J Ind Eng Int (2015) 11:555–574
DOI 10.1007/s40092-015-0121-y

ORIGINAL RESEARCH

A production-inventory model with permissible delay incorporating learning effect in random planning horizon using genetic algorithm

Mohuya B. Kar[1] · Shankar Bera[2] · Debasis Das[2] · Samarjit Kar[2]

Received: 12 August 2014 / Accepted: 16 December 2014 / Published online: 20 October 2015

Abstract This paper presents a production-inventory model for deteriorating items with stock-dependent demand under inflation in a random planning horizon. The supplier offers the retailer fully permissible delay in payment. It is assumed that the time horizon of the business period is random in nature and follows exponential distribution with a known mean. Here learning effect is also introduced for the production cost and setup cost. The model is formulated as profit maximization problem with respect to the retailer and solved with the help of genetic algorithm (GA) and PSO. Moreover, the convergence of two methods—GA and PSO—is studied against generation numbers and it is seen that GA converges rapidly than PSO. The optimum results from methods are compared both numerically and graphically. It is observed that the performance of GA is marginally better than PSO. We have provided some numerical examples and some sensitivity analyses to illustrate the model.

Keywords Inventory · Learning effect · Inflation · Random planning horizon · Permissible delay in payment · Stock-dependent demand · Deteriorating items · Genetic algorithm

✉ Samarjit Kar
kar_s_k@yahoo.com

Mohuya B. Kar
mohuyakar@yahoo.com

Shankar Bera
berashankar@gmail.com

Debasis Das
debasis_opt@yahoo.co.in

[1] Department of Computer Science, Heritage Institute of Technology, Kolkata, WB 700107, India

[2] Department of Mathematics, National Institute of Technology, Durgapur, WB 713209, India

Introduction

In the present competitive market, the supplier influences the customers in many different ways to capture the market. For this reason, they offer a delay period in payment without any extra charges. The wholesaler allows a time period after which retailers are to make the payment without paying any interest. But after the delay period, if it is not payed, an interest charged for the rest of the period. Goyal (1985) first established a single-item inventory model under permissible delay in payment. Later many researchers worked in this area. Chang et al. (2003) and Chung and Liao (2004) dealt with the problem of determining the EOQ for exponentially deteriorating items under permissible delay in payment. Das et al. (2010a, b) presented an EPQ model for deteriorating items under permissible delay in payment. Teng et al. (2011) developed an EOQ model for stock-dependent demand under supplier's trade credit offer with a progressive payment scheme. Min et al. (2012) developed an EPQ model with inventory-level-dependent demand and permissible delay in payment. Recently, Ouyang and Chang (2013) proposed an optimal production lot with imperfect production process under permission delay in payment and complete backlogging

Now—a—days, there is a stiff competition amongst the multi-nationals to influence the customers and to capture the market. Thus, in the recent competitive market, the inventory stock is decoratively exhibited and displayed through electronic media to attract the customers and thus to boost the sale. For this reason, a group of researchers have considered inventory control systems with stock-dependent demand in their research such as Datta and Pal (1992), Mandal and Phaujdar (1989a, b), Giri et al. (1996), Hou (2006), Roy et al. (2009a), and others. They considered linear form of stock-dependent demand.

Production cost of a manufacturing system depends upon the combination of different production factors. These factors are (a) raw materials, (b) technical knowledge, (c) production procedure, (d) firm size, (e) quantity of product, and so on. Normally, the cost of raw materials is imprecise in nature. So far, cost of technical knowledge, that is labor cost, has been usually assumed to be constant. However, as the employees perform the same task repeatedly, they learn how to provide repeatedly a standard level of performance efficiently. Therefore, the processing cost per unit product decreases in every cycle. Similarly, a part of the ordering cost may also decrease in every cycle. In the inventory control literature, this phenomenon is known as the learning effect. Although different types of learning effects in various areas have been studied (cf. Chiu and Chen 2005, Kuo and Yang 2006, Alamri and Balkhi 2007, etc.), it has rarely been studied in the context of inventory control problems.

Classical inventory models are usually developed over the infinite planning horizon. According to Gurnani (1985) and Chung and Kim (1989), the assumption of infinite planning horizon is not realistic due to several reasons such as variation of inventory costs, change in product specifications and designs, technological changes, etc. Moreover, for seasonal products like fruits, vegetables, warm garments, etc., business period is not infinite, rather fluctuates with each season. Hence, the planning horizon for seasonal products varies over years and may be considered as random with a distribution. Moon and Yun (1993) developed an EOQ model with the random planning horizon. Also Das et al. (2010a, b) presented an EPQ model with inflation random product life cycle. Till now, none has developed EPQ models considering the delay in payment under random planing horizon. Table 1 represents the summary of related literature for inventory models with random planning horizon.

Taking the above shortcomings into account, this paper presents some EPQ models for a deteriorating item considering delay in payment and linearly stock-dependent demand with a random planing horizon, that is, the life time of the product is assumed as random in nature and follows an exponential distribution with a known mean. Learning effect in setup cost and production cost is introduced, i.e., setup cost and production cost reduce successively with each cycle. Interest earned and interest paid vary with the permitted fixed period of permissible delay offered by the wholesaler. The model is formulated as a profit maximization problem and solved using Genetic Algorithm. The model is illustrated with numerical examples and some sensitivity analyses are performed.

In addition to the above considerations, the rest of the article is organized as follows. Section "Notations and assumptions" presents assumptions and notations that are used throughout this article. Section "Mathematical formulation" formulates a mathematical model in order to maximize the total profit of the retailer. Two different solution procedures (GA and PSO) are discussed in Sect. "Solution procedure". In Sect. "Numerical example and sensitivity analysis," several numerical examples are given to illustrate the solution procedure and sensitivity analyses are performed. The paper is concluded in Sect. "Conclusion and future scope." Finally the managerial insights are presented in Sect. "Managerial insights."

Notations and assumptions

The mathematical model of the proposed production-inventory problem is developed based on the following assumptions and notations :

Table 1 Summary of related literature for inventory models with random planning horizon

Author(s) and year	Single/two warehouse	Demand rate	Inflation	Learning effect	Delay in payment	Method
Moon and Yun (1993)	SW	Constant	No	No	No	Classical method
Maiti et al. (2006)	TW	Stock dependent	No	No	No	Classical method
Roy et al. (2007)	TW	Stock dependent	No	No	No	GA
Roy et al. (2009a)	SW	Stock dependent	Yes	Yes	No	FGA
Roy et al. (2009b)	SW	Stock dependent	Yes	No	No	GA
Das et al. (2012)	TW	Stock dependent	Yes	No	No	GA
Su et al. (2014)	SW	Constant	No	No	No	Classical method
Present paper	SW	Stock dependent	Yes	Yes	Yes	GA,PSO

