

# Source selection problem of competitive power plants under government intervention: a game theory approach

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**Abstract** Pollution and environmental protection in the present century are extremely significant global problems. Power plants as the largest pollution emitting industry have been the cause of a great deal of scientific researches. The fuel or source type used to generate electricity by the power plants plays an important role in the amount of pollution produced. Governments should take visible actions to promote green fuel. These actions are often called the governmental financial interventions that include legislations such as green subsidiaries and taxes. In this paper, by considering the government role in the competition of two power plants, we propose a game theoretical model that will help the government to determine the optimal taxes and subsidies. The numerical examples demonstrate how government could intervene in a competitive market of electricity to achieve the environmental objectives and how power plants maximize their utilities in each energy source. The results also reveal that the government's taxes and subsidiaries effectively influence the selected fuel types of power plants in the competitive market.

**Keywords** Game theory · Green electricity · Power plant · Bertrand game · Government intervention · Source selection

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## List of symbols

### Indexes

- $i, j$  The indexes of the competitive power plant  
 $k, l$  The indexes of the energy source used by the power plant

### Parameters

- $C_{ik}$  The unit production cost of the power plant  $i$  when using the energy source  $k$ ,  $C_{ik} > 0$   
 $F_{ik}$  The initial setup fee of the power plant  $i$  when using the energy source  $k$ ,  $F_{ik} > 0$   
 $w_{ik}$  The pollution amount that the power plant  $i$  produces when using the energy source  $k$ ,  $w_{ik} > 0$   
 $\tilde{\alpha}_{ikl}$  The stochastic market base for the power plant  $i$  when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$ .  $\tilde{\alpha}_{ikl}$  is defined as a random variable with the mean  $\bar{\alpha}_{ikl}$  and the variance  $\sigma_{ikl}^2$   
 $\lambda_{ikl}$  The constant absolute risk aversion (CARA) of the power plant  $i$  when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$ ,  $\lambda_{ikl} > 0$   
 $LB_G$  The lower bound of the government's profit  
 $R_i$  The reservation utility of power plant  $i$   
 $\tau$  The confidence level provided as an appropriate safety margin for the profit by the government  
 $\beta_{ikl}$  The demand sensitivity of power plant  $i$  to its own price when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$ ,  $\beta_{ikl} > 0$   
 $\gamma_{ikl}$  The demand sensitivity of power plant  $i$  to the rival's price, when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$ ,  $\gamma_{ikl} > 0$

### Variables

- $p_{ikl}$  The power plant  $i$ 's electricity price when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$

- $S_{ikl}$  The subsidy provided by the government for the power plant  $i$  when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$ ,  $S_{ikl} \geq 0$
- $T_{ikl}$  The tax imposed by the government on the power plant  $i$  when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$ ,  $T_{ikl} \geq 0$
- $\tilde{D}_{ikl}$  The demand of power plant  $i$ , when using the energy source  $k$  and the power plant  $j$  using the energy source  $l$

## Introduction

Electricity is the cornerstone of health care, sanitation services, and the educational, economic, scientific, and agricultural progresses that characterize a modern society. Indeed, there are few goods or services that do not directly depend on electricity in developing and developed countries (Nagurney et al. 2006). The electricity industry is growing and the global electricity consumption in 2025 is estimated to be around 23.1 trillion kWh. (See Casazza and Delea 2003; Singh 1999 and Zaccour 1998, for detailed information regarding electric power industry and its market).

Despite the major positive effects of electrical energy on economic growth, its heavy reliance on fossil fuel makes dipterous impacts on the environment. The scientific community is pointing to an increase in human-induced green house gases (GHGs) (such as CO<sub>2</sub>, CH<sub>4</sub>, NO<sub>x</sub>, and halo carbons) over the past century as the major cause of climatic change. Due to fossil fuel combustion, the electricity generation sector is a major producer of the GHGs (Palmer and Burtraw 2005). In general, Fossil fuels, including coal, are expected to be used in 36 % of electricity production in 2020. More than a third of the total carbon dioxide and nitrous oxide emission in the US is attributed for generating electricity. Similarly in China, the electric power sector currently accounts for more than one-third of its annual coal consumption, while such power plants generate over 75 % of the air pollution (cf. Pew Center). Given that global electricity demand is increasing by 2.4 % each year, and it is accompanied by rising global emissions of the GHGs, the current centralized generation system should be re-evaluated (Colson and Nehrir 2009). Therefore, several researches such as Poterba (1993) and Cline (1992) suggest that any policy in electricity industry should be made with regard to emissions of the generated GHGs and their effects on global warming and immense risks of unstable climate changes.

Natural Resources Canada (NRCAN) (2005) and European Environment Agency (EEA) (2007) have followed various GHGs reduction strategies. Some power plants have already started to build up and use sequestration

facilities to capture the generated carbon dioxide (NRCAN 2006). In addition, there has been a significant growth in the use of electricity generation technologies that utilize renewable energy sources, which are less-GHG-intensive (Tampier 2002; Canada 2003; EurObserv'ER 2007; EIA 2008). According to the Kyoto protocol in 1992 (Yoo and Kwak 2009), the governments should take actions to raise the percentage of green electricity supply. Green electricity is the energy that is generated from renewable sources such as solar power, wind power, small-scale hydroelectric power, tidal power, and biomass power. These sources mostly do not produce pollutants; hence, they are called environmentally friendly. Renewable energies are regarded as a key factor in tackling global climate changes and energy shortage crisis (Guler 2009). Renewable energies, such as wind, have globally experienced fast growth during the past decade (Li and Shi 2010; Saidur et al. 2010). Green energy sources involve employing the state-of-the art technologies; therefore, these energy sources are, generally, more costly than the fossil energy sources. To promote green electricity, the government should take visible actions to compensate for the extra production costs (Yoo and Kwak 2009). These actions are often called the governmental financial interventions. These interventions are usually in the form of legislations on green energy subsidiaries and tax tariffs. For example, the pollution taxes, in particular, carbon taxes are a powerful policy mechanism that can address market failures in energy industry (Wu et al. 2006). Painuly (2001) pointed to encouraging power generation from renewable sources such as solar and wind powers through the use of green credits issued by the government. Such credits are now being utilized in the European Union as well as in several states of the US (see RECS 1999; Schaeffer et al. 1999). Consistent with the leading role of the government, the organizations show growing interest in green energy production. On the other hand, raising the awareness of the electricity consumers leads to the genesis of the concept of green branding and green consumerism (Barari et al. 2012).

Specifically, this research uses the mathematical game theory model to answer the following questions:

1. Which financial instruments are effective in maximizing the impact of interventions of the government?
2. What are the competitive responses of the power plants against government financial interventions? (2) What are the best strategies that simultaneously optimize government objective and power plant's profit?

The remainder of this paper is organized as follows: "Literature review": briefly discusses the related literature. "The model" presents the proposed model and derives equilibrium solutions. In "Numerical example", a numerical example is presented. Concluding remarks and

suggestions for the future research are given in “[Summary and conclusion](#)”.

## Literature review

The struggle between economy and environment has led managerial researches to promote methodologies that their goal is to achieve profits by preserving the sustainability of the environment. The electricity-generating plants, as the largest pollution emitting industry, have encouraged many scientific researches. Accordingly, governments and policy makers have been concentrating on renewable energy, which is regarded as environmentally sustainable energy. The renewable energy creates several public benefits, such as environmental improvement (reduction of power plant greenhouse emissions and thermal and noise pollution), increased fuel diversity, reduction of the effects of energy price's volatility on the economy, and national economic security (Menegaki 2007).

There are innumerable research papers on competition between two power plants. Some papers deal with either quantity competition or price competition. Their primary focus is on applying the game theory to derive equilibrium under varied assumptions (Liu et al. 2007). Menniti et al. (2008) suggested the evolutionary game model to obtain near Nash equilibrium when more than two producers exist in the electricity market. Jia and Yokoyama (2003) discussed the cooperation of the independent power producers in retail market and calculated the profits of producers. They proposed a schema for allocation of extra profit of coalition among power producers. Some researchers applied game theory models in electricity market auction (see Gan et al. 2005a, b).

