The effect of supply network configuration on occurring chaotic behavior in the retailer's inventory

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Abstract: Today's market place is increasingly dynamic and volatile. In this area supply network issues recently have attracted a lot of attention from industrial practitioners and academics worldwide. Chaos theory is the study of complex, nonlinear, dynamic systems. For chaotic systems, a tiny change in conditions may result in an enormous change in system output, whereas substantial changes in conditions may be absorbed without significant effect to the system's output. In this paper, the researchers have investigated the effect of supply network design on the chaotic behavior of supply network from retailer's inventory point of view. The multi-level supply network is characterized by extension of the widely known beer distribution model. System dynamic approach is implemented to study the supply network behavior. Furthermore, we use Lyapunov exponents to quantify the system chaos. Simulation outputs and its analysis indicate that the number of distributors and wholesalers have a significantly effect on making the system behavior chaotic.

Keywords: *Supply network configuration; Chaotic systems; System dynamic; Retailer's inventory*

1. Introduction

Supply chain management (SCM) seems to be a growing area of interest amongst researchers and practitioners from varied disciplines. Supply chain (SC) has evolved from the era when issues related to materials flow were introduced by Forrester (1961), which later on became part of SCM. The SC members perform different functions or activities like logistics, inventory management, ordering, forecasting and product design involved in management of flow of goods, information and money (Arshinder Kanda and Deshmukh, 2008). By extending the supply chain from one partner in each level to more than one, terminology of 'supply network' is introduced in the literature.

Recent trends in global sourcing, production and distribution have both increased supply network complexity and reinforced the notion that supply network strategies and practices are essential elements of business strategy (Arshinder Kanda and Deshmukh, 2008; De Treville *et al.*, 2004; Vonderembse *et al.*, 2006). In a supply network context, for a given product and speed required by the customer, the coordination of logistics flows becomes more and more complicated when vertical, horizontal, spatial and relational complexities increase (Romano, 2009).

Network design includes decisions on the number, locations, and size of manufacturing plants and warehouses, the assignment of retail outlets to warehouses, and so forth. Network design is a strategic decision that has a long-lasting effect on the firm (Simchi-Levi *et al.*, 2008). Supply chain management literature recognizes the importance of supply network and business process configuration as a driver of performance (Romano, 2009). Based on the basic supply chain model, Cooper *et al.* (1997) argue that the way supply networks perform depends on how business processes are managed.

Chaos theory is the study of complex, non-linear, dynamic systems. The field was pioneered by Lorenz (1963), who was studying the dynamics of turbulent flow in fluids. Although we all recognize the swirls and vortices that characterize turbulent flow, the complexities of turbulent flow have confounded mathematicians for years (Levy, 1994). One of the major achievements of chaos theory is its ability to demonstrate how a simple set of deterministic relationships can produce patterned yet unpredictable outcomes. Chaotic systems never return to the same exact state, yet the outcomes are bounded and create patterns that embody mathematical constants (Feigenbaum, 1983). In chaotic systems, small disturbances multiply over time because of the nonlinear relationships and dynamic, repetitive nature of

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chaotic systems. As a result, such systems are extremely sensitive to initial conditions, which makes forecasting very difficult. In this enviro-nment long-term planning is very difficult (Levy, 1994). In other words, for chaotic systems, a tiny change in conditions may result in an enormous change in system output, whereas substantial change in conditions may be absorbed without significant effect on the system's output (Wilding, 1998). It is the promise of finding a fundamental order and structure behind complex events that probably explains the great interest chaos theory has generated in so many fields (Levy, 1994). There has been considerable interest in applying chaos theory to finance, economics and manage-ment studies. Studies in the domains of inventory management and supply chain management, how-ever, are still limited (Hwarng and Xie, 2008).

In fact, changing the supply network confi-guration is substantial and has long term effects on its performance. According to the above literature, chaotic behavior of supply network may be controllable by designing a right configuration for it. An important challenge in any supply network design project is to evaluate the impact of the network on inventory (Simchi-Levi *et al.*, 2008).

In this paper, the researchers have investigated the effect of supply network design on the chaotic behavior of supply network from retailer's inventory point of view. The structure of this paper is as follows:

Section 2 describes chaotic dynamic systems and relevant concepts. Section 3 demonstrates the framework, formulation and functional mechanisms of the under studying model. Meanwhile in this sec-tion, more detailed aspects of the supply network con-figuration will be considered. Section 4 required experiments will be designed and conducted. Sec-tion 5 presents the analysis of results and descry-bes the findings. Section 6 discusses the conclu-sions and recommendations for future work.