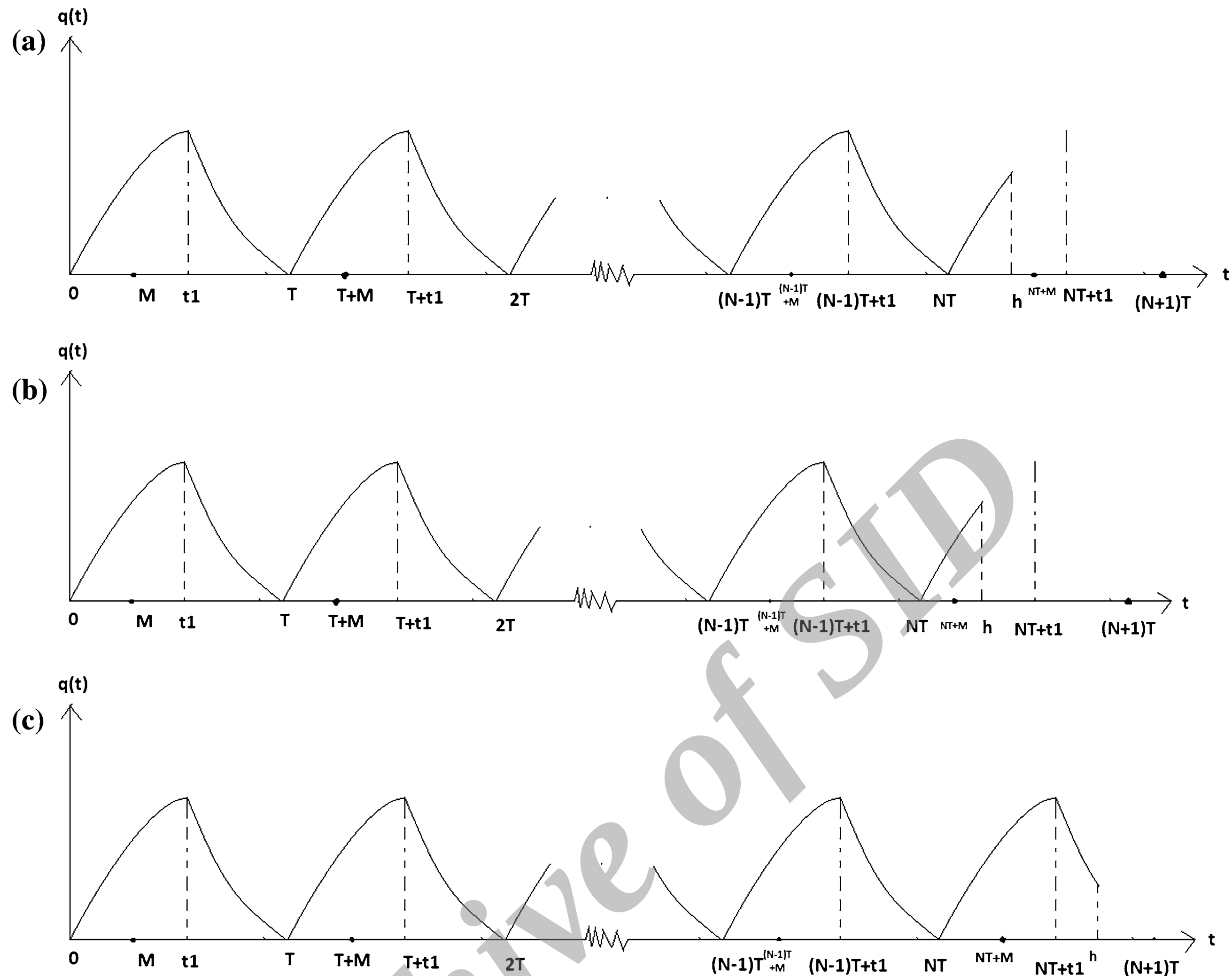

Fig. 1 **a** Pictorial representation for the inventory model for case 1(a). **b** Pictorial representation for the inventory model for case 1(b). **c** Pictorial representation for the inventory model for case 1(c)

Assumptions

1. Demand rate is stock dependent.
2. Time horizon is random.
3. Time horizon accommodates first N cycles and ends during $(N+1)$ cycles.
4. Setup time is negligible.
5. Production rate is known and constant.
6. Shortages are not allowed.
7. A constant fraction of on-hand inventory gets deteriorated per unit time.
8. Lead time is zero.
9. Production cost and setup cost decrease due to the learning effect.

Notations

The following notations have been used throughout this paper:

N = Number of fully accommodated cycles to be made during the prescribed time horizon.

$q(t)$ = On-hand inventory of a cycle at time t,$(j-1)T \leq t \leq jT$ (j = 1,2,...,N).

t_1 = Production period in each cycle.

P = Production rate in each cycle.

D = Demand rate in each cycle = $\alpha + \beta q(t)$.

C_1 = Holding cost per unit item per unit time.

$C_3^j = C_3 + C_3' e^{-\delta_j}$ is setup cost in j-th ($j = 1, 2, \ldots, N$) cycle, $\delta > 0$ (δ is the learning coefficient associated with setup cost).

$p_0e^{-\gamma_j}$ = Production cost in j-th $(j = 1, 2, \ldots, N)$ cycle, $p_0, \gamma > 0$ (γ is the learning coefficient associated with production cost).

$m_0p_0e^{-\gamma_j}$ = Selling price in j-th $(j = 1, 2, \ldots, N)$ cycle, $p_0, \gamma > 0$, $m_0 > 1$.

I_c = Interest charged per dollar per unit time.

I_e = Interest earned per dollar per unit time.

M = The period of cash discount for which the supplier does not charge any interest.

T = Duration of a complete cycle.

i = Inflation rate.

r = Discount rate.

$R = r - i$, the net discount rate of inflation is constant.

$P(N, T)$ = Total profit after completing N fully accommodated cycles.

H = Total time horizon (a random variable) and h is the real time horizon.

$m_1p_0e^{-\gamma(N+1)}$ = Reduced selling price for the inventory items in the last cycle at the end of time horizon, $p_0, \gamma > 0$, $m_1 < 1$.

θ = Deterioration rate of the produced item.

$E_k\{P(N,T)\}$ = Expected total profit from N complete cycles in k-th case, $k = 1, 2$.

$E_k\{TP_L(T)\}$ = Expected total profit from last cycle in k-th case, $k = 1, 2$.

$E_k\{TP(T)\}$ = Expected total profit from the planning horizon in k-th case, $k = 1, 2$.

Mathematical formulation

In this section, we formulate a production-inventory model for deteriorating items under inflation over a random planning horizon incorporating learning effect using permissible delay period. Here it is assumed that there are N full cycles during the real-time horizon h and the planning horizon ends within $(N+1)$th cycle, i.e., between the time $t = NT$ and $t = (N+1)T$. At the beginning of every jth $(j = 1, 2, \ldots, N+1)$ cycle, production starts at $t = (j-1)T$ and continues up to $t = (j-1)T + t_1$, and inventory gradually increases after meeting the demand due to production. Production thus stops at $t = (j-1)T + t_1$ and the inventory falls to zero level at the end of the cycle time $t = jT$, due to deterioration and consumption. The retailer pays the payment at time period M without any interest. At the end of this period, he/she starts paying for the interest charged on the items in stocks. Also during the time, the account is not settled, and generated sales revenue is deposited in an interest bearing account. This cycle repeats again and again. For the last cycle, some amounts may be left after the end of planning horizon. This amount is sold at a reduced price in a lot. Regarding the interest payed and interest earned based on the length of the jth$(1 \le j \le N)$ cycle time T, two different cases may arise:

Case 1: $(j-1)T + M \le (j-1)T + t_1 \le jT$
Case 2: $(j-1)T + t_1 \le (j-1)T + M \le jT$

Again for last cycle, according to the values of M, T, and t_1, we have three different subcases for each case, which are pictorially depicted in Figs. 1a, b, c, 2a, b, c, respectively. We discuss the detailed formulations in each subcases.

Here, it is assumed that the planning horizon H is a random variable and follows exponential distribution with probability density function (p.d.f) as

$$f(h) = \begin{cases} \lambda e^{-\lambda h}, & h \ge 0 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Formulation for N full cycles

The differential equations describing the inventory level $q(t)$ in the interval $(j-1)T \le t \le jT (1 \le j \le N)$, $j = 1, 2, \ldots, N$ are given by

$$\frac{dq(t)}{dt} = \begin{cases} P - (\alpha + \beta q(t)) - \theta q(t), & (j-1)T \le t \le (j-1)T + t_1 \\ -(\alpha + \beta q(t)) - \theta q(t), & (j-1)T + t_1 \le t \le jT, \end{cases} \tag{2}$$

where $P > 0$, $\alpha > 0$, $\beta > 0$, $\theta > 0$, and $0 < t_1 < T$, subject to the conditions that $q(t) = 0$ at $t = (j-1)T$, and $q(t) = 0$ at $t = jT$.

The solutions of the differential equations in (2) are given by

$$q(t) = \begin{cases} \dfrac{P-\alpha}{\theta+\beta}\left[1 - e^{(\theta+\beta)\{(j-1)T-t\}}\right], & (j-1)T \le t \le (j-1)T + t_1 \\ \dfrac{\alpha}{(\theta+\beta)}\left[e^{(\theta+\beta)(jT-t)} - 1\right], & (j-1)T + t_1 \le t \le jT. \end{cases} \tag{3}$$

Now at $t = (j-1)T + t_1$, from (3) we get,

$$\frac{P-\alpha}{\theta+\beta}\left[1 - e^{-(\theta+\beta)t_1}\right] = \frac{\alpha}{(\theta+\beta)}\left[e^{(\theta+\beta)(T-t_1)} - 1\right]$$
$$\Rightarrow t_1 = \frac{1}{\theta+\beta} ln\left[1 + \frac{\alpha}{P}\left(e^{(\theta+\beta)T} - 1\right)\right]. \tag{4}$$

Expected total profit from N full cycles

From the symmetry of every full cycle, present value of the expected total profit from N full cycles, $E_k\{P(N,T)\}$ in kth$(k = 1, 2)$ case is given by

$$E_k\{P(N,T)\} = \text{ESRN} + \text{EI}_k\text{EN} - \text{EPCN} - \text{EHCN} - \text{ETOCN} - \text{EI}_k\text{PN}. \tag{5}$$

where ESRN, EI_kEN, EPCN, EHCN, ETOCN, and EI_kPN are present values of expected total sales revenue, expected total interest earned, expected total production cost,

