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ORIGINAL RESEARCH

A mathematical model for the design of distributed layout by considering production planning and system reconfiguration over multiple time periods

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Abstract In this paper, we develop a new mathematical model that integrates layout configuration and production planning in the design of dynamic distributed layouts. The model incorporates a number of important manufacturing attributes such as demand fluctuation, system reconfiguration, lot splitting, work load balancing, alternative routings, machine capability and tooling requirements. In addition, the model allows several cost elements to be optimized in an integrated manner. These costs are associated with material handling, machine relocation, setup, inventory carrying, in-house production and subcontracting needs. Numerical examples of different sizes are presented to illustrate the nature of the developed model and shed light on several managerial insights.

Keywords Distributed layout · Dynamic reconfiguration · Production planning · Mixed integer linear programming

Introduction

Manufacturing systems that produce multiple components and function in highly volatile environments are increasingly challenged to meet consistently high levels of operational efficiency and flexibility. Such a challenge can be addressed partly by designing appropriate facility layouts. Planning good layouts is critical; in the United States alone over 250 billion USD is spent annually on plant layouts that require planning and replanning (Tompkins et al. 1996). Furthermore, between 20 and 50 % of costs within manufacturing are related to material handling. Effective and innovative facility planning can reduce material handling costs by 10–30 % (Tompkins et al. 1996). Not surprisingly, a large number of articles on facility layout have been published, with the majority focusing on product layout, functional layout, cellular layout or their variants. However, there is an emerging consensus that these layout types are not suitable for factories where multiple components are produced in highly volatile environments (Benjaafar et al. 2002). Generally, these layouts are developed assuming stable demand and product mix for a considerably long planning horizon.

Distributed layout has emerged as an alternative to conventional layouts. In a distributed layout, similar departments (machines) are distributed throughout the factory floor to increase access to these resources from different regions of the layout (Baykasoglu 2003). This type of layout minimizes material handling costs because it enables the identification of efficient routes for a large number of product mixes. The idea to disaggregate functional departments into individual machines and maximize distribution by placing them as far from each other as possible was first proposed by Montreuil and Venkatadri (1991). Urban and Russel (2000) proposed a model that does not require machines to be placed in a functional layout or in a cellular arrangement, but instead allows material flow requirements to dictate machine placement. Benjaafar and Sheikhzadeh (2000) explored layout configuration in stochastic environments and showed that there is a value in creating replicates of the same department and distributing them throughout the plant floor. Drolet (1989) investigated a distributed layout configuration where virtual cells are formed and temporarily devoted to job orders. The application of distributed layout in virtual cellular

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manufacturing systems was also illustrated in Baykasoglu (2003). Lahmer and Benjaafar (2005) presented a procedure for the design of distributed layouts in settings with multiple periods where product demand and product mix may vary from period to period. Using simulation, Krishnan et al. (2009) analyzed several performance parameters in distributed layouts under stochastic conditions.

Another aspect of facility design prescribed to address the challenges of meeting high operational efficiency and flexibility in highly volatile environments is dynamic system reconfiguration (DSR). An early paper on modeling a multi-period dynamic functional layout where departments can be relocated was published by Rosenblatt (1986). More recent works that attempt to solve dynamic functional layout problems include Balakrishnan et al. (2000), Dunker et al. (2005), Baykasoglu et al. (2006), McKendall and Shang (2006), McKendall et al. (2006) and Pillai et al. (2011). There is also substantial literature on DSR in the context of cellular manufacturing systems (CMS). For example, Chen (1998) and Balakrishnan and Cheng (2005) developed mathematical models for DSR of cellular manufacturing systems and proposed dynamic programming approaches to solve their respective models. Defersha and Chen (2006a) developed a comprehensive model that incorporates several design factors in addition to dynamic cell configuration. Later, in Defersha and Chen (2006b), the authors developed a genetic algorithm to solve the comprehensive model. The use of a genetic algorithm, simulated annealing and Tabu search for DSR of cellular manufacturing systems was also reported in Wicks and Reasor (1999), Mungwattana (2000), Tavakkoli et al. (2005b) and Tavakkoli et al. (2005a). A solution technique based on artificial neural network can be found in Saidi and Safaei (2006). Dynamic reconfiguration of distributed layouts has been reported in Lahmer and Benjaafar (2005). Some technological advances are also enabling DSR. For instance, a compact and mobile milling machine (TRAK QuikCell QCM-1) developed by Southwestern Industries (http://www.southwesternindustries.com) is small enough to fit through most doors and has a rigid frame that does not require re-leveling after each move. A shift to lighter machine tools is being driven by advances in materials and processing technologies (Heragu and Kochhar 1994). As well, there are now systems, such as robotic parkings (http://www.roboticparking.com) modular automated parking system, that allow easy storage and retrieval of large equipment and machine tools. Although originally designed for car parking garages, the technology is being used in manufacturing environments to store machine tools and retrieve them as needed (Benjaafar et al. 2002).

As emerging technologies increasingly support reconfiguration, the objective of layout design is shifting from long-term material handling efficiency to short-term responsiveness. Managers can focus on operational performance by reconfiguring layouts more frequently to relieve short-term congestion, and maximizing throughput for current product mix and demand. Thus, the nature of decisions on layout reconfiguration is becoming more tactical than strategic. In this light, integrating DSR with tactical decisions such as production planning is a sensible approach. Such an integration has been reported in the context of cellular manufacturing in Nsakanda et al. (2006), Defersha and Chen (2008) and Ahkioon et al. (2009). This research paper presents a comprehensive model that combines distributed layout, dynamic reconfiguration and production planning. The model incorporates these factors: sequence of operations, alternate part routings, machine capability, machine capacity, workload balancing and lot splitting. This work is also related to the growing body of literature on multi-cost-objective layout design. The majority of approaches that address distributed layout design problems tend to minimize material handling costs only. These include Benjaafar and Sheikhzadeh (2000), Baykasoglu (2003) and Lahmer and Benjaafar (2005). However, when systems reconfiguration and production planning are considered concurrently, the actual problem involves other costs associated with machine relocation, setup, inventory holding, in-house production and subcontracting needs (See Tables 1 and 2 for a comparison between this paper and recently published articles on distributed layout).

