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Solving a bi-objective project capital budgeting problem using a fuzzy multi-dimensional knapsack

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Abstract: In this paper, the researchers have proposed a multi-dimensional knapsack model for project capital budgeting problem in uncertain situation which has been modeled through fuzzy sets. The optimistic and pessimistic situations were considered and associated deterministic models were yielded. Numerical example has been supplied toillustrate the performance of proposed model. The results were promising in the sense of helping the decision makers.

Keywords: *Capital budgeting; Fuzzy multi-dimesional knapsack; Project selection; Portfolio selection*

1. Introduction

Optimum investment when we are facing a set of chances has both practical and theoretical importance. The selection should contain set investments to meet a high level profit and low risk. Generally, this is interpreted as capital budgeting which is a common paradigm with enough flexibility for standing in many areas.

Capital budgetinghas also attracted a large variety of research efforts due to its adaptability to real case conditions. Liang (2008) proposed chance programming models for capital budgeting in fuzzy environments. Huang (2008) developed mean-variance model for fuzzy capital budgeting. Chan *et al.* (2005) proposeda goal-seeking approach to capital budgeting. Steuer and Na (2003) proposed a categorized bibliographic study on multiple criteria decision making combined with finance. Bernardo (2001) developed capital budgeting and compensation with asymmetric information and moral hazard. Badri (2001) developed a comprehensive 0–1 goal programming model for project selection. Timothy and Kalu (1999) proposedan extended goal programming approach for capital budgeting under uncertainty. An empirical study of capital budgeting practices for strategic investments in CIM technologies has been accomplished by Slagmulder *et al.* (1995). Lee and Kim (1994) proposed capital budgeting model with flexible budget. Liang and Gao (2008) proposed dependent-chance programming models for capital budgeting in fuzzy environments.

Knapsackis assumed as a NP-hard problem (Fréville, 2004). Knapsackis fitted properly to a set of optimization and engineering problems as well as capital budgeting and project selection. Balev *et al.* (2008) proposed a dynamic programming based reduction procedure for the multidimensional 0–1 knapsack problem. Kaparis and Letchford (2008) proposed local and global lifted cover inequalities for the 0–1 multidimensional knapsack problem. Bektas and Oğuz (2007) developed separating cover inequalities for the multidimensional knapsack problem. Akbar (2006) proposed multidimensional multiple-choice knapsack problem. Fréville (2004) presented an overview of multidimensional 0–1 knapsack problem. Stavros (2007) proposed a partially ordered knapsack and applied it to scheduling problem.

In this paper, the researchers associated the capital budgeting problem as a multidimensional knapsack in an uncertain situation which has been modeled using fuzzy sets. Total available budget, net present value (NPV) of a project and the associated project profit are assumed to be positive Trapezoidal Fuzzy Numbers (TrFNs). Two objectives were considered (i.e. lowest cost and maximum profit).

The following sections are arranged as below. Section 2 is allocated to define the problem as a multidimensional knapsack. The proposed fuzzy model is presented in Section3. Experimental results are outlined in Section 4. The conclusions are represented in Section 5.

2. Problem definition

The classical multi-dimensional knapsackis reviewed briefly.Then, it will be associated tocapital budgeting problem.

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2.1. Multi-dimensional knapsack problem

We are given a set of n objects numbered 1, 2, …,n and a knapsack of total volume and weight capacity which are represented by Wand V, respectively. Each object i has weight w_i, volume v_i and utilityp_i. Let $X = [x_1, x_2 \ldots x_n]$ be a solution vector in which $x_i = 0$ if object i is not in the knapsack, and $x_i = 1$ if it is in the knapsack.

The goal is to find a subset of objects to put into the knapsack so that the total available volume and weight capacity is obeyed and the total utility of this set is maximized concurrently. The crisp zero-one representation of aforementioned problem is as Model (1):

$$\text{Max } \sum_{i=1}^{n} p_i x_i$$

Subject to:

$$\sum_{i=1}^{n} w_i x_i \le W \tag{1}$$

$$\sum_{i=1}^{n} v_i x_i \le V$$

$$x_i = 0,1$$

2.2. Fuzzy multi-dimensional knapsack problem

In real world problems, it is often impossible or non-realistic to gather a crisp value for the coefficient of the Model (1).