Some researchers have concentrated on the role of the green policy of the governments in polluting industries. Dong et al. (2010) presented a framework for analyzing the conflicts between a local government and a potentially pollution producer using the game theory. They investigated the effects of the subsidies and penalties policies on the implementation of cleaner production. Sheu and Chen (2012) analyzed the effects of the governmental financial intervention on green supply chain's competition using a three-stage game-theoretic model. Analytical results showed that governments should adopt green taxation and subsidization to ensure profitability of the production of green products.

Owing to occasional factors or events, market demand becomes highly uncertain across many industries. The retail price, market demand, and production cost are often uncertain when a firm determines the decisions in production planning and plant dimensioning (Alonso-Ayuso et al. 2005). A few papers added demand uncertainty to the

pricing models (Deneckere et al. 1997; Mantrala and Raman 1999; Dana 2001; Kunnumkal and Topaloglu 2008). We assumed demand uncertainty to electricity market base for the power plant.

In the power industry, Nagurney et al. (2006) developed a computational framework for determining the optimal carbon taxes in the context of electric power supply chain networks (generation/distribution/consumption). They developed three distinct types of carbon taxation environmental policies. For these taxation schemas, the behavior of the decision makers in the electric power supply chain network was analyzed via finite-dimensional variational inequality technique. The numerical results showed that the carbon taxation has been effective in achieving the desired result intended by the policy makers.

Table 1 demonstrates that the proposed approach of the paper covers new features in comparison with other existing models. To the best of the authors' knowledge, no research was found in the context of electricity market, which considers responses of the competitive power plants against the government's green policies. Due to this gap in the literature, there are three main contributions in this research. First, the government is regarded as the leading player to investigate the impacts of its green legislations and financial interventions on electricity market. Although the governmental economic incentives as promoting environmental protection have been investigated in some particular industries (Ulph 1996; Fullerton and Wu 1998; Walls and Palmer 2001), they have not been studied in the electricity industry. Based on the emerging worldwide green legislations, such as WEEE, and the Kyoto global warming agreement, the involvement of the governments in energy industry via coercive strategies, including legislations and economic incentives, is indispensable. Second, it is assumed that there is uncertainty regarding the demand of electricity because each power plant often take decisions about electricity planning, capacities and type of sources long before the real demand of electricity are resolved. Thus, our model explicitly considers that the electricity demand which is function of electricity prices is known by power plants with a level of uncertainty. The demand uncertainty brings about profit uncertainty of power plants and they assumed to be risk averse toward their profit uncertainty. Third, although some research has focused on green manufacturing/remanufacturing, few studies have addressed the coordination and competition of the power plants under the governments' policies. In this paper, the role of the government as the Stackelberg leader on the strategies of the power plants as the Stackelberg followers is, especially, investigated. A bi-level programming model is proposed for such hierarchical decision-making framework.

**Table 1** A comparison between previous studies and the current study

Ref.	Game theory	Cooperation	Government intervention	Considering government profit	Various power plants	Green issues	Key features	A methodology
Nagurney et al. (2006)	×	×	✓	×	✓	✓	Determination of optimal carbon taxes to electric power plants	NLP
Barari et al. (2012)	✓	×	✓	×	×	✓	Analysis of green supply chain contracts with the focus on maximizing economic profits	ESS NLP
Jia and Yokoyama (2003)	✓	✓	×	×	×	×	Profit allocation, power producers, retail market, cooperative Game theory	Mathematic solving
ZHU and Dou (2007)	✓	×	✓	✓	×	✓	Costs and benefits analyses, Core Enterprises and Governments, Evolutionary Game	ESS, mathematic solving
Dong et al. (2010)	✓	×	✓	×	×	✓	Optimal strategy, cleaner production policies, Chinese electroplating industry, game theory, microeconomics	Mathematic solving
Sheu and Chen (2012)	✓	✓	✓	×	×	✓	Analyses the effects of government financial intervention, competition among green supply chain	NLP
Zhao et al. (2012)	✓	×	✓	×	×	✓	Game theory, strategy selection for environmental risk, carbon emissions reduction, green supply chain	Mathematic solving
Zhang and Liu (2013)	✓	×	×	×	×	✓	Three-level green supply chain, coordination mechanism	Mathematic solving, NLP
Wang et al. (2007)	✓	×	×	×	×	×	Game theory, market price of electricity, demand uncertainty and unit reliability	LP, stochastic programming
Zhu and Huang (2013)	×	×	×	×	✓	✓	Renewable energy, stochastic uncertainty, management of electric power systems	Fractional programming
The proposed model (current study)	✓	✓	✓	✓	✓	✓	A game theory framework, Government taxes and subsidies on optimal, power plant, optimal prices, resource selection, green polices	NLP, stochastic programming

LP linear programming, NLP non-linear programming, ESS evolutionary stable strategy

## The model

### Problem description

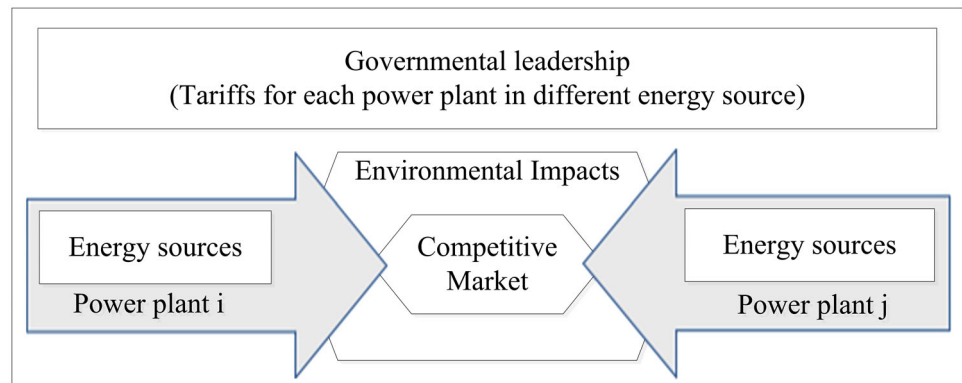
In this paper, two power plants in a competitive electricity market are considered, where each power plant has  $N$  options for energy source type. Government imposes different levels of subsidy or tax for the power plants regarding their selected energy source type. The government aims to reduce pollution of power plants with regard to specific budget. On the other hand, each power plant tries to maximize its profit. Figure 1 illustrates a conceptual flowchart of this game. The competition between the

power plants can be interpreted as a two-person non-zero sum game. The principal strategies of power plants are type of energy sources and their minor strategies are the electricity prices in the market. These strategies in competitive electricity market should be adopted with regard to government's tariffs and market responses.

The proposed models are established upon the following assumptions:

**Assumption 1** We assume that the competitive power plants follow the government's financial legislations and have the capability to produce electricity using different energy sources.

**Fig. 1** A conceptual framework of the game



**Assumption 2** The market competition environment between two power plants is consistent with assumptions of Bertrand’s model of oligopoly. The power plants jointly set the price that maximizes their own profits. The demand function for each power plant is contingency upon electricity prices which assumed continuous and linear.

**Assumption 3** The competitive power plants are able to set up facilities for generating electricity from the specific sources. When the power plants install and start up the corresponding power generations instruments, the production capacity is ample for market demand.

**Assumption 4** The production rate of the power plants is equal to the corresponding demand rate. Moreover, they have negligible internal consumption and waste rate.

**Assumption 5** The time order of this game is assumed as follows:

*Stage 1* The government determines their taxes and tariffs based on the goal of minimizing the pollution by speculating about the potential reactions of the power plants in the bargaining context.

*Stage 2* Both of the competing power plants determine the pricing strategies for the consumers. The power plants then determine the utility of each fuel type. Finally, based on the bargaining game, the equilibrium solutions are found.

