A new heuristic method based on CPM in SALBP

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Abstract: The task of balancing of assembly lines is well-known in mass production system but this problem is NP-hard even for the simple straight line. Therefore, utilizing heuristic methods for these problems is totally unquestionable. Furthermore, in line with balancing problems, heuristic methods are the foundation of the metaheuristic methods, thus it seems to be necessary to use more efficient heuristic methods. This paper presents a new heuristic method based on the famous critical path method (CPM) for the simple assembly line balancing problem (SALBP) that the only precedence constraints between tasks and cycle time have been considered. In this research regarding the objectives of minimizing the number of workstation, maximizing smoothness index and maximizing line performance, the efficiency of the suggested method and 11 popular methods are compared at SALBP-1. For comparative evaluation, 11 networks are collected from open literature, and are used with different cycle times. On the whole, 40 problems have been solved and it is found that the heuristic method based on critical path is more efficient than the others.

Keywords: *Assembly line balancing problem; Critical path method; Heuristic method*

1. Introduction

Salveson (1955) published the first article on the assembly line balancing problem. Since then, most of the researches in assembly line balancing have been devoted to modelling and solving the simple assembly line balancing problem (SALBP).

The SALBP consists of assigning tasks to stations in such a way that: the precedence relationships among the tasks are satisfied, the sum of performance task times assigned to each station does not exceed the cycle time and each task is assigned to one and only one station. Several problem versions arising from different objectives are as follows: SALBP-F is a feasibility problem which is to establish whether or not a feasible line balance exists for a given combination of number of stations and cycle time. SALBP-1 and SALBP-2 have a dual relationship, because the first one minimizes the number of station with a given fixed cycle time, while the second one minimizes the cycle time (maximizes the production rate) with a given number of station. SALBP-E is the most general problem version maximizing the line efficiency thereby simultaneously minimizing cycle time and the number of station considering their interrelationships (Scholl and Becker, 2006).

Assembly line balancing problem can be solved by different methods that we can divide them into two main parts: exact and approximated methods. Most of the exact techniques fall into two categories; dynamic programming and branch and bound methods (Baybars, 1986; Scholl, 1999). Scholl & Becker (2006), Erel and Sarin (1998) discussed about exact and heuristic solution methods in simple assembly line balancing. Talbot *et al.* (1986) divided heuristics for SALBP into four categories: single pass, composite, backtracking and time trapped optimizing approaches.

Well-known heuristic methods are divided into simple heuristics and metaheuristics. Helgeson and Birnie (1961) proposed a new approach as one of the first simple heuristic methods named rank positional weight (RPW), Boctor (1995) recommended several heuristic methods for SALBP, Scholl and Klein (1999) suggested several heuristic methods for U type line. some of the other heuristic rules can be found at the other papers like Talbot & Patterson (1984) and Arcus (1963).

Metaheuristics comprises various methods including Ant colony, Tabu Search, Genetic and Simulated annealing algorithms, etc. which are used to solve different line balancing problems. In most of the studies including SALBP, heuristic

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approaches are the foundation of metaheuristic approaches. Kim *et al.* (1998), Sabuncuoglu *et al.* (2000), Ponnambalam *et al.* (2000), Goncalves and De Almeida (2002), enlarged genetic algorithm based heuristic for SALBP. Baykasoglu (2006) proposed a SA algorithm using several heuristic rules for U shape and straight line. Baykasoglu and Dereli (2008) formulated the rank positional weight with ant colony for solving the two-sided assembly line.

This paper will discuss about an approximated method that is called critical path method. A new heuristic approach will be introduced and compared with 11 heuristic methods on several test problems and a case study about iron assembly line will be added. All of these methods have been tested on SALBP-1 where cycle time is given and the aim is minimizing the work station, but in this study a multi objective will be followed such as minimizing the number of workstation and mean absolute deviation, maximizing smoothness index and maximizing line efficiency simultaneously.

In the second section, a new method will be discussed and illustrated with two simple examples, in the third section an explanation about other heuristic rules will be given in this paper. In the fourth section performance indexes are described. At the end, the proposed algorithm is tested and compared with literature test problems that are related to SALBP-1. Finally, the efficiency of each method is evaluated by the several different indexes.

2. Describing suggested method based on critical path

The critical path method (Heerkens, 2001) (CPM) is very popular and it is used widely in project management problems. This method is used for two reasons:

1. Assembly line balancing problems and project management problems have similar network structures.

2. In project management all of tasks that are on critical path have a high priority for performance and each delay in their performance ends in postponing the whole project and it can have a similar meaning for the assembly line, somehow if a suitable task is not assigned to each work station we might have more work stations and consequently increasing expenses and more human resources.

The heuristic method is used to solve various assembly line balancing problems such as straight, U-shape, and parallel assembly lines. In this study, the heuristic method is used for straight assembly line balancing. The parameters used in this method are:

$T(s_i)$ Total time of each station.

$T(x)$ Time of each task.

CT Given cycle time.

N Number of tasks.

MN Number of work stations.

S Minimum feasible number of work stations.

MCT Minimum feasible cycle time.