2. Chaotic dynamic systems

Chaotic behaviors arise from nonlinear dynamic systems. Such systems exist in nature as well as in industries. One of the areas which attracts a lot of interest for detecting the existence of chaos is supply networks (Williams, 1997).

As a matter of fact, supply network dynamics are established based on negative and positive feedback loops. Positive feedback loops, e. g. the effect of receiving an inventory on inventory position, present systems which cause exponential deduction or augmentation, with a self-reinforcing mechanism. Negative loops, e. g. the effect of the delivering an inventory on inventory position, present processes by which systems gradually approach to a definite value. There is a difference between the ideal and current circumstances in such processes. If both of them are not equal together, the negative loop will be activated in order to lead the system to ideal circumstances. Thus, it can be said that the processes with negative loops have a self-correcting mechanism by which they lead systems to stability and durability. Furthermore, negative loops may cause to emerge unstable conditions as well. Such a condition happens when negative loop operation encounters delay (Baumol *et al.*, 1989; Brock *et al.*, 1992).

In other words, negative loop process which in fact plays the role of rectifier in a system, operates with delay and does not return to its place on time. Such a delay in the operation of negative loop causes the emergence of cycles with extremely high or low dimensions. These cycles according to the system structure may be appeared like dam-ped, explosive, stable and regulate or stable and haphazard shapes. The interesting point in the behavior of system whose operation with negative loop are faced is the delay. Whenever one or more processes with positive loop are added to the system, the instability of system become much more perilous and strange patterns are initiated. In fact, augmenter tension which exists in positive loops, makes the intensity of imbalance forces much stronger so that the situation of instability and imbalance in system gets more complicated. Systems which have such characteristics often produce chaotic processes (Baumol *et al.*, 1989; Dockner and Schittenkop, 2001).

Systems which have chaotic processes make oscillations with basically infinite periods. In other words, a chaotic system constructs cycles which are not repeated in the consideration period. The reason, by which such unrepeated cycles are created, is the existence of nonlinear boundaries in the system. They make the motion be pulled backward and forward in such a manner that does not conform to their previous routes. Such back-ward and forward going motions make a chaotic system extremely sensitive to its initial conditions. If one or more initial conditions slightly change, the new route in the system diverges from the previous route exponentially. This feature has a substantial role in conducting tests for detecting chaotic processes. One of these experiments which has been widely used by researchers and has been proved that is the most reliable one, is

lyapunov exponent (Hwarng and Xie, 2008; Williams, 1997).

Lyapunov exponent test is initiated based on the chaotic series feature which implies that the adjacent points in such series move away from each other gradually and get diverged. Lyapunov exponent measures such a divergence with an exponential function. Lyapunov exponent calculation is done through the rate of stretching or folding which occurs in the system motion. In fact, in this method, the average velocity by which the two primarily near point trajectories get diverged exponentially is calculated.

If the greatest lyapunov exponent is positive, the system behavior will be chaotic and vice versa. The calculation method is as follows (Williams, 1997):

If $x_{n+1} = f(x_n)$ we can show the distance between $x_0, x_0 + \varepsilon$ with ε and the distance between $f^n(x_0), f^n(x_0 + \varepsilon)$ with $\varepsilon e^{n\lambda(x_0)}$ which is an exponential function. In other words, $\varepsilon e^{n\lambda(x_0)}$ may be calculated by Equation 1.

$$\varepsilon e^{n\lambda(x_0)} = \left| f^n(x_0 + \varepsilon) - f^n(x_0) \right| \quad (1)$$

While in fact, $\varepsilon e^{n\lambda(x_0)}$ shows the difference between adjacent points in each iteration. λ is the so called lyapunov exponent. The limit of the above relation is as follows:

$$\lambda(x_0) = \lim_{\varepsilon \to 0} \lim_{n \to \infty} \frac{1}{n} \log \left| \frac{f^n(x_0 + \varepsilon) - f_n(x_0)}{\varepsilon} \right| \quad (2)$$

Or,

$$\lambda(x_0) = \lim_{\varepsilon \to 0} \frac{1}{n} \log \left| \frac{df^n(x_0)}{dx_0} \right| \quad (3)$$

In this paper, the researchers have used Equation 3 to calculate lyapunov exponents for each experiment under each scenario.