The indexes, parameters and variables used in the model formulae are given in list of symbols.

**Power plant’s model**

*Demand*

In the new deregulated environment, the price of the electricity is no longer set by the regulators; instead, it is set by the market forces (Skantze and Gubina 2000). The Demand function for each power plant is assumed continuous which takes the following forms:

$$\tilde{D}_{ikl} = \tilde{\alpha}_{ikl} - \beta_{ikl}P_{ikl} + \gamma_{jkl}P_{jkl} \quad (1)$$

for  $i \neq j, j, s \in \{1, 2\}$  and  $k, l = 1, \dots, n;$

This function is a class of a more general linear demand functions used in many of the previous studies (McGuire and Staelin 1983; Choi 1991; Shy 2003). The differentiation parameters of the two power plants,  $\beta_{ikl}$  and  $\gamma_{jkl}$  are independent and positive values. The power plant with larger  $\tilde{\alpha}_{ikl}$  has a relative advantage of accessing customer due to a better brand, position, quality, reputation and so on. The function (1) means that the market demand of each power plant is an increasing function of its rival price, but a decreasing function of its own price. Moreover,  $\tilde{\alpha}_{ikl}$  and  $\tilde{\alpha}_{jkl}$  are assumed independent stochastic variables, thus  $cov(\tilde{\alpha}_{ikl}, \tilde{\alpha}_{jkl}) = 0$ .

*Profit function*

The profit function for each power plant is formulated as follows:

$$\tilde{\Pi}_{ikl} = (p_{ikl} - C_{ik} - T_{ikl} + S_{ikl})\tilde{D}_{ikl} - F_{ik} \quad (2)$$

$i = 1, 2, k, l = 1, \dots, n$

This function shows that each power plant’s profit depends on the amount of its demand, price the initial setup and the unit production cost, as well as the government’s tax and subsidy. Note that tax reduces marginal profit, however, subsidy increases the marginal profit of the power plant.

*Utility function*

It is known that the power plants would be at risk from demand of new fuel system. By considering the risk sensitivity of the power plants, it is assumed that each power plant assesses its utility via the following Mean–Variance value function of its random profit (Agrawal and Seshadri 2000; Tsay 2002; Gan et al. 2005a, b; Lee and Schwarz 2007; Xiao and Yang 2008):

$$U(\tilde{\Pi}_{ikl}) = E(\tilde{\Pi}_{ikl}) - \lambda_{ikl} \text{var}(\tilde{\Pi}_{ikl}) \quad i = 1, 2 \quad k, l = 1, \dots, n \tag{3}$$

where the second term is the risk cost of the power plant  $i$ , and  $\lambda_{ikl}$  reflects the attitude of the power plant  $i$  towards uncertainty. Equation (3) means that the power plant  $i$  will make a trade-off between the mean and the variance of its random profit. The larger the CARA,  $\lambda_{ikl}$ , of the power plant  $i$ , the more conservative its behavior will be. Based on the Eqs. (2) and (3), we have:

$$U(\tilde{\Pi}_{ikl}) = (p_{ikl} + S_{ikl} - T_{ikl} - C_{ik})(\bar{\alpha}_{ikl} - \beta_{ikl} p_{ikl} + \gamma_{jkl} p_{jkl}) - \lambda_{ikl} (p_{ikl} + S_{ikl} - T_{ikl} - C_{ik})^2 \sigma_{ikl}^2 - F_{ik} \quad i = 1, 2, k, l = 1, \dots, n \tag{4}$$

In this study, this utility is considered as the payoff of power plant. The game is a non-zero-sum game, because a gain by one player does not necessarily correspond with a loss by another.

**Government’s model**

Government as well as the other player aims to take measures which optimizes the pollution level and the profit. The proposed model for the government can be expressed as:

$$\begin{aligned} &\min (w_{ik} \tilde{D}_{ikl} + w_{jl} \tilde{D}_{jkl}) \quad i \neq j, i, j \in \{1, 2\}, k, l \in \{1, \dots, n\} \\ &\text{s.t Pr}\{(T_{ikl} - S_{ikl}) \tilde{D}_{ikl} + (T_{jkl} - S_{jkl}) \tilde{D}_{jkl} \geq Lb_G\} \geq \tau \\ &(p_{ikl} - C_{ik} - T_{ikl} + S_{ikl}) \bar{D}_{ikl} - \lambda_{ikl} (p_{ikl} - C_{ik} - T_{ikl} + S_{ikl})^2 \sigma_{ikl}^2 - F_{ik} \geq R_i \\ &(p_{jkl} - C_{jl} - T_{jkl} + S_{jkl}) \bar{D}_{jkl} - \lambda_{jkl} (p_{jkl} - C_{jl} - T_{jkl} + S_{jkl})^2 \sigma_{jkl}^2 - F_{jl} \geq R_j \\ &T_{ikl}, T_{jkl}, S_{ikl}, S_{jkl} \geq 0 \end{aligned} \tag{5}$$

In this optimization problem, the objective function represents the total pollution emitted by power plants. According to green policy, the government would minimize the total emitted pollution. The first constraint assures the government’s revenue from the power plants dose not lower than a lower bound with probability  $\tau$ . The second and third constraints are individual rational constraints (IR) under which the power plants would like to accept

government’s tariffs; otherwise, the power plants will reject the tariffs and withdraw from the electricity market. In other words, IR constraints guarantee that the power plants would like to have long-term relationship with the government.

**Nash equilibrium point**

The fundamental solution concept in the game theory is a Nash equilibrium (NE) point where each agent’s strategy is the best response to the strategies of the others. Each player has no motivation to deviate from the NE strategy, because it would lead to a decrease of its expected payoff. The NE of the game is formally defined as follows (Krause et al. 2006):

In a  $n$ -person game, the strategy profile  $p^* = (p_1^*, \dots, p_n^*)$  is a NE if for all  $i \in \{1, \dots, n\}$  we have:

$$U_i(p_1^*, \dots, p_n^*) \geq U_i(p_1^*, \dots, p_{i-1}^*, p_i, p_{i+1}^*, \dots, p_n^*) \tag{6}$$

Several algorithms have been developed for computing NE. The interested reader may refer to Krause et al. (2004) and Porter et al. (2004). In this study, we use NE approach for the Bertrand game to calculate the price equilibrium of electricity in a competitive market.

**Bertrand game**

Consider the simultaneous-move game where the power plant  $i$  ( $i = 1, 2$ ) independently chooses  $p_i$ . Let  $(p_1^*, p_2^*)$  be the Bertrand equilibrium prices which are obtained from the first-order condition given by  $\partial U_i(p_i, p_j) / \partial p_i = 0$  ( $i = 1, 2, i \neq j$ ) (Vives 1985).

**Lemma 1** According to Eq. 4, using the Bertrand equation, the equilibrium price for each one of the power plants under given government policies could be found as follows

$$\begin{aligned} p_{ikl}^* &= M_{ikl}^* - S_{ikl} + T_{ikl} + C_{ik} \\ p_{jkl}^* &= M_{jkl}^* - S_{jkl} + T_{jkl} + C_{jl} \end{aligned} \tag{7}$$

where

$$\begin{aligned} M_{ikl}^* &= \frac{\left\{ 2(\beta_{jkl} + \lambda_{jkl} \sigma_{jkl}^2) [\bar{\alpha}_{ikl} + \beta_{ikl} (S_{ikl} - T_{ikl} - C_{ik}) - \gamma_{jkl} (S_{jkl} - T_{jkl} - C_{jl})] \right\}}{\left\{ 4(\beta_{ikl} + \lambda_{ikl} \sigma_{ikl}^2) (\beta_{jkl} + \lambda_{jkl} \sigma_{jkl}^2) - \gamma_{jkl} \gamma_{ikl} \right\}}, \\ M_{jkl}^* &= \frac{\left\{ 2(\beta_{ikl} + \lambda_{ikl} \sigma_{ikl}^2) [\bar{\alpha}_{jkl} + \beta_{jkl} (S_{jkl} - T_{jkl} - C_{jk}) - \gamma_{ikl} (S_{ikl} - T_{ikl} - C_{il})] \right\}}{\left\{ 4(\beta_{ikl} + \lambda_{ikl} \sigma_{ikl}^2) (\beta_{jkl} + \lambda_{jkl} \sigma_{jkl}^2) - \gamma_{jkl} \gamma_{ikl} \right\}}. \end{aligned} \tag{8}$$

Proofs of all the Lemmas and propositions are given in Appendix 1.