In this method, firstly precedence network is used to find the critical path, and then those tasks that are located on this path have a higher selection priority. Those tasks that are not on the critical path will be assigned to a work station by two reasons:

1. To preserve succession and precedence priorities,

2. If work station capacities are not used completely yet. If we have more than one task in the candidate assignment list, the task with greatest time will be chosen. Assigning tasks to work station is completed when all tasks are assigned to work stations. In this method, another criterion is used instead of cycle time named θ, calculating as follows:

$$S = \sum_{i=0}^{n} T(x) / CT \tag{1}$$

If S is not integer we round it up.

$$\mathrm{MCT} = \sum_{i=0}^{n} T(x) / S \tag{2}$$

$$\theta = [(MCT + CT) / 2] \tag{3}$$

θ Is between MCT and CT ($MCT < \theta < CT$) so that θ minimizes the number of workstations and disorders of work capacity simultaneously. Although CT can be substituted by each value between CT and MCT, but it has been shown by experience that θ offers better results. To obtain desired conditions, the following relations should be maintained:

$$T(s_i) = \sum_{x \in s_i} T(x) \le CT \qquad i = 1, \dots, M \qquad (4)$$

If $x \, p \, y$ (x precedes y) and ($x \in s_i$) and ($y \in s_j$) then ($i \le j$)   (5)

Relation (4) above means that the sum of the times assigned to one station should not be more than cycle. Relation (5) shows that if activity x precedes y and activity x is done in i-th station and y is done in j-th station, then $i \le j$, means that x is done before y or in the same station with y.

2.1. Algorithm of the straight assembly line balancing problem by heuristic method based on CPM

1. Calculating minimum feasible number of workstion S and the minimum feasible cycle time MCT and the adjusted value of:

$\theta = [(MCT + CT) / 2]$.

2. Creating a new work station, finding the critical path and then identifying activities permitted for assign.

3. Assign activities are on the critical path with high priority, if it cannot be assigned the other possible activities that have a greater processing time can be used of.

4. Computing the remained time for the current station, if the mentioned station has enough time for any feasible unassigned tasks, again the new critical path will be found then we go to step 3, otherwise go to step 2.

Note that:

- When assigning work to the critical path, those works that are not on the path will be assigned only for preserving priority or to fill work capaci-ties of the stations, if necessary.

- θ Obtained above is as the upper bound of work capacity for the stations instead of CT.

- This order in each stage is continued with finding the critical path, until all the activities will be assigned to the work stations.

- In this approach AON (activity-on-node) has been used for precedence network.

For better understanding of proposed method, a simple illustrative example is solved in the next sub-section and then a case study example is of-fered.

2.2. Illustrative examples

2.2.1. Example 1

Firstly, an example is used that has been considered in literature by Jackson. Assumption is that $CT = 21$ then we calculate S, MTC and finally θ. And the initial critical path is indicated in Figure1.

Here we have brief results without the modified network in each step:

1. Calculating $S = 46/21 = 2.19$ and after rounding up it is 3 and MCT = $46/3 = 15.33$, therefore

2. $\theta = [(21 + 15.33) / 2] = 18.16$, so $15.33 < 18.16 < 21$, if all of tasks times are not integer we can make it round.

3. Creating work station 1, finding critical path and critical activities for the first time , this phase has been shown in figure 1. so we assign task 1 according to the CPM and after finding the new critical tasks, we have two different choices, tasks 2, 4. In this situation we select a task with greater processing time here it is task 4 and then at new critical path task 2 will be assigned, following, in accordance with the remain cycle time for current station and critical activities merely task 6 can be assigned and total station time is:

$$T(s_1) = 6 + 7 + 2 + 2 = 17.$$

4. Creating work station 2, Critical activities on the critical path are 3, 7, 9 and 11 we assign task 3 as a result, critical path changes to 8, 10 and 11 so task 8 will be assigned, with the same method assigning activities 5, 7. According to the remained satition time and activities, no more activities cannot be assigned to this station. total station time equals:

$$T(s_1) = 5 + 6 + 1 + 3 = 15.$$

5. Creating work station 3, Critical activities on the critical path are 9,10 and 11 acording to the percedance relashenship and processing time there is not any difference between tasks 9 and 10, so task 9,10 will be assigned and finally 11. Here the algorithm will be finished because all the activi-ties are assigned. Total process time for this station is:
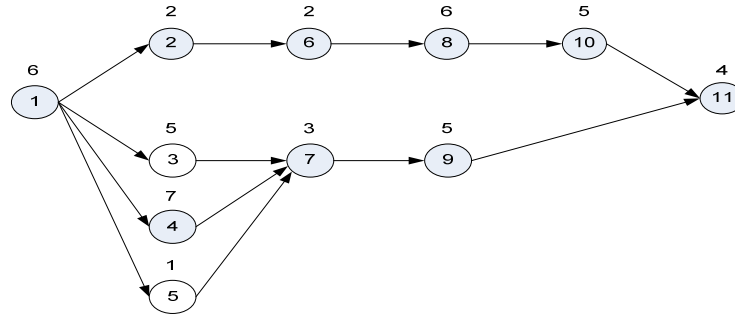
$$T(s_3) = 5 + 5 + 4 = 14.$$

Figure 1: The network of Jackson problem.

2.2.2. Example 2: Iron assembly line balancing: Case study

In this section first we need to explain about our case study and then with an illustrative example the new heuristic will be explained. This case study was in an iron factory in Iran and all data are real and the duration of tasks are not integer because these times have been collected by means of stop watch method, but it is possible to use it with integer time in future researches.

All the activities in this assembly line were assigned to 6 work stations. Of course, some of the activities are done as preassembly ones out of production line before beginning assembly. An operator is working at each work station and current cycle time is 74 seconds. One of the aims of this problem, as well as minimizing work stations, is to assign all preassembly activities to the work stations in such a way that all activies are fulfilled on production line consequently. All the activities, times, and their prerequisites are shown in Table 5. To solve this problem, first a precedence network of activities is drawn using Table 5, and the initial critical path is indicated in Figure 2.