3. Model

3.1. Generality of model

We have considered a supply network which is a set of partners whose behaviors are based upon the so called beer distribution supply chain. In this system, orders emit from customers to retailers and consequently from retailers to wholesalers

and so on. Our supply chain of interest has some differences with traditional beer distribution model especially in its configuration.

First of all, with respect to the objective of this paper, as it is shown in Figure 1 the proposed model is comprised of more than one partner on each network level. Moreover, between each two levels an agent has been embedded for receiving and dispatching orders as well as products. We called that agent Orders Receiving-Dispatching (ORD) agent or Products Receiving-Dispatching (PRD) agent, depending on its functionality.

Orders propagate from customers to the factories. Each retailer estimates its associated customers' demands and places an order with the first ORD agent. This agent then decides how many orders should be assigned to each wholesaler. Then each wholesaler according to the received orders and its previous situation releases an order for the second ORD agent. Sequentially, again similar process performed by this agent is determined assignment decisions. These ordering processes continue up to upstream level of the supply network.

Conversely, products flow from the factory to the customers. The factory ships the goods to the distributors, if it has sufficient inventory. Each distributor receives the goods and distributes them to the wholesalers, according to the second PRD agent decisions. Sequentially, each wholesaler gets its goods and ships them to the retailers, conforming to the third agent decisions. Finally, the retailers deliver goods to their customers.

3.2. Ordering mechanisms

The ordering heuristic used in this study is an anchoring and adjustment heuristic for stock management which applies a feedback mechanism. To facilitate the model description, the following notations are introduced:

$\hat{L}_t$	The expected demand at time t,
L_t	The actual demand at time t,
S_t	The actual stock level at time t,
S^*	The desired stock level,
AS_t	The adjustment for the stock level at time t,
SL_t	The actual supply line (orders placed but not yet received) at time t,
SL^*	The desired supply line,

ASL_t The adjustment for the supply line at time t,

θ A constant which determines how fast expectations are updated, $0 \le \theta \le 1$,

α_S The rate at which the discrepancy between actual and desired stock levels is eliminated, $0 \le \alpha_S \le 1$,

α_{SL} The rate at which the discrepancy between actual and desired supply line is eliminated, $0 \le \alpha_{SL} \le 1$,

O_t The order quantity at time t.

Assuming that the decision makers have adaptive expectations, the expected demand at time t can be defined as follows:

$$L_t = \theta L_{t-1} + (1-\theta)L_{t-1}, \quad 0 \le \theta \le 1 \tag{4}$$

To regulate the stock and supply line, a negative feedback mechanism is used. The adjustment is linear in the discrepancy between the desired stock S^* and the actual stock S_t and in the discrepancy between the desired supply line SL^* and the actual supply line SL_t. That is:

$$AS_t = \alpha_S(S^* - S_t) \tag{5}$$

$$ASL_t = \alpha_{SL}(SL^* - SL_t) \tag{6}$$

Usually, $\alpha_{SL} \le \alpha_S$ as it is logical to pay more attention to the inventory than the supply line. Therefore, the generic decision rule for each supply chain level for the order quantity at time *t* is defined as below:

$$O_t = \max(O, \hat{L}_t + AS_t + ASL_t) \tag{7}$$

We adopt the above ordering heuristic for the following reasons: (1) the rule explains the subjects' behavior well; (2) the negative feedback mechanism applied in this heuristic is widely adopted in actual ordering or replenishment decisions, e.g., the continuous review policy and the periodic review policy; and (3) with the changes of the two parameters, α_S and α_{SL}, the heuristic can represent many possible ordering decisions. A wide variety of ordering decisions can thus reflect many realistic situations and assist us in drawing more valid conclusions with regard to the effect of the supply chain factors on the system behavior in the supply chain (Brian Hwarng and Xie, 2008).

3.3. Agents' mechanisms

In this section we describe the working mechanisms for ORD and PRD agents in above model. To describe this function following notations are considered. Note that the amount of variables will be revised in each period t.