**Proposition 1** The optimal power plants' utility at the equilibrium price for the given government's policies is formulated as follows:

$$U^*(\tilde{\Pi}_{ikl}) = (\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)M_{ikl}^{*2} - F_{ik}, \quad i = 1, 2, \quad k, l = 1, \dots, n. \quad (9)$$

Government model in equilibrium prices

To obtain the government's optimal policy, the government's model at the equilibrium prices should be solved. According to Eqs. (1), (5), (7) and (9), we derive the following Lemma:

**Lemma 2** The government's model at the equilibrium prices is formulated as follows:

The final model has a linear objective function and a set of non-linear constraints. The first constraint is stochastic constraint which can be transformed into a certain constraint.

**Lemma 3** The government's first constraint is equal to

$$\left\{ \begin{aligned} & (T_{ikl} - S_{ikl})\tilde{\alpha}_{ikl} + (T_{jkl} - S_{jkl})\tilde{\alpha}_{jkl} \\ & + ((T_{jkl} - S_{jkl})\gamma_{ikl} - (T_{ikl} - S_{ikl})\beta_{ikl})(\Delta_{ikl} + (\theta_{ikl} - 1)) \\ & (S_{ikl} - T_{ikl} - C_{ik}) + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \\ & + ((T_{ikl} - S_{ikl})\gamma_{jkl} - (T_{jkl} - S_{jkl})\beta_{jkl})(\Delta_{jkl} + (\theta_{jkl} - 1)) \\ & (S_{jkl} - T_{jkl} - C_{jl}) + \chi_{ikl}(S_{ikl} - T_{ikl} - C_{kl}) \\ & - \phi^{-1}(\tau)\sqrt{(T_{ikl} - S_{ikl})^2\sigma_{ikl}^2 + (T_{jkl} - S_{jkl})^2\sigma_{jkl}^2} \end{aligned} \right\} \geq Lb_G, \quad (16)$$

where  $\phi$  is the standardized normal distribution.

$$\min \Gamma + \eta_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) + \eta_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \quad i \neq j, \quad i, j \in \{1, 2\}, \quad k, l \in \{1, \dots, n\}$$

$$\text{s.t Pr} \left\{ \left\{ \begin{aligned} & ((T_{ikl} - S_{ikl})\tilde{\alpha}_{ikl} + (T_{jkl} - S_{jkl})\tilde{\alpha}_{jkl} \\ & + ((T_{jkl} - S_{jkl})\gamma_{ikl} - (T_{ikl} - S_{ikl})\beta_{ikl})(\Delta_{ikl} + (\theta_{ikl} - 1)(S_{ikl} - T_{ikl} - C_{ik}) \\ & + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl})) + \\ & ((T_{ikl} - S_{ikl})\gamma_{jkl} - (T_{jkl} - S_{jkl})\beta_{jkl})(\Delta_{jkl} + (\theta_{jkl} - 1)(S_{jkl} - T_{jkl} - C_{jl}) \\ & + \chi_{ikl}(S_{ikl} - T_{ikl} - C_{kl})) \end{aligned} \right\} \geq Lb_G \right\} \geq \tau$$

$$\Delta_{ikl} + \theta_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \geq \sqrt{\frac{R_i + F_{ik}}{(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)}}$$

$$\Delta_{jkl} + \theta_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) + \chi_{ikl}(S_{ikl} - T_{ikl} - C_{kl}) \geq \sqrt{\frac{R_j + F_{jl}}{(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2)}}$$

$$T_{ikl}, T_{jkl}, S_{ikl}, S_{jkl} \geq 0 \quad (10)$$

where

$$\Gamma = w_{ik}\tilde{\alpha}_{ikl} + w_{jl}\tilde{\alpha}_{jkl} + (w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl})\Delta_{ikl} + (w_{ik}\gamma_{jkl} - w_{jl}\beta_{jkl})\Delta_{jkl} \quad (11)$$

$$\eta_{ikl} = (\theta_{ikl}(w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl}) + \chi_{ikl}(w_{ik}\gamma_{jkl} - w_{jl}\beta_{jkl}) - (w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl})) \quad (12)$$

$$\Delta_{ikl} = \frac{2(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2)\tilde{\alpha}_{ikl} + \gamma_{jkl}\tilde{\alpha}_{jkl}}{(4(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})} \quad (13)$$

$$\theta_{ikl} = \frac{(2\beta_{ikl}(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})}{(4(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})} \quad (14)$$

$$\chi_{jkl} = \frac{(-2\gamma_{jkl}(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) + \gamma_{jkl}\beta_{jkl})}{(4(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})} \quad (15)$$

Bargaining game

The goal of the Nash bargaining game, as a cooperative game, is dividing the benefits or utility between two players based on their competition in the market place. The Nash bargaining game model (Nash 1950) requires the feasible set to be compact and convex. It contains some payoff vectors, so that each individual payoff is greater than the individual breakdown payoff. Breakdown Payoffs are the starting point for bargaining which represent the possible payoff pairs obtained if one player decides not to bargain with the other player. In this study, the equilibrium point of the game is achieved using the Nash bargaining game. It is believed that a power plant does not stay in the business unless it can meet its minimum needs; therefore,

**Table 2** Utility function of both power plants using the three different energy sources

Energy source	Power plant 2		
	Solar	Gas	Diesel fuel
<b>Power plant 1</b>			
Solar	$(U^*(\Pi_{111}), U^*(\Pi_{211}))$	$(U^*(\Pi_{112}), U^*(\Pi_{212}))$	$(U^*(\Pi_{113}), U^*(\Pi_{213}))$
Gas	$(U^*(\Pi_{121}), U^*(\Pi_{221}))$	$(U^*(\Pi_{122}), U^*(\Pi_{222}))$	$(U^*(\Pi_{123}), U^*(\Pi_{223}))$
Diesel fuel	$(U^*(\Pi_{131}), U^*(\Pi_{231}))$	$(U^*(\Pi_{132}), U^*(\Pi_{232}))$	$(U^*(\Pi_{133}), U^*(\Pi_{233}))$

where  $U^*(\tilde{\Pi}_{ikl}) = (\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)M_{ikl}^{*2} - F_{ik}$ ,  $i = 1, 2$ ,  $k, l = 1, \dots, n$ .

**Table 3** Data of power plant 1 and power plant 2

Energy source	Solar-solar	Solar-gas	Solar-diesel	Gas-solar	Gas-gas	Gas-diesel	Diesel-solar	Diesel-gas	Diesel-diesel
$C_{1k}$	6	6	6	8	8	8	10	10	10
$C_{2l}$	6	9	11	6	9	11	6	9	11
$F_{1k}$	800	800	800	300	300	300	150	150	150
$F_{2l}$	1,200	350	200	1,200	350	200	1,200	350	200
$W_{1k}$	6.3	6.3	6.3	3.4	3.4	3.4	48	48	48
$W_{2l}$	5.7	30.8	48.14	5.7	30.8	48.14	5.7	30.8	48.14
$\lambda_{1kl}$	0.3	0.33	0.35	0.28	0.21	0.3	0.25	0.2	0.22
$\lambda_{2kl}$	0.32	0.27	0.26	0.34	0.23	0.22	0.36	0.24	0.22
$\beta_{1kl}$	30	24	21	35	32	30	39	37	35
$\beta_{2kl}$	29	32	40	27	33	38	28	31	34
$\gamma_{1kl}$	42	38	36	43	36	45	47	43	29
$\gamma_{2kl}$	41	45	49	37	33	40	38	39	31
$\bar{\alpha}_{1kl}$	1,200	1,250	1,270	26	1,210	1,220	1,130	1,160	1,100
$\bar{\alpha}_{2kl}$	1,810	1,850	1,800	1,820	1,825	1,890	1,860	1,850	1,920
$\sigma_{1kl}^2$	20	24	38	26	30	46	40	44	50
$\sigma_{2kl}^2$	21	29	24	19	25	28	16	23	40

the breakdown point of the game for each plant depends on its individual policy.