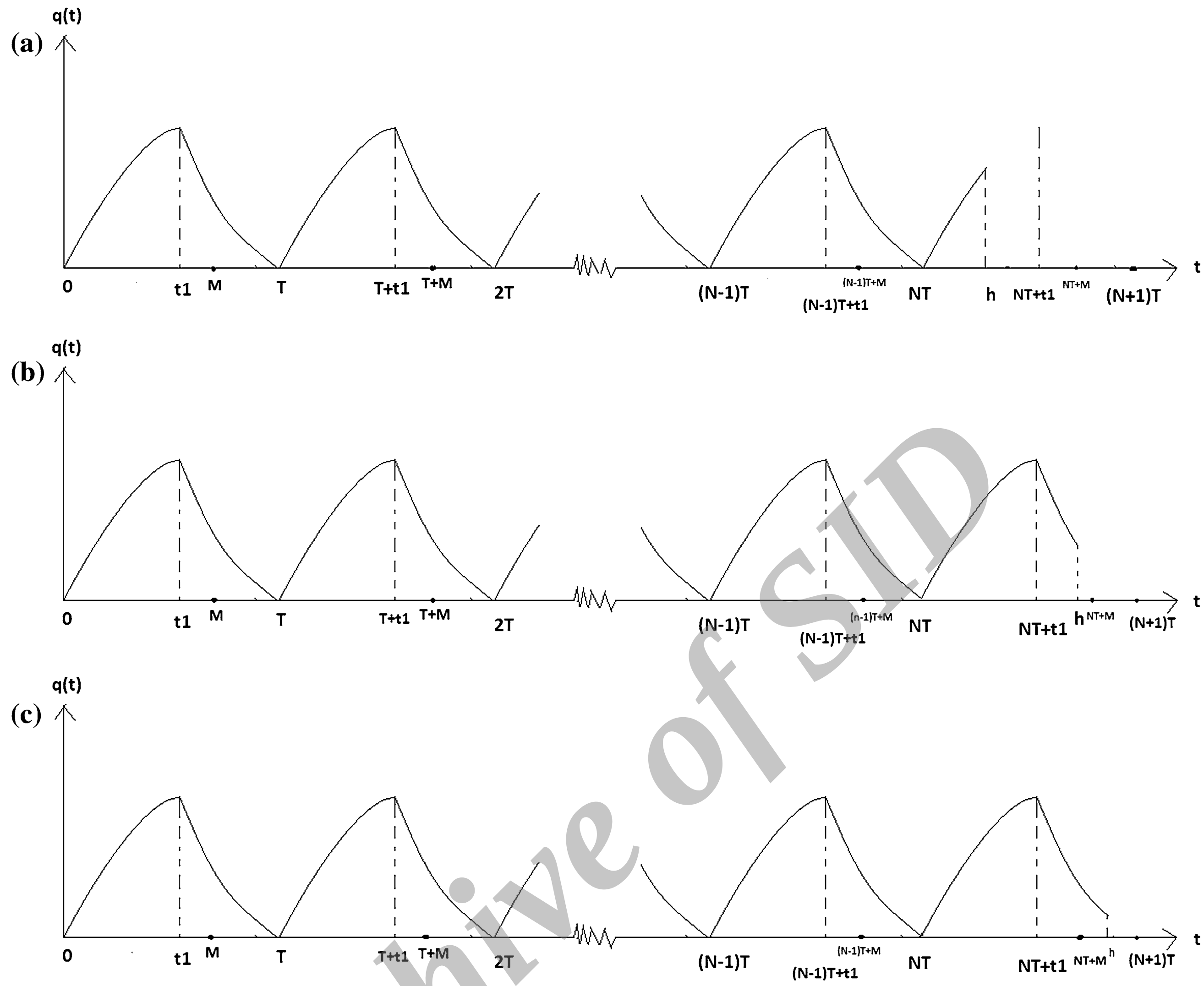

Fig. 2 **a** Pictorial representation for the inventory model for case 2(a). **b** Pictorial representation for the inventory model for case 2(b). **c** Pictorial representation for the inventory model for case 2(c)

expected holding cost, expected total ordering cost, and expected total interest paid, respectively, from N full cycles and their expressions are derived in Appendix 1 [see equations (22), (25), (31), (16), (13), (19), (28), (34), respectively].

Formulation for last cycle

Duration of the last cycle is $[NT, h]$, where h is the real-time horizon corresponding to the random time horizon H.

Here two different cases may arise depending upon the last cycle length.

Case-I: $NT < h \le NT + t_1$ and
Case-II: $NT + t_1 < h \le (N+1)T$.

The differential equations describing the inventory level $q(t)$ in the interval $NT < t \le h$ are given by

$$\frac{dq(t)}{dt} = \begin{cases} P - (\alpha + \beta q(t)) - \theta q(t) & NT \le t \le NT + t_1 \\ -(\alpha + \beta q(t)) - \theta q(t), & NT + t_1 \le t \le (N+1)T. \end{cases} \tag{6}$$

subject to the conditions that,

$q(NT) = 0$ and $q\{(N+1)T\} = 0.$

The solutions of the differential equations in (6) are given by

$$q(t) = \begin{cases} \dfrac{P-\alpha}{\theta+\beta}\left[1 - e^{(\theta+\beta)(NT-t)}\right], & NT \le t \le NT + t_1 \\ \dfrac{\alpha}{\theta+\beta}\left[e^{(\theta+\beta)\{(N+1)T-t\}} - 1\right], & NT + t_1 \le t \le (N+1)T. \end{cases} \tag{7}$$

Expected total profit from last cycle

Present value of expected total profit from last cycle, $E_k\{\mathrm{TP}_L(T)\}$, in kth$(k=1,2)$ case is given by

$$E_k\{\mathrm{TP}_L(T)\} = \mathrm{ESR}_L + \mathrm{ERSP}_L + \mathrm{EI}_k E_L - \mathrm{EHC}_L - \mathrm{EPC}_L - \mathrm{EOC}_L - \mathrm{EI}_k P_L, \tag{8}$$

where $\mathrm{ESR}_L, \mathrm{ERSP}_L, \mathrm{EI}_k E_L, \mathrm{EHC}_L, \mathrm{EPC}_L, \mathrm{EOC}_L, \mathrm{EI}_k P_L$ are present values of expected sales revenue, expected reduced selling price, expected interest earned, expected holding cost, expected production cost, expected ordering cost, and expected interest paid, respectively, from the last cycle and their expressions are derived in Appendix 2 [see equations (45), (49), (60), (68), (41), (44), (48), (61), (69), respectively].

Expected total profit from the system

Now, expected total profit from the complete time horizon, $E_k\{\mathrm{TP}(T)\}$, in kth$(k=1,2)$ case is given by

$$E_k\{\mathrm{TP}(T)\} = E_k\{P(N,T)\} + E_k\{\mathrm{TP}_L(T)\}. \tag{9}$$

Problem formulation

When the resultant effects of inflation and discounting (R) are crisp in nature, then our problem is to determine T from

$$\begin{aligned} &\text{Max} \quad E_k(TP),\\ &\text{subject to } \mathrm{T} \geq 0.\\ &k = 1,2. \end{aligned} \tag{10}$$

Solution procedure

Genetic algoritm (GA)

The discovery of genetic algorithms (GA) by Holland (1975) is further described by Goldberg (1998). GA is a randomized global search technique that solves problems imitating processes observed from natural evolution. GA continually exploits new and better solutions without any pre-assumptions such as continuity and unimodality. GA has been successfully adopted in many complex optimization problems and shows its merits over traditional optimization methods, especially when the system under study has multiple local optimal solutions. A GA normally starts with a set of potential solutions (called initial population) of the decision making problem under consideration. Individual solutions are called chromosomes. Crossover and mutation operations happen among the potential solutions to get a new set of solutions and it continues until terminating conditions are encountered. Michalewicz (1992) proposed a GA named contractive mapping genetic algorithm (CMGA) and proved the asymptotic convergence of the algorithm by Banach's fixed-point theorem. In CMGA, movement from old population to new takes place only when average fitness of new population is better than the old one. The algorithm is presented below. In the algorithm, p_c, p_m are probability of crossover and probability of mutation, respectively, T is the generation counter, and $P(T)$ is the population of potential solutions for generation T. M is iteration counter in each generation to improve $P(T)$ and M_0 is the upper limit of M. Initializing ($P(1)$) function generates the initial population $P(1)$ (initial guess of solution set). Objective function value due to each solution is taken as fitness of the solution. Evaluating ($P(T)$) function evaluates fitness of each member of $P(T)$.

GA algorithm

1. Set generation counter $T=1$, iteration counter in each generation $M=0$.
2. Initialize probability of crossover p_c, probability of mutation p_m, upper limit of iteration counter M_0, population size N.
3. Initialize ($P(T)$).
4. Evaluate ($P(T)$).
5. While ($M < M_0$).
6. Select N solutions from P(T) for mating pool using roulette-wheel selection process Michalewicz (1992). Let this set be $P'(T)$.
7. Select solutions from $P'(T)$, for crossover depending on p_c.
8. Make crossover on selected solutions.
9. Select solutions from $P'(T)$, for mutation depending on p_m.
10. Make mutation on selected solutions for mutation to get population $P_1(T)$.
11. Evaluate ($P_1(T)$).
12. Set $M = M + 1$.
13. If average fitness of $P_1(T) >$ average fitness of $P(T)$, then
14. Set $P(T+1) = P_1(T)$.
15. Set $T = T + 1$.
16. Set $M = 0$.
17. End if
18. End while
19. Output: Best solution of $P(T)$.
20. End algorithm.

The above model is solved by using GA approach, discussed in article-2. Our GA consists of parameters, population size = 50, probability of crossover = 0.6, probability of mutation = 0.2, and maximum generation = 50. A real number presentation is used here. In this representation, each chromosome X is a string of n numbers of GA, which denote the decision variable. For each chromosome X, every gene, which represents the independent variables, is randomly generated between their boundaries until it is feasible. In this GA, arithmetic crossover and random mutation are applied to generate new offsprings.

