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ORIGINAL RESEARCH

Developing a new stochastic competitive model regarding inventory and price

Reza Rashid · Ali Bozorgi-Amiri · S. M. Seyedhoseini

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Abstract Within the competition in today's business environment, the design of supply chains becomes more complex than before. This paper deals with the retailer's location problem when customers choose their vendors, and inventory costs have been considered for retailers. In a competitive location problem, price and location of facilities affect demands of customers; consequently, simultaneous optimization of the location and inventory system is needed. To prepare a realistic model, demand and lead time have been assumed as stochastic parameters, and queuing theory has been used to develop a comprehensive mathematical model. Due to complexity of the problem, a branch and bound algorithm has been developed, and its performance has been validated in several numerical examples, which indicated effectiveness of the algorithm. Also, a real case has been prepared to demonstrate performance of the model for real world.

Keywords Competitive location problem · Multi-product inventory · Location inventory · Queuing theory

List of symbols

I:	Set of supplier nodes index by i
J:	Set of potential distributer nodes index by j
K:	Set of customer nodes index by k
F_j	Fixed costs of establishing retailer
H_i	Holding cost of product i
E_i	Shortage cost for product i
λ_{ik}	Demand rate of product i in customer k
N	Number of candidate nodes for retailers
M	Number of products
K	Number of customers
U_{ik}	Utility of other companies for customer k
λ_j	Service rate of suppliers for retailer j
$Q_{min,i}$	Minimum size of order
O	Ordering cost
n	Number of business days in a year
c_{ij}	Preparation costs of a good i in retailer j
d_{jk}	Distance between retailer j and customer k
A, B, C	Specific parameters for utility model
T_j	1 if retailer j establish and 0 otherwise
Q_{ij}	Size of orders for product i in retailer j
R_{ij}	Order point for product i in retailer j
D_{ij}	Demand of product i in retailer j
π_{ij0}	Steady-state probability of state zero for product i in retailer j
I_{ij}	Expected value of inventory for retailer j and product i
p_{ij}	Selling price of a product i in retailer j
U_{ikj}	Utility of facility j for customer k for product i

R. Rashid (✉) · S. M. Seyedhoseini
Department of Industrial Engineering, Iran University of Science and Technology, Narmak, 16844 Tehran, Iran
e-mail: reza.rashid69@gmail.com

A. Bozorgi-Amiri
School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

Introduction

Nowadays, supply chains play an undeniable role to meet diverse needs of customers. A supply chain is a network of organizations that work together to control and manage materials and information from suppliers to the customers (Aitken 1998). Location analysis and network design are two major research areas in supply chain optimization;

location problems deal with the decisions of where to optimally locate facilities, whereas network design involves activating optimal links (Contreras and Fernández 2012). This paper has considered inventory costs and it has extended the competitive location problem.

For a supply chain, competition between firms involves attracting as many customers as possible, and the factors which could be important for costumers are price, closeness, and quality of service; consequently, a retailer has a defined utility for a specific customer. In this paper, two factors of price and closeness have been considered to construct a function for customer's utility.

In summary, it is clear that despite many contributions in the location problems, there is little consideration due to competitive location problems under retailer's inventory. In this paper, a competitive location problem has been considered, which can represent following key questions; where retailers should be located? And what is the optimal inventory control policy for each retailer? These questions are obviously interconnected, for example, location and numbers of retailers affect interval flows for each retailer and consequently it would influence inventory costs; for that matter, a mixed-integer nonlinear model has been presented.

The main contributions of this paper can be summarized as follows:

- A mathematical model has been developed for multi-product inventory location problem, which has been considered in a stochastic competitive environment.
- An (*R, Q*) inventory model with stochastic parameters has been developed by queuing theory, and to prepare a simple model, all of the steady-state probabilities have been solved in terms of one state.
- To solve the model, a branch and bound algorithm has been proposed.

This model can be applied for companies that want to establish new facilities in stochastic and competitive environments and would like to regard inventory decisions with regard to location decisions.

The reminder of this paper is organized as follows: In "Problem description", first we represented an inventory model and then a mathematical model has been constructed for competitive location. A branch and bound algorithm has been proposed in "Solution approach", and the model has been validated in "Computational results". We conclude our study in "Conclusion and future directions".

Literature review

Day by day, the number of people who have been attracted by supply chain network design (SCND) among supply chain researchers is increasing. Hiremath et al. (2012) proposed the design of an innovative and hybrid outbound logistics network for an automotive manufacturing supply chain. Their model's objectives were to minimize the total network cost, maximize the unit fill rate, and maximize the resource (facility) utilization subject to a host of capacity, demand, flow, and resource constraints. Singh et al. (2012) incorporated operational risks with Design of global supply chain network design. They proposed an integrated model based on a set of risk factors such as distribution risk, demand risk, supply risk, and interaction risk to evaluate the location of the plants and warehouses.

Babazadeh et al. (2012) proposed a new network design mathematical model for an agile supply chain. Melo et al. (2009) worked on optimization of supply chain performance by determining optimal location. Mousavi et al. (2013) considered a network design problem for a three-level supply chain and proposed a new mathematical model, where their aims were to determine the number of located distribution centers, their locations, capacity level, and allocating customers to distribution centers. Liu et al. (2010) proposed a non-linear programming to find the location of warehouses in supply chain; their problem objective was to minimize inventory costs with regard to online demand.

