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ORE extraction and blending optimization model in poly-metallic open PIT mines by chance constrained one-sided goal programming

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Abstract: Determination a sequence of extracting ore is one of the most important problems in mine annual production scheduling. Production scheduling affects mining performance especially in a poly-metallic open pit mine with considering the imposed operational and physical constraints mandated by high levels of reliability in relation to the obtained actual results. One of the important operational constraints for optimization is the uniformity of metallic minerals grade after the blending process. This constraint directly affects the performance of the mineral processing plant. The sequence of extracting ore is usually determined by the order of pushbacks, which should be mined. Metallic minerals' grade in each pushback is stochastic in nature that comes from some statistical errors committed by the sampling. In such situations, decision making about the order of pushbacks for extraction as an exact defined process is not possible. Moreover, the decision-maker should maximize the total Net Present Value NPV as the main objective of mining operations by considering the high performance of mineral processing plant. To deal with such cases, this research proposes a model based on the chance-constrained one-sided goal-programming and the obtained results from this procedure confirms the model's reliability and correctness.

Keywords: *ORE blending; Poly-metallic open pit mine; Chance-constrained programming; Goal programming*

1. Introduction

The production-scheduling models directly affect the quality of ore entering the mineral processing plant mined from the pushbacks. Ore entering the mineral processing plant is named run-of-mine ore or feed. Maximizing the net present value is usually the main objective function of mine production scheduling. To maximize the net present value, a set of the operational and physical constraints should be considered (Lerchs and Grossmman, 1965; Whittle, 1997). The number of technical constraints parameters, i.e. the number of both operational and physical constraints in the poly-metallic mines, such as copper-molybdenum and gold mines, are too many and frequently stochastic in nature. In mining operations, the mineralized zone or orebody is generally divided into three-dimensional rectangular representative blocks. Each block is assigned by attributes of the deposits such as the grade of minerals or elements and the block mass. Mineral's grade and other attributes should be obtained and estimated by the sampling process from drill cores. Some attributes are applied for calculation of the net economic value of each block, i.e., value of the metal(s) contained in the ore minus the costs of mining, mineral processing, refinery, tax, etc. (Abrishamifar, 2003; Dimitrakopoulos and Ramazan, 2004; Ramazan, 2007). The positive net economic value refers to the "ore" blocks. The "waste" blocks have the negative economic value, which is the cost of their removing or mining costs. Determining and choosing those blocks within the orebody, i.e. pushbacks, especially in poly-metallic open pit mines must be mined in appropriate years to maximize the total NPV, is a decision-making problem. Integer programming is usually a suitable method for application in decision-making problems when the number of integer variables is limited. But in mining field especially for the proposed model application of the integer programming to decide which blocks should be mined in what sequence and in which year is not usually possible in practice because the number of the blocks within the orebody is too numerous.

The proposed model for this study is based on goal programming. Goal programming model allows taking into consideration several objectives in a given problem simultaneously in order to choose the most satisfactory solution in conditions at hand. One can find a solution with minimizing

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the deviations between the achievement levels of the objectives and the goals set for them by using of the goal-programming model. When the goals are surpassed, the deviations will be positive and in the case of under achievement of the goals, the deviations will be negative.

Goal-programming developed by Charnes *et al*. (1955), Charnes and Cooper (1961), and applied by Lee (1973), and Lee and Clayton (1972). Goal-programming models apply in diverse fields such as management of solid waste, accounting and financial aspect of stock management, marketing, quality control, transportation, site selection, and some other technical fields (Aouni *et al.*, 2001). Although the applications of goal-programming are very limited in mining field, but it has been applied and developed successfully in open pit mining production scheduling (Abrishamifar, 2005; Esfandiari *et al.*, 2004; Esfandiari *et al.*, 2003).

Figure 1 shows Sarcheshmeh-Copper Mine as the poly-metallic open pit mine that several ore and waste blocks have been mined according to a three-dimensional rectangular block model.