1. Calculating S with $CT = 74$, $S = 336.26 / 74 = 4.54$ and after rounding up it is 5 and $MCT = 336.26 / 5 = 67.25$.

2. Calculating $\theta = [(74 + 67.25) / 2] = 7.62$, so

$67.25 < 70.62 < 74$.

3. Creating workstation 1, finding critical path for first time and after each assignment, initial critical activities in this phase are 1, 2, 8, 9, 17, 19, 21, 23, 24, 25, 26, 29 at first assigning activities are on the critical path according to the precedence relationship and remained time in the current station like preceding explanations so activities 1, 2, 3, 4, 5, 11, 12 will be assigned and calculated as $T(s_1) = 70.29$.

4. Creating work station 2, finding new critical path for each assignment, assigning activities 6,7,8,10,13,14,15,18 and calculating $T(s_2) = 59.46$.

5. Creating work station 3, finding new critical path for each assignment, assigning activities 9,16,17,19, 20 and calculating $T(s_3) = 65.78$.

The assigning process continues until all the activities will be assigned to the 5 work stations, according to Table 1.

3. Review of heuristic methods

In this section some heuristic methods will be introduced that have been used in this paper for comparing with proposed method. The parameters used in these methods are:

t_i Assembly time required to complete task *i*.

i, j Task index.

C Station cycle time.

IS_i Set of immediate successors of *i*.

N Number of tasks to be balanced into stations.

IP_i Set of immediate predecessors of *i*.

S_i Set of all successors of *i*.

P_i Set of all predecessors of *i*.

UB_i Upper bound on the station, to which *i* may be assigned.

LB_i Lower bound on the station to which *i* may be assigned.

$[X]^+$ Smallest integer greater than or equal to *X*.

The relations of calculating UB_i and LB_i are as follows:

$$UB_i = N + 1 - [(t_i + \sum_{j \in s_i} tj)/C]^+ \qquad (6)$$

$$LB_i = [(t_i + \sum_{j \in p_i} tj)/C]^+ \qquad (7)$$

4. Performance indexes

There are several indexes on literature but some of them that are more efficient for SALBP-1 have been selected, therefore four commonly used measures for our aim are as follows:

1. Number of Work Station (NWS): Less the index, show decreasing the station and better distribution of tasks.

2. Smoothness Index (SI): The smoothness index is an index for the relative smoothness of a given assembly line. A smoothness index of 0 indicates a perfect balance. A smaller SI results in a smoother line, thereby reducing the in-process inventory (Ponnambalam *et al.*, 2000).

$$SI = \sqrt{\frac{\sum_{i=1}^{m}\left(T(s_{max}) - T(s_i)\right)^2}{M}} \qquad (8)$$

3. Mean Absolute Deviation (MAD): The mean absolute deviation is an index for comparing the difference between expected value and actual value and in assembly line MAD of 0 indicates an excellent balancing where that total time assigned tasks to each station equals cycle time (Rachamadugu and Talbot, 1991).

$$MAD = \frac{1}{M} \sum_{x \in s_i} |T(s_i) - MTC| \qquad (9)$$

4. Line Efficiency (LE): The line efficiency is the ratio between total station time to the product of cycle time and the number of work stations, represented as a percentage. The greatest LE results in a efficient line, it shows the percentage use of the line and is expressed as: (Ponnambalam *et al.*, 2000).

$$LE = \frac{\sum_{i=1}^{m} T(s_i)}{M \times C} \times 100 \qquad (10)$$

Above indexes have been selected to evaluate the final answer of 11 heuristic methods and proposed method for several test problems. In the next part the quality of different methods using these indexes will be evaluated.

5. Comparing the results of new method against other methods

In order to compare the efficiency of the proposed algorithm, 11 test problems with various cycle time that are available in the literature taken from http:// www.bwl.tu-darmstadt.de/ bwl3/ forsch/projekte/alb/albdata.htm. In general, 40 problems have been solved. Furthermore, the proposed heuristic and all of the other 11 methods have been programmed in Macromedia Flash professional 8 supported by Java Script programming language that it can be used as a new software for proposed heuristic method. In table 3, the efficiency of 11 solving methods and the proposed method has been shown by these indexes: NWS, SI, MAD and LE. In table 3 the optimal results in all indexes that are obtained from heuristic methods are bolted. It is considerable that the first objective is more important than the other objectives. Therefore, after comparing the first objective and getting a good answer the other objectives will be compared.

Comparison of the results of new method with other calculated methods in Table 3 has been shown in Table 4 regarding assigned activities to workstations with multi objectives.

In Table 3, all indexes show the superiority of the proposed heuristic method in assembly line balancing over other 11 methods and according to the final results in Table 4 it has been clearly shown that the proposed method (number 12) has a better situation than the other applied methods.

6. Conclusion

According to the results of performance indexes, we can easily conclude that heuristic method based on critical path gives better results for assigning tasks and minimizing the number of work stations. Although some of other methods minimize work stations, but they do not give considerable results for other indexes. Though this new heuristic method is an approximated method like the other heuristic methods, it has a better efficient in general.