The competitive location problem is a renowned problem, in which costumers have many choices with different utilities. In this domain, Huff (1964) was the first who proposed spatial interaction models. He considered closeness as a factor for utility function of costumers. Ten years later Nakanishi and Cooper (1974) considered more aspects, and five years later, Jain and Mahajam (1979) differentiated between those aspects. They proposed two categorized, the first kind of aspects that were independent to costumers will, for example quality of service. In addition the second kind of aspects that were dependent to costumers will, for example closeness.

In this domain, Rahim et al. (2003) dealt with a competitive location production problem. Their goal was to examine how firms should select their production sites, capacities, and their quantities under competition.

In recent years Saidani et al. (2012) considered competitive facility location problem, in which a probabilistic Huff-like model has been used to prepare a mathematical model. One year later, Lüer-Villagra and Marianov (2013) considered price and location; they proposed a competitive hub location and pricing problem for the air passenger industry.

In the literature, there are some researchers who considered th e problem of location inventory. For instance, Rudi et al. (2001) proposed two location inventory models with transshipment. In these models, effects of transshipments between two independent locations also have been examined. Shen et al. (2003) dealt with a joint location problem, where a single supplier and multiple retailers have been considered.

Generally, in a supply chain, most of the parameters are not deterministic. For this reason, some researchers used queuing theory to construct their stochastic models. Pulut and Ulengin (2011) coordinated the inventory policies in a two-stage decentralized supply chain, where each supplier has been considered as an *M/M/1* queue and the manufacture has been assumed *GI/M/1*. Babai et al. (2010), considered demand and lead time as stochastic parameters and analyzed a single-echelon single-item inventory system by means of queuing theory (Seyedhoseini and R. Teimoury 2014) considered poison demand for customer in a cross-docking problem and used queuing theory to prepare a stochastic model.

Isotupa (2006) analyzed a lost sales (*s, Q*) inventory system with two types of ordinary and priority customers and exponentially distributed lead time. She considered two independent Poisson processes with different parameters for each type of customers. Then he used queuing theory to derive the expression of the long-run expected cost rate. Considering effectiveness of queuing theory in inventory problems, we also used queuing theory to develop a stochastic inventory control model.

Problem description

In this paper, the basic supply chain elements consist of a network with retailers, and customers, where retailers deliver costumers orders. Material flow in which network implemented is illustrated in Fig. 1. The considered problem deals with the decisions of where to optimally locate retailers, and the objective function is maximizing profits

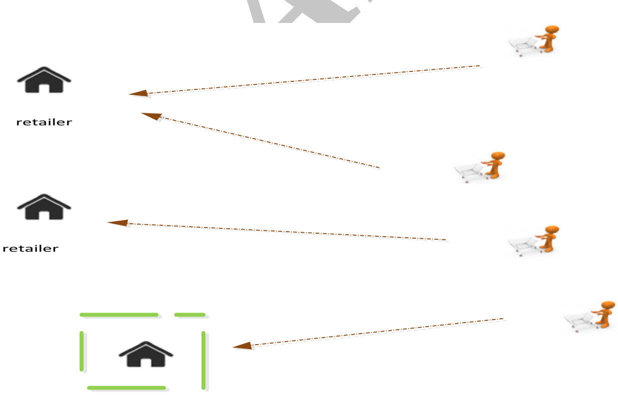

Fig. 1 A sample of proposed problem

by considering price of goods with regard to costs associated with establishing and inventory costs.

The main assumptions can be summarized as follows:

1. Distance and price are major factors in utility function of customers.
2. Supply chain proposes different kinds of products for customers.
3. Demand of customer k for product i has been assumed Poisson with rate of λ_{ik}.
4. Lead time for retailer j has been assumed exponentially distribution with mean value of $\frac{1}{\lambda_j}$.
5. Retailers use (*R, Q*) inventory control policy.
6. Candidate nodes for establishing retailers are fixed.
7. Locations of customers are fixed.

In a competitive environment, if there would be many retailers with different distances, they would have different utility for a costumer. To construct a perfect model we extended the Haf's utility to Eq. 1:

$$U_{kj} = A \times p_j^{-B} \times d_{jk}^{-C}, \tag{1}$$

where A, B, and C are constant, p_j represents price of retailer j, and d_{jk} represents distance between retailer j and costumer k. By considering Eq. (1), the probability of providing goods of product i from retailer j for costumer k can be defined as follows:

$$\frac{U_{kj}}{U_k + \sum_{q=1}^{N} U_{kq}}, \tag{2}$$

where U_k represents utility of other companies for customer k, and it has been assumed constant.

Considering following assumptions, a queue of inventories occurs in each retailer; for better description Fig. 2 demonstrates transition diagram for inventory system, and four lemmas have been represented to calculate different parameters of inventory system.

Henceforth, let π_i denotes steady-state probability of state I, D denotes retailer's demand rate, and λ denotes lead time rate. For this system, lemma 1 prepares steady state probabilities for the problem, when $0 < R < Q$.