Particle swarm optimization (PSO)

PSO is a population-based stochastic optimization technique developed by Eberhart and Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. A swarm of m particles moving about in an n-dimensional real-valued search space, the ith particle is a n-dimensional vector, denoted as $X_i = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in})$, $i = 1, 2, 3, \ldots, n$. The ith particle's velocity is also a n-dimensional vector, denoted as $V_i = (v_{i1}, v_{i2}, v_{i3}, \ldots, v_{in})$, $i = 1, 2, 3, \ldots, n$. Denote the best position of the ith particle as $P_{\text{besti}} = (p_{i1}, p_{i2}, p_{i3}, \ldots, p_{in}), i = 1, 2, 3, \ldots, n$, and the best position of the colony $P_{\text{gbest}} = (p_{g1}, p_{g2}, p_{g3}, \ldots, p_{gn})$, $i = 1, 2, 3, \ldots, n$. Each particle of the population modified its position and velocity according to the following mathematical expression:

$$V_i^{t+1} = w * V_i^t + c_1 * \text{rand} * (P_{\text{besti}} - X_i^t) + c_2 * \text{rand} * (P_{\text{gbest}} - X_i^t)$$
$$X_i^{t+1} = X_i^t + V_i^{t+1}$$

where t is the current generation number, w is the inertia weight factor, c_1 and c_2 are learning factors, determining the influence of P_{besti} and P_{gbest}, rand and Rand are random numbers uniformly distributed in the range (0, 1), V_i^t and X_i^t are the current velocity and position of the particle, respectively, P_{besti} is the best solution this particle has reached, and P_{gbest} is the current global best solution of all the particles.

The first part of the first formula is the inertia velocity of particle, which reflects the memory behavior of particle; the second part (the distance between the current position and the best position of the ith particle) is "cognition" part, which represents the private thinking of the particle itself; the third part (the distance between the current position of the ith particle and the best position of the colony) is the "social" part, showing the particle's behavior stem from the experience of other particles in the population. The particles find the optimal solution by cooperation and competition.

Numerical example and sensitivity analysis

Let us consider a numerical example with the following numerical data:

Table 2 The sensitivity analysis of the demand parameter when $\theta = 0.1$, and $\lambda = 0.01$

α	β	Case-1		Case-2	
		Total profit	t_1	Total profit	t_1
	0.20	3884.8872	5.02	4160.5547	2.540
55	0.25	4060.8811	5.02	4271.1099	2.702
	0.30	4227.2261	5.02	4385.8486	2.780
	0.20	4276.6484	5.02	4594.0981	2.540
60	0.25	4441.4120	5.02	4695.0381	2.552
	0.30	4595.8433	5.02	4798.8311	2.780
	0.20	4684.2881	5.02	5030.7891	2.540
65	0.25	4834.4668	5.02	5121.1382	2.552
	0.30	4974.2280	5.02	5211.9111	2.714

Table 3 The sensitivity analysis of the deterioration parameter when $\alpha = 55$, and $\lambda = 0.01$

β	θ	Case-1		Case-2	
		Total profit	t_1	Total profit	t_1
	0.075	3955.4363	5.02	4184.4292	2.540
0.20	0.100	3884.8872	5.02	4160.5547	2.540
	0.125	3817.8208	5.02	4139.3906	2.282
	0.075	4130.3584	5.02	4299.8169	2.756
0.25	0.100	4060.8811	5.02	4271.1099	2.702
	0.125	3994.7908	5.02	4244.6841	2.552
	0.075	4295.7216	5.02	4419.2905	2.906
0.30	0.100	4227.2261	5.02	4385.8486	2.780
	0.125	4161.9863	5.02	4354.9688	2.756

$C_3 = 50$, $C_3' = 40$, $C_1 = 0.75$, $P = 95$, $m_0 = 1.8$, $\lambda = 0.01$, $r = 0.1$, $i = 0.05$, $R = 0.05$, $\gamma = 0.05$, $p_0 = 4$, $I_e = 0.15$, $I_c = 0.2$, $m_1 = 0.8$, $\delta = 0.5$ in appropriate units.

According to the proposed computational procedures (GA and PSO), the results listed in Table 1 are obtained for different values of $M = 4.0, 4.2, 4.4, 4.5, 4.6, 4.8$ of case 1 and 2.

From the above numerical illustration, it is observed that for fixed α, β , and θ as M increases, total profit also increases in both cases. Results are as per expectation.

Sensitivity analysis

Sensitivity analyses are performed using GA to study the effect of changes in different values of α, β, θ, λ, δ, R, and γ which are executed through the Tables 2, 3, 4, 5, 6, 7. It is observed that if θ and λ are fixed for different values of α as β increases, total profit increases. If α and λ are fixed for different values of β as θ increases, total profit decreases. And for fixed values of α, β , and θ as λ increases, total

Table 4 The sensitivity analysis of λ when $\alpha = 55$, $\beta = 0.2$ and $\theta = 0.1$

λ	Case-1		Case-2	
	Total profit	t_1	Total profit	t_1
0.007	3994.9146	5.02	4316.1123	2.540
0.008	3957.1479	5.02	4263.0098	2.540
0.009	3920.4871	5.02	4211.1714	2.540
0.010	3884.8872	5.02	4160.5547	2.540
0.011	3850.3074	5.02	4111.1191	2.540
0.012	3816.7051	5.02	4062.8242	2.540
0.013	3784.0439	5.02	4016.8979	2.282

Table 5 The sensitivity analysis of the learning coefficient δ associated with setup cost

α	δ	Case-1		Case-2	
		Total profit	t_1	Total profit	t_1
	0.4	3875.7031	5.02	4148.0098	2.54
55	0.5	3884.8872	5.02	4160.5547	2.54
	0.6	3891.9399	5.02	4169.6669	2.54
	0.4	4267.4639	5.02	4581.5532	2.54
60	0.5	4276.6484	5.02	4594.0981	2.54
	0.6	4283.7007	5.02	4603.2099	2.54
	0.4	4675.1040	5.02	5018.2441	2.54
65	0.5	4684.2881	5.02	5030.7891	2.54
	0.6	4691.3403	5.02	5039.9004	2.54

Table 6 The sensitivity analysis of R when $\alpha = 55$, $\beta = 0.2$ and $\theta = 0.1$

R	Case-1		Case-2	
	Total profit	t_1	Total profit	t_1
0.047	4057.3096	5.02	4319.6108	2.540
0.048	3998.2109	5.02	4265.3291	2.540
0.049	3940.7598	5.02	4212.3247	2.540
0.050	3884.8872	5.02	4160.5547	2.540
0.051	3830.5315	5.02	4109.9785	2.540
0.052	3777.6306	5.02	4061.0032	2.282
0.053	3726.1289	5.02	4014.2515	2.282

profit decreases. It is also observed that for different values of α as δ increases, total profit increases, and for fixed values of α, β, and θ as R and γ increase, total profit decreases. All these observations agree with the reality.

Comparison of results using GA and PSO

From Table 8, it is observed that in all cases, GA gives the better results than PSO. For comparison, we consider ten different generations in 50 runs of both the algorithms and

Table 7 The sensitivity analysis of γ when $\alpha = 55$, $\beta = 0.2$ and $\theta = 0.1$

γ	Case-1		Case-2	
	Total profit	t_1	Total profit	t_1
0.047	3933.0518	5.02	4245.4448	2.282
0.048	3916.9043	5.02	4215.8638	2.282
0.049	3900.8491	5.02	4187.4551	2.540
0.050	3884.8872	5.02	4160.5547	2.540
0.051	3869.0166	5.02	4133.9204	2.540
0.052	3853.2368	5.02	4107.5469	2.540
0.053	3837.5469	5.02	4081.4312	2.540

Table 8 Optimal solutions of illustrated examples for case 1 and 2 using GA and PSO

M	α	β	θ		Case-1		Case-2	
					Total profit	t_1	Total profit	t_1
4.0	55	0.2	0.1	GA	3318.6824	5.02	3688.8113	2.54
				PSO	3290.6721	4.96	3652.3742	2.47
4.2	55	0.2	0.1	GA	3538.1899	5.02	3874.9792	2.54
				PSO	3502.2781	4.96	3838.4371	2.47
4.4	55	0.2	0.1	GA	3767.0151	5.02	4064.5139	2.54
				PSO	3699.1854	4.96	3998.4203	2.47
4.5	55	0.2	0.1	GA	3884.8872	5.02	4160.5547	2.54
				PSO	3816.3524	4.96	4086.3254	2.47
4.6	55	0.2	0.1	GA	4005.0486	5.02	4257.4502	2.54
				PSO	3928.7324	4.96	4178.3794	2.47
4.8	55	0.2	0.1	GA	4252.1855	5.02	4453.8202	2.54
				PSO	8166.4732	4.96	4361.7524	2.47

perform a t-test to study the convergence. The result of t- test is shown in Table 9. In view of that the performance of GA is acceptable. Also it is clear that there is no significant difference in the mean with the two optimization algorithms. In addition, PSO can also provide a more stable and reliable solution, because it yields significantly smaller standard deviation.