Since heuristic approaches are the foundation of metaheuristic methods, so suggested method can increase the effectiveness of metaheuristic approaches like simulated annealing, genetic algorithm and ant colony optimization at SALBP. Applying proposed method in parallel, U-shaped, and other types of production lines can be considered in future researches.
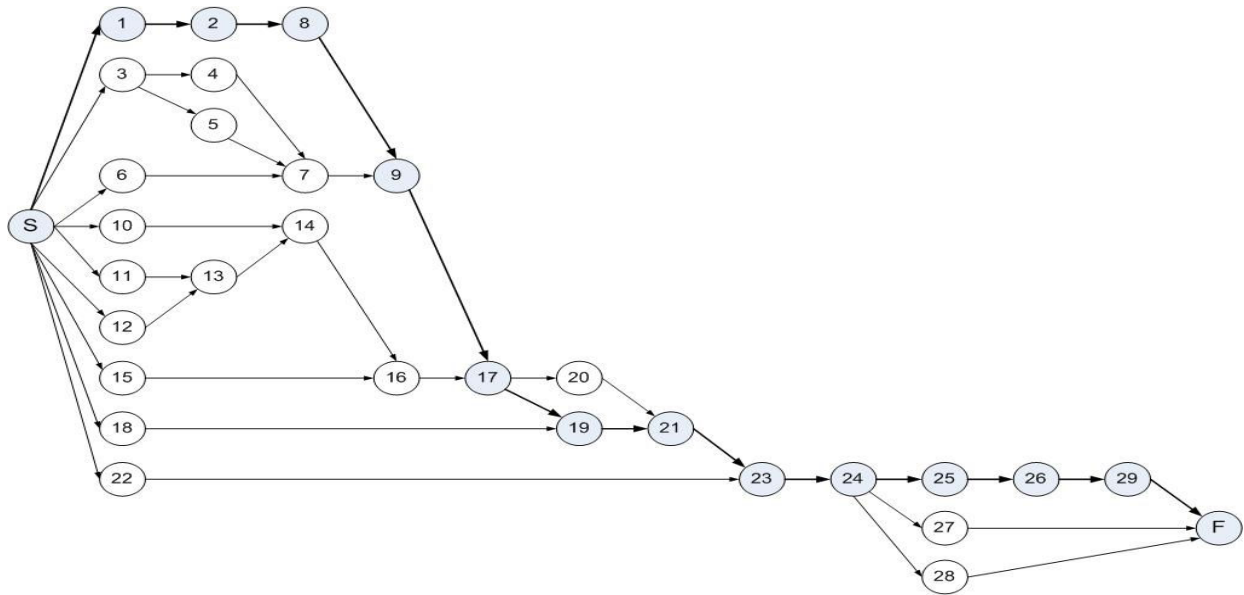
Figure 2: Succession and precedence network of activities of iron assembly line using Table 5.

Table 1: Results of assigning activities by new heuristic method.

Work stations	x_1	x_2	x_3	x_4	x_5
	1	10	9	22	25
	2	13	16	21	26
	11	6	17	23	29
Activities	12	8	19	24	27
	3	7	20	28	
	5	14			
	4	15			
		18			
Total time of each station	70.29	59.46	65.78	70.42	70.31

Table 2: List of heuristic rules considered in this paper.

Rule No	Rule Name	Symbol	Definition	Reference
1	Longest Processing Time	LPT	t_i	(Talbot and Patterson, 1984)
2	Shortest Processing Time	SPT	t_i	(Baykasoglu, 2006)
3	Maximum Number of Immediate Successors after task i	NIS	$|IS_i|$	(Tonge, 1960)
4	Random Priority	RND	random	(Arcus, 1963)
5	Smallest Task Number	STN	i	(Arcus, 1963)
6	Maximum Ranked Positional Weight	RPW	$t_i + \sum_{j \in S_i} tj$	(Helgeson and Birnie, 1961)
7	Greatest (Processing Time Divided by the Upper Bound)	G_PT_UB	t_i / UB_i	(Baykasoglu, 2006)
8	Smallest Lower Bound	SLB	$[(t_i + \sum_{j \in P_i} tj)/C]^+$	(Talbot and Patterson, 1984)
9	Minimum Slack	MSLK	$UB_i - LB_i$	(Talbot and Patterson, 1984)
10	Maximum Number of Immediate Predecessors	NIP	$|IP_i|$	(Baykasoglu, 2006)
11	Smallest Upper Bound	SUB	$N+1-[(t_i + \sum_{j \in S_i} tj)/C]^+$	(Talbot and Patterson, 1984)
12	Critical Path Method (Proposed Method)	CPM-LPT	according to section two	Proposed method

Table 3: Results for SALBP problems using task assignment rules given in an order as defined in Table 2.