The Model (1) can be developed in a fuzzy environment as Model (2).

$$\text{Max } \sum_{i=1}^{n} \tilde{p}_i x_i$$

Subject to:

$$\sum_{i=1}^{n} \tilde{w}_i x_i \le \tilde{W} \tag{2}$$

$$\sum_{i=1}^{n} \tilde{v}_i x_i \le \tilde{V}$$

$$x_i = 0,1$$

In Model (2) the expected profit of a project, weight of objects, objects volume and total available resources are assumed to be a fuzzy number.

3. Proposed fuzzy capital budgeting model

Consider there are several investment chances in a full uncertain situation where there are no exact information about requirement and outputs of investment. A DM likes to invest in a way that the total profit of investment is maximized and the total available budget for investment is obeyed. It is a capital budgeting problem which may be solved optimally by fuzzy multidimensional knapsack formulation.

In this section, a fuzzy capital budgeting model will be developed. Without loss of generality, the main requirement of an investment is human resources, facilities/ machines and a raw. The investment output is all of its tangible, intangible, and probable losses as well as its profits. DM has no clear sense about the amount of these requirements or output in a deterministic way. This vagueness can be reported in TrFNs. Let, describe the notations and fuzzy parameters as TrFNs. Suppose, we are facing with *n* projects with the following indices and parameters.

j	Number of projects, $j = 1, 2, \ldots, n$.
i	Type of human resources, $i = 1, 2, \ldots, m$.
k	Machine kind, $k = 1, 2, \ldots, s$.
o	Type of raw material, $o = 1, 2, \ldots, z$.
$\tilde{H}_i$	Available human resource of type *i*.
$\tilde{h}_{ij}$	Requirement of human resource I in project *j*.
$\tilde{M}_k$	Available machine- hour of type *k*.
$\tilde{m}_{ij}$	Requirement of machine- hour of type kind project *j*.
$\tilde{R}_o$	Maximum available raw material of type *o*.
$\tilde{r}_{oj}$	Requirement of raw material o in project *j*.
$\tilde{B}_j$	Maximum available budget for project *j*.
$\tilde{C}_i$	Per hour cost of human resource *i*.
$\tilde{C}_k$	Per hour cost of machine type *k*.
$\tilde{C}_o$	Unit cost material.
$\tilde{p}_j$	Total net profit of project *j*.

The decision variable of the model is considered as below:

$$x_j = \begin{cases} 1 & if\ project\ j\ is\ selected\ for\ investment \\ 0 & otherwise \end{cases}$$

By these definitions the proposed fuzzy 0-1 programming is as Model (3).

Max $\Phi =$

$$\sum_{j=1}^{n} x_j \left[\tilde{p}_j - \left(\sum_{i=1}^{m} \tilde{h}_{ij}.\tilde{C}_i + \sum_{k=1}^{s} \tilde{m}_{kj}.\tilde{C}_k + \sum_{o=1}^{z} \tilde{r}_{oj}.\tilde{C}_o \right) \right]$$

Subject to:

$$\sum_{j=1}^{n} \tilde{h}_{ij} x_j \le \tilde{H}_i, \qquad i = 1,2,...,m \qquad (3)$$

$$\sum_{j=1}^{n} \tilde{m}_{kj} x_j \le \tilde{M}_k, \qquad k = 1,2,...,s$$

$$\sum_{j=1}^{n} \tilde{r}_{oj} x_j \le \tilde{R}_o, \qquad o = 1,2,...,z$$

$$(\sum_{i=1}^{m} \tilde{h}_{ij}.\tilde{C}_i + \sum_{k=1}^{s} \tilde{m}_{kj}.\tilde{C}_k + \sum_{o=1}^{z} \tilde{r}_{oj}.\tilde{C}_o) x_j \le \tilde{B}_j, \quad j = 1,2,...,n,$$

$$(\sum_{i=1}^{m} \tilde{h}_{ij}.\tilde{C}_i + \sum_{k=1}^{s} \tilde{m}_{kj}.\tilde{C}_k + \sum_{o=1}^{z} \tilde{r}_{oj}.\tilde{C}_o) x_j < \tilde{P}_j, \quad j = 1,2,...,n,$$

$$\sum_{j=1}^{n} x_j \ge 1$$

$$x_j = 0,1 \quad j = 1,2,...,n$$

Where, all fuzzy parameters are TFNs in left and right spread formats.