In the model proposed in this paper, we use the concept of resource elements (REs) to capture alternative routings for processing parts. The concept was first introduced in Gindy et al. (1996) as a means of defining alternative routings by analyzing shared and unique capabilities of machine tools. In the REs approach, a potential machining operation is called a form generating schema (FGS). An FGS is a technologically meaningful combination of a cutting tool with a specific geometry, a set of relative motions between the part and the cutting tool, and the typical levels of technological output that can be associated with using that combination of tool and relative motion (Gindy et al. 1996; Baykasoglu 2003). Each resource element represents a collection of FGSs, and a machine tool is identified by the set of REs it possesses. Machine tools which possess a resource element required by a particular operation are considered as alternative routing to process this operation.

In addition to capturing alternative routing, we innovatively use the concept of REs as a basis to impose workload balancing among resources. In most previous studies considering workload balancing, a workload had to be evenly divided among machines that were deemed similar (though not necessarily identical). We approach workload balancing in a different way. A workload calling for a particular resource element is to be evenly divided among the machines that have this RE. For example, consider a system that has four machine tools (M1, ..., M4) having a total of five resource elements (RE1,…,RE5). Now assume machine tools M1 and M2 have RE1 and RE2; machine tool M3 has RE1, RE2, RE3 and RE4; and machine tool M4 has RE3, RE4 and RE5. In our approach, we impose a constraint such that the workload using RE1 and RE2 is to be evenly distributed among M1, M2 and M3 which have these resource elements; a workload using RE3 and RE4 is to be evenly divided among M3 and M4; and finally, a workload using RE5 is to be entirely performed on M4 as this is the only machine having RE5. Therefore, the workloads of the two identical machines (M1 and M2) are balanced; the workload performed on M3 is greater than the individual loads on M1 and M2 because M3 has more Res; and there is not a workload balancing constraint between pairs of dissimilar machines (say M1 and M4). In other words, in our model workload balancing is (1) fully enforced among identical machines, (2) partially enforced among machines having some shared capabilities, and (3) not enforced among dissimilar machines.

The remainder of this paper is organized as follows: in Sect. 2, we provide the problem description and the proposed mathematical model. Numerical examples are presented in Sect. 3 to illustrate the features of the developed model. The discussion and conclusion make up Sect. 4.

Mathematical model

Problem description

Consider a manufacturing system processing P products in T number of equal planning periods where the demand for the products may vary from period to period deterministically. The system consists of M machines to be distributed over N distinct locations $(N = M)$ and reconfiguration may take place at the beginning of each planning period. There are a total of R resource elements, and each machine has some of these REs, representing the capabilities it shares with other machines, as well as those that are unique to it. Processing a part requires a set of operations to be performed in a given sequence. A particular operation can be performed using a given resource element, and machines possessing this element are considered as alternative routes for this operation. The processing time for each operation is known. In a given time period, a demand for a part can be satisfied by producing it in-house, subcontracting its production, or using inventory carried over from the previous period. Without loss of generality, we assume a part inventory is zero at the beginning of the first period and at the end of the last period. A production lot of a part may be split into smaller sublots that are to be processed independently. The material flow cost of a part is linearly related to the distance it travels using the material handling system. The cost to relocate a machine is also assumed to be linearly related to the relocation distance. However, we assume that the distance between a pair of locations when moving a part is not the same as the distance between the same pair of locations when relocating a machine. This is because parts are moved using a material handling system (e.g., AGV with a specified path), whereas machines are relocated in a different way. The workload of the system in a given time period is evenly distributed among the machine tools that share the particular resource element being used. The overall objective is to minimize the total

Table 1 List of manufacturing attributes

1. Alternative routing	8. Production planning
2. Demand fluctuation	9. Setup cost
3. Dynamic system reconfiguration	10. Movement of parts (material handling cost)
4. Workload balancing	11. Machine capacity
5. Lot splitting	12. Subcontracting cost
6. Types of tools required by a part	13. Operation cost
7. Types of tools available on a machine	

Table 2 Attributes used in the present study and in a sample of recently published articles

Article/Attributes	1	2	3	4	5	6	7	8	9	10	11	12	13
Present study (this paper)	×	×	×	×	×	×	×	×	×	×	×	×	×
Nageshwaraniyer et al. (2013)		×	×							×			
Hamedi et al. (2012)		×				×	×			×	×		
Lahmer and Benjaafar (2005)		×	×							×	×		×
Baykasoglu (2003)		×				×	×			×	×		
Urban and Russel (2000)		×								×	×		
Benjaafar and Sheikhzadeh (2000)		×	×							×	×		×

Attributes' names are referred in Table 1 costs associated with material handling, machine relocation, subcontracting, setup, inventory holding and internal part production.

Notation

The problem described in the previous section is formulated as a mixed integer linear programming. The notations used in this formulation are presented below.