Conclusion and future scope

In this paper, a realistic production-inventory model for deteriorating items has been considered under inflation and permissible delay in payments with stock-dependent demand, over a random planning horizon. Also learning effect on production and setup costs is incorporated in an economic production quantity model. The model is formulated as a nonlinear programing problem and solved numerically by both GA and PSO and the compaired. Sensitivity analyses are also performed for different parameters to study the effect of the decision variable. Here, for the first time, trade credit is allowed in an inventory model with random planning horizon which is obtained in the case fashionable goods.

Finally, for future research, one can incorporate more realistic assumptions in the proposed model considering shortages, variable deterioration rate, stochastic nature of demand, and production rate. The similar problems can be formulated with multi-items with budget and space constraints.

Managerial insights

From the Table 8, it is observed that profit increases with trade credit. Therefore, a retailer will try to avoid the maximum trade credit from the supplier. In Table 2, higher α and β furnish more profits. Specially, as β increases, profit increases. Hence, a retailer will adopt more display of goods for more sales and profits. From Tables 5 and 7, the profit increases with δ and γ. Thus, manager should always employ experience workers to get the benefit of their experience.

Table 9 Optimal solutions of the Model for different generations of case-1 and case-2 using GA and PSO

	M		Case-I		Case-II	
			T1	Total Profit	T1	Total Profit
GEN-5	4.5	GA	4.88	3873.9843	2.42	4149.0486
		PSO	4.85	3808.2343	2.32	4078.3434
GEN-10	4.5	GA	4.89	3874.2342	2.42	4150.7432
		PSO	4.85	3808.6031	2.32	4079.1364
GEN-15	4.5	GA	4.91	3874.8439	2.45	4151.2436
		PSO	4.85	3809.0012	2.36	4080.0006
GEN-20	4.5	GA	4.91	3875.4311	2.46	4151.9033
		PSO	4.87	3809.3433	2.38	4080.2329
GEN-25	4.5	GA	4.92	3876.0821	2.49	4151.9033
		PSO	4.87	3810.1045	2.42	4080.2329
GEN-30	4.5	GA	4.95	3876.9807	2.49	4152.0134
		PSO	4.89	3811.2430	2.42	4080.8792
GEN-35	4.5	GA	4.95	3878.1205	2.49	4152.7657
		PSO	4.90	3812.1236	2.44	4081.0806
GEN-40	4.5	GA	4.96	3880.0283	2.50	4152.7657
		PSO	4.91	3813.4326	2.45	4082.9801
GEN-45	4.5	GA	4.97	3882.1838	2.51	4158.7657
		PSO	4.91	3815.1062	2.45	4085.0018
GEN-50	4.5	GA	5.02	3884.8872	2.54	4160.5547
		PSO	4.96	3816.3524	2.47	4086.3254

Mean of GA is 4.9722 and SD of GA is 0.03153. Mean of PSO is 4.9344 and SD of PSO is 0.03126 and t − cal = 2.69003, t − tab = 2.872 at 18 d.f. with 1 % level of significance

So we failed to reject H0 at 1 % level of significance

Appendix 1

[Calculation for expected sales revenue for N full cycles]: present value of holding cost of the inventory for the jth$(1 \le j \le N)$ cycle, (HC_j), is given by

$$\begin{aligned}\mathrm{HC}_j &= C_1\int_{(j-1)T}^{(j-1)T+t_1} q(t)e^{-Rt}\mathrm{d}t + C_1\int_{(j-1)T+t_1}^{jT} q(t)e^{-Rt}\mathrm{d}t \\ &= \left[\frac{C_1(P-\alpha)}{(\theta+\beta)R}\left(1-e^{-Rt_1}\right) - \frac{C_1(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\right. \\ &\quad \times\left(1-e^{-(\theta+\beta+R)t_1}\right) - \frac{C_1\alpha}{(\theta+\beta)(\theta+\beta+R)} \\ &\quad \times\left(e^{-RT}-e^{(\theta+\beta)T-(\theta+\beta+R)t_1}\right) \\ &\quad \left.+\frac{C_1\alpha}{(\theta+\beta)R}\left(e^{-RT}-e^{-Rt_1}\right)\right]e^{-R(j-1)T}. \end{aligned} \tag{11}$$

Total holding cost from N full cycles, (HCN), is given by

$$\begin{aligned}\mathrm{HCN} &= \sum_{j=1}^{N}\mathrm{HC}_j \\ &= \left[\frac{C_1(P-\alpha)}{(\theta+\beta)R}\left(1-e^{-Rt_1}\right) - \frac{C_1(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\right. \\ &\quad \times\left(1-e^{-(\theta+\beta+R)t_1}\right) - \frac{C_1\alpha}{(\theta+\beta)(\theta+\beta+R)} \\ &\quad \times\left(e^{-RT}-e^{(\theta+\beta)T-(\theta+\beta+R)t_1}\right) \\ &\quad \left.+\frac{C_1\alpha}{(\theta+\beta)R}\left(e^{-RT}-e^{-Rt_1}\right)\right]\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right). \end{aligned} \tag{12}$$

So, the present value of expected holding cost from N complete cycles, (EHCN), is given by

$$\begin{aligned}\text{EHCN} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\text{HCN}.\,f(h)\,\mathrm{d}h\\ &= \left[\frac{C_1(P-\alpha)}{(\theta+\beta)R}\left(1-e^{-Rt_1}\right)-\frac{C_1(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\right.\\ &\quad\times\left(1-e^{-(\theta+\beta+R)t_1}\right)-\frac{C_1\alpha}{(\theta+\beta)(\theta+\beta+R)}\\ &\quad\times\left(e^{-RT}-e^{(\theta+\beta)T-(\theta+\beta+R)t_1}\right)\\ &\quad\left.+\frac{C_1\alpha}{(\theta+\beta)R}\left(e^{-RT}-e^{-Rt_1}\right)\right]\frac{e^{-\lambda T}}{1-e^{-(R+\lambda)T}}.\end{aligned} \tag{13}$$

Present value of production cost for the jth$(1\le j\le N)$ cycle, (PC_j), is given by

$$\begin{aligned}\text{PC}_j &= p_0e^{-\gamma j}.P\int_{(j-1)T}^{(j-1)T+t_1}e^{-Rt}dt\\ &= \frac{p_0e^{-\gamma j}.P}{R}\left(1-e^{-Rt_1}\right)e^{-R(j-1)T}.\end{aligned} \tag{14}$$

Present value of total production cost from N full cycles, (PCN), is given by

$$\begin{aligned}\text{PCN} &= \sum_{j=1}^{N}\text{PC}_j\\ &= \frac{p_0}{R}.P.e^{RT}.(1-e^{-Rt_1}).e^{-(\gamma+RT)}.\left(\frac{1-e^{-N(\gamma+RT)}}{1-e^{-(\gamma+RT)}}\right).\end{aligned} \tag{15}$$

Present value of expected total production cost from N full cycles, (EPCN), is given by

$$\begin{aligned}\text{EPCN} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\text{PCN}.\,f(h)\,\mathrm{d}h\\ &= \frac{p_0}{R}.P.e^{RT}.(1-e^{-Rt_1}).e^{-(\gamma+RT)}.\left\{\frac{e^{-\lambda T}}{(1-e^{-(\gamma+RT+\lambda T)})}\right\}.\end{aligned} \tag{16}$$

Present value of ordering cost for the jth$(1\le j\le N)$ cycle, C_3^j, is given by

$$C_3^j = \{C_3+C_3'.e^{-\delta j}\}.e^{-R(j-1)T},\ C_3, C_3', \delta>0. \tag{17}$$

Present value of total ordering cost from N full cycles, (TOCN), is given by

$$\begin{aligned}\text{TOCN} &= \sum_{j=1}^{N}C_3^j\\ &= C_3\left(\frac{1-e^{-NRT}}{1-e^{-RT}}\right)+C_3'.e^{-\delta}.\left(\frac{1-e^{-N(\delta+RT)}}{1-e^{-(\delta+RT)}}\right).\end{aligned} \tag{18}$$

Present value of expected total ordering cost from N full cycles, (ETOCN), is given by

$$\begin{aligned}\text{ETOCN} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\text{TOCN}.\,f(h)\,\mathrm{d}h\\ &= \frac{C_3e^{-\lambda T}}{(1-e^{-(\lambda+R)T})}+C_3'.e^{-\delta}.\frac{e^{-\lambda T}}{(1-e^{-(\delta+RT+\lambda T)})}.\end{aligned} \tag{19}$$

Present value of selling price for the jth$(1\le j\le N)$ cycle, (SR_j), is given by

$$\begin{aligned}\text{SR}_j &= m_0.p_0.e^{-\gamma j}\int_{(j-1)T}^{jT}\{\alpha+\beta q(t)\}.e^{-Rt}\mathrm{d}t\\ &= m_0.p_0.e^{-\gamma j}\left[\frac{\alpha.\theta+P\beta}{(\theta+\beta)R}\left(1-e^{-Rt_1}\right)\right.\\ &\quad\left.+\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left(e^{-(\theta+\beta+R)t_1}-1\right)\right]\\ &\quad+\frac{\alpha.\beta}{(\theta+\beta)(\theta+\beta+R)}\left(e^{(\theta+\beta)(T-t_1)-Rt_1}-e^{-RT}\right)\\ &\quad+\left.\frac{\alpha.\theta}{(\theta+\beta)R}\left(e^{-Rt_1}-e^{-RT}\right)\right]e^{-R(j-1)T}.\end{aligned} \tag{20}$$