The objective function is a multi-objective function which tries to maximize the net profit of the selected projects and minimize their costs, simultaneously. These objectives are assumed to have the same weights and priorities so they have been combined with a simple additive weighting method. The first set of constraints, which should be held for all projects and all human resources of the projects, insures that human resources availability is met during project selection procedure. The second and third sets of constraints have the same description of first set of constraints but they are applied for machine-hour and raw materials, respectively. The fourth set of constraints holds the budget availability for each project in the project selection procedure. The fifth set of constraints checks if total cost of a selected project is less than its profit. The sixth set of constraints insures that at least one project is selected and finally the zero-one orientation of decision variable of the model is reserved in the seventh set of constraints.

Considering the α-cut concept for fuzzy parameters of Model (3), an interval 0-1 programming model is represented as Model (4).

Max $\Phi =$

$$\sum_{j=1}^{n} x_j ([p_j^L - p_j^U]_\alpha - (\sum_{i=1}^{m} [C_i^L - C_i^U]_\alpha.[h_{ij}^L - h_{ij}^U]_\alpha$$

$$+ \sum_{k=1}^{s} [C_k^L - C_k^U]_\alpha.[m_{kj}^L - m_{kj}^U]_\alpha$$

$$+ \sum_{o=1}^{z} [C_o^L - C_o^U]_\alpha.[r_{oj}^L - r_{oj}^U]_\alpha)), 0 \le \alpha \le 1.$$

Subject to: (4)

$$\sum_{j=1}^{n} [h_{ij}^L - h_{ij}^U]_\alpha x_j \le [H_i^L - H_i^U]_\alpha,$$
$$i = 1,2,...,m, \quad 0 \le \alpha \le 1$$

$$\sum_{j=1}^{n} [m_{kj}^L - m_{kj}^U]_\alpha x_j \le [M_k^L - M_k^U]_\alpha,$$
$$k = 1,2,...,s, \quad 0 \le \alpha \le 1$$

$$\sum_{j=1}^{n} [r_{oj}^L - r_{oj}^U]_\alpha x_j \le [R_o^L - R_o^U]_\alpha,$$
$$o = 1,2,...,z, \quad 0 \le \alpha \le 1$$

$$(\sum_{1}^{m} [C_i^L - C_i^U]_\alpha.[h_{ij}^L - h_{ij}^U]_\alpha$$

$$+ \sum_{k=1}^{s} [C_k^L - C_k^U]_\alpha.[m_{kj}^L - m_{kj}^U]_\alpha$$

$$+ \sum_{o=1}^{z} [C_o^L - C_o^U]_\alpha.[r_{oj}^L - r_{oj}^U]_\alpha x$$

$$j \le [B_j^L - B_j^U]_\alpha, j = 1,2,...,n, 0 \le \alpha \le 1$$

$$(\sum_{i=1}^{m}[C_i^L - C_i^U]_\alpha.[h_{ij}^L - h_{ij}^U]_\alpha$$

$$+\sum_{k=1}^{s}[C_k^L - C_k^U]_\alpha.[m_{kj}^L - m_{kj}^U]_\alpha$$

$$+\sum_{o=1}^{z}[C_o^L - C_o^U]_\alpha.[r_{oj}^L - r_{oj}^U]_\alpha x_j$$

$$<[P_j^L - P_j^U]_\alpha, \quad j=1,2,...,n, 0\le\alpha\le 1$$

$$\sum_{j=1}^{n} x_j \ge 1$$

$$x_j = 0,1 \quad j=1,2,...,n$$

Following the interval programming optimistic Model (5) and pessimistic Model (6) will be emerged.