Indexes and input data

- T Number of equal planning periods where planning periods are indexed by $t = 1, 2, \ldots, T$.
- P Number of products where products are indexed by $p = 1, 2, \ldots, P$.
- O_p Number of operations required by product p where operations are indexed by $o = 1, 2, \ldots, O_p$.
- N_p Maximum number of sublots of product p in a given time period where production sublots are indexed by $n = 1, 2, \ldots, N_p$.
- M Number of machines in the manufacturing facility where machines are indexed by $m = 1, 2, \ldots, M$.
- R Number of resource elements in the manufacturing facility where resource elements are indexed by $r = 1, 2, \ldots, R$.
- L Number of locations at which machines are installed, where locations are indexed by $l = 1, 2, \ldots, L$.
- J Number of groups of machines with similar functionality where groups are indexed by $j = 1, 2, \ldots, J$.
- C Length of a planning period in terms of available work time in minutes.
- $D_{p,t}$ Demand quantity for product p in time period t.
- Θ_p Unit cost of producing product p in-house (not including setup).
- $\hat{\Theta}_p$ Unit cost of subcontracting product p.
- H_p Unit inventory holding cost per period for product p.
- F_p Material handling cost per unit distance for one unit of product p.
- $U_{o,p}$ Unit processing time for operation o of product p.
- $A_{r,m}$ A binary datum which equals 1 if resource element r is available on machine m; 0 otherwise.
- $B_{r,o,p}$ A binary datum which equals 1 if resource element r is required by operation o of product p; 0 otherwise. An operation requires only a single resource element and machines having this resource element are considered as alternative routing for this operation.
- $K_{o,p,m}$ A binary datum which equals 1 if operation o of product p can be processed on machine m; 0 otherwise. $K_{o,p,m} = \sum_{r=1}^{R} (A_{r,m} \times B_{r,o,p})$.
- $E_{l,l'}$ Machine relocation distance between locations l and l'.
- $\tilde{E}_{l,l'}$ Material handling distance between locations l and l'.
- G_m Relocation cost per unit distance for machine m.
- S_p Setup cost for processing a sublot of product p.
- Υ Workload balancing factor in (0, 1). This factor is chosen to be very close to 1 to impose workload balancing.
- Ω Large positive number.

Variables:

Continuous Variables:

- $v_{p,t}$ Production lot size of product p in time period t.
- $b_{n,p,t}$ The size of the nth sublot of product p in time period t.
- $\hat{v}_{p,t}$ Volume of product p subcontracted in time period t.
- $\delta_{o,n,p,m,t}$ The time elapsed in processing operation o of the nth sublot of product p on machine m in time period t.
- $h_{p,t}$ Inventory level of product p at the beginning of period t.
- $d_{o,n,p,t}$ Distance between the locations where operations o and $o+1$ of nth sublot of product p are processed multiplied by the sublot size $b_{n,p,t}$ in time period t.
- $e_{m,t}$ Distance between the location of machine m in period $t-1$ and its location in period t.

Binary Variables:

- $\alpha_{m,l,t}$ A binary variable equal to 1 if machine m is located at location l in time period t; 0 otherwise.
- $\gamma_{o,n,p,m,t}$ A binary variable equal to 1 if operation o of the nth sublot of product p is processed by machine m in time period t; 0 otherwise.
- $y_{n,p,t}$ A binary variable equal to 1 if nth sublot of product p is created and processed in time period t; 0 otherwise.

Objective function and constraints

Following the problem description and notation given in Sects. 2.1 and 2.2, the comprehensive mathematical model for distributed layout manufacturing system design is presented below.

Minimize:

$$Z=\sum_{t=2}^{T}\sum_{m=1}^{M}(G_m\cdot e_{m,t})+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{n=1}^{N_p}\sum_{o=1}^{O_p-1}(F_p\cdot d_{o,n,p,t})$$
$$+\sum_{t=1}^{T}\sum_{p=1}^{P}(H_p\cdot h_{p,t})+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{n=1}^{N_p}(S_p\cdot y_{n,p,t})$$
$$+\sum_{t=1}^{T}\sum_{p=1}^{P}(\Theta_p\cdot v_{p,t})+\sum_{t=1}^{T}\sum_{p=1}^{P}(\hat{\Theta}_p\cdot \hat{v}_{p,t}) \quad (1)$$

Subject to:

$$e_{m,t}\geq E_{l,l'}+\Omega(\alpha_{m,l,t-1}+\alpha_{m,l',t})-2\Omega;$$
$$\forall(m,t,l,l')|t>1 \quad (2)$$

$$e_{m,t}\leq E_{l,l'}-\Omega(\alpha_{m,l,t-1}+\alpha_{m,l',t})+2\Omega;$$
$$\forall(m,t,l,l')|t>1 \quad (3)$$

$$d_{o,n,p,t}\geq \widetilde{E}_{l,l'}\cdot b_{n,p,t}+\Omega\big(\alpha_{m,l,t}+\gamma_{o,n,p,m,t}+\alpha_{m',l',t}+\gamma_{o+1,n,p,m',t}\big)-4\Omega;$$
$$\forall(o,n,p,t,m,m',l,l')|(o<O_p\,\&\,K_{o,p,m}\times K_{o+1,p,m'}=1) \quad (4)$$

$$d_{o,n,p,t}\leq \widetilde{E}_{l,l'}\cdot b_{n,p,t}-\Omega\big(\alpha_{m,l,t}+\gamma_{o,n,p,m,t}+\alpha_{m',l',t}+\gamma_{o+1,n,p,m',t}\big)+4\Omega;$$
$$\forall(o,n,p,t,m,m',l,l')|(o<O_p\,\&\,K_{o,p,m}\times K_{o+1,p,m'}=1) \quad (5)$$

$$v_{p,1}+\hat{v}_{p,1}=D_{p,1}+h_{p,2};\quad \forall(p) \quad (6)$$

$$v_{p,t}+h_{p,t}+\hat{v}_{p,t}=D_{p,t}+h_{p,t+1};\quad \forall(p,t)|(1<t<T) \quad (7)$$

$$v_{p,T}+h_{p,T}+\hat{v}_{p,T}=D_{p,T};\quad \forall p \quad (8)$$

$$\sum_{p=1}^{P}\sum_{n=1}^{N_p}\sum_{o=1}^{O_p}\delta_{o,n,p,m,t}\leq C;\quad \forall(m,t) \quad (9)$$

$$\delta_{o,n,p,m,t}\geq U_{o,p}\cdot b_{n,p,t}+\Omega\cdot(\gamma_{o,n,p,m,t}-1);\quad \forall(o,n,p,m,t)|(K_{o,p,m}=1) \quad (10)$$

$$\delta_{o,n,p,m,t}\leq U_{o,p}\cdot b_{n,p,t}-\Omega\cdot(\gamma_{o,n,p,m,t}-1);\quad \forall(o,n,p,m,t)|(K_{o,p,m}=1) \quad (11)$$