N_d Number of partners in downstream level,

N_u Number of partners in upstream level,

i Index of N_d,

j Index of N_u,

TO Total orders from downstream level,

RO_i Received order from i th partner of downstream level,

AO_j Assigned order to j th partner of upstream level,

λ_j Random number between 0 and 1,

TB Total backorders of upstream level,

BO_j Amount of backorders of j th partner of upstream level,

SL_i The actual supply line (order in way) for i th partner of downstream level,

RD_j Received delivery from j th partner of upstream level,

TD Total deliveries from partners of upstream level,

TS Total supply line of partners of downstream level,

AD_i Assigned delivery to ith partner of downstream level.

3.3.1. ORD agents

As stated above Orders Receiving-Dispatching agent act for aggregating received orders from downstream level of the supply network then decides how many orders should be assigned to each partner in upstream level of the supply network. The total of received orders from downstream level and total of backorders in upstream level could be calculated from Equations 8 and 9.

$$TO = \sum_{i=1}^{N_d} RO_i \tag{8}$$

$$TB = \sum_{j=1}^{N_u} BO_j \qquad (9)$$

$$TS = \sum_{i=1}^{N_d} SL_i \qquad (13)$$

In our model we assume there is information sharing between each pair of partners except between customers and retailers. So, assigning mechanisms in first ORD and other ORDs will be different. Assignments in first ORD, simply are performed by uniformly random dispatching. So ORD1 acts by Equation 10. Assignments for other ORDs are performed based on the amount of backorders of each partner in upstream level. Specifically ORD2 to ORD4 agents act by Equation 11.

Unlike ORDs, all of PRD agents work similarly. As described in model, we don't need to implement PRD agent between customers and retailers, because each retailer delivers products to its customer. So, all PRD agents assign deliveries according to Equation 14.

$$AD_i = \frac{SL_i}{\sum_{i=1}^{d}(SL_i)} \times TD \qquad (14)$$

$$AO_j = \frac{\lambda_j}{\sum_{j=1}^{N_u} \lambda_j} \times TO \qquad (10)$$

4. Design of experiments and simulation

4.1. Experimental design

$$AO_j = \frac{\left(\dfrac{TB}{BO_j}\right)}{\sum_{j=1}^{N_u}\left(\dfrac{TB}{BO_j}\right)} \times TO \qquad (11)$$

3.3.2. PRD agents

Similar to ORD agents, Products Receiving-Dispatching (PRD) agents act for aggregating received deliveries from upstream level of the supply network then decides how many products should be assigned to each partner in downstream level of the supply network. Total received orders from downstream level and total of backorders in upstream level could be calculated from Equations 12 & 13.

Four major supply network configuration factors, namely, number of factories, number of wholesalers, number of distributers and number of retailers, which have direct impacts on the supply network configuration and dynamics, are considered in the model. In order to study the impacts of considered factors under various ordering decisions, we perform simulation with two minor factors, namely, the adjustment parameter for inventory (α_S) and the adjustment parameter for supply line (α_{SL}). The supposed levels for each factor are illustrated in Table 1.

In order to approach the research target, the researchers perform scenario-based simulation. In their studying, scenarios are designed regarding that condition that number of upstream partners is less than that of downstream one (i.e. $N_f < N_d < N_w < Nr$). Table 2 shows scenario formation and coding used in this paper.

$$TD = \sum_{j=1}^{N_u} RD_j \qquad (12)$$

Table 1: Experimental design.

Factors	Levels
Number of factories(N_f)	[1 , 2]
Number of wholesalers(N_d)	[1, 3, 5]
Number of distributers(N_w)	[1, 5, 7]
Number of retailers(Nr)	[1, 10, 30, 50]
Adjustment parameter for inventory (α_S)	0.00 −1.00, an increment of 0.05
Adjustment parameter for supply line (α_{SL})	0.00 −1.00, an increment of 0.05

Table 2: Scenario formation and coding.