Lemma 1 *Steady-state probabilities could be calculated as follows:*

$$\pi_I = \frac{\lambda}{D}\left(\frac{D+\lambda}{D}\right)^{I-1}(\pi_0) \quad \forall 1 \le I \le R+1 \tag{3}$$

$$\pi_I = \frac{\lambda}{D}\left(\frac{D+\lambda}{D}\right)^{R}(\pi_0) \quad \forall R+2 \le I \le Q \tag{4}$$

$$\pi_I = \sum_{k=0}^{R+Q-I}\left(\frac{\lambda}{D}\right)^2\left(\frac{D+\lambda}{D}\right)^{R-k-1}(\pi_0) \quad \forall Q+1 \le I \le R+Q \tag{5}$$

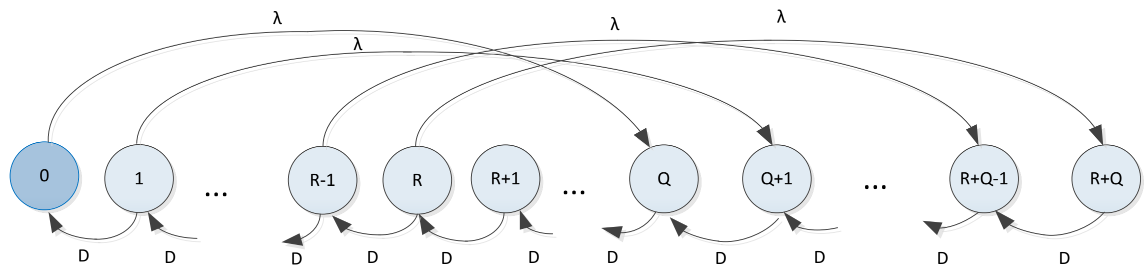

Fig. 2 Transition diagram for (R, Q) inventory system

Proof Equation (1) is clear but for I between $R + 2$ and Q there is

$$D(\pi_{R+1}) = D(\pi_{R+2}) = \cdots = D(\pi_Q) \tag{6}$$

Considering Eq. (1) and (4), Eq. (2) will be proved. When I is bigger than Q, interval flow must come from state $I + 1$ or $I - Q$; consequently the following expressions are true:

$$D(\pi_I) = \lambda(\pi_{I-Q}) + D(\pi_{I+1}) \tag{7}$$

$$D(\pi_{R+Q}) = \lambda(\pi_R) \tag{8}$$

Considering Eqs. (5) and (6), Eq. (7) could be derived, which proves Eq. (3).

$$D(\pi_I) = \sum_{k=0}^{R+Q-I} \frac{\lambda}{D}(\pi_{R-k}) \tag{9}$$

Also for this queue, lemma 2 calculates π_0.

Lemma 2 *In this queue π_0 is equal to $\pi_0 = \frac{1}{B}$, where B is equal to Eq. 10, and x is equal to $\frac{D+\lambda}{D}$.*

$$B = \left(1 + \frac{\lambda}{D}\left(\frac{1 - x^{\mathrm{R}}}{1 - x}\right) + (Q - R)\frac{\lambda}{D}.x^{\mathrm{R}}\right) + \left(\frac{\lambda}{D}\right)^2 \cdot \left(\frac{1 - x^{\mathrm{R}}}{(1 - x)^2} - \frac{\mathrm{R}\cdot x^{\mathrm{R}}}{1 - x}\right) \tag{10}$$

Proof It is clear that Eq. (10) is true.

$$\sum_{I=0}^{R+Q} \pi_I = 1 \tag{11}$$

When I is lesser than $R + 1$ Eq. (12) is true.

$$\sum_{I=0}^{Q} \pi_I = \left(1 + \frac{\lambda}{D}\left(\frac{1 - x^{\mathrm{R}}}{1 - x}\right) + (Q - R)\frac{\lambda}{D}\cdot x^{\mathrm{R}}\right)\pi_0 \tag{12}$$

And also when I is bigger than Q Eq. (13) is true.

$$\sum_{I=Q+1}^{R+Q} \pi_I = \sum_{j=1}^{R} j.\left(\frac{\lambda}{D}\right)^2 x^{j-1}\cdot\pi_0 = \left(\frac{\lambda}{D}\right)^2 \cdot \left(\frac{1 - x^{\mathrm{R}}}{(1 - x)^2} - \frac{\mathrm{R}\cdot x^{\mathrm{R}}}{1 - x}\right) \tag{13}$$

Considering Eqs. (10), (11), and (12) lemma 2 can be proved.For this queue, expected value for inventory can be calculated by lemma 2.