$$\delta_{o,n,p,m,t}\leq \Omega\cdot\gamma_{o,n,p,m,t};\quad \forall(o,n,p,m,t)|(K_{o,p,m}=1) \quad (12)$$

$$\gamma_{o,n,p,m,t}\leq K_{o,p,m};\quad \forall(o,n,p,m,t) \quad (13)$$

$$\sum_{p=1}^{P}\sum_{n=1}^{N_p}\sum_{o=1}^{O_p}B_{r,o,p}\times\delta_{o,n,p,m,t}\geq\left(\frac{\sum_{m''=1}^{M}\sum_{p=1}^{P}\sum_{n=1}^{N_p}\sum_{o=1}^{O_p}B_{r,o,p}\times\delta_{o,n,p,m'',t}}{\sum_{m'=1}^{M}A_{r,m'}}\right)\times\Upsilon;$$
$$\forall(r,m,t) \quad (14)$$

$$\sum_{m=1}^{M}\gamma_{o,n,p,m,t}=y_{n,p,t};\quad \forall(o,n,p,t) \quad (15)$$

$$b_{n,p,t}\leq\Omega\cdot y_{n,p,t};\quad \forall(n,p,t) \quad (16)$$

$$\sum_{n=1}^{N_p}b_{n,p,t}=v_{p,t};\quad \forall(p,t) \quad (17)$$

$$\sum_{l=1}^{L}\alpha_{m,l,t}=1;\quad \forall(m,t) \quad (18)$$

$$\sum_{m=1}^{M}\alpha_{m,l,t}=1;\quad \forall(l,t) \quad (19)$$

$$\alpha_{m,l,t}, *\gamma_{\omega,\nu,\pi,\mu,\tau},\ \theta_{\nu,\pi,\tau}\ \alpha\rho\varepsilon\ \ \beta\iota\nu\alpha\rho\theta\bullet \quad (20)$$

The objective function in Eq. (1) consists of six cost terms: machine relocation, material handling, inventory holding, machine setup, in-house production, and subcontracting needs in that order. The constraints in Eqs. (2) and (3) are to equate the variable $e_{m,t}$ to the distance $E_{l,l'}$ if machine m is relocated from location l to location l' at the beginning period t. The value of the variable $d_{o,n,p,t}$ is equal to the product $\widetilde{E}_{l,l'}\cdot b_{n,p,t}$ if operations o and $o+1$ of nth sublot of product p are processed on machines m at location l and m' at location l', respectively, in period t. This requirement is enforced by Eqs. (4) and (5). The constraints in Eqs. (6), (7) and (8) are for inventory balance. Equation (9) guarantees that the workload on machine m in time period t is less than or equal to the available time C. Equations (10) and (11) state that the time $\delta_{o,n,p,m,t}$ elapsed in processing operation o of the nth sublot of product p on machine m in time period t is equal to the product $U_{o,p}\cdot b_{n,p,t}$ if this operation is assigned to this machine in this time period. Otherwise, the value of this variable is set to zero by Eq. (12). The constraint in Eq. (13) permits the processing of operation o of sublot n of product p on machine m in time period t if and only if operation o of product p can be assigned on machine m. The workload balancing constraint is in Eq. (14). The left-hand side of this equation is the amount of workload performed by machine m in period t using resource element r. The right-hand side of this constraint is expressed as (i) the total workload of all the machines using resource element r which equal to $\sum_{m''=1}^{M}\sum_{p=1}^{P}\sum_{n=1}^{N_p}\sum_{o=1}^{O_p}B_{r,o,p}\times\delta_{o,n,p,m'',t}$ (ii) divided by the number of machines having this resource element $\sum_{m'=1}^{M}A_{r,m'}$ and (iii) multiplied by a factor $\Upsilon\in(0,1)$. If this factor is set very close to 1, the workload of the system in using resource element r will be evenly distributed among the machines having this resource element. Equation (15) ensures the assignment of the oth operation of the

Table 3 Comparison between distributed and functional layouts in Problem 1

Objective function values				
	Levels of sharing processing capabilities(REs)			
	Case 1	Case 2	Case 3	Case 4
DL1	322,120	399,395	830,240	764,405
DL2	286,450	308,695	764,215	829,820
DL3	389,805	465,485	746,280	805,985
DL4	276,190	344,705	796,730	746,250
DL5	288,405	365,225	591,335	794,100
Average	312,549	376,701	745,760	788,112
Functional	857,625	882,465	907,890	904,005
Percentage saving	63.65	57.31	17.85	12.82

*n*th sublot of part p in time period t to one of the machines if the sublot is created. The constraint in Eq. (16) ensures that the production quantity of each sublot in each time period, $b_{n,p,t}$, is equal to 0 if this sublot is not created (i.e., $y_{n,p,t} = 0$). The constraint in Eq. (17) enforces that the sum of the sizes of the sublots of a given product should be equal to the production lot size of that particular product in each period. The constraints in Eqs. (18) and (19) ensure that each location is assigned to only one machine and each machine is assigned to only one location. Equation (20) is the integrality constraint on the binary variables.

Numerical examples

Since the comprehensive problem addressed in this paper has not been previously presented, we have no comparable examples from the literature to use. Therefore, we generated several data sets to illustrate the problem and demonstrate the performance of the proposed solution procedure. One of these data sets (referred to as Problem-1) is provided in detail in Appendix 1. In this data set, we considered a system composed of 20 resource elements and 22 machine tools. Table 8 shows four different cases in which each of 20 REs is available on one or more machines. More specifically, case 1 represents a situation in which a particular RE is available on several machine tools; case 4 represents a situation where most of the machines have unique capabilities; and cases 2 and 3 lie in between the two extremes. The average number of machines per RE in these four cases is 4.55, 2.65, 1.5, and 1.1, respectively. In Table 9 are the model parameters (Θ_p, $\hat{\Theta}_p$, H_p, F_p, S_p, N_p, O_p), the index of the required resource element r for each operation, and the processing time $U_{o,p}$. The demands for the parts in four planning periods are provided in Table 10. The relocation cost G_m for each machine type m is in Table 11.