If  $U_{ikl}^*$  is the optimal utility function for the power plant  $i$  and  $U_{jkl}^*$  is the optimal utility function for the player  $j$ , they will maximize  $(U_{ikl}^* - R_i)(U_{jkl}^* - R_j)$ , where  $R_i$  and  $R_j$ , are reservation utilities (breakdown points) for the power plants. That is the power plants would withdraw from the competitive market, if they obtain optimal utilities lower than the reservation utilities (i.e.,  $U_{ikl}^* \leq R_i$  and  $U_{jkl}^* \leq R_j$ ). Therefore, according to the Nash bargaining procedure, the source selection model of the power plants is given by

$$\begin{aligned} & \max (U_{ikl}^* - R_i)(U_{jkl}^* - R_j) \quad i \neq j \\ & \text{s.t. } U_{ikl}^* > R_i \\ & \quad U_{jkl}^* > R_j \end{aligned} \tag{17}$$

The algorithm of the game procedure is as follows: by solving problem (10–15) first, the optimal government policy  $T_{ikl}^*$ ,  $T_{jkl}^*$ ,  $S_{ikl}^*$  and  $S_{jkl}^*$  are achieved. Afterwards, the power plants should determine electricity prices for all possible type of sources with regard to government’s taxes and subsidies. From problem (7) the equilibrium prices for

each pair of source types, i.e.,  $p_{ikl}^*$  and  $p_{jkl}^*$  are calculated. Then, the optimal strategies for energy sources concerning the optimal government’s tariffs and equilibrium prices are obtained from the Nash bargaining problem (17).

### Numerical example

In this section, we provide the numerical examples to discuss how the theoretical results in this paper can be applied in practice. It is supposed that there are three types of energy sources for each power plant which include solar, gas and diesel gas. To demonstrate how the government’s revenue affects the market equilibrium, we consider three different examples. These examples are distinctive according to the minimum acceptable level of government’s revenue. The payoff matrix for two-person non-zero sum game between the plants is shown in Table 2. Moreover, data for this numerical example are presented in Table 3.

All the calculations are done with MATLAB 14. According to this data, the values for all the variables were



**Table 4** Numerical results for example 1

Energy source	Solar-solar	Solar-gas	Solar-diesel	Gas-solar	Gas-gas	Gas-diesel	Diesel-solar	Diesel-gas	Diesel-diesel
$T_{1kl}$	0.925	27.745	119.677	333.357	5.657	9.433	144.29	35.372	267.671
$T_{2kl}$	0.005	103.437	141.298	0.003	0.333	41.836	0.103	6.034	273.347
$S_{1kl}$	0	55.068	124.896	169.914	0	24.092	0.272	0.188	0
$S_{2kl}$	0.379	0.002	0	5.884	3.896	0.03	5.36	19.371	0
$p_{1kl}$	55.6715	105.5519	131.1695	175.8069	44.0521	43.0772	157.6612	65.8186	281.4594
$p_{2kl}$	61.4076	140.8418	156.9571	140.2412	47.1325	73.4309	137.7303	61.5796	289.1927
$U(\Pi_{1kl})$	8.4754e+004	5.1302e+005	5.8232e+005	502.5792	3.5086e+004	1.0805e+005	500.4246	1.9344e+004	502.4488
$U(\Pi_{2kl})$	1.0995e+005	3.1795e+004	800.5179	6.5569e+005	6.7020e+004	1.8590e+004	6.3231e+005	1.5834e+005	803.6828

computed. The details of calculated values for all the variables are given in Appendix 2.

**Example 1** For the first numerical example, it is supposed that the government takes account of  $Lb_G = 1,000$ , and power plants 1 and 2 consider the reservation utility  $R_1 = 500$  and  $R_2 = 800$ , respectively. The government model will be, first, solved to get the subsidies and taxes. Using these values, the equilibrium price is obtained. Finally, using the Nash bargaining game, the non-zero sum game will be solved. The calculated values for this example are summarized in Table 4. The results of optimal, taxes, subsidies, electricity prices, and utility value of power plant are given in the rows of the table, respectively. These values are provided for the nine possible combinations of the power plants' recourses.

**Table 5** Numerical results for the Nash bargaining game model in example 1. ( $10^6$ )

Energy source	Power plant 2		
	Solar	Gas	Diesel fuel
Power plant 1			
Solar	9,196.2	15,885	0
Gas	1.4110	2,290.2	1,913.3
Diesel fuel	0	2,968.6	0.0000064064

**Table 6** Numerical results for example 2

Energy source	Solar-solar	Solar-gas	Solar-diesel	Gas-solar	Gas-gas	Gas-diesel	Diesel-solar	Diesel-gas	Diesel-diesel
$T_{1kl}$	3.152	27.576	119.04	166.467	8.558	9.522	146.838	38.508	267.672
$T_{2kl}$	1.662	108.336	143.739	2.374	1.076	45.016	52.066	6.506	376.338
$S_{1kl}$	0.071	53.802	122.862	0	0	22.069	0	0.193	0
$S_{2kl}$	0.154	0	0	6.717	1.905	0.033	55.732	18.081	102.99
$p_{1kl}$	58.5190	109.6974	133.8829	178.8311	47.0091	45.9637	160.4813	68.8856	281.4603
$p_{2kl}$	64.1997	145.7515	159.3987	143.1020	50.0759	76.7115	140.6245	64.3994	289.1936
$U(\Pi_{1kl})$	8.7199e+004	5.3800e+005	5.9415e+005	502.6213	3.5217e+004	1.1146e+005	500.4272	1.9223e+004	502.4241
$U(\Pi_{2kl})$	1.1360e+005	3.1815e+004	800.7171	6.6815e+005	6.7698e+004	1.8780e+004	6.4442e+005	1.6347e+005	803.6486

The minimum utility of the first and second power plants for different strategies is 500.4246 and 800.5179, respectively. It is assumed that the power plants would withdraw from the competitive market; if the power plants obtain the utility lower than these values (In the real application of the model, different value for breakdowns can be considered regarding the individual preferences). Thus, we have  $R_1 = 500.4246$  and  $R_2 = 800.5179$ .

Regarding these breakdown values, the bargaining game model can be shown in Table 5.

Therefore, the equilibrium strategy is solar-gas. In other words, if power plant 1 uses solar source and power plant 2 uses gas source, the power plants would obtain maximum utility.

**Example 2** In this example, it is supposed that the government considers  $Lb_G = 10,000$  and as in the previous example, power plant 1 considers  $R_1 = 500$  and power plant 2 considers  $R_2 = 800$ . Then, Table 6 demonstrates the optimal values for different strategies of power plants.

Similar to the previous example, it is considered that  $R_1 = 500.4272$  and  $R_2 = 800.7171$ .

Values of the bargaining game model are shown in Table 7:

Therefore, the equilibrium strategies for the first and second power plants are solar and gas, respectively.

**Table 7** Numerical results for the Nash bargaining game model in example 2. ( $10^6$ )

Energy source	Power plant 2		
	Solar	Gas	Diesel fuel
Power plant 1			
Solar	9,779.5	16,670	0
Gas	1.4642	2,322.4	1,995
Diesel fuel	0	3,045.6	0.0000058539

**Example 3** In this example, it is supposed that the government raises its minimum acceptable revenue to  $Lb_G = 100,000$ . Other parameters remain the same. The detailed results of the example are shown in Table 8.