Sample Names	CT		Rules Number											
			1	2	3	4	5	6	7	8	9	10	11	12
Mitchell	39	NWS	3.00	3.00	3.00	3.00	3.00	3.00	3.00	**3.00**	3.00	3.00	3.00	3.00
		SI	5.16	3.46	6.38	3.46	3.70	6.38	5.16	**1.29**	5.48	5.20	3.46	2.58
		MAD	2.67	2.67	4.67	2.67	2.00	4.67	2.67	**0.67**	3.33	4.00	2.67	1.33
		LE	89.74	89.74	89.74	89.74	89.74	89.74	89.74	**89.74**	89.74	89.74	89.74	89.74
	35	NWS	**3.00**	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
		SI	**0.00**	7.66	16.52	16.04	13.59	16.52	16.07	16.04	13.59	14.33	7.66	6.54
		MAD	**0.00**	5.13	12.13	11.63	9.63	12.13	11.63	11.63	9.63	9.63	5.13	4.63
		LE	**100.00**	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00
	26	NWS	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	**5.00**
		SI	6.21	5.02	6.94	5.02	6.94	7.99	8.40	4.22	6.94	6.21	6.21	**1.18**
		MAD	3.20	2.40	3.60	2.40	3.60	4.80	5.20	2.40	3.60	3.20	3.20	**0.40**
		LE	80.77	80.77	80.77	80.77	80.77	80.77	80.77	80.77	80.77	80.77	80.77	**80.77**
	21	NWS	**5.00**	6.00	6.00	6.00	6.00	6.00	**5.00**	6.00	6.00	6.00	6.00	6.00
		SI	**0.00**	5.12	7.38	5.93	6.10	7.38	**0.00**	6.75	6.10	5.12	5.12	1.96
		MAD	**0.00**	3.00	4.83	3.50	4.00	4.83	**0.00**	4.33	4.00	3.00	3.00	1.00
		LE	**100.00**	83.33	83.33	83.33	83.33	83.33	**100.00**	83.33	83.33	83.33	83.33	83.33
Jackson	21	NWS	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	**3.00**
		SI	9.81	7.51	9.81	9.81	9.81	9.81	9.81	9.81	9.81	9.81	9.81	**3.27**
		MAD	7.56	4.22	7.56	7.56	7.56	7.56	7.56	7.56	7.56	7.56	7.56	**1.78**
		LE	73.02	73.02	73.02	73.02	73.02	73.02	73.02	73.02	73.02	73.02	73.02	**73.02**
	14	NWS	4.00	**4.00**	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	5.00
		SI	3.24	**0.71**	3.24	3.08	3.08	3.08	3.24	3.08	3.08	3.24	3.24	3.95
		MAD	2.00	**0.50**	2.00	1.50	1.50	1.50	2.00	1.50	1.50	2.00	2.00	2.16
		LE	82.14	**82.14**	82.14	82.14	82.14	82.14	82.14	82.14	82.14	82.14	82.14	65.71
	13	NWS	4.00	**4.00**	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	5.00
		SI	2.24	**0.71**	2.12	2.24	2.24	2.24	2.24	2.24	2.24	2.12	2.12	3.95
		MAD	1.50	**0.50**	1.25	1.50	1.50	1.50	1.50	1.50	1.50	1.25	1.25	2.16
		LE	88.46	**88.46**	88.46	88.46	88.46	88.46	88.46	88.46	88.46	88.46	88.46	70.77
	10	NWS	6.00	6.00	**5.00**	6.00	6.00	6.00	6.00	6.00	6.00	**5.00**	**5.00**	6.00
		SI	3.16	2.94	**1.10**	2.08	3.21	3.16	3.16	3.21	3.21	**1.10**	**1.10**	2.08
		MAD	1.78	1.67	**0.64**	1.44	2.00	1.78	1.78	2.00	2.00	**0.64**	**0.64**	1.44
		LE	76.67	76.67	**92.00**	76.67	76.67	76.67	76.67	76.67	76.67	**92.00**	**92.00**	76.67
Mertens	18	NWS	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	**2.00**
		SI	3.54	4.95	4.95	4.95	4.95	4.95	3.54	4.95	4.95	4.95	4.95	**0.71**
		MAD	2.50	3.50	3.50	3.50	3.50	3.50	2.50	3.50	3.50	3.50	3.50	**0.50**
		LE	80.56	80.56	80.56	80.56	80.56	80.56	80.56	80.56	80.56	80.56	80.56	**80.56**
	15	NWS	**2.00**	3.00	**2.00**	**2.00**	3.00	**2.00**	**2.00**	3.00	3.00	**2.00**	**2.00**	3.00
		SI	**0.71**	4.40	**0.71**	**0.71**	4.76	**0.71**	**0.71**	4.40	4.76	**0.71**	**0.71**	6.03
		MAD	**0.50**	2.44	**0.50**	**0.50**	3.11	**0.50**	**0.50**	2.44	3.11	**0.50**	**0.50**	3.78
		LE	**96.67**	64.44	**96.67**	**96.67**	64.44	**96.67**	**96.67**	64.44	64.44	**96.67**	**96.67**	64.44
	10	NWS	4.00	4.00	**3.00**	4.00	4.00	**3.00**	4.00	4.00	4.00	**3.00**	4.00	4.00
		SI	3.35	2.50	**0.58**	2.50	3.35	**0.58**	3.35	2.50	3.35	**0.58**	2.50	2.50
		MAD	1.75	1.75	**0.44**	1.75	1.75	**0.44**	1.75	1.75	1.75	**0.44**	1.75	1.75
		LE	72.50	72.50	**96.67**	72.50	72.50	**96.67**	72.00	72.50	72.50	**96.67**	72.50	72.50

Table 3: Results for SALBP problems using task assignment rules given in an order as defined in Table 2 (continued).