Lemma 3 *Expected value for length of proposing queue is*

$$I = \left[\left(\frac{(Q - R - 1)(Q + R)}{2}\right)\frac{\lambda}{D}x^{\mathrm{R}} + \frac{\lambda}{D}(Q) + \frac{\lambda}{\mathrm{D}}\left(1 + \frac{\lambda}{\mathrm{D}}(\mathrm{Q})\right)\cdot\left(\frac{1 - x^{\mathrm{R}}}{(1 - x)^2} - \frac{\mathrm{R}\cdot x^{\mathrm{R}}}{1 - x}\right) + \mathrm{Q}\cdot\left(\frac{\lambda}{\mathrm{D}}\right)^2 \cdot \frac{1 - x^{\mathrm{R}}}{1 - x} + \frac{1}{2}\cdot\left(\frac{\lambda}{\mathrm{D}}\right)^2 \cdot \frac{2 - (R+1)(R+2)x^R + 2(R)(R+2)x^{R+1} - (R+1)Rx^R}{(1 - x)^3}\right]\pi_0 \tag{14}$$

Proof From lemma 1, for I between Q and $R + Q$, we have

$$\pi_I = \frac{\lambda}{D}\sum_{j=I-Q}^{R} \pi_j \tag{15}$$

Consequently, Eq. (15) is true.

$$\sum_{I=0}^{R+Q} I\cdot\pi_I = \left(\frac{(Q-R-1)(Q+R)}{2}\right)\frac{\lambda}{D}x^R\cdot\pi_0 + \frac{\lambda}{D}(Q\cdot\pi_0) + \sum_{I=1}^{R}\left(I + \frac{\lambda}{D}\cdot\left((I+1)\cdot Q + \frac{I(I+1)}{2}\right)\right)\frac{\lambda}{D}x^{I-1}\cdot(\pi_0) \tag{16}$$

Considering Eq. (17), Eq. (14) can be derived.

$$\sum_{I=1}^{R} \frac{\lambda}{D}((I^2 + I)/2)\frac{\lambda}{D}\left(\frac{D+\lambda}{D}\right)^{I-1} = \frac{1}{2}\cdot\left(\frac{\lambda}{D}\right)^2 \cdot \frac{d}{dx^2}\left(\frac{x^2 - x^{R+2}}{1 - x}\right) \tag{17}$$

Lemma 4 *Expected value for shortage of retailer can be computed as follows:*

$$\mathrm{S_j} = \pi_0 \times \left(\frac{D}{\lambda}\right) \tag{18}$$

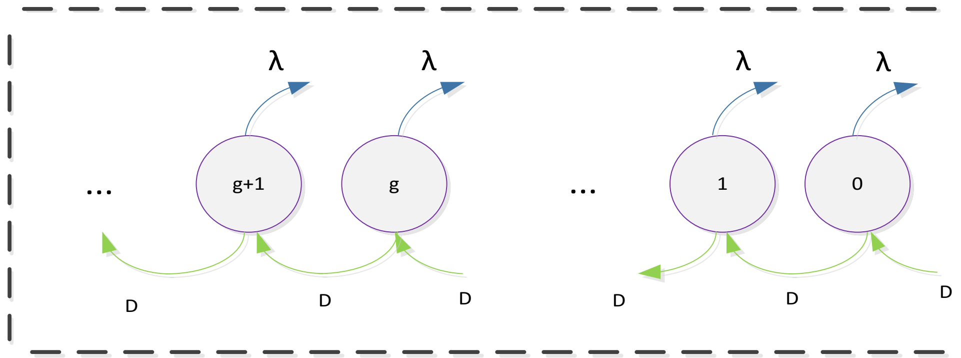

Fig. 3 Transition diagram for shortages states

Proof The shortage occurs when retailer has no inventory. In this section g has been used for representing state of shortage, and for computing expected value of shortage we decompose state zero to Fig. 3. For this queue, if π_g^s represents steady-state probability of g shortages, then it would be clear that

$$(D+\lambda)\pi_{g+1}^s = (D)\pi_g^s \quad \forall g \geq 0, \tag{19}$$

where π_g demonstrates steady-state probability for g shortages. Considering Eq. (19) π_g could be computed as follows:

$$\pi_g^s = \left(\frac{D}{D+\lambda}\right)^g \pi_0^s \tag{20}$$

By considering Eq. (20), Eq. (21) could be derived.

$$\sum_{g=0}^{\infty}\left(\frac{D}{D+\lambda}\right)^g \pi_0^s = \pi_0 \tag{21}$$

Also average shortage is equal to

$$\begin{aligned} S_j &= \pi_0^s\left(\left(\frac{D}{D+\lambda}\right)+2\left(\frac{D}{D+\lambda}\right)^2+\cdots+k\left(\frac{D}{D+\lambda}\right)^k+\ldots\right) \\ &= \pi_0^s z\left(1+2(z)^1+\cdots+k(z)^{k-1}+\cdots\right), \end{aligned} \tag{22}$$

where z is equal to $\frac{D}{D+\lambda_j}$. Also it is known that

$$\begin{aligned} &\left(1+2(z)^1+\cdots+k(z)^{k-1}+\cdots\right) \\ &= \frac{d}{dz}\left(z+(z)^2+\cdots+(z)^k+\cdots\right) = \frac{d}{dz}\left(\frac{z}{1-z}\right) \\ &= \frac{1}{(1-z)^2} \end{aligned} \tag{23}$$

By considering Eqs. (22) and (23), Eq. (18) could be derived. For this queue when Corruption rate be considered, when μ denotes corruption rate, shortage can be calculated by lemma 4, but for lemmas 1, 2, and 3, $\mu + D$ must be replaced with D.For this inventory system, if n represents number of business days in a year, demand of a year can be calculated by $n{\cdot}D$. This section presents a mathematical model to solve the problem described above.