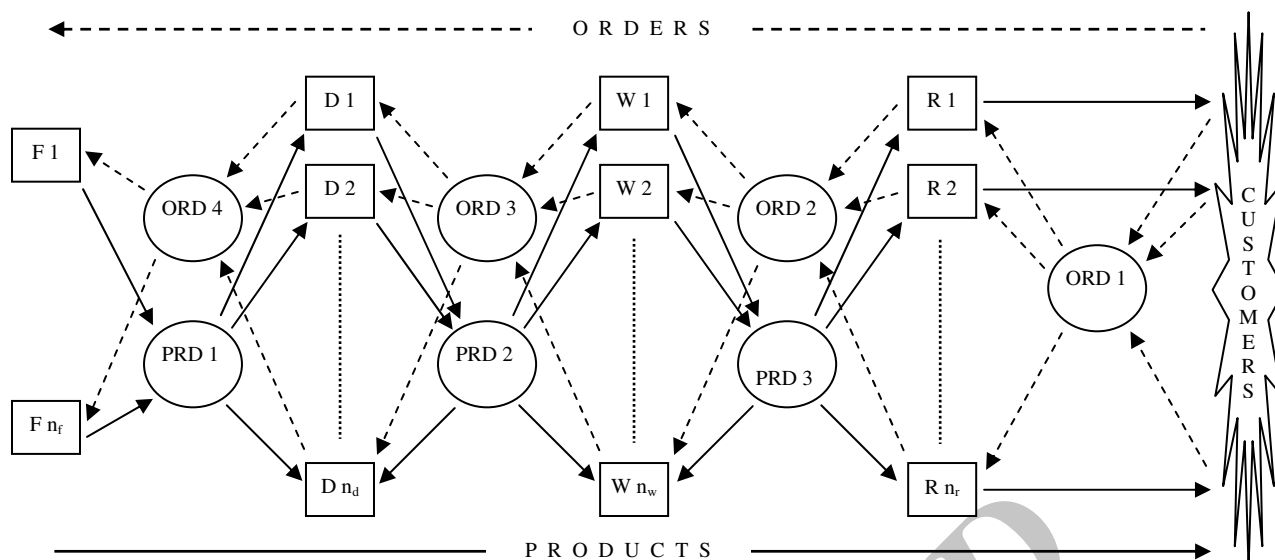| Scenarios	1	2	3	4	5	6	7	8	9	10	11	12	13	14
N_f	1	1	1	1	1	1	1	1	1	1	1	1	1	1
N_d	1	1	1	1	1	1	1	1	1	1	3	3	3	3
N_w	1	1	1	1	5	5	5	7	7	7	5	5	5	7
Nr	1	10	30	50	10	30	50	10	30	50	10	30	50	10
Scenarios	15	16	17	18	19	20	21	22	23	24	25	26	27	28
N_f	1	1	1	1	1	2	2	2	2	2	2	2	2	2
N_d	3	3	5	5	5	3	3	3	3	3	3	5	5	5
N_w	7	7	7	7	7	5	5	5	7	7	7	7	7	7
Nr	30	50	10	30	50	10	30	50	10	30	50	10	30	50

Figure 1: The configuration of the supply network.
(F= Factory, D= Distributer, W= Wholesaler, R= Retailer)

Table 3: Initial values of variables in each level.

Variable	Retailer	Wholesaler	Distributer	Factory
Total inventory	12	12	12	12
Order transmission delay	4	4	4	4
Shipment delay	-	4	4	4
Desired inventory level	20	40	50	60
Desired supply line	10	20	30	-

Table 4: Values of parameters.

Variable	Values
Customer order in each period (fixed)	12
Updating parameter for expectations (θ)	0.25
Time periods for simulation	2000
Difference between adjacent points of total inventory to calculate lyapunov exponent	0.001

4.2. Simulation design

A simulation model was developed to survey the system behavior in a supply network model which is characterized in section 3. The supply chain system is simulated under the 28 scenarios described in Section 4.1. The researchers have adopted the same initial conditions as those used by Hwarng and Xie (2008). The initial values of variables and parameters are presented in Tables 3 and 4, respectively. The model was built using well known system dynamics simulation software, Vensim DSS (2003).

In the simulation, θ is set to 0.25 for all levels. The two decision parameters, α_S and α_{SL}, are varied to study different system behaviors. For simplicity, the same total initial inventory, initial order transmission delay, initial shipment delay, θ, α_S and α_{SL} are used at different supply network levels. But for desired inventory level and the desired supply line, we suppose different goals at different supply network levels. It is considerable that total inventory in each level equally divided among the partners of that level. Finally, for each parameter set, 21 different values with an increment of 0.05 from 0.00 to 1.00 are used to simulate various ordering decisions. Since, $\alpha_{SL} < \alpha_S$, there are 210 parameter sets or ordering decisions in total for each scenario. The run length for each simulation scenario is 2000 time periods.