The layout showing potential machine locations in Problem-1 is provided in Fig. 3. Although the proposed model can address any type of layout shape and material handling system, we prefer to adopt a system served by automated guided vehicles (AGVs) arranged in tandem configuration. AGVs are preferable to stationary material handling robots because of their mobility, and to conveyors because of their flexibility (Asef and Laporte 2005). An AGV system can be reconfigured to accommodate changes in production volume, product mix, product routing, and equipment interface requirements more readily than most other material handling systems (Goetz and Egbelu 1990). In Table 12, we provide the locations of machines in an arbitrarily generated functional layout (where similar machines are placed in close proximity) and five arbitrarily generated distributed layouts (DL1, ..., DL5). The material handling and machine relocation distances between each pair of locations are shown in Tables 13 and 14, respectively.

In Problems 2 to 6, we considered the processing of 35, 50, 65, 80, and 120 parts, respectively. The maximum number of operations per part was six (in Problems 2, 3 and 4) and eight (in Problems 5 and 6). However, because

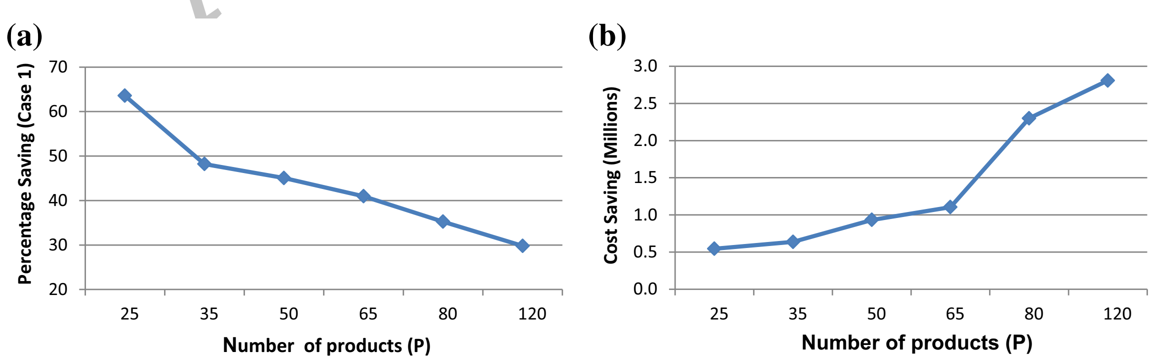

Fig. 1 Cost saving in moving from functional to distributed layout in Problems 1–6 under case 1. **a** Percentage saving, **b** saving in monetary units

Table 4 Dynamic versus static distributed layouts in Problem 1

Total costs				
	Levels of sharing machines capabilities (REs)			
	Case 1	Case 2	Case 3	Case 4
SDL	254,135	305,605	412,680	495,315
DDL	239,582	290,827	313,988	335,991
Saving %	5.76	4.83	23.91	32.16

SDL static distributed layout, *DDL* dynamic distributed layout

Table 5 Dynamic versus static distributed layouts in Problems 2–6

Problem no.	Case 1	Case 4
2	0.0	21.3
3	2.4	17.1
4	2.6	21.0
5	0.0	13.0
6	1.3	16.3

Problems 2 to 6 were similar in nature with Problem 1, we do not provide their detailed data sets in this article. Moreover, because our main aim in this paper is to present a comprehensive model for the design of distributed layouts, we do not include the simulated annealing algorithm that was used to solve the problems.

Functional versus distributed layout

The aim of this section is to illustrate the greater effectiveness of using distributed layouts compared to using a functional layout in a situation where there are machine tools with overlapping capabilities. To draw a fair comparison between these two arbitrarily generated layouts, we did not optimize machine allocations in either case. Machines that share capabilities (having common REs) were placed in close proximity in the functional layout, and were distributed arbitrarily in the distributed layouts. Recall that our intention in solving Problem-1, which poses four levels of overlapping capabilities (Table 8) and six layouts (Table 12), was to optimize material handling and other cost elements. Table 3 indicates that using distributed layouts results in significant savings. It is important to note the remarkably large cost reduction in case 1. These savings reflect the significant reduction in material handling costs that results when several machine tools with a number of shared capabilities are distributed, making their capabilities easily accessible from different regions of the layout. As we expected, the reduction in cost savings decreases as we move from case 1 to case 4. Our study thus shows that distributed layouts would be highly desirable in situations where there are many machine tools with several shared capabilities. Given that many modern manufacturing facilities contain a variety of machine tools with similar and overlapping capabilities able to produce a wide spectrum of components (Gindy et al. 1996), distributed layouts are more relevant than ever.

The cost savings under case 1 in Problems 2 to 6 appear in Fig. 1. The first graph (graph-a) shows that the percentage of savings decreases as the number of parts increases when using distributed layouts. However, since larger problems incur higher production costs, the monetary value of the savings rapidly increases as problems grow in size (see graph-b), making distributed layouts very appealing.

Static versus dynamic distributed layout

In this section, we compare static versus dynamically reconfigured distributed layouts in four different cases of Problem-1 (as described in the previous section) and several other problems. We solved the problems by prohibiting dynamic reconfiguration. Hence, in a static distributed layout, machine allocation is optimized to provide a robust layout which remains unchanged for the entire planning horizon. Table 4 provides the values of the objective function in the four cases of Problem-1, and the percentage

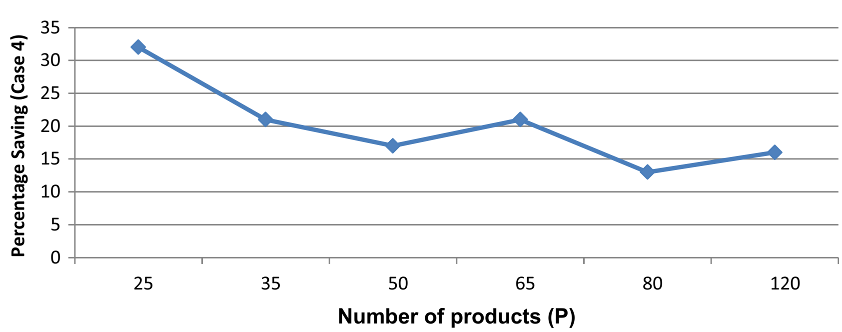

Fig. 2 Cost saving percentage from dynamic reconfiguration as the problem size increases