The proposed models should be solved for a predefined α-cut level in order to complete a full analysis. In the next section a full analysis will be represented for an illustrative instance.

$$Max\ \Phi^U = \sum_{j=1}^{n} x_j \left[[p_j^U]_\alpha - \left[\sum_{i=1}^{m}[C_i^L]_\alpha.[h_{ij}^L]_\alpha \right.\right.$$

$$\left.\left. + \sum_{k=1}^{s}[C_k^L]_\alpha.[m_{kj}^L]_\alpha + \sum_{o=1}^{z}[C_o^L]_\alpha.[r_{oj}^L]_\alpha \right]\right],$$

$$0\le\alpha\le 1$$

Subject to: (5)

$$\sum_{j=1}^{n}[h_{ij}^L]_\alpha x_j \le [H_i^U]_\alpha, \ i=1,2,...,m, \quad 0\le\alpha\le 1$$

$$\sum_{j=1}^{n}[m_{kj}^L]_\alpha x_j \le [M_k^U]_\alpha, \ k=1,2,...,s, \quad 0\le\alpha\le 1$$

$$\sum_{j=1}^{n}[r_{oj}^L]_\alpha x_j \le [R_o^U]_\alpha, \ o=1,2,...,z, \quad 0\le\alpha\le 1$$

$$(\sum_{i=1}^{m}[C_i^L]_\alpha.[r_{oj}^L]_\alpha + \sum_{k=1}^{s}[C_k^L]_\alpha.[m_{kj}^L]_\alpha$$

$$+\sum_{o=1}^{z}[C_o^L]_\alpha.[r_{oj}^L]_\alpha)x_j \le [B_j^U]_\alpha,$$

$$j=1,2,...,n, \quad 0\le\alpha\le 1$$

$$(\sum_{i=1}^{m}[C_i^L]_\alpha.[r_{oj}^L]_\alpha + \sum_{k=1}^{s}[C_k^L]_\alpha.[m_{kj}^L]_\alpha$$

$$+\sum_{o=1}^{z}[C_o^L]_\alpha.[r_{oj}^L]_\alpha)x_j < [P_j^U]_\alpha,$$

$$j=1,2,...,n, \quad 0\le\alpha\le 1$$

$$\sum_{j=1}^{n} x_j \ge 1$$

$$x_j = 0,1 \quad j=1,2,...,n$$

$$\text{Max } \Phi^L = \sum_{j=1}^{n} x_j \left[[p_j^L]_\alpha - \left[\sum_{i=1}^{m}[C_i^U]_\alpha.[h_{ij}^U]_\alpha \right.\right.$$

$$\left.\left. + \sum_{k=1}^{s}[C_k^U]_\alpha.[m_{kj}^U]_\alpha + \sum_{o=1}^{z}[C_o^U]_\alpha.[r_{oj}^U]_\alpha \right]\right],$$

$$0\le\alpha\le 1$$

Subject to: (6)

$$\sum_{j=1}^{n}[h_{ij}^U]_\alpha x_j \le [H_i^L]_\alpha, \ i=1,2,...,m, \quad 0\le\alpha\le 1$$

$$\sum_{j=1}^{n}[m_{kj}^U]_\alpha x_j \le [M_k^L]_\alpha, \ k=1,2,...,s, \quad 0\le\alpha\le 1$$

$$(\sum_{i=1}^{m}[C_i^U]_\alpha.[h_{ij}^U]_\alpha + \sum_{k=1}^{s}[C_k^U]_\alpha.[m_{kj}^U]_\alpha$$

$$+\sum_{o=1}^{z}[C_o^U]_\alpha.[r_{oj}^U]_\alpha)x_j \le [B_j^L]_\alpha,$$

$$j=1,2,...,n, \quad 0\le\alpha\le 1$$

$$(\sum_{i=1}^{m}[C_i^U]_\alpha.[h_{ij}^U]_\alpha + \sum_{k=1}^{s}[C_k^U]_\alpha.[m_{kj}^U]_\alpha$$

$$+\sum_{o=1}^{z}[C_o^U]_\alpha.[r_{oj}^U]_\alpha)x_j < [P_j^L]_\alpha,$$

$$j=1,2,...,n, \quad 0\le\alpha\le 1$$

$$\sum_{j=1}^{n} x_j \ge 1$$

$$x_j = 0,1 \quad j=1,2,...,n$$

4. Results

In this section, the proposed model is tested. A full analysis is performed with both optimistic and pessimistic models.

Table 1: Available resources.