We concentrate on studying the system behavior exhibited in the total effective inventory at

retailers. Effective inventory is defined as the inventory level after fulfilling the backlog. In each scenario, the researchers have summed the effective inventory of each retailer and then calculate the largest LE from the total effective inventory. Totally, 210 LEs are calculated for each scenario. In the other word, 210 multiplied by 28 (i.e. 5880) LEs are calculated in whole of simulation.

5. Results and analyses

Results of model simulation indicate that there are chaotic behaviors in some cases. For example, Figure 2 shows the total retailer's inventory for scenario 16 assuming $\alpha_S = 0.85$ and $\alpha_{SL} = 0.15$. In these figures just three initial values (12.00, 12.05, 12.10) for total retailer's inventory have been illustrated, while in our simulation 21 values had been studied. In fact, Figure 2 shows a tiny change in initial values result in an enormous change in the total retailer's inventory at the end of simulation. This fact indicates to be a chaotic behavior.

To investigate the effect of the supply network configuration factors on the system behavior, we need to analyze the largest LEs calculated from the total effective inventories. Most famous technique for this purpose is analysis of variance (ANOVA). For using ANOVA techniques the distribution of residual errors must be normally distributed. Unfortunately, this most important assumption in our model could not be assumed. So, to perform the above study we should choose another suitable technique. By widely exploring of existent techniques in this area, we recognize that binary logistic regression technique is suitable to analyze our model.

Logistic regression considers the relationship between a response variable and one or more factors. Contrarily with least squares regression, logistic regression techniques are used with categorical response variables (McCullagh and Nelder, 1992). Logistic regression procedures could be used to assess the relationship between one or more factor variables and a categorical response variable. There are three types of logistic regression, binary, ordinal and nominal. Binary logistic regression is used when response variable consists of two categories. Both ordinal and nominal logistic regressions are used when response variable is comprised of more than two categories. Where natural ordering of the levels is important we use ordinal whenever nominal logistic regression is used. Binary logistic regression is suitable to perform logistic regression on a binary response variable. A binary variable only has two possible values, such as presence or

absence of a chaotic behavior. A model with one or more factors is fit using an iterative-reweighted least squares algorithm to obtain maximum likelyhood estimates of the parameters (McCullagh and Nelder, 1992). Binary logistic regression has also been used to classify observations into one of two categories, and it may give fewer classification errors than discriminated analysis for some cases (Fienberg, 1987; Press and Wilson, 1978). Summarily, we perform binary logistic regression to analyze the model outputs.

5.1. Analytical analysis

After running the simulation and obtaining all of the 5880 LEs, the researchers categorized them to presence or absence of a chaotic behavior. They perform this classification based on this fact that if the greatest lyapunov exponent is positive, the system behavior will be chaotic otherwise it will not be chaotic. Therefore, they defined two states for the system; chaotic (1) and non-chaotic (0) in order to use the BLR after preparing data.

The sparsity-of-effects principle states that a system is usually dominated by the main effects and low-order interactions. Thus it is most likely that main (i.e. single factor) effects and two-factor interactions are the most significant responses. In other words, higher order interactions such as three-factor interactions are very rare.

Based on this principle, three-factor and other higher-order interactions are negligible and thus can be combined as an estimate of error (Montgomery, 2001). So, the researchers have tested the main effects and two-factor interactions. Table 5 shows logistic regression outputs which include, estimated coefficients, standard error of the coefficients, z-values, p-values, odds ratio and a 95% confidence interval for the odds ratio.

From the output, we can see that the estimated coefficients for distributer (z = -12.76, p-value=0), wholesaler (z = -8.09, p-value = 0.000), interaction between factory and distributer (z = -2.98, p-value = 0.003) and interaction between distributer and wholesaler (z = 14.58, p-value = 0.000) have p-values less than 0.05, indicating that there is sufficient evidence that the coefficients are not zero using an α-level of 0.05. In other words, only the above factors and interactions have significant effects on making system behavior chaotic.