Mathematical model

Objective:

$$\begin{aligned} \max z = &\sum_{i=1}^{M}\sum_{j=1}^{N}(p_{ij}-c_{ij}){\cdot}n{\cdot}D_{ij} - \sum_{j=1}^{N} T_j \times F_j \\ &- \sum_{i=1}^{M}\sum_{j=1}^{N} H_i \times I_{ij} - \sum_{i=1}^{M}\sum_{j=1}^{N} E_i \times \pi_{ij0} \times \left(\frac{D_{ij}}{\lambda_j}\right) \\ &- \sum_{i=1}^{M}\sum_{j=1}^{N} O \times \frac{D_{ij} \times n}{Q_{ij}} \end{aligned} \tag{24}$$

St:

$$\begin{aligned} \pi_{ij0} = 1/\Bigg[&\left(\left(1+\frac{\lambda_j}{D_{ij}}\left(\frac{1-x_{ij}^{R_{ij}}}{1-x_{ij}}\right)+(Q_{ij}-R_{ij})\frac{\lambda_j}{D_{ij}}\cdot x_{ij}^{R_{ij}}\right)\right. \\ &+\left(\frac{\lambda_j}{D_{ij}}\right)^2\cdot\left(\frac{1-x_{ij}^{R_{ij}}}{(1-x_{ij})^2}-\frac{R_{ij}.x_{ij}^{R_{ij}}}{1-x_{ij}}\right)\Bigg] \end{aligned} \tag{25}$$

$$\begin{aligned} I_{ij} = \Bigg[&\left(\frac{(Q_{ij}-R_{ij}-1)(Q_{ij}+R_{ij})}{2}\right)\frac{\lambda_j}{D_{ij}}x_{ij}^{R_{ij}}+\frac{\lambda_j}{D_{ij}}(Q_{ij}) \\ &+\frac{\lambda_j}{D_{ij}}\left(1+\frac{\lambda_j}{D_{ij}}(Q_{ij})\right)\cdot\left(\frac{1-x_{ij}^{R_{ij}}}{(1-x_{ij})^2}-\frac{R_{ij}.x_{ij}^{R_{ij}}}{1-x_{ij}}\right) \\ &+Q_{ij}\cdot\left(\frac{\lambda_j}{D_{ij}}\right)^2\cdot\frac{1-x_{ij}^{R_{ij}}}{1-x_{ij}}+\frac{1}{2}\cdot\left(\frac{\lambda_j}{D_{ij}}\right)^2 \\ &\cdot\frac{2-(R_{ij}+1)(R_{ij}+2)x_{ij}^{R_{ij}}+2(R_{ij})(R_{ij}+2)x_{ij}^{R_{ij}+1}-(R_{ij}+1)R_{ij}.x_{ij}^{R_{ij}}}{(1-x_{ij})^3}\Bigg]\pi_{ij0} \end{aligned} \tag{26}$$

$$x_{ij} = \frac{D_{ij}+\lambda_j}{D_{ij}} \tag{27}$$

$$D_{ij} = \sum_{k=1}^{K}\lambda_{ik}\left(\frac{U_{ikj}\times T_j}{U_{ik}+\sum_{j=1}^{N}U_{ikj}\times T_j}\right) \tag{28}$$

$$U_{ikj} = A \times p_{ij}^{-B} \times d_{jk}^{-C} \tag{29}$$

$$Q_{\min,i} \times \mathrm{T_j} \leq \mathrm{Q_{ij}} \tag{30}$$

$$T_j \leq R_{ij} \leq Q_{ij} \times T_j \tag{31}$$

Objective function has composed of five sections. First section relates to retailer's selling and preparation costs of goods, second section is for establishing costs, third section is for holding costs, fourth section relates to shortage costs, and fifth section calculates ordering costs for facilities.

Constraint 24 calculates steady-state probability of being in state zero for different products in retailer j. Constraints 25 and 26 compute steady-state inventory of item i in retailer j. Constraints 27 and 28 calculate demand of product i in retailer j. Constraint 29 ensures that if retailer j established, then its ordering quantity must be bigger than a specific value. Constraint 30 ensures that reordering point of a retailer must be smaller than ordering quantity.

Solution approach

Considering nonlinearity of the proposed model, a branch and bound algorithm has been proposed to solve the model.

Henceforth, let L denote set of potential locations which have been planned, and M denote set of potential locations which have not planned yet. If SI_{ij} denotes sum of holding costs, shortage costs, and ordering costs for product i in retailer j, "Appendix A" can be used to find minimum SI_{ij}. So Eq. (31) can prepare an upper bound for the model.

$$\sum_i \sum_j p_{ij} \cdot D_{ij} - \sum_{j \in A} T_j \times F_j - \sum_i \sum_{j \in A} \min\left(SI_{ij}\right) \tag{32}$$

The algorithm consists of a finite number of steps. In the first step, we construct a possible solution which is used as the initial lower bound. In this algorithm $(T_1,\ldots,T_j,\ldots,T_N)$ has been considered as a vector for establishing retailer.