Human Resource	H.R Type1	(5, 7, 11, 15)
	H.R Type2	(32, 40, 44, 50)
	H.R Type3	(41, 43, 47, 48)
	H.R Type4	(8, 16, 26, 32)
	H.R Type5	(10, 18, 23, 32)
Machines	Machine Type1	(35, 44, 52, 59)
	Machine Type2	(47, 53, 55, 63)
	Machine Type3	(32, 35, 36, 42)
	Machine Type4	(9, 17, 23, 24)
	Machine Type5	(37, 45, 48, 48)
Raw Materials	Raw Material 1	(43, 43, 50, 57)
	Raw Material 2	(3, 5, 12, 15)
	Raw Material 3	(6, 9, 13, 20)
	Raw Material 4	(25, 27, 28, 33)
	Raw Material 5	(18, 21, 26, 35)

Table 2: Unit cost for resources.

Human Resource	H.R Type1	(5, 7, 11, 15)
	H.R Type2	(32, 40, 44, 50)
	H.R Type3	(41, 43, 47, 48)
	H.R Type4	(8, 16, 26, 32)
	H.R Type5	(10, 18, 23, 32)
Machines	Machine Type1	(35, 44, 52, 59)
	Machine Type2	(47, 53, 55, 63)
	Machine Type3	(32, 35, 36, 42)
	Machine Type4	(9, 17, 23, 24)
	Machine Type5	(37, 45, 48, 48)
Raw Materials	Raw Material 1	(43, 43, 50, 57)
	Raw Material 2	(3, 5, 12, 15)
	Raw Material 3	(6, 9, 13, 20)
	Raw Material 4	(25, 27, 28, 33)
	Raw Material 5	(18, 21, 26, 35)

Table 3: Available budget & net profit.

Project No.	Available Budget	Net Profit
Project 1	(39304, 39307, 39363, 39372)	(7238, 7260, 7265, 7284)
Project 2	(14140, 14151, 14152, 14192)	(7065, 7110, 7143, 7188)
Project 3	(5789, 5857, 5894, 5913)	(3559, 3602, 3620, 3627)
Project 4	(47219, 47237, 47239, 47251)	(7977, 8018, 8033, 8045)
Project 5	(26336, 26340, 26418, 26419)	(8558, 8567, 8580, 8607)
Project 6	(40169, 40180, 40186, 40186)	(6770, 6774, 6836, 6892)
Project 7	(22964, 22987, 23002, 23038)	(1607, 1669, 1687, 1752)
Project 8	(24694, 24728, 24735, 24780)	(8209, 8262, 8274, 8284)
Project 9	(2239, 2250, 2312, 2331)	(4275, 4309, 4312, 4313)
Project 10	(22029, 22068, 22069, 22092)	(4573, 4575, 4581, 4598)

Table 4: Human resource requirements.

Project No.	H.R. Type 1	H.R. Type 2	H.R. Type 3	H.R. Type 4	H.R. Type 5
Project 1	(1, 5, 7, 9)	(3, 5, 5, 7)	(5, 13, 20, 26)	(7, 8, 11, 12)	(6, 14, 15, 17)
Project 2	(3, 7, 8, 8)	(3, 11, 16, 17)	(4, 6, 11, 12)	(0, 3, 10, 12)	(1, 2, 5, 6)
Project 3	(2, 3, 9, 9)	(2, 2, 5, 12)	(9, 12, 13, 13)	(5, 12, 16, 22)	(1, 4, 10, 10)
Project 4	(8, 14, 15, 19)	(9, 11, 17, 17)	(8, 8, 9, 11)	(8, 12, 12, 13)	(1, 3, 4, 5)
Project 5	(1, 2, 2, 4)	(2, 2, 3, 6)	(9, 17, 23, 26)	(3, 7, 9, 10)	(3, 6, 10, 11)
Project 6	(8, 8, 10, 18)	(4, 6, 9, 14)	(3, 9, 11, 11)	(1, 3, 8, 14)	(0, 1, 8, 10)
Project 7	(8, 9, 9, 10)	(9, 9, 14, 14)	(3, 3, 11, 13)	(3, 7, 8, 12)	(9, 10, 11, 12)
Project 8	(8, 12, 12, 13)	(7, 9, 13, 17)	(3, 4, 4, 8)	(0, 2, 8, 13)	(5, 11, 11, 15)
Project 9	(3, 6, 8, 15)	(4, 9, 9, 14)	(2, 3, 3, 3)	(10, 17, 17, 20)	(0, 5, 9, 14)
Project 10	(4, 5, 5, 9)	(8, 8, 11, 13)	(8, 11, 11, 12)	(3, 10, 10, 13)	(10, 19, 24, 26)

Table 5: Machine requirements.