Present value of total selling price from N full cycles, (SRN), is given by

$$\begin{aligned}\text{SRN} &= \sum_{j=1}^{N}\text{SR}_j\\ &= m_0.p_0.e^{-\gamma j}\left[\frac{\alpha.\theta+P\beta}{(\theta+\beta)R}\left(1-e^{-Rt_1}\right)\right.\\ &\quad\left.+\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left(e^{-(\theta+\beta+R)t_1}-1\right)\right]\\ &\quad+\frac{\alpha.\beta}{(\theta+\beta)(\theta+\beta+R)}\left(e^{(\theta+\beta)(T-t_1)-Rt_1}-e^{-RT}\right)\\ &\quad+\left.\frac{\alpha.\theta}{(\theta+\beta)R}\left(e^{-Rt_1}-e^{-RT}\right)\right].e^{-\gamma}\left(\frac{1-e^{-N(\gamma+RT)}}{1-e^{-(\gamma+RT)}}\right).\end{aligned} \tag{21}$$

Present value of expected total selling price from N full cycles, (ESRN), is given by

$$\begin{aligned}\text{ESRN} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\text{SRN}.\,f(h)\,\mathrm{d}h\\ &= m_0.p_0\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}(1-e^{-Rt_1})\right.\\ &\quad-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}(1-e^{-(\theta+\beta+R)t_1})\\ &\quad+\frac{\alpha\beta}{(\theta+\beta)(\theta+\beta+R)}(e^{(\theta+\beta)(T-t_1)-Rt_1}-e^{-RT})\\ &\quad\left.+\frac{\theta\alpha}{(\theta+\beta)R}(e^{-Rt_1}-e^{-RT})\right]\frac{e^{-(\gamma+\lambda T)}}{1-e^{(\gamma+\lambda T+RT)}}.\end{aligned} \tag{22}$$

Calculation for interest earned and interest paid in jth cycle

The retailer pays the payment at M without any interest, and at the end of this period, He/She starts paying for the interest charges on the items in stocks. Also during the time the account is not settled, generated sales revenue is deposited in an interest bearing account. Regarding the interest pay and interest earned based on the length of the jth $(1 \le j \le N)$ cycle time T, two different cases may arise:

Case-1 $(j-1)T + M \le (j-1)T + t_1 \le jT$

Present value of Interest earned in jth cycle, I_1E_j is given by

$$\begin{aligned} I_1E_j &= I_e.m_0.p_0.e^{-\gamma j}\int_{(j-1)T}^{(j-1)T+M}\left\{\alpha+\beta q(t)\right\}.\\ &\quad\times\left\{(j-1)T+M-t\right\}e^{-Rt}\,\mathrm{d}t\\ &= I_e.m_0.p_0.e^{-\gamma j}\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M+\frac{\alpha\theta+P\beta}{(\theta+\beta)R^2}(e^{-RM}-1)\right.\\ &\quad-\frac{M\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\\ &\quad\left.-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\{e^{-(\theta+\beta+R)M}-1\}\right].e^{-R(j-1)T}. \end{aligned} \tag{23}$$

Total interest earned during N full cycles, I_1EN is given by

$$\begin{aligned} I_1EN &= \sum_{j=1}^{N} I_1E_j\\ &= I_e.m_0.p_0.\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M+\frac{\alpha\theta+P\beta}{(\theta+\beta)R^2}(e^{-RM}-1)\right.\\ &\quad-\frac{M\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\\ &\quad\left.\times\{e^{-(\theta+\beta+R)M}-1\}\right].\frac{1-e^{-(\gamma+RT)N}}{1-e^{-(\gamma+RT)}}.e^{-\gamma}. \end{aligned} \tag{24}$$

Present expected value of total interest earned during N full cycles, EI_1EN, is given by

$$\begin{aligned} EI_1EN &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} I_1EN.f(h)\,\mathrm{d}h\\ &= I_e.m_0.p_0.\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M+\frac{\alpha\theta+P\beta}{(\theta+\beta)R^2}(e^{-RM}-1)\right.\\ &\quad-\frac{M\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\\ &\quad\left.\times\{e^{-(\theta+\beta+R)M}-1\}\right].\frac{e^{-(\gamma+\lambda T)}}{1-e^{-(\gamma+\lambda T+RT)}}. \end{aligned} \tag{25}$$

Present value of interest paid to the wholeseller for the jth $(1 \le j \le N)$ cycle, (I_1P_j), is given by

$$\begin{aligned} I_1P_j &= I_c.p_0.e^{-\gamma j}\int_{(j-1)T+M}^{jT} q(t).e^{-Rt}\,\mathrm{d}t\\ &= I_c.p_0.e^{-\gamma j-R(j-1)T}.\left[\frac{P-\alpha}{\theta+\beta}\left\{\frac{1}{R}\left(e^{-RM}-e^{-Rt_1}\right)\right.\right.\\ &\quad\left.-\frac{1}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1}\right)\right\}\\ &\quad+\frac{\alpha}{\theta+\beta}\left\{\frac{1}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)t_1+(\theta+\beta)T}-e^{-RT}\right)\right.\\ &\quad\left.\left.-\frac{1}{R}\left(e^{-Rt_1}-e^{-RT}\right)\right\}\right]. \end{aligned} \tag{26}$$

Total interest paid during N full cycles, I_1PN, is given by

$$\begin{aligned} I_1PN &= \sum_{j=1}^{N} I_1P_j\\ &= I_c.p_0.\left[\frac{P-\alpha}{\theta+\beta}\left\{\frac{1}{R}\left(e^{-RM}-e^{-Rt_1}\right)\right.\right.\\ &\quad\left.-\frac{1}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1}\right)\right\}\\ &\quad+\frac{\alpha}{\theta+\beta}\left\{\frac{1}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)t_1+(\theta+\beta)T}-e^{-RT}\right)\right.\\ &\quad\left.\left.-\frac{1}{R}\left(e^{-Rt_1}-e^{-RT}\right)\right\}\right]\frac{1-e^{-(\gamma+RT)N}}{1-e^{-(\gamma+RT)}}e^{-\gamma}. \end{aligned} \tag{27}$$

Present expected value of total interest paid during N full cycles, EI_1PN, is given by

$$\begin{aligned} EI_1PN &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T} I_1PN.f(h)\,\mathrm{d}h\\ &= I_c.p_0.\left[\frac{P-\alpha}{\theta+\beta}\left\{\frac{1}{R}\left(e^{-RM}-e^{-Rt_1}\right)\right.\right.\\ &\quad\left.-\frac{1}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1}\right)\right\}\\ &\quad+\frac{\alpha}{\theta+\beta}\left\{\frac{1}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)t_1+(\theta+\beta)T}-e^{-RT}\right)\right.\\ &\quad\left.\left.-\frac{1}{R}\left(e^{-Rt_1}-e^{-RT}\right)\right\}\right].\frac{e^{-(\gamma+\lambda T)}}{1-e^{-(\gamma+\lambda T+RT)}}. \end{aligned} \tag{28}$$

Case-2 $(j-1)T + t_1 < (j-1)T + M \le jT$

Present value of Interest earned in jth cycle, I_2E_j, is given by

$$I_2E_j = I_e.m_0.p_0.e^{-\gamma j}\int_{(j-1)T}^{(j-1)T+M}\left\{\alpha+\beta q(t)\right\}.$$

$$\left\{(j-1)T+M-t\right\}e^{-Rt}\,\mathrm{d}t = I_e.m_0.p_0.e^{-\gamma j-R(j-1)T}$$

$$\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M+\frac{P\beta}{(\theta+\beta)R}(t_1-M)e^{-Rt_1}\right.$$
$$+\frac{\alpha\theta}{(\theta+\beta)R^2}(e^{-RM}-1)+\frac{P\beta}{(\theta+\beta)R^2}(e^{-Rt_1}-1)$$
$$-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}.M+\frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)}$$
$$\times(t_1-M)e^{-(\theta+\beta+R)t_1}(1-e^{(\theta+\beta)T})$$
$$-\frac{P\beta}{(\theta+\beta)(\theta+\beta+R)}(t_1-M)e^{-(\theta+\beta+R)t_1}$$
$$-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}(e^{-(\theta+\beta+R)t_1}-1)$$
$$\left.+\frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)^2}e^{(\theta+\beta)T}(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1})\right]. \tag{29}$$