1. (First step).
1.1 Put $j = 1$ and go to step 1.2.
1.2 Put $T_j = 1$ and, considering pervious established retailers use "Appendix A" to find optimal costs and profits and calculate $z\ (j)$ with Eq. (23).
1.3 If $z\ (j)$ is bigger than $z\ (j+1)$ go to step 1.3, and if it is lesser than $z\ (j+1)$, put $T_j = 0$, and then go to step 1.3.
1.4 If J is lesser than N add one to it and go to step 1.2, and if it is equal to N consider $z\ (N)$ as algorithms upper bound, and go to second step.
2. (The main step).
2.1 Put $t = 1$.
2.2 Prepare $2 \times N$ branches, and two branches emerge for retailer j, where T_j takes 1 or 0 and move retailer j from set M to set L. Go to 2.3.
2.3 Use Eqs. (32) and (33) to find demand of each retailer for set L and M for each branch. Go to 2.4.

$$D_{if} = \sum_{k=1}^{K} \lambda_{ik} \left(\frac{U_{ikf} \times T_f}{U_{ik} + \sum_{j \in A} U_{ikj} \times T_j + \sum_{j \in B} U_{ikj}} \right) f \in L \tag{33}$$

$$D_{if} = \sum_{k=1}^{K} \lambda_{ik} \left(\frac{U_{ikf}}{U_{ik} + \sum_{j \in A} U_{ikj} \times T_j + \sum_{j \in B} U_{ikj}} \right) f \in M \tag{34}$$

2.4 Considering demand of retailers for set L, find minimum SI_{ij} for product i and retailer j which is in set L. Go to 2.5.
2.5 Use Eq. (31) to calculate upper bound, and go to 2.6.
2.6 For branches that upper bound is lesser than lower bound, cut the branch; otherwise, go to step 2.7.
2.7 If $t = N$, the best solution is the branch with maximum lower bound, and go to Step 3. Otherwise, add one to t and go to step 2.8.
2.8 Choose the branch with minimum lower bound and prepare $2 \times (N-t-1)$ branches that could be emerging from it, in which another retailer takes $T = 1$ or 0 and moves from set M to L. Go to 2.3.
3. (Stop)

Lemma 5 *For location j in set M, if the following condition be satisfied, it could be moved to set L and T_j takes value of zero:*

$$\sum_i p_j \cdot D_{ij}^* - F_j - \sum_i \min\left(SI_{ij}\right) \leq 0, \tag{35}$$

where $\min(SI_{ij})$ uses Eq. 32 for its demand and D_j^* could be calculated by Eq. (35).

$$D_{ij}^* = \sum_{k=1}^{K} \lambda_k \left(\frac{U_{ikj}}{U_{ik} + U_{ikj} + \sum_{f \in A} U_{ikf} \times T_f} \right) \tag{36}$$

Proof If retailer j be established his demand will be lesser than Eq. (35) and also his demand will be more than Eq. (31); consequently, his selling profits could be smaller than $\sum_i p_j.D_{ij}^*$ and his inventory costs could be bigger than $\sum_i \min(SI_{ij})$. Lemma 5 has two major effects: first it reduces branches; second it could improve upper bound by increasing non established retailers.

Computational results

In this paper, using some numerical examples, performance of the proposed inventory model and the proposed branch and bound algorithm has been evaluated, and then efficiency of the model has been examined for a real case.