2. Chance-constrained goal programming

The goal programming model minimizes the undesired deviations through the following functions:

$$minimize:$$

$$Z=\sum_{t=1}^{T}(D_t^+ + D_t^-)+\sum_{u=1}^{U}D'^{+}_{u}+\sum_{d=1}^{D}D''^{-}_{d} \quad (1)$$

$$s.t. \quad F_t(x)-D_t^+ + D_t^- = G_t\ ; \quad t=1,...,T \quad (2)$$

$$H_u(x)-D'^{+}_{u}+D'^{-}_{u}=G'_u\ ; \quad u=1,...,U \quad (3)$$

$$Y_d(x)-D''^{+}_{d}+D''^{-}_{d}=G''_d\ ; \quad d=1,...,D \quad (4)$$

$$\forall x\in X\geq 0 \quad ; \qquad D_t^+, D_t^-, D'^{+}_{u}, D'^{-}_{u}, D''^{+}_{d}, D''^{-}_{d}\geq 0 \quad (5)$$

Where T is the number of two-sided goals, U is the number of one-sided goals for upward deviations, and D is the number of one-sided goals for downward deviations.

$D_t^+, D_t^-, D'^{+}_{u}, D'^{-}_{u}, D''^{+}_{d}, D''^{-}_{d}$ are the positive and negative deviations. G_t, G'_u, G''_d represent the levels of expected aspirations or goals associated with the objectives $F_t(x)$, $H_u(x)$, and $Y_d(x)$ respectively.

In mining fields, the vector of objective function coefficients and some of the constraints are represented in probabilistic forms. Therefore, stochastic goal programming, or chance-constrained goal programming, may be used as an acceptable and feasible method to determine the production scheduling in poly-metallic open pit mines. The first formulation of the stochastic goal-programming model was defined by Contini's work (Contini, 1968). The selected goals have been considered as uncertain variables with a normal distribution. The model maximizes the probability that the consequences of the decision-making will belong to a certain region encompassing the uncertain goals by generating a solution close to the uncertain goals.

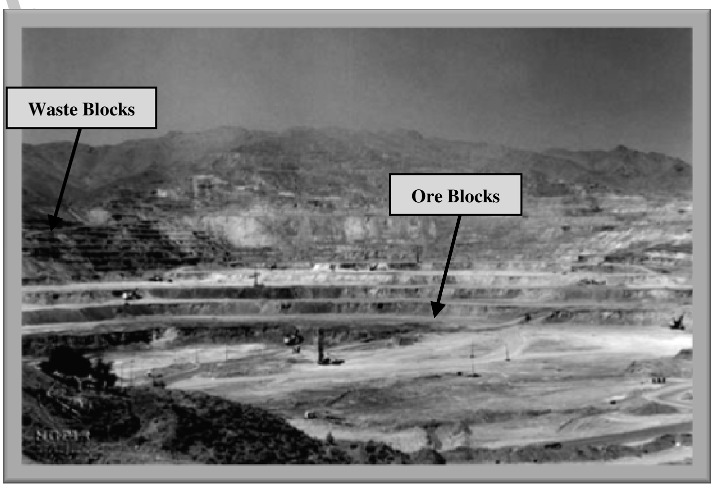

Figure 1: Some ore and waste blocks have been mined in previous years in Sarcheshmeh-Copper Mine.