Therefore, as in the previous examples, it is considered that  $R_1 = 500.4336$  and  $R_2 = 800.9081$ .

Values of the bargaining game model are shown in Table 9:

Thus, the optimal strategies for first and second power plants are solar and gas, respectively.

**Numerical examples analysis**

In the numerical examples, we analyses three levels of 1,000, 10,000 and 100,000 for minimum level of government’s revenue. The competitive market condition becomes more sever, as the government raises the minimum level of revenue. The results show that the equilibrium strategies for plants 1 and 2 in these numerical examples are solar and gas, respectively. This is very close to the government’s green policy, thus, it is desirable. This model is provided for the government’s policy so that the power plants under this policy follow the green policy in each condition.

Figures 2 and 3 illustrate the change in the net government’s tariffs,  $(T_{ikl} - S_{ikl})$ , in each energy source type versus  $Lb_G$  for the power plants 1 and 2, respectively.

Figures 2 and 3 show that the government’s penalties for the non-green energy sources, always are higher than penalties

**Table 9** Numerical results for the Nash bargaining game model in example 3. ( $10^6$ )

Energy source	Power plant 2		
	Solar	Gas	Diesel fuel
Power plant 1			
Solar	16,128	24,382	0
Gas	1.9011	2,650.2	2,827.2
Diesel fuel	0	4,095.7	0.0000054901

for the green energy sources. The increase in  $Lb_G$  will raise the imposed penalty for the non-green energy sources. Hence, high tax levels for the non-green energy sources will encourage the power plants to use the green energy sources.

To draw detailed comparisons among different government’s revenue policy, government’s optimal tax and subsidy are indicated in Table 10.

A simple comparison, between three different conditions, shows that when the government increases its minimum revenue, it would increases the taxes on non-green energy sources and decreases the subsidies. On contrary, for the power plants with green energy sources, the government would raises the subsidies and reduces the taxes. For example in the non-green case (diesel–gas) the government increases the taxes, and in the green case (solar–gas), it almost sets the high value for the offered subsidies. Using this methodology, the government would be able to set the optimum level of subsidies and taxes in competitive electricity market. On the other hand, the competitive power plants choose energy sources such that their utility values become maximum.

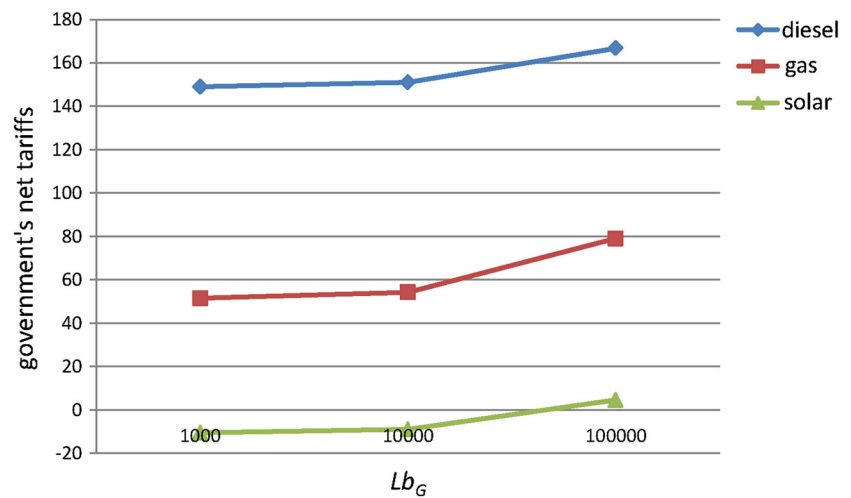
**Summary and conclusions**

This study is a contribution to the growing research on the development of rigorous mathematical and game theory frameworks for environmental-energy modeling. The proposed computational framework helps the governmental policy

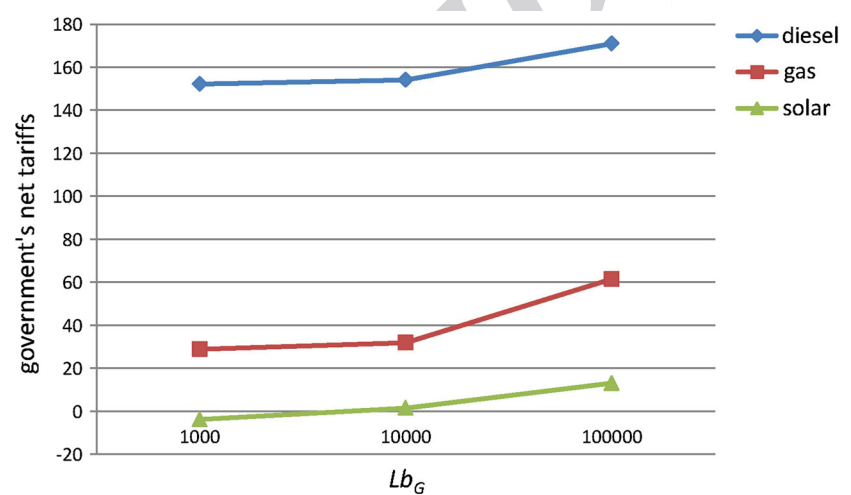
**Table 8** Numerical results for example 3

Energy source	Solar–solar	Solar–gas	Solar–diesel	Gas–solar	Gas–gas	Gas–diesel	Diesel–solar	Diesel–gas	Diesel–diesel
$T_{1kl}$	23.682	25.894	8.897	194.169	37.872	13.659	174.204	66.667	267.672
$T_{2kl}$	24.236	151.39	165.965	9.745	26.738	77.411	14071.085	8.114	273.347
$S_{1kl}$	2.291	42.504	0.009	0	1.4	7.386	1.555	0.034	0
$S_{2kl}$	5.88	0.275	0	0	1.178	4.034	14060.189	0.524	0
$p_{1kl}$	83.1406	145.9197	158.5796	206.5350	75.4869	71.7145	186.2924	97.6813	281.4601
$p_{2kl}$	88.6842	188.6242	181.6264	169.3080	78.4564	106.0076	167.1148	92.3828	289.1930
$U(\Pi_{1kl})$	1.1111e+005	7.8127e+005	7.0737e+005	502.8494	3.6548e+004	1.4423e+005	500.4336	2.0130e+004	502.3446
$U(\Pi_{2kl})$	1.4661e+005	3.2029e+004	800.9081	7.8776e+005	7.4321e+004	2.0471e+004	7.6060e+005	2.0945e+005	803.7810

**Fig. 2** Change in the government's net tariffs for the power plant 1 versus  $Lb_G$



**Fig. 3** Change in the government's net tariffs for the power plant 2 versus  $Lb_G$



**Table 10** A comparison between the performance of the solar–gas and diesel–gas

	Energy sources	$T_{1kl}$	$T_{2kl}$	$S_{1kl}$	$S_{2kl}$
$Lb_G = 1,000$	Solar–gas	27.745	103.437	55.068	0.002
	Diesel–gas	35.372	6.034	0.188	19.371
$Lb_G = 10,000$	Solar–gas	27.576	108.336	53.802	0
	Diesel–gas	38.508	6.506	0.193	18.081
$Lb_G = 100,000$	Solar–gas	25.894	151.39	42.504	0.275
	Diesel–gas	66.667	8.114	0.034	0.524

makers to determine the optimal tariffs on the power plants in a competitive electricity market. To be more specific, the model allows the governmental policy makers to determine the optimal taxes and subsidies for each individual electric power plant regarding the emitted pollutants. These values depend upon the minimum level of expected utility of the power plants and the government's green policy. The model provides the best

strategy for energy sources of power plants. Three numerical examples were presented to illustrate the model's performance in three different levels of the government's revenue. These numerical examples also demonstrate how the policy makers could determine the optimal taxes and subsidies to achieve the desired environmental objectives and how the power plants could maximize their utility in each energy source.