The odds ratio is a measure of effect size, describing the strength of association or non-independence between two binary data values. It is used as a descriptive statistic, and plays an important role in logistic regression. Unlike other measures of association for paired binary data

such as the relative risk, the odds ratio treats the two variables being compared symmetrically, and can be estimated using some types of non-random samples (Mosteller, 1968). An odds ratio of 1 indicates that the condition or event under study is equally likely to occur in both levels. An odds ratio greater or less than 1 indicates that the condition or event is more likely to occur differently among the two groups. Calculated odds ratios in Table 5 and their confidence intervals, again support the findings from p-value analysis.

Table 6 illustrates the observed and expected frequencies. This table allows us to see how well the model fits the data by comparing the observed and expected frequencies. In order to test the goodness of fit the researchers implemented two common tests, namely, Pearson and Deviance Tests. Table 7 illustrates the results of Pearson and Deviance Tests. The goodness-of-fit tests, with p-values ranging from 0.999 and 0.997, indicate that there is insufficient evidence to claim that the model does not fit the data adequately. If the p-value is less than our accepted α-level (i.e. 0.05), the test would reject the null hypothesis of an adequate fit. These findings increase the validity of the discussed conclusions from binary logistic regression. In Summary, findings from analytical analysis indicate that for more analysis we just need to focus on the effects of distributer, wholesaler, interaction factory against distributer and interaction distributer against wholesaler.
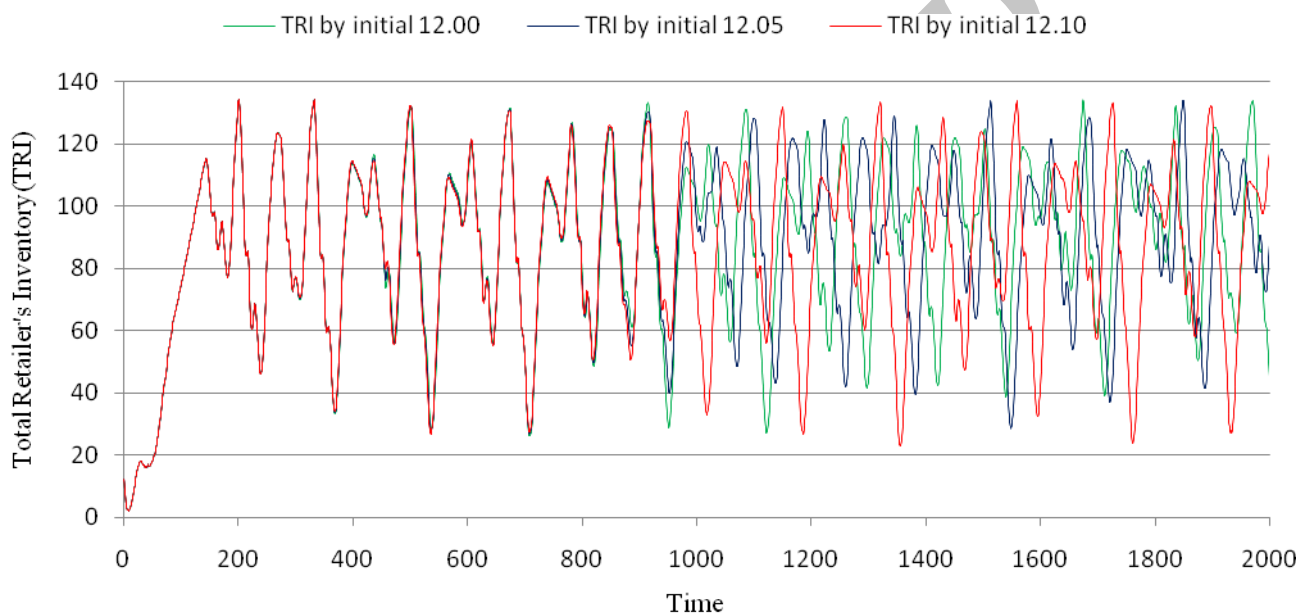

Figure 2: Total retailer's inventory level vs. Simulation time.

Table 5: Binary logistic regression results.