Several other techniques have been proposed to formulate and solve the stochastic goal-programming model. The most widely used technique is the chance-constrained programming developed by Charnes and Cooper (1952; 1959; 1963). In chance-constrained programming, some parameters of the constraints are introduced as random variables with a minimum probability. The following equations show the outline and layout of chance-constrained programming (Glynn and Robinson, 1997; Taha, 2007):

$$maximize: Z = \sum_{j=1}^{n} C_j x_j \tag{6}$$

$$\begin{aligned} s.t. \quad & P\left[\left[\sum_{j=1}^{n} a_{ij} x_j \le b_i\right]\right] \ge (1-\alpha_i); \\ & i = 1,\dots,m \ ; \quad x_j \ge 0 \ ; \quad 0 < \alpha_i < 1 \end{aligned} \tag{7}$$

According to the conditions of a_{ij} and b_i for all i and j, three situations can occur: a) only a_{ij} is random variable; b) only b_i is random; and c) both a_{ij} and b_i are random variables. In mining studies the model may be defined by the use of the last situations, i.e. both a_{ij} and b_i are random. By assuming $S_i = \sum_{j=1}^{n}(a_{ij}x_j - b_i)$ and consideration of a_{ij} and b_i to be as normal distribution, then S_i will also be a normal distribution and the model can be rewritten as follows:

$$P[[S_i \le 0]] \ge (1-\alpha_i) \tag{8}$$

$$E(S_i) = \sum_{j=1}^{n}(E(a_{ij})x_j - E(b_i)) \tag{9}$$

$$\begin{aligned} VAR(S_i) &= \sum_{j=1}^{n} VAR(a_{ij})x_j^2 \\ &+ \sum_{j}\sum_{k \ne j} COV(a_{ij}, a_{ik})x_j x_k + VAR(b_i) \end{aligned} \tag{10}$$

Since in mining operations the parameters (a_{ij}, a_{ik}) are independent, the $VAR(S_i)$ is reduced as follows:

$$VAR(S_i) = \sum_{j=1}^{n} VAR(a_{ij})x_j^2 + VAR(b_i) \tag{11}$$

$$p[[S_i \le 0]] \ge (1-\alpha_i) \rightarrow$$

$$P\left[\left[\frac{S_i - E(S_i)}{\sqrt{VAR(S_i)}} \le \frac{0 - E(S_i)}{\sqrt{VAR(S_i)}}\right]\right] \ge (1-\alpha_i) \tag{12}$$

Where $\frac{S_i - E(S_i)}{\sqrt{VAR(S_i)}}$ is standard normal with the mean of zero and variance of one. This means that $p[[S_i \le 0]] = F\left(\frac{0 - E(S_i)}{\sqrt{VAR(S_i)}}\right)$. F represents the cumulative distribution function of a standard normal distribution. If $F_z^{-1}(\alpha_i)$ be the standard normal value, the statement $p[[S_i \le 0]] \ge (1-\alpha_i)$ is realized if and only if $\frac{0 - E(S_i)}{\sqrt{VAR(S_i)}} \ge F_z^{-1}(\alpha_i)$. This condition yields the following nonlinear deterministic constraints:

$$E(S_i) + F_z^{-1}(\alpha_i)\sqrt{VAR(S_i)} \le 0 \tag{13}$$

Equation (13) can be put in a separable programming scheme by using the following substitution:

$$\varphi_i(x) = \sqrt{\sum_{j=1}^{n} Var(a_{ij})x_j^2 + VAR(b_i)} \tag{14}$$

Therefore, the chance-constrained programming can be modified as follows:

$$\begin{aligned} & minimize: \\ & Z = \sum_{j=1}^{n} C_j x_j \end{aligned} \tag{15}$$

$s.t.$

$$\sum_{j=1}^{n}(E(a_{ij})x_j - E(b_i)) + F_z^{-1}(\alpha_i)\varphi_i(x) \le 0 \tag{16}$$

$$\sum_{j=1}^{n} VAR(a_{ij})x_j^2 + VAR(b_i) - \varphi_i^2(x) = 0 \tag{17}$$

$$\forall x_j \ge 0 \ ; \quad \forall \varphi_i(x) \ge 0 \tag{18}$$

If only the a_{ij} variable is random, all the above equations (8) through (16) must be modified by substituting $E(b_i) = 0$ and $VAR(b_i) = 0$, and all the zero number must be changed to b_i.