Table 6 Illustration of workload balancing

Υ	Workload of RE-1 on machines 1–6						Total
	1	2	3	4	5	6	
0.99	1,131	1,131	1,200	1,131	1,131	1,131	6,855
0.00	0	0	6,350	0	750	0	7,070

of savings obtained by changing from static to dynamic distributed layout. The table shows that dynamic reconfiguration can lead to significant cost savings when the manufacturing system has more unique machines with less shared capabilities, as in case 4. Conversely, there is less need for system reconfiguration when a manufacturing facility has machine tools with several shared capabilities, as in case 1. As can be seen in Table 5, we found similar results in several other problems. Figure 2 shows that when using dynamic reconfiguration, the percentage of savings tends to decrease as problem size increases. However, the actual manufacturing cost in larger problems is very high, and even a small percentage in savings can imply a very significant monetary value.

Other model features

In this section, we illustrate the benefits of incorporating workload balancing, production planning and subcontracting in the proposed comprehensive model. The sample results in Table 6 show the distribution of a workload that requires the use of resource element-1 (RE-1), which is available on each of machines 1 to 6. In the first row in this table, workload balancing ($\Upsilon = 0.99$) results in a workload that is evenly distributed among all the machines. In the second row, in contrast, the workload is unevenly distributed on the six machines when the workload factor Υ is set to zero. These results reflect the importance of incorporating a workload balancing constraint in the proposed model. As Table 7 shows, incorporating one or both of production planning and subcontracting typically results in a substantial decrease in the objective function, indicating their significance in economic terms. More importantly, the incorporation of these attributes affects several objective function terms, further signifying the value of utilizing a comprehensive model in manufacturing system analysis. A model consisting of different aspects of the system can help us to understand the problem better. An integrated system approach can minimize the possibility that certain important aspects of the system will be overlooked while other issues are being studied.

Discussion and conclusion

The design and operation of production systems in the current era of global competition is becoming a very complex and difficult task. Modeling and optimization of such complex systems is of paramount importance in achieving competitive advantages. In this work, we developed a new mathematical model that integrates layout configuration and production planning in the design of dynamic distributed layouts. This type of layout is emerging as a remedy to the challenges faced by manufacturing systems that produce multiple components in today's highly volatile environments. The model incorporates a number of important manufacturing attributes such as demand fluctuation, system reconfiguration, lot splitting, work load balancing, alternative routings, machine capability and tooling requirements. In addition, the model allows the optimization of several cost elements in an integrated manner. These costs include material handling, machine relocation, setup, inventory carrying, in-house production and subcontracting needs. Numerical examples revealed that distributed layouts are

Table 7 Effects of production planning and subcontracting

Cost	Production planning/subcontracting			
	Without/without	With/without	Without/with	With/with
Relocation cost	23,220	30,640	9,100	10,900
Material handling cost	263,105	207,415	86,555	58,360
Inventory holding cost	0	23,850	0	10,150
Setup cost	19,200	13,050	11,050	9,900
In-house production cost	138,500	138,500	100,400	103,200
Subcontracting cost	0	0	157,350	147,450
Total cost	444,025	413,455	364,455	339,960

highly desirable in a situation where there are many machine tools with several shared or similar capabilities. Given that today's modern manufacturing facilities exhibit this type of situation, distributed layouts are becoming more and more relevant. On the other hand, we observed that dynamic reconfiguration can lead to significant cost savings when a manufacturing system consists of more unique machines with less shared capabilities, illustrating that the need for system reconfiguration can be lessened by having machine tools with several shared capabilities and distributing them on the shop floor. Furthermore, we demonstrated how looking at several pragmatic aspects of the manufacturing system can significantly affect manufacturing costs. Thus, we illustrated the value of using a comprehensive model in manufacturing system analysis. In future research in this area, we will enhance our model to account for uncertainties in product demand and mix. Moreover, we plan to develop a scheduling model for manufacturing systems based on distributed layouts, an area in which existing research is very limited.

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Appendix 1: Input data for Problem 1

See Fig. 3 and Tables 8, 9, 10, 11, 12, 13, 14.

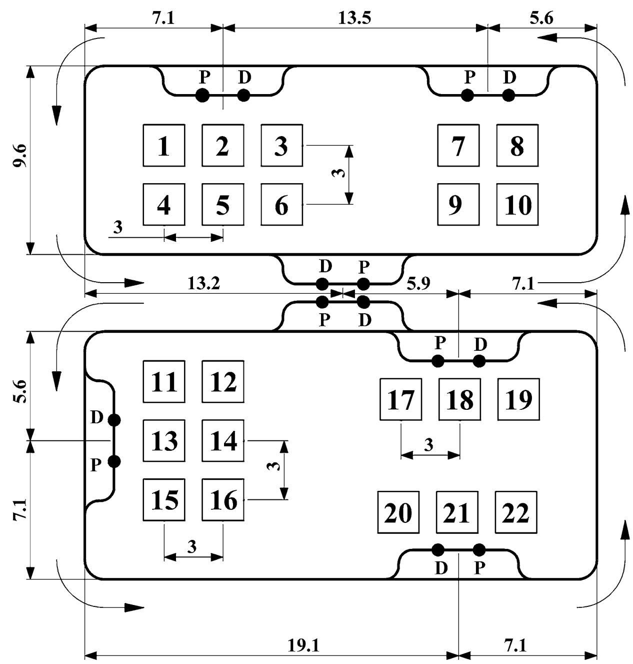

Fig. 3 Layout showing AGV paths and locations for machines-dimensions are in unit distance