Factors	Coefficient	SE Coefficient	Z	P-Value	Odds Ratio	95% CI Lower	95% CI Upper
Factory	-0.499321	1.142900	-0.44	0.662	0.61	0.06	5.70
Distributer	-5.51589	0.432151	-12.76	0.000	0.00	0.00	0.01
Wholesaler	-1.28235	0.158449	-8.09	0.000	0.28	0.20	0.38
Retailer	0.0186530	0.009919	1.88	0.060	1.02	1.00	1.04
Factory*Distributer	-0.323751	0.108694	-2.98	0.003	0.72	0.58	0.90
Distributer*Wholesaler	0.882105	0.060493	14.58	0.000	2.42	2.15	2.72
Wholesaler*Retailer	0.0002954	0.001630	0.18	0.856	1.00	1.00	1.00
Factory*Wholesaler	0.300691	0.170819	1.76	0.078	1.35	0.97	1.89
Factory*Retailer	-0.0024805	0.007422	-0.33	0.738	1.00	0.98	1.01
Distributer*Retailer	-0.0000582	0.002542	-0.02	0.982	1.00	0.99	1.00

Table 6: Observed and expected frequencies.

Value	Type	Group 1	2	3	4	5	6	7	8	9	10	Total
1	Observed	0	0	0	0	3	12	64	269	421	501	1270
	Expected	0.0	0.1	0.7	2.9	10.0	30.7	84.4	201.5	391.1	548.5	
0	Observed	588	588	588	585	588	576	524	319	167	87	4610
	Expected	588.0	587.9	587.3	585.1	578.0	557.3	503.6	386.5	196.9	39.5	
Total		588	588	588	588	588	588	588	588	588	588	5880
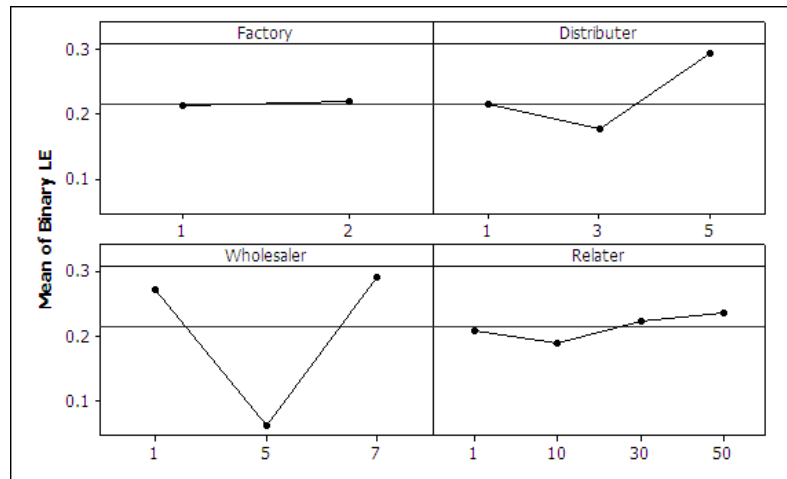
Figure 3. Main effects plot.

Table 7: Goodness-of-Fit Tests.

Test	Chi-Square	DF	P-Value
Pearson	3368.84	5867	0.999
Deviance	2594.19	5867	0.997

5.2. Intuitive effects

For studying intuitive effects of previously mentioned factors, the researchers implemented "main effects" and "interaction effects" plots.

Main Effects Plot shows data means when we have multiple factors. The points in the plot are the means of the response variable at the various levels of each factor, with a reference line drawn at the grand mean of the response data. In fact, we use the main effects plot for comparing magnitudes of main effects. Figure 3 displays "main effects" plot of factory, distributer, wholesaler and retailer.

As shown in Figure 3, by changing the level of the effective factors, distributer and wholesaler, we see explicit non-linear impact on a chaotic behavior. For both factors, at first and third level the researchers saw more chaotic behavior, but at the second level chaotic behavior decreased significantly. Consideration of such a fact demonstrates that for preventing from chaotic behavior, we need to determine the appropriate number of partner for each effective factor.

6. Conclusion

This research studies the chaotic behaviors exhibited at effective inventory of retailers in a supply network under the influence of changing its configuration. The multi-level supply network is characterized by extension of the widely known beer distribution model. System dynamic approach is implemented to study the supply network behavior. We use Lyapunov exponents which are calculated based on inventories to quantify system chaos.

According to the simulation outputs, binary logistic regression is performed. Findings from analytical analysis indicate that for more analysis we just need to focus on the effects of distributer, wholesaler, interaction factory against distributer and interaction distributer against wholesaler. These results are supported by "main effects" and "interaction effects" plots, intuitively.

As shown in this study, to prevent chaotic behavior, we need further research to determine the appropriate number of partner for each effective factor.

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