There are several directions and suggestions for future research. First of all, the proposed model can be easily extended to the case where more than two power plants exist with different environmental effects. Second, tax and subsidy may be incorporated in the prices of electricity, thus customers are persuaded to use green electricity owing to their competitive price. In this case, the demand functions should be changed appropriately. Moreover, we assume that the government reduces environmental impacts with regard to specific revenue. However, it is extremely appealing to investigate the effects of other objectives of the government such as revenue seeking behavior. Eventually, it would be

very interesting, but challenging to consider the uncertainty on other model parameters such as electricity production costs or reservation utility of power plants.

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**Appendix 1**

*Proof of lemma 1* The first-order conditions of utility of power plants are

$$\frac{\partial U(\Pi_{ikl})}{\partial p_{ikl}} = 0 \tag{18}$$

$$\begin{aligned} &\Rightarrow (\bar{\alpha}_{ikl} - \beta_{ikl}p_{ikl} + \gamma_{jkl}p_{jkl}) + (-\beta_{ikl})(p_{ikl} - C_{ik} - T_{ikl} + S_{ikl}) \\ &\quad - 2\lambda_{ikl}(p_{ikl} - C_{ik} - T_{ikl} + S_{ikl})\sigma_{ikl}^2 \\ &= \bar{\alpha}_{ikl} - \beta_{ikl}(p_{ikl} + S_{ikl} - T_{ikl} - C_{ik}) + -\beta_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) \\ &\quad + \gamma_{jkl}(p_{jkl} + S_{jkl} - T_{jkl} - C_{jl}) - \gamma_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \\ &\quad + (-\beta_{ikl})(p_{ikl} + S_{ikl} - T_{ikl} - C_{ik}) \\ &\quad - 2\lambda_{ikl}(p_{ikl} + S_{ikl} - T_{ikl} - C_{ik})\sigma_{ikl}^2 = 0 \end{aligned} \tag{19}$$

Now, let us define the following variables:

$$M_{ikl} = (p_{ikl} + S_{ikl} - T_{ikl} - C_{ik}), \tag{20}$$

$$M_{jkl} = (p_{jkl} + S_{jkl} - T_{jkl} - C_{jl}). \tag{21}$$

Using  $M_{ikl}$  and  $M_{jkl}$ , we rewrite first-order conditions as

$$\bar{\alpha}_{ikl} + \beta_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) - \gamma_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) - M_{ikl}(2\beta_{ikl} + 2\lambda_{ikl}\sigma_{ikl}^2) + \gamma_{jkl}M_{jkl} = 0, \tag{22}$$

$$\bar{\alpha}_{jkl} + \beta_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) - \gamma_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) - M_{jkl}(2\beta_{jkl} + 2\lambda_{jkl}\sigma_{jkl}^2) + \gamma_{ikl}M_{ikl} = 0, \tag{23}$$

By solving (22) and (23) simultaneously, we have

$$M_{ikl}^* = \frac{\begin{cases} 2(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2)[\bar{\alpha}_{ikl} + \beta_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) - \gamma_{jkl}(S_{jkl} - T_{jkl} - C_{jl})] \\ + \gamma_{jkl}[\bar{\alpha}_{jkl} + \beta_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) - \gamma_{ikl}(S_{ikl} - T_{ikl} - C_{ik})] \end{cases}}{(4(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})}. \tag{24}$$

$M_{jkl}^*$  can be obtained in a similar manner. The  $p_{ikl}^*$  and  $p_{jkl}^*$  obtained from Eq. (7) are the optimum prices if the utility functions are concave on  $p_{ikl}$  and  $p_{jkl}$ . The second derivatives of the function are as follows

$$\begin{cases} \frac{\partial^2 U(\Pi_{ikl})}{(\partial p_{ikl})^2} = -\beta_{ikl} - \beta_{ikl} - 2\lambda_{ikl}\sigma_{ikl}^2 = -2\beta_{ikl} - 2\lambda_{ikl}\sigma_{ikl}^2 \leq 0 \\ \frac{\partial^2 U(\Pi_{jkl})}{(\partial p_{jkl})^2} = -\beta_{jkl} - \beta_{jkl} - 2\lambda_{jkl}\sigma_{jkl}^2 = -2\beta_{jkl} - 2\lambda_{jkl}\sigma_{jkl}^2 \leq 0 \end{cases} \tag{25}$$

Therefore, the utilities are concave functions on electricity prices. □

*Proof of Proposition 1* By substituting  $M_{ikl}^*$  obtained from Lemma 1 into Eq. (4), after some mathematical manipulations, the utility function can be simplified into  $U^*(\tilde{\Pi}_{ikl}) = (\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)M_{ikl}^{*2} - F_{ik}$ . □

*Proof of Lemma 2* The second and the third constraints are constraints for the power plants' utility that are straightforward from proposition 1. Thus, we only discuss the objective function and the first constraint. Let us define the following notations

$$\Delta_{ikl} = \frac{2(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2)\bar{\alpha}_{ikl} + \gamma_{jkl}\bar{\alpha}_{jkl}}{(4(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})} \tag{26}$$

$$\theta_{ikl} = \frac{(2\beta_{ikl}(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})}{(4(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})} \tag{27}$$

$$\chi_{jkl} = \frac{(-2\gamma_{jkl}(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\beta_{jkl})}{(4(\beta_{ikl} + \lambda_{ikl}\sigma_{ikl}^2)(\beta_{jkl} + \lambda_{jkl}\sigma_{jkl}^2) - \gamma_{jkl}\gamma_{ikl})} \tag{28}$$

Substituting Eqs. (26)–(28) into Eq. (24), we have:

$$M_{ikl} = \Delta_{ikl} + \theta_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl}). \tag{29}$$

Moreover, substituting demand function (1) into government problem (5) we have:

$$\begin{aligned} &\min w_{ik}\tilde{\alpha}_{ikl} + w_{jl}\tilde{\alpha}_{jkl} + (w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl})p_{ikl} + (w_{ik}\gamma_{jkl} - w_{jl}\beta_{jkl})p_{jkl}, \\ &\text{s.t Pr}\{(T_{ikl} - S_{ikl})\tilde{\alpha}_{ikl} + (T_{jkl} - S_{jkl})\tilde{\alpha}_{jkl} + ((T_{jkl} - S_{jkl})\gamma_{ikl} \\ &\quad - (T_{ikl} - S_{ikl})\beta_{ikl})p_{ikl} + ((T_{ikl} - S_{ikl})\gamma_{jkl} \\ &\quad - (T_{jkl} - S_{jkl})\beta_{jkl})p_{jkl} \geq Lb_G\} \geq \tau. \end{aligned} \tag{30}$$

Using Lemma 1, the problem (30) is transformed into

$$\begin{aligned} &\min w_{ik}\tilde{\alpha}_{ikl} + w_{jl}\tilde{\alpha}_{jkl} - (w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl})(S_{ikl} - T_{ikl} - C_{ik}), \\ &\quad - (w_{ik}\gamma_{jkl} - w_{jl}\beta_{jkl})(S_{jkl} - T_{jkl} - C_{jl}) + (w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl})M_{ikl} + (w_{ik}\gamma_{jkl} - w_{jl}\beta_{jkl})M_{jkl} \\ &\text{s.t Pr}\left\{ \begin{aligned} &(T_{ikl} - S_{ikl})\tilde{\alpha}_{ikl} + (T_{jkl} - S_{jkl})\tilde{\alpha}_{jkl} \\ &+ ((T_{jkl} - S_{jkl})\gamma_{ikl} - (T_{ikl} - S_{ikl})\beta_{ikl})(M_{ikl} - S_{ikl} + T_{ikl} + C_{ik}) \\ &+ ((T_{ikl} - S_{ikl})\gamma_{jkl} - (T_{jkl} - S_{jkl})\beta_{jkl})(M_{jkl} - S_{jkl} + T_{jkl} + C_{jl}) \end{aligned} \right\} \geq \tau. \end{aligned} \tag{31}$$