Combination the goal-programming model with the chance-constrained programming is named as the chance-constrained goal programming CCGP. The general form of the chance-constrained goal-programming model can be defined as follows (Ruszczynski and Shapiro, 2003; Zhigljavsky and Zilinskas, 2008; Boland *et al.*, 2008):

$$Min\,Z = \sum_{t=1}^{T}(D_t^{+} + D_t^{-}) + \sum_{u=1}^{U} D'^{+}_{u} + \sum_{d=1}^{D} D''^{-}_{d} \quad (19)$$

$$s.t.$$

$$p\left[\left[G_t - D_t^{-} \le F_t(x) \le G_t + D_t^{+}\right]\right] \ge (1-\alpha_t); \quad t = 1,\dots,T \quad (20)$$

$$p\left[\left[H_u(x) \le G'_u + D'^{+}_u\right]\right] \ge (1-\alpha_u); \quad u = 1,\dots,U \quad (21)$$

$$p\left[\left[Y_d(x) \ge G''_d + D''^{-}_d\right]\right] \ge (1-\alpha_d); \quad d = 1,\dots,D \quad (22)$$

$$P\left[\left[\sum_{j=1}^{m} a_{ij} x_j\right]\right] \ge (1-\beta_i) \quad (23)$$

$$\forall x \in X \ge 0 \quad ;$$
$$D_t^{+}, D_t^{-}, D'^{+}_u, D'^{-}_u, D''^{+}_d, D''^{-}_d \ge 0 \quad (24)$$
$$0 < \alpha_t, \alpha_u, \alpha_d, \beta_i < 1$$

In mining operations, application of the two-sided goal is not usually applied in practice and they are almost as one of the forms of the one-sided goals, i.e. upward or downward deviations.

The deterministic equivalent for one-sided goals with upward and downward deviations can be defined as follows respectively:

$$E(H_u(x)) - D'^{+}_u + F_z^{-1}(1-\alpha_u)\delta_u(x) \le G'_u \quad (25)$$

$$E(Y_d(x)) - D''^{-}_d + F_z^{-1}(\alpha_d)\theta_d(x) \ge G''_d \quad (26)$$

$$\delta_u(x) = \sqrt{VAR(H_u(x))} \quad (27)$$

$$\theta_d(x) = \sqrt{VAR(Y_d(x))} \quad (28)$$

Deterministic equivalent transformation of chance constrained goal-programming causes the appearance of some the constraints as a non-linearity property in the model. The model can be solved by various non-linear programming techniques such as the generalized reduced gradient technique. Here the proposed model is solved by "LINGO" software.

3. Description the proposed model

In the proposed model, the goals can be divided into two categories: a) economic goal, and b) mineral processing and refinery goals. The decision-maker needs to know the amount of maximized net present value under "ideal conditions" for mining operations. Ideal conditions for a poly-metallic open pit mine refers to the set of conditions, which directly affect mining operations in practice. However, the decision-maker and mine designer wish that the set of conditions never occurs because they cause the reduction of the net present value. The maximized net present value is somehow unknown for the ideal mining operations. The decision-maker is concerned and interested in approaching these amounts at real conditions. However, the maximized net present value in the ideal mining operations is much more than the amounts of actual net present value by considering all real conditions as technical constraints. The maximized net present value in the ideal mining operations can be obtained and interpreted by the application of a chance-constrained programming model and maximizing the minimum obtained income in the years of planning through the following relations:

$$Maximize:\ Z = MNPV^{\otimes} \quad (29)$$

$$\left\{Inc(t) - MNPV^{\otimes}\right\}_t \ge 0 \quad (30)$$

$$MNPV^{\otimes} \ge 0 \quad (31)$$

"Program I" as follows, refers and shows the real conditions by considering all technical constraints.