Project No.	Machine Type 1	Machine Type 2	Machine Type 3	Machine Type 4	Machine Type 5
Project 1	(0, 1, 1, 1)	(6, 6, 6, 12)	(9, 10, 11, 11)	(3, 3, 4, 4)	(7, 10, 18, 19)
Project 2	(3, 4, 6, 8)	(1, 1, 6, 9)	(2, 11, 18, 18)	(7, 8, 12, 13)	(1, 1, 1, 4)
Project 3	(6, 8, 14, 18)	(2, 4, 9, 9)	(6, 8, 9, 10)	(10, 13, 15, 16)	(3, 5, 8, 13)
Project 4	(7, 10, 11, 11)	(4, 5, 9, 16)	(8, 16, 18, 22)	(1, 1, 3, 3)	(1, 3, 4, 6)
Project 5	(7, 9, 9, 12)	(4, 5, 5, 6)	(3, 7, 9, 10)	(9, 15, 15, 15)	(9, 10, 12, 13)
Project 6	(5, 9, 9, 11)	(10, 13, 14, 17)	(9, 12, 12, 13)	(8, 14, 17, 17)	(5, 11, 16, 20)
Project 7	(2, 9, 9, 16)	(2, 4, 9, 15)	(8, 9, 11, 16)	(4, 6, 7, 7)	(7, 7, 10, 14)
Project 8	(9, 9, 12, 17)	(9, 16, 20, 20)	(5, 7, 9, 10)	(5, 10, 14, 16)	(9, 12, 12, 20)
Project 9	(1, 6, 7, 12)	(6, 9, 10, 15)	(7, 8, 8, 12)	(0, 2, 3, 5)	(7, 7, 8, 9)
Project 10	(7, 9, 9, 13)	(1, 2, 9, 11)	(2, 9, 10, 14)	(5, 7, 15, 17)	(7, 12, 19, 20)

Table 6: Raw material requirements.

Project No.	R. M. Type 1	R. M. Type 2	R. M. Type 3	R. M. Type 4	R. M. Type 5
Project 1	(5, 6, 10, 10)	(8, 8, 8, 9)	(7, 8, 10, 11)	(5, 7, 7, 8)	(5, 9, 15, 21)
Project 2	(2, 5, 5, 7)	(1, 1, 9, 10)	(8, 10, 13, 15)	(2, 2, 3, 11)	(6, 8, 13, 19)
Project 3	(6, 7, 8, 14)	(3, 9, 13, 17)	(9, 13, 14, 14)	(1, 2, 2, 2)	(4, 6, 8, 10)
Project 4	(8, 9, 15, 22)	(8, 13, 15, 15)	(2, 3, 3, 5)	(0, 8, 10, 12)	(6, 7, 8, 10)
Project 5	(10, 12, 13, 13)	(7, 15, 20, 21)	(7, 9, 12, 16)	(7, 11, 16, 16)	(0, 3, 3, 7)
Project 6	(4, 5, 8, 9)	(2, 6, 14, 15)	(3, 7, 8, 11)	(2, 2, 2, 7)	(2, 2, 9, 9)
Project 7	(2, 5, 5, 7)	(5, 10, 10, 13)	(9, 10, 15, 16)	(6, 13, 15, 21)	(4, 9, 9, 10)
Project 8	(3, 4, 4, 10)	(7, 9, 10, 11)	(4, 7, 9, 10)	(1, 6, 10, 11)	(7, 9, 9, 15)
Project 9	(10, 12, 15, 20)	(4, 7, 14, 17)	(6, 7, 13, 13)	(5, 8, 12, 21)	(4, 8, 8, 9)
Project 10	(6, 7, 8, 11)	(10, 12, 21, 22)	(1, 4, 4, 9)	(9, 10, 11, 11)	(10, 11, 12, 16)

4.1. Test problem

Consider 10, 5, 5, 5 as available projects, human resource kinds, machine kinds and raw material types, respectively. The data of the numerical example are presented in Tables 1 to 6.