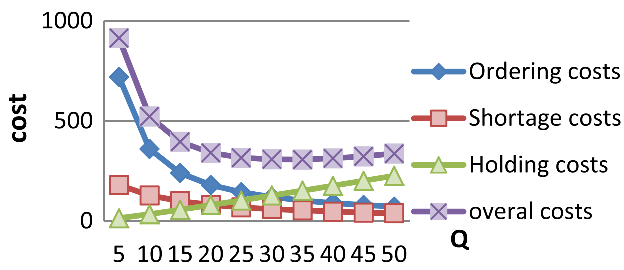

Fig. 4 Sensitivity of the model due to Q

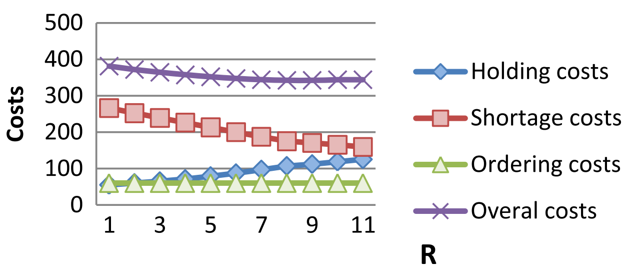

Fig. 5 Sensitivity of the model due to R

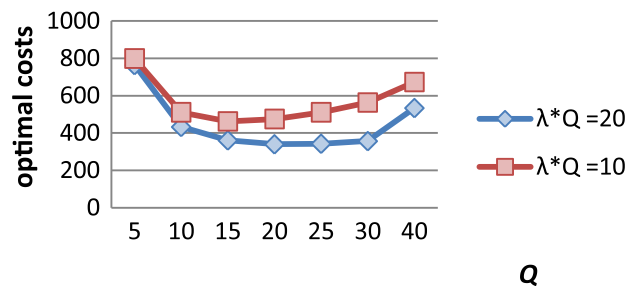

Fig. 6 Behavior of the model for constant $\lambda \times Q$

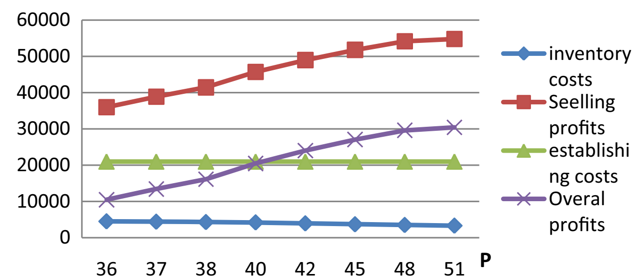

Fig. 7 Selling price effects

Numerical examples

For better description of model, an example has been produced where (H, P, D, O, R, λ) is equal to (10, 28, 10, 10, 3, 1), and behavior of inventory system has been examined for different values of Q in Fig. 4. For this example, shortage costs and ordering costs have negative gradients due to Q, but holding costs has positive gradient due to Q.

Previous example has been considered and behavior of the inventory system due to R has been examined in Fig. 5, when Q is equal to 20.

Sometimes value of λ depends on Q. For this reason, previous example has been considered when R is a variable and $\lambda \times Q$ is a constant value, and for different values of Q optimal costs have been illustrated in Fig. 6.

To demonstrate performance of the model and our B&B algorithm an example has been produced in "Appendix B". For this example, only the first retailer needs to be established, and optimal price of its item is equal to 51. Figure 7 also has been represented to analyze costs and profits due to selling price. Increasing in selling price decreases demand; consequently inventory costs would decrease. But selling profits is influenced by price and demand. So before $P = 51$, increase in selling price increases overall profits; however, overall profits decreases after $P = 51$.

In this research, proposed branch and bound has been coded in C++ software, and its performance has been evaluated in Table 1 For each size five examples have been proposed, where all parameters have been selected randomly while creating the data set, where $p_j \in [1, 10]$, $F_j \in [1, 10]$, $\lambda_j \in [1, 10]$, $H \in [1, 10]$ and $E \in [1, 10]$, and $O \in [10, 20]$. The results indicate the effectiveness of the proposed algorithm.

Real example

In this section, a real case has been represented. One of tangible examples for proposing model is clothing business. Albasco Company has 21 shops in Iran where two of them are located in Mashhad city. In Mashhad city there are two business competitors for Albasco that compete on price and quality. Albasposh Company has three shops in Mashhad and Poshiran Company has two shops in Mashhad city. In this competition, Albasco has better position, because its factory is located in Mashhad and its ordering costs is lesser than the others.

Albasco sells different kinds of clothing products, but each kind has constant price in different shops. For this reason we divided them into four categories: sports, menswear, ladies wear, and children clothes, and for each type of categories, we used average price, average demand, average holding cost, average shortage cost, and average ordering costs to apply the model. There are 48 metropolitan areas in Mashhad city, and demand densities are specific for each metropolitan area; for this reason center of each area has been assumed as a customer.

Experts in Albasco defined eight potential locations for their new shops in Mashhad city. For this case, the

Table 1 Performance of proposed branch and bound

Size	Average processing time (second)	Maximum processing time (second)
5	0.9	1
10	5.03	14
15	63.80	330.02
20	470.85	1,132.74

proposed model has been used, and Fig. 8 illustrates optimal locations for new shops regarding current shops.

For this problem two facilities are needed to be established in addition the previous shops. In these shops profits of selling goods are equal to 35353960000 and their inventory costs are equal to 505590000. If Albasco Company establishes these two shops, it would attract more than 50 % of demands. For more explanation, inventory results for this solution are condensed in Table 2.

Conclusion and future directions

In this paper, a competitive location model regarding inventory costs has been constructed, where two factors of distance and price have been considered for utility functions of customers. In this problem, location of facilities and their price affect the demand of each facility; subsequently, it affects inventory costs. So we prepared a model which could optimize location of facilities and inventory systems simultaneously.

In one idea, location problem is a strategic decision, and it could not be considered with inventory decisions. But it is obvious that they could impact each other in long-run planning. For this reason, an (*R*, *Q*) inventory model has been developed, and to prepare a cohesive model, demand and lead time have been considered as stochastic parameters, and queuing theory has been used to calculate average inventory costs. Because of its nonlinearity, the proposed