Sample Names	CT		Rules Number											
			1	2	3	4	5	6	7	8	9	10	11	12
	8	NS	5.00	6.00	5.00	5.00	5.00	5.00	5.00	**5.00**	5.00	6.00	6.00	**5.00**
		SI	2.57	1.35	2.57	2.57	1.41	2.57	2.57	**1.41**	2.57	1.35	1.35	**1.41**
		MAD	1.04	0.56	1.04	1.04	0.64	1.04	1.04	**0.64**	1.04	0.56	0.56	**0.64**
		LE	72.50	60.42	72.50	72.50	72.50	72.50	72.50	**72.50**	72.50	60.42	60.42	**72.50**
Lutz1	2828	NWS	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	**6.00**
		SI	955.95	697.63	884.14	953.24	974.05	970.11	955.95	974.05	633.54	884.14	862.97	**364.69**
		MAD	634.22	412.22	554.89	634.22	634.22	634.22	634.22	634.22	412.22	554.89	554.89	**206.89**
		LE	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	**83.33**
	2357	NWS	7.00	**7.00**	7.00	7.00	8.00	7.00	7.00	8.00	7.00	7.00	7.00	7.00
		SI	498.29	**427.80**	484.81	483.02	716.92	540.17	498.29	716.92	498.20	484.81	484.56	532.06
		MAD	268.57	**222.29**	268.57	259.43	328.38	322.86	268.57	328.38	258.29	268.57	268.57	292.00
		LE	85.70	**85.70**	85.70	85.70	74.99	85.70	85.70	74.99	85.70	85.70	85.70	85.70
	2020	NWS	8.00	9.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	**8.00**	9.00
		SI	320.18	557.12	352.83	366.80	366.80	293.51	320.18	366.80	361.27	352.83	**303.74**	505.19
		MAD	204.25	330.96	204.25	204.25	204.25	146.50	204.25	204.25	180.75	204.25	**179.25**	282.27
		LE	87.50	77.78	87.50	87.50	87.50	87.50	87.50	87.50	87.50	87.50	**87.50**	77.78
	1768	NWS	9.00	10.00	9.00	10.00	9.00	9.00	9.00	9.00	9.00	9.00	**9.00**	9.00
		SI	248.49	528.82	225.89	494.49	230.73	269.82	248.49	230.73	1768.00	225.89	**203.31**	251.98
		MAD	132.49	264.80	139.65	266.40	103.65	153.63	132.49	103.65	92.21	139.65	**105.43**	128.74
		LE	88.86	79.98	88.86	79.98	88.86	88.86	88.86	88.86	11.14	88.86	**88.86**	88.86
Heskiaoff	342	NWS	4.00	4.00	4.00	4.00	4.00	3.00	3.00	4.00	4.00	4.00	4.00	4.00
		SI	139.14	104.05	136.24	131.42	139.60	1.15	1.15	139.14	139.60	135.57	135.63	139.11
		MAD	92.00	66.00	92.00	92.00	92.00	0.89	0.89	92.00	92.00	92.00	92.00	92.00
		LE	74.85	74.85	74.85	74.85	74.85	99.81	99.81	74.85	74.85	74.85	74.85	74.85
	324	NWS	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	**4.00**
		SI	126.40	104.05	126.30	124.60	125.17	126.32	126.40	125.11	125.70	109.77	102.51	**67.00**
		MAD	92.00	66.00	92.00	92.00	92.00	92.00	92.00	92.00	92.00	75.50	66.00	**50.00**
		LE	79.01	79.01	79.01	79.01	79.01	79.01	79.01	79.01	79.01	79.01	79.01	**79.01**
	256	NWS	5.00	5.00	5.00	5.00	5.00	**4.00**	5.00	5.00	5.00	5.00	5.00	5.00
		SI	87.89	60.26	82.50	57.40	85.89	**0.00**	87.89	67.64	86.83	65.10	55.06	55.01
		MAD	60.24	38.24	53.12	32.32	53.44	**0.00**	60.24	40.24	57.04	42.72	32.32	39.12
		LE	80.00	80.00	80.00	80.00	80.00	**10.00**	80.00	80.00	80.00	80.00	80.00	80.00
	216	NWS	5.00	6.00	5.00	6.00	5.00	5.00	5.00	6.00	5.00	5.00	6.00	**5.00**
		SI	22.42	62.43	16.28	61.84	18.41	24.17	22.42	67.53	16.81	13.39	64.95	**11.63**
		MAD	15.52	35.67	9.84	33.78	13.04	17.12	15.52	40.00	10.32	7.84	34.00	**8.32**
		LE	94.81	79.01	94.81	79.01	94.81	94.81	94.81	79.01	94.81	94.81	79.01	**94.81**
Roszieg	32	NWS	5.00	5.00	5.00	5.00	5.00	**4.00**	**4.00**	5.00	5.00	5.00	5.00	5.00
		SI	12.69	10.06	13.98	10.47	12.12	**1.12**	**1.12**	12.12	12.12	12.69	12.69	12.12
		MAD	8.40	6.40	9.60	6.80	8.40	**0.75**	**0.75**	8.40	8.40	8.40	8.40	8.40
		LE	78.13	78.13	78.13	78.13	78.13	**97.66**	**97.66**	78.13	78.13	78.13	78.13	78.13
	25	NWS	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	**6.00**
		SI	7.58	6.84	8.34	7.58	5.15	6.89	7.58	5.93	5.18	6.84	7.20	**1.58**
		MAD	4.61	3.94	5.28	4.61	2.94	3.94	4.61	2.94	2.94	3.94	4.28	**0.89**
		LE	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	83.33	**83.33**
	21	NWS	7.00	7.00	7.00	7.00	7.00	**6.00**	**6.00**	7.00	**6.00**	7.00	7.00	7.00
		SI	6.57	6.61	6.28	5.48	6.61	**0.41**	**0.41**	6.55	**0.41**	6.52	6.19	3.80
		MAD	3.96	4.20	3.92	3.59	4.20	**0.28**	**0.28**	3.96	**0.28**	3.96	3.67	2.49
		LE	85.03	85.03	85.03	85.03	85.03	**99.21**	**99.21**	85.03	**99.21**	85.03	85.03	85.03

Table 3: Results for SALBP problems using task assignment rules given in an order as defined in Table 2 (continued).