Program I: The probability of the net content value of each mined ore block and the income in the tth year of planning can define through the following relationship:

$$P\left[\left\{Inc(t) - \sum_p\sum_i\sum_j\sum_k\left(\left(\left((V_{i,j,k} - Cr_{i,j,k}) * R_k * g^{p}_{i,j,k} - Cp\right) * \psi_c{}^{p,t}_{i,j}\right) * Cm * \psi_m{}^{p,t}_{i,j}\right) - F\right\}_t \ge 0\right] \ge 1-\alpha_1 \quad ; t = 1,\dots,T \quad (32)$$

The mass balance probability between the k^{th} metallic mineral in feed in t^{th} year and $(t+1)^{th}$ year of planning must be presented as the follows:

$$P\left[\sum_{\|\vec{\lambda}\|}\left\{g_{\vec{\lambda}}^{k} * R_k * \psi_{c\ \vec{\lambda}}^{\ t} - g_{\vec{\lambda}}^{k} * R_k * \psi_{c\ \vec{\lambda}}^{\ t+1}\right\}_t \geq 0\right] \geq 1-\alpha_2 \ ;\ t=1,...,T-1 \tag{33}$$

The probability of maximum usage of refinery plants' capacity for k^{th} metallic mineral is shown as follows:

$$P\left[\sum_{\|\vec{\lambda}\|}\left\{g_{\vec{\lambda}}^{k} * R_k * \psi_{c\ \vec{\lambda}}^{\ t}\right\}_{k,t} - Rc_k \leq 0\right] \geq 1-\alpha_3 \quad t=1,...,T \ ;\ K=1,...,K \tag{34}$$

Total tonnage mined ore and waste blocks should be balanced with mining capacity through the following relationship:

$$\sum_{\|\vec{\lambda}\|}\left\{\psi_{m\ \vec{\lambda}}^{\ t} - Mc\right\}_t \leq 0 \ ;\ t=1,...,T \tag{35}$$

Total tonnage of feed should be balanced with mineral processing plant capacity by the following equation:

$$\sum_{\|\vec{\lambda}\|}\left\{\psi_{c\ \vec{\lambda}}^{\ t} - Pc\right\}_t \leq 0;\ t=1,...,T \tag{36}$$

Total tonnage of mined ore and waste blocks in each pushback should be balanced with its ore reserve as follows:

$$\sum_{\|\vec{\Omega}\|}\left\{\psi_{m\ \vec{\Omega}}^{\ p} - Or_p\right\}_p = 0 \ ;\ p=1,...,P \tag{37}$$

The mass balancing between tonnage of mined blocks with feed and waste tonnages can be shown as follows:

$$\left\{\psi_{m\ \vec{\lambda}}^{\ t} - \psi_{c\ \vec{\lambda}}^{\ t} - \psi_{W\ \vec{\lambda}}^{\ t}\right\}_{\|\vec{\lambda}\|,t} = 0 \ ; \quad t=1,...,T;\ \|\vec{\lambda}\| \in \langle p|i|j \rangle \tag{38}$$

Income sorting in each year and its relation to the amount in the next year of planning are shown as follows:

$$\{Inc(t) - Inc(t+1)\}_t \geq 0 \ ; t=1,...,T \tag{39}$$

$$Inc(t) \geq 0 \ ;\ \psi_m \geq 0;\ \psi_c \geq 0;\ \psi_W \geq 0 \tag{40}$$

Where $\|\vec{\lambda}\| \in \langle p|i|j \rangle$ is the vector space of p^{th} pushback, i^{th} grade range of principle mineral, and j^{th} grade range of the most valuable of minor mineral. $\|\vec{\Omega}\| \in \langle i|j|t \rangle$ is the vector space of i^{th} grade range of principle mineral, j^{th} grade range of the most valuable minor mineral, and t^{th} year of planning. k is kth metallic mineral, and T is the time required for the considered planning period. ψ_m is the total mined materials, i.e. ore & waste tonnage. ψ_c is the tonnage of ore should be sent to mineral processing plant, i.e. run-of-mine ore or feed. ψ_W is the tonnage of waste should be sent to waste dump. $1-\alpha_1, 1-\alpha_2$ and $1-\alpha_3$ are the minimum probability values. *Mc, Pc* and *Rc* are the capacities of mining, mineral processing plant, and refinery plants respectively. *R* is the recovery, *g* is the mineral grade, and *V* is the metallic mineral value. *Cm*, *Cp* and *Cr* are the costs of mining, mineral processing, and refinery respectively. *F* is the fixed cost. *Inc* is the income. *Or* is the ore reserve in each pushback. The expression $MNPV^{\otimes}$ is the maximized net present value under ideal conditions of mining operations. The net present value in each year at ideal conditions for mining operations is usually more than the maximized net present value at real conditions. Therefore, the economic goal is defined in the proposed model as achieving the net present value at real conditions with respect to the maximized net present value at ideal conditions. One can predict that the economic goal may not be achieved in other situations that are realistic. The optimized planning with minimum deviation between the net present values at the real conditions with the maximized net present value at the ideal conditions is obtained by the application of the proposed model in the suggested manner. The mineral processing and refinery goals may be defined as follows:

a) minimum harmful or gangue minerals in the feed;

b) maximum usage of the mining capacity;
c) maximum usage of the mineral processing plant capacity; and
d) maximum usage of refiner plant capacity.

"Program II" shows a part of proposed model based on stochastic goal-programming as a closed form.

Program II: Minimizing the deviations of one-sided goals can be presented as:

$$Minimize: Y = \sum_t \left\langle D(t)_1^- + D(t)_3^- + D(t)_4^- + D(t)_5^- \right\rangle + \sum_t D(t)_2^+ \quad (41)$$

The mathematical relations for the probability of economic goal, the probability of minimum gangue minerals in feed, the probability of goal of the mining capacity, the probability of goal of mineral processing plant's capacity, and the probability of goal of refinery plant capacity are presented respectively as follows:

$$P\left[\left\{Inc(t) \geq MNPV^{\otimes} - D(t)_1^-\right\}_t\right] \geq 1-\alpha_1^{Economic} \quad (42)$$

$$P\left[\left\{\sum_p \sum_i \sum_j \sum_k \left\langle 1-g_{i,j,k}^{p}\right\rangle * \psi_c{}_{i,j}^{p,t}\right\}_t \leq 0 + D(t)_2^+\right] \geq 1-\alpha_2^{GangueMinerals} \quad (43)$$

$$P\left[\sum_{\|\bar{\lambda}\|}\left\{\psi_c{}_{\bar{\lambda}}^{t} - \psi_m{}_{\bar{\lambda}}^{t}\right\}_t - 0.95Mc \geq 0 - D(t)_3^-\right] \geq 1-\alpha_3^{MiningCapacity} \quad (44)$$

$$P\left[\sum_{\|\bar{\lambda}\|}\left\{\psi_c{}_{\bar{\lambda}}^{t}\right\}_t - 0.95Pc \geq 0 - D(t)_4^-\right] \geq 1-\alpha_4^{ProcessingCapacity} \quad (45)$$

$$P\left[\sum_{\|\bar{\lambda}\|}\left\{g_{\bar{\lambda}}^{k} * R_k * \psi_c{}_{\bar{\lambda}}^{t}\right\}_{t,k} - 0.95Rc \geq 0 - D(t)_5^-\right] \geq 1-\alpha_5^{RefineryCapacity} \quad (46)$$

Indeed the main structure of the proposed model is established by combination of "Program I" and "Program II" through the following relations:

$$\text{Program I + Program II} \quad (47)$$

$$Y \geq 0; \quad D(t)_1^-, D(t)_2^+, D(t)_3^-, D(t)_4^-, D(t)_5^- \geq 0 \quad (48)$$

Where Y is the optimized summation of upward and downward deviations. The set of

$$\left\{D(t)_1^-, D(t)_2^+, D(t)_3^-, D(t)_4^-, D(t)_5^-\right\}$$

are the unwanted deviations and the set of

$$\left\{1-\alpha_1^{Economic}, 1-\alpha_2^{GangueMinerals}, 1-\alpha_3^{MiningCapacity}, 1-\alpha_4^{ProcessingCapacity}, 1-\alpha_5^{RefineryCapacity}\right\}$$

are the minimum probability values for economic, gangue minerals in the feed, mining capacity, processing capacity, and the refinery capacity goals for both of the above sets respectively.