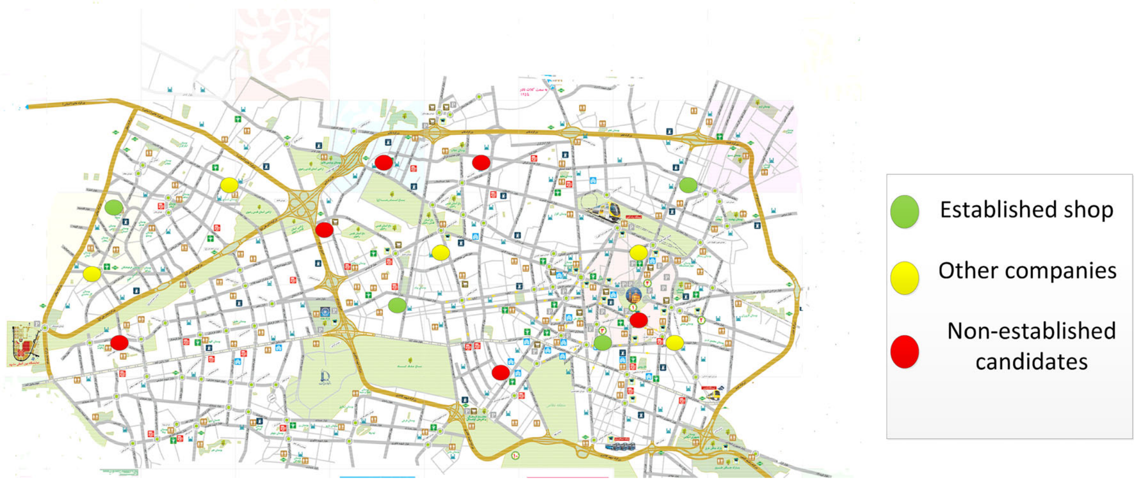

Fig. 8 Solution of the model regarding to current shops

Table 2 Description of inventory system for each established shop

Established facilities	Facility 1	Facility 2	Facility 3	Facility 4
D1	300	220	350	180
D2	200	150	230	125
D3	150	110	190	90
D4	250	150	280	145
R_{1j}	122	67	161	26
Q_{1j}	1,481	1,223	1,631	1,200
R_{2j}	64	33	85	20
Q_{2j}	1,087	914	1,184	820
R_{3j}	33	12	58	4
Q_{3j}	914	761	1,053	678
R_{4j}	87	25	108	23
Q_{4j}	1,322	973	1,418	954
Optimal inventory costs ($\times 10^4$)	13,866	11,173	15,315	10,205
Profit of facility ($\times 10^4$)	883,634	682,327	1,369,185	600,250

model is also hard to solve; consequently, we represented a branch-and-bound algorithm to find optimal solution.

In this paper, the inventory model has been analyzed due to Q and R, and their behaviors have been discovered. Also behavior of the inventory system has been examined when $\lambda \times Q$ is a constant value. We also proposed a branch and bound algorithm to solve the problem, and the results demonstrated efficiency of the algorithm.

In this paper, Albasco Company has been used as a real case to evaluated performance of the model in real-word, which demonstrated efficiency of our model. For future studies, this research can be extended by considering back order shortage; this may increase complexity of the problem but the model would become more realistic. Another extension of this research is possible by considering rate of corruption for perishable inventories.

Appendix A

In the continuous space SI_{ij} is convex due to Q_{ij} and R_{ij}; for this reason we supposed that R_{ij} and Q_{ij} are continuous and optimal Q_{ij} and R_{ij} have been calculated with the Steepest Ascent method. If Q_l and R_l denote solution of Gradient Search Procedure, then Fig. 9 demonstrates the optimal solution in discrete space.

Considering Fig. 9, for the discrete problem, nearest points in any direction to the optimal solution are $(|Q_l|, |R_l|)$, $(|Q_l|, |R_l| + 1)$, $(|Q_l| + 1, |R_l|)$, $(|Q_l| + 1, |R_l| + 1)$. Consequently, a near-optimal solution for discrete problem can be found in these points.

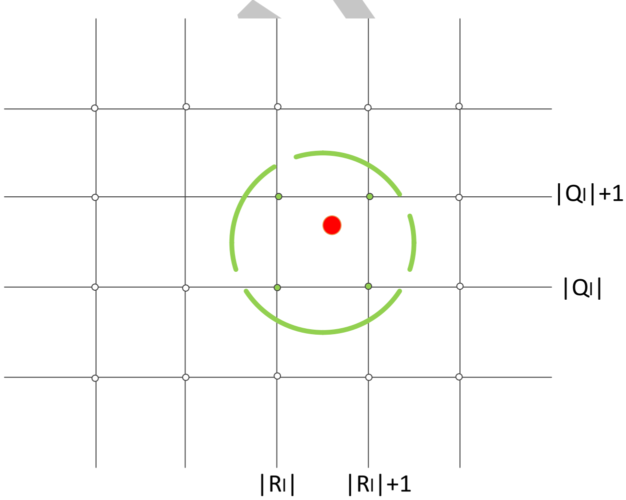

Fig. 9 Solution of discrete problem

Appendix B

See Tables 3, 4 and 5.

Table 3 Inventory data of each retailer

Retailers	1	2	3
F	21,000	24,000	23,000
H	20	7	15
E	48	30	35
λ_j	13	11	12
O	100	110	95

Table 4 Distance between each pair of customers and retailers

Distance among customers and retailers	Customers	1	2	3	4
Retailers	1	1	1	2	2
	2	3	2	3	1
	3	3	1	2	2

Table 5 Parameters of price model

A	1,000
B	3
C	2

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