Total interest earned during N full cycles, I_2EN, is given by

$$I_2EN=\sum_{j=1}^{N}I_2E_j$$
$$=I_e.m_0.p_0.\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M+\frac{P\beta}{(\theta+\beta)R}(t_1-M)e^{-Rt_1}\right.$$
$$+\frac{\alpha\theta}{(\theta+\beta)R^2}(e^{-RM}-1)+\frac{P\beta}{(\theta+\beta)R^2}(e^{-Rt_1}-1)$$
$$-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}.M+\frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)}$$
$$\times(t_1-M)e^{-(\theta+\beta+R)t_1}(1-e^{(\theta+\beta)T})$$
$$-\frac{P\beta}{(\theta+\beta)(\theta+\beta+R)}(t_1-M)e^{-(\theta+\beta+R)t_1}$$
$$-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}(e^{-(\theta+\beta+R)t_1}-1)$$
$$\left.+\frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)^2}e^{(\theta+\beta)T}(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1})\right]$$
$$\times\frac{1-e^{-(\gamma+RT)N}}{1-e^{-(\gamma+RT)}}e^{-\gamma}. \tag{30}$$

Present expected value of total interest earned during N full cycles, EI_2EN, is given by

$$EI_2EN=\sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}I_2EN.f(h)\,\mathrm{d}h$$
$$=I_e.m_0.p_0.\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M+\frac{P\beta}{(\theta+\beta)R}(t_1-M)e^{-Rt_1}\right.$$
$$+\frac{\alpha\theta}{(\theta+\beta)R^2}(e^{-RM}-1)+\frac{P\beta}{(\theta+\beta)R^2}(e^{-Rt_1}-1)$$
$$-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}.M+\frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)}$$
$$\times(t_1-M)e^{-(\theta+\beta+R)t_1}(1-e^{(\theta+\beta)T})$$
$$-\frac{P\beta}{(\theta+\beta)(\theta+\beta+R)}(t_1-M)e^{-(\theta+\beta+R)t_1}$$
$$-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}(e^{-(\theta+\beta+R)t_1}-1)$$
$$\left.+\frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)^2}e^{(\theta+\beta)T}(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1})\right]$$
$$\times\frac{e^{-(\gamma+\lambda T)}}{1-e^{-(\gamma+\lambda T+RT)}}. \tag{31}$$

Present value of interest paid to the wholeseller for the jth$(1\le j\le N)$ cycle, (I_2P_j), is given by

$$I_2P_j=I_c.p_0.e^{-\gamma j}\int_{(j-1)T+M}^{jT}q(t).e^{-Rt}\,\mathrm{d}t$$
$$=I_c.p_0.e^{-\gamma j-R(j-1)T}.\frac{\alpha}{\theta+\beta}\left[\frac{1}{\theta+\beta+R}\right.$$
$$\times\left(e^{(\theta+\beta)T-(\theta+\beta+R)M}-e^{-RT}\right)$$
$$\left.-\frac{1}{R}\left(e^{-RM}-e^{-RT}\right)\right]. \tag{32}$$

Present expected value of total interest paid during N full cycles, EI_2PN, is given by

$$I_2PN=\sum_{j=1}^{N}I_2P_j$$
$$=I_c.p_0.\frac{\alpha}{\theta+\beta}\left[\frac{1}{\theta+\beta+R}\left(e^{(\theta+\beta)T-(\theta+\beta+R)M}-e^{-RT}\right)\right.$$
$$\left.-\frac{1}{R}\left(e^{-RM}-e^{-RT}\right)\right].\frac{1-e^{-(\gamma+RT)N}}{1-e^{-(\gamma+RT)}}e^{-\gamma}. \tag{33}$$

Present expected value of total interest paid during N full cycles, EI_2PN, is given by

$$EI_2PN = \sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} I_2PN.f(h)\,\mathrm{d}h$$
$$= I_c.p_0.\frac{\alpha}{\theta+\beta}\left[\frac{1}{\theta+\beta+R}\left(e^{(\theta+\beta)T-(\theta+\beta+R)M} - e^{-RT}\right) - \frac{1}{R}\left(e^{-RM} - e^{-RT}\right)\right].\frac{e^{-(\gamma+\lambda T)}}{1-e^{-(\gamma+\lambda T+RT)}}. \tag{34}$$

Appendix 2

[Calculation for expected sales revenue for last cycle] *part-I* ($NT < h \leq NT + t_1$): present value of holding cost of the inventory for the last cycle is given by

$$\mathrm{HC}_{L1} = C_1 \int_{NT}^{h} q(t)e^{-Rt}\mathrm{d}t = \frac{C_1(P-\alpha)}{\theta+\beta}\left[\frac{1}{R}\left(e^{-RNT} - e^{-Rh}\right) - \frac{e^{(\theta+\beta)NT}}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)NT} - e^{-(\theta+\beta+R)h}\right)\right]. \tag{35}$$

Present value of production cost is given by

$$\mathrm{PC}_{L1} = p_0.e^{-\gamma(N+1)}.P\int_{NT}^{h} e^{-Rt}\mathrm{d}t = \frac{p_0.e^{-\gamma(N+1)}.P}{R}\left[e^{-RNT} - e^{-Rh}\right]. \tag{36}$$

Present value of ordering cost = $\{C_3 + C'_3.e^{-\delta(N+1)}\}$ e^{-NRT}.

Present value of sales revenue is given by

$$\mathrm{SR}_{L1} = m_0.p_0.e^{-\gamma(N+1)}\int_{NT}^{h}\{\alpha+\beta q(t)\}e^{-Rt}\mathrm{d}t = m_0.p_0.e^{-\gamma(N+1)}\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}\left(e^{-RNT} - e^{-Rh}\right) - \frac{\beta(P-\alpha)e^{(\theta+\beta)NT}}{(\theta+\beta)(\theta+\beta+R)}\left(e^{-(\theta+\beta+R)NT} - e^{-(\theta+\beta+R)h}\right)\right]. \tag{37}$$

Part-II ($NT + t_1 < h \leq (N+1)T$): Present value of holding cost of the inventory for the last cycle is given by

$$\mathrm{HC}_{L2} = C_1\int_{NT}^{h} q(t)e^{-Rt}\mathrm{d}t = \frac{C_1(P-\alpha)}{\theta+\beta}\left[\frac{1}{R}\left\{e^{-RNT} - e^{-R(NT+t_1)}\right\} - \frac{e^{(\theta+\beta)NT}}{(\theta+\beta+R)}\left\{e^{-(\theta+\beta+R)NT} - e^{-(\theta+\beta+R)(NT+t_1)}\right\}\right] + \frac{c_1\alpha}{\theta+\beta}\left[\frac{e^{(\theta+\beta)(N+1)T}}{(\theta+\beta+R)}\left\{e^{-(\theta+\beta+R)(NT+t_1)} - e^{-(\theta+\beta+R)h}\right\} - \frac{1}{R}\left\{e^{-R(NT+t_1)} - e^{-Rh}\right\}\right]. \tag{38}$$

Present value of production cost is given by

$$\mathrm{PC}_{L2} = p_0.e^{-\gamma(N+1)}.P\int_{NT}^{NT+t_1} e^{-Rt}\mathrm{d}t = \frac{p_0.e^{-\gamma(N+1)}.P}{R}\left[e^{-RNT} - e^{-R(NT+t_1)}\right]. \tag{39}$$

Present value of ordering cost= $\{C_3+C'_3.e^{-\delta(N+1)}\}.e^{-RNT}$.

Present value of sales revenue is given by

$$\mathrm{SR}_{L2} = m_0.p_0.e^{-\gamma(N+1)}\int_{NT}^{h}\{\alpha+\beta q(t)\}e^{-Rt}\mathrm{d}t = m_0.p_0.e^{-\gamma(N+1)}\left[e^{-RNT}\left\{\frac{\alpha\theta+P\beta}{(\theta+\beta)R}(1-e^{-Rt_1}) - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left(1-e^{-(\theta+\beta+R)t_1}\right)\right\} + \frac{\alpha\theta}{(\theta+\beta)R}\left(e^{-R(NT+t_1)} - e^{-Rh}\right) + \frac{\beta\alpha e^{(\theta+\beta)(N+1)T}}{(\theta+\beta)(\theta+\beta+R)}\left(e^{-(\theta+\beta+R)(NT+t_1)} - e^{-(\theta+\beta+R)h}\right)\right]. \tag{40}$$

