-648.

[34] Shen, Zuo-Jun Max and Xuanming, S., 2007, Customer behavior modeling in revenue management and auctions: A review and new research opportunities. *Production and operations management*, 16(6), 713-728.

[35] Talluri, K. T., Van Ryzin, G. J., 2004, *The Theory and Practice of Revenue Management*", Kluwer Academic Publishers.

[36] Tsay, A., 1999, The quantity flexibility contract and supplier-customer incentives. *Management Science*, 45, 1339 -1358.

[37] Tsay, A. A., Nahmias, S. and Agrawal, N., 1998, Modeling supply chain contracts: A review, Quantitative models for supply chain management (Chapter 10), *Kluwer Academic Publishers*, 299 -336.

[38] Vincke, Ph., 1999, Robust solutions and methods in decision aid. *Journal of Multi-Criteria Decision Analysis,* 8, 181-187.

[39] Vulcano, G., Van Ryzin, G. and Maglaras, C., 2002, Optimal dynamic auctions for revenue management. *Management Science*, 48(11), 1388-1407.

[40] Wu, D. J., Kleindorfer, P. and Zhang, J. E., 2002, Optimal bidding and contracting strategies for capital-intensive goods. *European Journal of Operational Research*, 137(3), 657-676.

[41] Zapfel, G and Gfrerer, H., 1995, Hierarchical model for production planning in the case of uncertain demand. *European Journal of Operational Research*, 86 (1), 142-161.