Sample Names	CT		Rules Number											
			1	2	3	4	5	6	7	8	9	10	11	**12**
	18	NS	8.00	9.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	**8.00**
		NS	4.49	4.50	2.89	3.10	5.06	5.06	4.49	5.06	5.06	3.66	2.89	**2.15**
		MAD	3.06	2.81	1.47	1.72	2.91	2.91	3.06	2.91	2.91	2.06	1.47	**1.31**
		LE	86.81	77.16	86.81	86.81	86.81	86.81	86.81	86.81	86.81	86.81	86.81	**86.81**
Buxey	54	NWS	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	**7.00**
		SI	13.85	8.09	13.44	10.20	13.53	13.53	13.85	10.20	13.53	8.02	8.02	**5.58**
		MAD	8.53	4.53	7.59	5.67	7.88	7.88	8.53	5.67	7.88	4.24	4.24	**3.31**
		LE	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	**85.71**
	47	NWS	8.00	**8.00**	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
		SI	11.93	**3.97**	11.37	9.76	10.06	10.06	11.93	10.71	10.06	6.86	6.86	10.06
		MAD	8.50	**1.38**	7.75	5.50	6.50	6.50	8.50	6.50	6.50	3.25	3.25	6.50
		LE	86.17	**86.17**	86.17	86.17	86.17	86.17	86.17	86.17	86.17	86.17	86.17	86.17
	41	NWS	**8.00**	9.00	9.00	9.00	9.00	9.00	**8.00**	9.00	9.00	9.00	9.00	9.00
		SI	**0.87**	6.61	5.99	7.42	8.08	8.08	**0.87**	5.92	8.08	6.66	6.66	5.73
		MAD	**0.63**	4.00	2.67	4.44	4.67	4.67	**0.63**	2.89	4.67	4.00	4.00	3.33
		LE	**98.78**	87.80	87.80	87.80	87.80	87.80	**98.78**	87.80	87.80	87.80	87.80	87.80
	36	NWS	10.00	11.00	10.00	10.00	10.00	10.00	10.00	11.00	10.00	11.00	11.00	**10.00**
		SI	4.90	8.16	6.78	4.02	5.90	5.90	4.90	8.77	5.90	7.99	7.99	**2.68**
		MAD	2.80	5.06	3.84	1.48	3.52	3.52	2.80	5.24	3.52	4.61	4.61	**1.72**
		LE	90.00	81.82	90.00	90.00	90.00	90.00	90.00	81.82	90.00	81.82	81.82	**90.00**
Sawyer	75	NWS	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	**5.00**
		SI	14.67	13.07	19.49	11.33	13.88	20.21	14.67	8.92	13.48	13.36	13.22	**6.24**
		MAD	11.04	7.84	13.12	6.72	7.44	13.92	11.04	4.64	8.64	8.64	8.24	**4.24**
		LE	86.40	86.40	86.40	86.40	86.40	86.40	86.40	86.40	86.40	86.40	86.40	**86.40**
	54	NWS	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	**7.00**
		SI	17.86	9.02	8.60	11.93	12.86	15.37	17.86	12.86	14.69	14.68	10.60	**6.55**
		MAD	11.22	5.02	4.82	7.02	8.16	9.22	11.22	8.16	8.65	8.65	5.31	**4.16**
		LE	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	85.71	**85.71**
	47	NWS	8.00	9.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	9.00	**8.00**
		SI	11.62	8.01	11.57	9.72	11.77	13.78	11.62	11.59	11.62	10.93	16.09	**10.28**
		MAD	6.50	5.11	6.63	5.13	7.00	8.25	6.50	6.25	6.50	6.13	8.89	**6.88**
		LE	86.17	76.60	86.17	86.17	86.17	86.17	86.17	86.17	86.17	86.17	76.60	**86.17**
	41	NWS	9.00	9.00	9.00	9.00	9.00	9.00	9.00	10.00	9.00	9.00	9.00	**9.00**
		SI	11.52	8.01	9.48	8.96	9.13	7.08	11.52	10.60	8.69	9.04	8.20	**6.58**
		MAD	6.44	5.11	5.33	4.89	5.33	4.67	6.44	5.92	5.33	5.11	5.33	**4.22**
		LE	87.80	87.80	87.80	87.80	87.80	87.80	87.80	79.02	87.80	87.80	87.80	**87.80**
Gunther	81	NWS	7.00	8.00	**7.00**	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
		SI	28.42	24.09	**17.18**	17.21	20.41	29.14	28.42	20.41	20.91	17.61	20.45	18.85
		MAD	18.00	12.78	**9.14**	9.14	14.86	18.57	18.00	14.86	15.43	10.00	14.86	13.71
		LE	85.19	74.54	**85.19**	85.19	85.19	85.19	85.19	85.19	85.19	85.19	85.19	85.19
	69	NWS	8.00	9.00	8.00	8.00	8.00	8.00	8.00	9.00	8.00	8.00	9.00	**8.00**
		SI	13.34	16.51	13.86	13.76	13.26	13.11	13.34	16.40	13.73	13.15	18.42	**6.13**
		MAD	8.44	8.74	9.19	9.19	8.44	9.19	8.44	8.30	9.19	9.19	9.11	**3.28**
		LE	87.50	77.78	87.50	87.50	87.50	87.50	87.50	77.78	87.50	87.50	77.78	**87.50**
	61	NWS	9.00	10.00	10.00	10.00	9.00	9.00	9.00	10.00	9.00	10.00	10.00	**9.00**
		SI	10.02	15.87	17.01	15.74	9.51	10.52	10.02	16.01	10.09	17.01	15.85	**9.45**
		MAD	5.93	7.90	9.50	8.10	5.04	6.81	5.93	8.70	5.93	9.50	8.16	**6.15**
		LE	87.98	79.18	79.18	79.18	87.98	87.98	87.98	79.18	87.98	79.18	79.18	**87.98**
	54	NWS	**9.00**	11.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
		SI	**0.58**	11.53	7.85	6.95	7.18	7.88	9.50	7.18	7.69	7.85	6.83	6.16