Now, let us define the following notations

$$\eta_{ikl}(\theta_{ikl}(w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl}) + \chi_{ikl}(w_{ik}\gamma_{jkl} - w_{jk}\beta_{jkl}) - (w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl})), \tag{32}$$

$$\Gamma = w_{ik}\tilde{\alpha}_{ikl} + w_{jl}\tilde{\alpha}_{jkl} + (w_{jl}\gamma_{ikl} - w_{ik}\beta_{ikl})\Delta_{ikl} + (w_{ik}\gamma_{jkl} - w_{jl}\beta_{jkl})\Delta_{jkl} \tag{33}$$

Substituting Eqs. (29), (32) and (33) in problem (31), we have

It is noted that  $y \leq 0$  and  $\frac{y-E(y)}{\sqrt{\text{Var}(y)}} \sim \text{Normal}(0, 1)$ . That is  $\frac{y-E(y)}{\sqrt{V(y)}} \leq \frac{-E(y)}{\sqrt{V(y)}}$ . The chance constraint of the government model is equivalent to  $\text{Pr}\{Z \leq \frac{-E(y)}{\sqrt{V(y)}}\} \geq \tau$ , where  $Z$  is the standardized normally distributed variable. Then, the chance constraint of the government model holds if and only if  $\phi^{-1}(\tau) \leq \frac{-E(y)}{\sqrt{V(y)}}$ , where  $\phi$  is the standardized normal

$$\begin{aligned} & \min \Gamma + \eta_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) + \eta_{jkl}(S_{jkl} - T_{jkl} - C_{jl}), \\ & \text{s.t Pr} \left\{ \left\{ \begin{aligned} & (T_{ikl} - S_{ikl})\tilde{\alpha}_{ikl} + (T_{jkl} - S_{jkl})\tilde{\alpha}_{jkl} \\ & + ((T_{jkl} - S_{jkl})\gamma_{ikl} - (T_{ikl} - S_{ikl})\beta_{ikl})(\Delta_{ikl} + \theta_{ikl} - 1)(S_{ikl} - T_{ikl} - C_{ik}) + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \\ & + ((T_{ikl} - S_{ikl})\gamma_{jkl} - (T_{jkl} - S_{jkl})\beta_{jkl})(\Delta_{jkl} + \theta_{jkl} - 1)(S_{jkl} - T_{jkl} - C_{jl}) + \chi_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) \end{aligned} \right\} \geq Lb_G \right\} \geq \tau. \end{aligned} \tag{34}$$

*Proof of Lemma 3* Since  $\tilde{\alpha}_{ikl}$  and  $\tilde{\alpha}_{jkl}$  are assumed to be independently and normally distributed variables, the value of  $y = -(T_{ikl} - S_{ikl})\tilde{\alpha}_{ikl} - (T_{jkl} - S_{jkl})\tilde{\alpha}_{jkl}$  is also normally distributed variable. It is equal to

$$\begin{aligned} y = & -((T_{jkl} - S_{jkl})\gamma_{ikl} - (T_{ikl} - S_{ikl})\beta_{ikl})(\Delta_{ikl} + (\theta_{ikl} - 1)) \\ & \times (S_{ikl} - T_{ikl} - C_{ik}) + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \\ & - ((T_{ikl} - S_{ikl})\gamma_{jkl} - (T_{jkl} - S_{jkl})\beta_{jkl})(\Delta_{jkl} + (\theta_{jkl} - 1)) \\ & \times (S_{jkl} - T_{jkl} - C_{jl}) + \chi_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) + Lb_G \end{aligned} \tag{35}$$

Therefore, we have

$$\begin{aligned} E(y) = & -(T_{ikl} - S_{ikl})E[\tilde{\alpha}_{ikl}] - (T_{jkl} - S_{jkl})E[\tilde{\alpha}_{jkl}] \\ & - ((T_{jkl} - S_{jkl})\gamma_{ikl} - (T_{ikl} - S_{ikl})\beta_{ikl})(\Delta_{ikl} + (\theta_{ikl} - 1)) \\ & \times (S_{ikl} - T_{ikl} - C_{ik}) + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \\ & - ((T_{ikl} - S_{ikl})\gamma_{jkl} - (T_{jkl} - S_{jkl})\beta_{jkl})(\Delta_{jkl} + (\theta_{jkl} - 1)) \\ & \times (S_{jkl} - T_{jkl} - C_{jl}) + \chi_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) + Lb_G \\ \text{Var}(y) = & (T_{ikl} - S_{ikl})^2 \text{Var}[\tilde{\alpha}_{ikl}] + (T_{jkl} - S_{jkl})^2 \text{Var}[\tilde{\alpha}_{jkl}] \end{aligned} \tag{36}$$

distribution. With some mathematical simplifications we have

$$\begin{aligned} & (T_{ikl} - S_{ikl})\tilde{\alpha}_{ikl} + (T_{jkl} - S_{jkl})\tilde{\alpha}_{jkl} \\ & + ((T_{jkl} - S_{jkl})\gamma_{ikl} - (T_{ikl} - S_{ikl})\beta_{ikl})(\Delta_{ikl} + (\theta_{ikl} - 1)) \\ & (S_{ikl} - T_{ikl} - C_{ik}) + \chi_{jkl}(S_{jkl} - T_{jkl} - C_{jl}) \\ & + ((T_{ikl} - S_{ikl})\gamma_{jkl} - (T_{jkl} - S_{jkl})\beta_{jkl})(\Delta_{jkl} + (\theta_{jkl} - 1)) \\ & (S_{jkl} - T_{jkl} - C_{jl}) + \chi_{ikl}(S_{ikl} - T_{ikl} - C_{ik}) \\ & - \phi^{-1}(\tau)\sqrt{(T_{ikl} - S_{ikl})^2\sigma_{ikl}^2 + (T_{jkl} - S_{jkl})^2\sigma_{jkl}^2} \geq Lb_G. \end{aligned} \tag{37}$$

□

### Appendix 2

Table 11

**Table 11** Values for all the variables in the numerical example

Energy source	Solar-solar	Solar-gas	Solar-diesel	Gas-solar	Gas-gas	Gas-diesel	Diesel-solar	Diesel-gas	Diesel-diesel
$\eta_{1kl}$	-91.4895	-512.1604	-630.6906	369.4945	-47.7980	-823.1405	805.3662	59.2742	232.6993
$\eta_{2kl}$	-109.0353	175.7468	672.0377	-421.2105	-82.8378	-152.0057	-413.5794	-493.0216	168.5081
$\chi_{1kl}$	-0.5085	-0.6354	-0.5615	-0.3631	-0.3093	-0.3390	-0.3109	-0.3270	-0.2293
$\chi_{2kl}$	-0.5155	-0.4485	-0.3741	-0.5239	-0.3381	-0.4366	-0.5740	-0.4683	-0.2369
$\Delta_{1kl}$	46.7425	54.1624	44.9004	35.9672	32.4313	30.8835	30.4241	31.2910	22.0292
$\Delta_{2kl}$	52.8162	49.0607	36.9422	50.3076	38.6132	37.1350	48.7253	43.7502	29.8931
$\theta_{1kl}$	0.1231	0.0598	0.0389	0.1847	0.2721	0.1431	0.1754	0.2045	0.3006
$\theta_{2kl}$	0.1070	0.0986	0.2140	0.1701	0.2822	0.2575	0.1983	0.2319	0.3195
$\phi^{-1}(\tau)$	1.645	1.645	1.645	1.645	1.645	1.645	1.645	1.645	1.645
$\Gamma$	2.5145 e+004	8.5612 e+004	1.0680 e+005	7.2053 e+004	1.0210 e+005	1.5044 e+005	9.7137 e+004	1.3866 e+005	1.3453 e+005

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