Table 8 Resource elements data

Resource element r	Indices of machines having resource element r			
	Case 1	Case 2	Case 3	Case 4
1	(1, 2, 3, 4, 5, 6)	(1, 2, 4, 6)	(1, 6)	(1)
2	(1, 2, 3, 4, 5, 6)	(1, 3, 5)	(2, 5)	(2)
3	(1, 2, 3, 4, 5, 6)	(2, 4, 6)	(3)	(3)
4	(1, 2, 3, 4, 5, 6)	(1, 3, 5, 6)	(4)	(4)
5	(7, 8, 9, 10)	(7, 8, 10)	(8)	(5)
6	(7, 8, 9, 10)	(9, 10)	(7, 10)	(6)
7	(7, 8, 9, 10)	(7, 8, 10)	(9)	(7)
8	(7, 8, 9, 10)	(8, 9)	(7, 10)	(8)
9	(11, 12, 13, 14, 15, 16)	(11, 14, 15)	(14, 15)	(9)
10	(11, 12, 13, 14, 15, 16)	(11, 13, 16)	(11, 16)	(10)
11	(11, 12, 13, 14, 15, 16)	(12, 14, 16)	(16)	(11)
12	(11, 12, 13, 14, 15, 16)	(11, 13, 15)	(12, 13)	(12)
13	(11, 12, 13, 14, 15, 16)	(11, 12, 14)	(11, 15, 16)	(13)
14	(17, 18, 19)	(18, 19)	(18)	(14)
15	(17, 18, 19)	(17, 19)	(17)	(15)
16	(17, 18, 19)	(18, 19)	(19)	(16)
17	(20, 21, 22)	(20, 22)	(22)	(17)
18	(20, 21, 22)	(21, 22)	(20)	(18, 21)
19	(20, 21, 22)	(20, 21)	(21)	(19, 22)
20	(20, 21, 22)	(20, 22)	(20, 22)	(20)

Table 9 Processing data for the parts

Part	Θ_p	$\hat{\Theta}_p$	H_p	F_p	S_p	N_p	O_p	Operation data (r, U_p)				
								$o = 1$	$o = 2$	$o = 3$	$o = 4$	$o = 5$
1	6	12	5	2	300	2	2	(14, 1)	(8, 3)			
2	10	40	5	3	200	2	3	(17, 2)	(19, 1)	(1, 2)		
3	6	18	2	2	150	2	4	(4, 2)	(12, 3)	(4, 2)	(2, 3)	
4	4	16	4	1	200	2	2	(14, 1)	(18, 2)			
5	6	18	4	2	300	2	2	(6, 2)	(14, 1)			
6	8	24	3	2	150	2	2	(13, 3)	(14, 2)			
7	8	24	3	2	350	2	4	(3, 2)	(1, 1)	(16, 3)	(2, 1)	
8	10	30	3	1	250	2	5	(0, 2)	(9, 1)	(13, 3)	(1, 2)	(4, 2)
9	4	16	5	3	400	2	5	(6, 2)	(4, 2)	(10, 2)	(3, 2)	(18, 2)
10	6	18	3	1	350	2	4	(18, 2)	(6, 1)	(19, 1)	(19, 2)	
11	2	6	2	3	150	2	3	(12, 1)	(8, 1)	(1, 3)		
12	4	12	5	2	350	2	3	(6, 3)	(15, 1)	(4, 2)		
13	4	12	4	3	250	2	3	(11, 2)	(9, 2)	(12, 2)		
14	2	6	4	2	350	2	4	(16, 3)	(14, 2)	(7, 2)	(14, 1)	
15	4	8	3	3	250	2	4	(17, 2)	(4, 1)	(13, 2)	(7, 2)	
16	10	40	3	2	200	2	4	(2, 2)	(12, 2)	(13, 2)	(3, 3)	
17	2	6	4	3	200	2	3	(16, 2)	(12, 3)	(1, 2)		
18	10	30	3	1	250	2	2	(19, 2)	(11, 2)			
19	8	16	6	2	350	2	5	(3, 2)	(17, 1)	(14, 3)	(6, 1)	(1, 1)
20	4	16	4	2	200	2	3	(16, 2)	(9, 2)	(6, 2)		
21	6	18	4	3	300	2	4	(10, 2)	(6, 1)	(6, 2)	(18, 3)	
22	6	12	3	1	350	2	4	(14, 3)	(18, 2)	(1, 1)	(10, 3)	
23	8	24	2	3	400	2	4	(18, 2)	(17, 1)	(6, 2)	(10, 1)	
24	4	12	3	2	350	2	4	(6, 1)	(10, 3)	(2, 2)	(7, 2)	
25	4	16	3	2	350	2	3	(15, 2)	(7, 2)	(10, 2)		

Operation data (r, U_p) is the index of the required resource element r and unit processing time U_p for the corresponding operation

Table 10 Demand data for the parts

Part	Demand $D_{p,t}$			
	$t = 1$	$t = 2$	$t = 3$	$t = 4$
1	50	100	0	650
2	0	50	550	200
3	150	300	300	0
4	400	0	150	350
5	0	100	450	450
6	250	600	0	0
7	550	0	0	200
8	0	100	400	100
9	650	150	700	100
10	0	350	0	0
11	550	250	0	350
12	450	0	0	0
13	0	450	200	50
14	100	650	600	0
15	400	150	0	0
16	0	100	700	250
17	750	0	300	200
18	200	700	700	0
19	0	0	200	0
20	150	0	100	200
21	150	0	0	500
22	700	700	150	450
23	700	450	250	300
24	600	100	450	200
25	500	450	350	0

Table 11 Machine relocation cost per unit distance

m	G_m	m	G_m	m	G_m	m	G_m
1	80	7	100	13	80	18	80
2	80	8	80	14	80	19	60
3	60	9	80	15	100	20	80
4	80	10	80	16	80	21	100
5	60	11	80	17	60	22	80
6	80	12	100				

Table 12 Machine locations for the functional and five arbitrary generated distributed layouts