Table 3: Results for SALBP problems using task assignment rules given in an order as defined in Table 2 (continued).

Sample Names	CT		Rules Number											
			1	2	3	4	5	6	7	8	9	10	11	12
		MAD	**0.44**	5.55	5.04	3.50	3.90	5.04	5.98	3.90	4.44	5.04	3.30	3.50
		LE	**99.38**	81.31	89.44	89.44	89.44	89.44	89.44	89.44	89.44	89.44	89.44	89.44
Bowman	20	NWS	**5.00**	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
		SI	**2.97**	3.87	6.31	5.92	5.92	5.92	2.97	5.92	5.92	3.87	3.87	3.87
		MAD	**1.60**	2.00	3.60	2.80	2.80	2.80	1.60	2.80	2.80	2.00	2.00	2.00
		LE	**75.00**	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00	75.00
Iron case study	74	NWS	5.00	6.00	6.00	5.00	5.00	5.00	5.00	5.00	5.00	6.00	6.00	**5.00**
		SI	14.03	21.21	20.03	8.00	8.58	13.94	14.03	8.64	8.58	20.03	20.03	**5.32**
		MAD	9.89	11.82	11.44	5.41	5.27	9.89	9.89	5.46	5.27	11.44	11.44	**3.71**
		LE	90.88	75.73	75.73	90.88	90.88	90.88	90.88	90.88	90.88	75.73	75.73	**90.88**
	70	NWS	6.00	**6.00**	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
		SI	18.35	**14.45**	18.35	19.91	19.96	26.34	18.35	19.84	19.96	18.35	18.35	24.49
		MAD	10.88	**8.84**	10.88	13.93	12.97	16.77	10.88	12.97	12.97	10.88	10.88	15.81
		LE	80.06	**80.06**	80.06	80.06	80.06	80.06	80.06	80.06	80.06	80.06	80.06	80.06
	67	NWS	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	**6.00**
		SI	15.37	14.45	15.02	13.96	15.99	21.54	15.37	14.74	15.99	15.02	15.02	**8.55**
		MAD	9.84	8.84	9.71	8.80	11.06	13.91	9.84	8.80	11.06	9.71	9.71	**5.40**
		LE	83.65	83.65	83.65	83.65	83.65	83.65	83.65	83.65	83.65	83.65	83.65	**83.65**

Table 4: Result of comparing the suggested method with other methods.

Rules Number	1	2	3	4	5	6	7	8	9	10	11	**12**
Total optimal answer	6	5	4	1	0	6	6	2	1	3	4	**19**
Total problems	40	40	40	40	40	40	40	40	40	40	40	**40**
Percentage	15%	12.50%	10%	2.50%	0%	15%	15%	5%	2.50%	7.50%	10%	**47.50%**

Table 5: Activities, times, and predecessors.

Activity no.	Activity name	Required time (sec.)	Predecessors
S	Beginning of assembly line	0	-
1	Setting heat-shield on the table	2.62	-
2	Attaching heat-shield on the soleplate	47.1	1
3	Taking out the cord from the bag	0.78	-
4	Putting locks around the cord	0.89	3
5	Setting clips for cable	7.9	3
6	Putting lower inside bottom	4.29	-
7	Putting the cord inside handle, lower and bottom	6.53	4,5,6
8	Installing terminal on the soleplate	10.12	2
9	Installing neon lamp and putting handle on the soleplate	20.19	7,8
10	Preassembling the right-side of spray set	10.3	-
11	Preassembling the left-side of spray set	6.4	-
12	Preassembling hose and cover	4.6	-
13	Attaching parts 10 and 11	9.56	11,12
14	Attaching parts 9 and 12	8.95	10,13
15	Preassembling shock set	6.59	-
16	Installing 13 and 14 on the water tank	14.97	14,15
17	Installing water tank on heat-shield	8.5	9,16
18	Assembly of crick crag spring and crick crag pin	3.12	-
19	Installing part 17 on water tank	11.12	17,18
20	Putting extension on water tank	11	17
21	Putting thermostat knob on the water tank	12.95	19,20
22	Installing shock-steam knob and spray knob on the body	24.85	-
23	Installing body on water tank and handle	17.24	21,22
24	Putting fixing part on the body	6.78	23
25	Installing cable press big	17	24
26	Installing lamp-holder	10.79	25
27	Installing slider	5.72	24
28	Installing spray support	8.6	24
29	Installing cable spring, bottom, and cover	36.8	26
F	Completed iron	0	-

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