4. Verifying the proposed model

Mathematical logic of the proposed model is verified by applying it to solve the proved hypothetical complex case study for precious metals in poly-metallic minerals. The mine is simulated as such to have very difficult conditions of the feed characters for planning very near characters of some pushbacks in Sarcheshmeh-Copper Mine. In this study, two pushbacks are considered for planning by the use of the proposed model. The first pushback has gold accompanying copper and molybdenum, with 112.77 million tons of ore reserve. The second pushback has a slight difference in its characters compared to the first. The second pushback does not have gold and has 24.85 million tons of ore reserve. By simulation procedure, the copper grade ranges for the first pushback is between (0-1.2) percentage, and for the second pushback is (0.31-0.7) percentage. In addition, the molybdenum grade ranges in both pushbacks are between (0-0.15) percentage and (0.026-0.1) percentage respectively. The gold grade range in the first pushback is (0-0.048) oz/ton. The statistical error entered the sampling is about $\pm 2.5\%$.

Table 1: Annual production scheduling and goals achievement.

No.	Description	Amount	Unit	Goal Achieved	Prediction of Goal Achievement
1	Optimum duration	12	Years	-	-
2	Net profit	106,418,302.1	U.S. dollars	No	Yes
3	Mined materials	11,468,330	Tons/Year	-	-
4	Feed	7,175,833	Tons/Year	-	-
5	Mining capacity usage	99.725	(%)	Yes	Yes
6	Mineral processing capacity usage	96.422	(%)	Yes	Yes
7	Refinery capacity usage	95.202	(%)	Yes	Yes
8	Average Cu grade in mined materials	0.4195	(%)	Yes	Yes
9	Average Cu grade in feed	0.6704	(%)	Yes	Yes
10	Average Mo grade in mined materials	0.0322	(%)	Yes	Yes
11	Average Mo grade in feed	0.0845	(%)	Yes	Yes
12	Average Au grade in mined materials	0.0166	Ounces/Ton	Yes	Yes
13	Average Au grade in feed	0.0265	Ounces/Ton	Yes	Yes

By applying the proposed model, the net present value at 20% discount rate is obtained $1,277,019,625. Optimum obtained operational life is 12 years with net profit per year $106,418,302.1. The obtained results show an improvement of about 22.5% by applying the proposed model. Table 1 shows the obtained results for annual production scheduling and goals achievement.

5. Conclusion

As one of the main advances in the recent years in the mining field, one can directly point to the development and introduction of new proposed design methods and planning techniques for mining. Many factors play significant roles in the design and planning of mining operations and this makes up the entire subject of mining design. Hence, it is hardly surprising that there are not enough mathematical tools available as algorithms, which are capable to find an optimal solution to this task or problem. The available algorithms; however, offer more or less functional alternatives, such as a wide set of parameters have been "implicitly or explicitly fixed" and non-changing under the supervision of a mine planner and depending on the quantity and quality of the input parameters that the model could handle and accept. To show the proposed model versatility and ability, the required tests must consider too much volume of different kinds of random data. The proposed evolved and expanded model uses the chance-constrained one-sided goal programming algorithm. The proposed model can be applied to provide an optimized planning scheme by considering all important factors affecting optimum planning. The most important property of the proposed model is the standardization of metal grade being sent to the mineral processing plant. In addition, there are not any significant inconsistencies for all of the effective factors in the mine planning and design either, so the proposed model provides increased flexibility for decision makers in evaluating different options and alternatives available at hand.

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