Machine index m at location l = 1 to 22					
Functional layout $l = m$	Distributed layouts				
	DL1	DL2	DL3	DL4	DL5
1	10	4	19	12	22
2	4	5	16	15	11
3	20	12	2	4	5
4	2	3	14	8	10
5	5	13	18	17	1
6	18	8	15	20	18
7	7	20	1	1	2
8	19	17	13	19	21
9	22	6	21	3	13
10	13	11	7	14	20
11	15	9	12	22	15
12	16	21	3	5	6
13	6	7	11	7	8
14	11	19	4	2	7
15	3	1	17	18	14
16	8	16	5	16	19
17	21	10	6	11	4
18	14	22	20	9	3
19	17	2	9	10	12
20	12	15	10	6	9
21	1	18	8	21	16
22	9	14	22	13	17

Table 13 Material handling distance between locations l and l', $E_{l,l'}$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	0	3	6	3	6	9	61	61	64	64	55	58	52	55	55	58	108	105	108	81	78	81
2	3	0	3	6	3	6	58	58	61	61	52	55	48	52	52	55	105	102	105	78	75	78
3	6	2	0	9	6	3	61	61	64	64	55	58	52	55	55	58	108	105	108	81	78	81
4	3	6	9	0	3	6	64	64	67	67	58	61	55	58	58	61	111	108	111	84	81	84
5	6	3	6	3	0	3	61	61	64	64	55	58	52	55	55	58	108	105	108	81	78	81
6	9	6	3	6	3	0	64	64	67	67	58	61	55	58	58	61	111	108	111	84	81	84
7	17	14	17	20	17	20	0	3	3	6	65	68	62	65	65	68	118	115	118	91	88	91
8	17	14	17	20	17	20	3	0	6	3	65	68	62	65	65	68	118	115	118	91	88	91
9	20	17	20	23	20	23	3	6	0	3	68	71	65	68	68	71	121	118	121	94	91	94
10	20	17	20	23	20	23	6	3	3	0	68	71	65	68	68	71	121	118	121	94	91	94
11	107	104	107	110	107	110	90	90	93	93	0	3	3	6	6	9	59	56	59	32	29	32
12	110	107	110	113	110	113	93	93	96	96	3	0	6	3	9	6	62	59	62	35	32	35
13	104	101	104	107	104	107	87	87	90	90	3	6	0	3	3	6	56	53	56	29	26	29
14	107	104	107	110	107	110	90	90	93	93	6	3	3	0	6	3	59	56	59	32	29	32
15	107	104	107	110	107	110	90	90	93	93	6	9	3	6	0	3	59	56	59	32	29	32
16	110	107	110	113	110	113	93	93	96	96	9	6	6	3	3	0	62	59	62	35	32	35
17	54	51	54	57	54	57	37	37	40	40	31	34	28	31	31	34	0	3	6	57	54	57
18	51	48	51	54	51	54	34	34	37	37	28	31	25	28	28	31	3	0	3	54	51	54
19	54	51	54	57	54	57	37	37	40	40	31	34	28	31	31	34	6	3	0	57	54	57
20	81	78	81	84	81	84	64	64	67	67	58	61	55	58	58	61	33	30	33	0	3	6
21	78	75	78	81	78	81	61	61	64	64	55	58	52	55	55	58	30	27	30	3	0	3
22	81	78	81	84	81	84	64	64	67	67	58	61	55	58	58	61	33	30	33	6	3	0

Table 14 Machine relocation distance between locations l and l', $E'_{l,l'}$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	0	3	6	3	4	7	15	18	15	18	12	12	15	15	18	18	18	20	22	22	24	25
2	3	0	3	4	3	4	12	15	12	15	12	12	15	15	18	18	16	18	20	21	22	24
3	6	3	0	7	4	3	9	12	10	12	13	12	16	15	19	18	14	16	18	20	21	22
4	3	4	7	0	3	6	15	18	15	18	9	10	12	12	15	15	16	18	21	20	22	24
5	4	3	4	3	0	3	12	15	12	15	10	9	10	12	15	15	13	16	18	18	20	22
6	7	4	3	6	3	0	10	12	9	12	11	10	13	12	16	15	12	13	16	17	18	20
7	15	12	9	15	12	10	0	3	3	4	19	17	21	19	24	22	13	13	13	19	19	19
8	18	15	12	18	15	12	3	0	4	3	22	19	24	21	26	24	14	13	13	13	19	19
9	15	12	10	15	12	9	3	4	0	3	18	15	19	17	21	19	10	10	10	16	16	16
10	18	15	12	18	15	12	4	3	3	0	20	18	22	19	24	22	12	10	10	17	16	16
11	12	12	13	9	10	11	19	22	18	20	0	3	3	4	6	7	12	15	18	14	16	19
12	12	12	12	10	9	10	17	19	15	18	3	0	4	3	7	6	9	12	15	11	14	16
13	15	15	16	12	10	13	21	24	19	22	3	4	0	3	3	4	12	15	18	13	15	18
14	15	15	15	12	12	12	19	21	17	19	4	3	3	0	4	3	9	12	15	10	13	15
15	18	18	19	15	15	16	24	26	21	24	6	7	3	4	0	3	13	16	19	12	15	18
16	18	18	18	15	15	15	22	24	19	22	7	6	4	3	3	0	10	13	16	9	12	15
17	18	16	14	16	13	12	13	14	10	12	12	9	12	9	13	10	0	3	6	6	6	8
18	20	18	16	18	16	13	13	13	10	10	15	12	15	12	16	13	3	0	3	6	6	6
19	22	20	18	21	18	16	13	13	10	10	18	15	18	15	19	16	6	3	0	8	6	6
20	22	21	20	20	18	17	19	13	16	17	14	11	13	10	12	9	6	6	8	0	3	6
21	24	22	21	22	20	18	19	19	16	16	16	14	15	13	15	12	6	6	6	3	0	3
22	26	24	22	24	22	20	19	19	16	16	19	16	18	15	18	15	8	6	6	6	3	0

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