4.2. Experimental results

The described problem was solved optimally by LINGO solver through a Branch & Bound algorithm. The optimistic and the pessimistic model were solved for different α-cut levels. The obtained results are summarized in following tables. The results show that some projects lie in optimum portfolio for all α-cut levels in boths proposed models. These projects have high priority for investement in these ambiguous conditions. Some projects don't lie in optimum portfolio at any condition. These projects are not proposed for investment at all. All remained project are ranked subject to decreasing order of their total selection frequency in both tables 8 and 10, respectively. These projects are selected for invesment due to their calculated ranks.

5. Conclusion

In this paper the researchers have developed a 0-1 programming model for capital budgeting in which the process of project selection has been accommodated in a full fuzzy environement. We developed a fuzzy 0-1 programming in a situation which an organization is faced several investment opportunities. The developed model consisted of two major kinds of constraints. The first one guaranteed the requirement resources for a candidate investment would not exceed the quantity of total available resources while the second one held on the spended cost for each investment under total available amount of considered budget for that project. The objective funtion of the model was a multi one with two major parts (profit and cost) which were combined by simple additive weighting method fairly. The researchers used TrFNs to represent the vagueness. Using α-cut level concepts, the researchers developed 2 different models, one for optimistic and the other for pessimistic condition. The proposed models were coded in LINGO and a numerical example was supplied. The obtained results show that the proposed procedure is efficient and viable.

The procedure helps decision makersto selecta set of investment plan among several ones in full ambiguous conditions. By selecting different α-cut levels, the decision maker may gain a suitable vision about the outcome of his/her chosen investment.

Table7: Optimisstic programming.

Run	α-cut	O.F.V.	State
1	0	42923.00	Global Optimum
2	0.1	41657.67	Global Optimum
3	0.2	40449.68	Global Optimum
4	0.3	39204.03	Global Optimum
5	0.4	37920.72	Global Optimum
6	0.5	34744.50	Global Optimum
7	0.6	33536.60	Global Optimum
8	0.7	32296.10	Global Optimum
9	0.8	31023.00	Global Optimum
10	0.9	29717.30	Global Optimum
11	1	28379.00	Global Optimum

Table 8: Runs of optimistic programming.

		Project No.									
		1	2	3	4	5	6	7	8	9	10
runs	1	1	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	0	1	1	1
	3	1	1	1	1	1	1	0	1	1	1
	4	1	1	1	1	1	1	0	1	1	1
	5	1	1	1	1	1	1	0	1	1	1
	6	1	1	1	1	1	1	0	1	0	1
	7	1	1	1	1	1	1	0	1	0	1
	8	1	1	1	1	1	1	0	1	0	1
	9	1	1	1	1	1	1	0	1	0	1
	10	1	1	1	1	1	1	0	1	0	1
	11	1	1	1	1	1	1	0	1	0	1
	Total	11	11	11	11	11	11	1	11	5	11

Table 9: Pessimistic programming.

Run	α-cut	O.F.V.	State
1	0	3280.000	Global Optimum
2	0.1	4475.890	Global Optimum
3	0.2	5652.560	Global Optimum
4	0.3	6881.970	Global Optimum
5	0.4	8246.880	Global Optimum
6	0.5	9588.250	Global Optimum
7	0.6	10906.08	Global Optimum
8	0.7	12200.37	Global Optimum
9	0.8	13471.12	Global Optimum
10	0.9	14718.33	Global Optimum
11	1	15942.00	Global Optimum

Table 10: Runs of optimistic programming.

		Project No.									
		1	2	3	4	5	6	7	8	9	10
runs	1	1	1	0	1	1	0	0	1	0	0
	2	1	1	0	1	1	0	0	1	0	0
	3	1	1	0	1	1	0	0	1	0	0
	4	1	1	0	1	1	1	0	1	0	0
	5	1	1	0	1	1	1	0	1	0	0
	6	1	1	0	1	1	1	0	1	0	0
	7	1	1	0	1	1	1	0	1	0	0
	8	1	1	0	1	1	1	0	1	0	0
	9	1	1	0	1	1	1	0	1	0	0
	10	1	1	0	1	1	1	0	1	0	0
	11	1	1	0	1	1	1	0	1	0	0
	Total	11	11	0	11	11	8	0	11	0	0

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