Present value of expected holding cost for the last cycle is given by

$$\mathrm{EHC}_L = \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\mathrm{HC}_L.f(h)\mathrm{d}h = \sum_{N=0}^{\infty}\int_{NT}^{NT+t_1}\mathrm{HC}_{L1}.f(h)\mathrm{d}h + \sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T}\mathrm{HC}_{L2}.f(h)\mathrm{d}h, = \mathrm{EHC}_{L1} + \mathrm{EHC}_{L2}. \tag{41}$$

where

$$\begin{aligned}\mathrm{EHC}_{L1} &= \sum_{N=0}^{\infty}\int_{NT}^{NT+t_1}\mathrm{HC}_{L1}.f(h)\mathrm{d}h\\ &= \frac{c_1(P-\alpha)}{\theta+\beta}\left[\frac{\theta+\beta}{R(\theta+\beta+R)}(1-e^{-\lambda t_1})\right.\\ &\quad -\frac{\lambda}{R(R+\lambda)}\left(1-e^{-(R+\lambda)t_1}\right)+\frac{\lambda}{(\theta+\beta+R)(\theta+\beta+R+\lambda)}\\ &\quad \left.\times\left(1-e^{-(\theta+\beta+R+\lambda)t_1}\right)\right]\frac{1}{1-e^{-(R+\lambda)T}}.\end{aligned} \tag{42}$$

and

$$\begin{aligned}\mathrm{EHC}_{L2} &= \sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T}\mathrm{HC}_{L2}.f(h)\mathrm{d}h\\ &= \left[\frac{c_{1(P-\alpha)}}{\theta+\beta}(e^{-\lambda t_1}-e^{-\lambda T})\left\{\frac{1}{R}(1-e^{-Rt_1})\right.\right.\\ &\quad \left.-\frac{1}{\theta+\beta+R}\left(1-e^{-(\theta+\beta+R)t_1}\right)\right\}\\ &\quad +\frac{c_1\alpha}{\theta+\beta}\left\{\left(\frac{e^{(\theta+\beta)T-(\theta+\beta+R)t_1}}{\theta+\beta+R}-\frac{e^{-Rt_1}}{R}\right)(e^{-\lambda t_1}-e^{-\lambda T})\right.\\ &\quad -\frac{\lambda}{(\theta+\beta+R)(\theta+\beta+R+\lambda)}\\ &\quad \left(e^{(\theta+\beta)T-(\theta+\beta+R+\lambda)t_1}-e^{-(R+\lambda)t_1}\right)\\ &\quad \left.\left.+\frac{\lambda}{R(R+\lambda)}\left(e^{-(R+\lambda)t_1}-e^{-(R+\lambda)T}\right)\right\}\right]\frac{1}{1-e^{-(R+\lambda)T}}.\end{aligned} \tag{43}$$

Present value of expected production cost for the last cycle is given by

$$\begin{aligned}\mathrm{EPC}_L &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\mathrm{PC}_L.f(h)\mathrm{d}h\\ &= \sum_{N=0}^{\infty}\int_{NT}^{NT+t_1}\mathrm{PC}_{L1}.f(h)dh+\sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T}\mathrm{PC}_{L2}.f(h)\mathrm{d}h\\ &= \frac{p_0.e^{-\gamma}.P}{R}\left[(1-e^{-\lambda t_1}).\frac{1}{(1-e^{-(RT+\lambda T+\gamma)})}+\frac{\lambda}{(R+\lambda)}\right.\\ &\quad \left.\times\left\{\frac{e^{-(R+\lambda)t_1}-1}{(1-e^{-(RT+\lambda T+\gamma)})}\right\}\right]+\frac{p_0.e^{-\gamma}.P}{R}\\ &\quad \left[(1-e^{-Rt_1}).(e^{-\lambda t_1}-e^{-\lambda T}).\frac{1}{(1-e^{-(RT+\lambda T+\gamma)})}\right].\end{aligned} \tag{44}$$

Present value of expected sales revenue from the last cycle is given by

$$\begin{aligned}\mathrm{ESR}_L &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\mathrm{SR}_L.f(h)\mathrm{d}h\\ &= \sum_{N=0}^{\infty}\int_{NT}^{NT+t_1}\mathrm{SR}_{L1}.f(h)\mathrm{d}h\\ &\quad +\sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T}\mathrm{SR}_{L2}.f(h)\mathrm{d}h\\ &= \mathrm{ESR}_{L1}+\mathrm{ESR}_{L2}.\end{aligned} \tag{45}$$

where

$$\begin{aligned}\mathrm{ESR}_{L1} &= \sum_{N=0}^{\infty}\int_{NT}^{NT+t_1}\mathrm{SR}_{L1}.f(h)\mathrm{d}h\\ &= m_0.p_0\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}\left\{(1-e^{-\lambda t_1})\right.\right.\\ &\quad \left.-\frac{\lambda}{R+\lambda}\left(1-e^{-(R+\lambda)t_1}\right)\right\}-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\\ &\quad \left.\left\{(1-e^{-\lambda t_1})-\frac{\lambda}{\theta+\beta+R+\lambda}\left(1-e^{-(\theta+\beta+R+\lambda)t_1}\right)\right\}\right]\\ &\quad \frac{e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.\end{aligned} \tag{46}$$

and

$$\begin{aligned}\mathrm{ESR}_{L2} &= \sum_{N=0}^{\infty}\int_{NT+t_1}^{(N+1)T}\mathrm{SR}_{L2}.f(h)\mathrm{d}h\\ &= m_0.p_0\left[\left\{\frac{\alpha\theta}{(\theta+\beta)R}+\frac{P\beta}{(\theta+\beta)R}(1-e^{-Rt_1})\right.\right.\\ &\quad \left.-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left(1-e^{-(\theta+\beta+R)t_1}\right)\right\}(e^{-\lambda t_1}-e^{-\lambda T})\\ &\quad -\frac{\lambda\alpha\theta}{(\theta+\beta)R(R+\lambda)}\left(e^{-(R+\lambda)t_1}-e^{-(R+\lambda)T}\right)\\ &\quad +\frac{\beta\alpha e^{(\theta+\beta)T}}{(\theta+\beta)(\theta+\beta+R)}\left\{e^{-(\theta+\beta+R)t_1}(e^{-\lambda t_1}-e^{-\lambda T})\right.\\ &\quad \left.\left.-\frac{\lambda}{\theta+\beta+R+\lambda}\left(e^{-(\theta+\beta+R+\lambda)t_1}-e^{-(\theta+\beta+R+\lambda)T}\right)\right\}\right]\\ &\quad \times\frac{e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.\end{aligned} \tag{47}$$

Present value of expected ordering cost for the last cycle is given by

$$\begin{aligned}\mathrm{EOC}_L &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\left\{C_3+C_3'.e^{-\delta(N+1)}\right\}.e^{-NRT}f(h)\mathrm{d}h\\ &= C_3\frac{(1-e^{-\lambda T})}{(1-e^{-(\lambda+R)T})}+C_3'.e^{-\delta}.\frac{(1-e^{-\lambda T})}{(1-e^{-(\delta+\lambda T+RT)})}.\end{aligned} \tag{48}$$

Present value of expected reduced selling price from the last cycle is given by

$$\begin{aligned}\mathrm{ERSP}_L &= m_1p_0\sum_{N=0}^{\infty}e^{-\gamma(N+1)}\int_{NT}^{(N+1)T}e^{-Rh}q(h).f(h)\mathrm{d}h\\ &= m_1p_0e^{-\gamma}\sum_{N=0}^{\infty}e^{-\gamma N}\int_{NT}^{NT+t_1}e^{-Rh}q(h).f(h)\mathrm{d}h\\ &\quad +m_1p_0e^{-\gamma}\sum_{N=0}^{\infty}e^{-\gamma N}\int_{NT+t_1}^{(N+1)T}e^{-Rh}q(h).f(h)\mathrm{d}h\\ &= \mathrm{ERSP}_{L1}+\mathrm{ERSP}_{L2}.\end{aligned} \tag{49}$$

where

$$
\begin{aligned}
\mathrm{ERSP}_{L1} &= m_1 p_0 e^{-\gamma} \sum_{N=0}^{\infty} e^{-\gamma N} \int_{NT}^{NT+t_1} e^{-Rh} q(h)\,.f(h)\mathrm{d}h \\
&= m_1 .p_0 \frac{P-\alpha}{\theta+\beta}\left[\frac{1}{R+\lambda}\left(1-e^{-(R+\lambda)t_1}\right)\right. \\
&\quad \left. -\frac{1}{\theta+\beta+R+\lambda}\left(1-e^{-(\theta+\beta+R+\lambda)t_1}\right)\right] \\
&\quad \frac{\lambda e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.
\end{aligned} \tag{50}
$$

and

$$
\begin{aligned}
\mathrm{ERSP}_{L2} &= m_1 p_0 e^{-\gamma} \sum_{N=0}^{\infty} e^{-\gamma N} \int_{NT+t_1}^{(N+1)T} e^{-Rh} q(h)\,.f(h)\mathrm{d}h \\
&= m_1 p_0 \frac{\alpha}{\theta+\beta}\left[\frac{1}{\theta+\beta+R+\lambda}\right. \\
&\quad \times \left(e^{(\theta+\beta)T-(\theta+\beta+R+\lambda)t_1} - e^{-(R+\lambda)T}\right) \\
&\quad \left. -\frac{1}{R+\lambda}\left(e^{-(R+\lambda)t_1} - e^{-(R+\lambda)T}\right)\right] \frac{\lambda e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.
\end{aligned} \tag{51}
$$

Calculations for interest earned and interest paid in last cycle

Case 1 $(NT+M \le NT+t_1 \le (N+1)T)$

Subcase (1a) $(NT \le t \le NT+M)$

Present value of interest earned in last cycle, IEL_{11}, is given by

$$
\begin{aligned}
\mathrm{IEL}_{11} &= I_e.m_0.p_0 e^{-\gamma(N+1)} \int_{NT}^{h} \left\{\alpha+\beta q(t)\right\}(h-t)e^{-Rt}\mathrm{d}t \\
&= I_e.m_0.p_0 e^{-\gamma(N+1)} \left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}(h-NT)e^{-RNT}\right. \\
&\quad + \frac{\alpha\theta+P\beta}{(\theta+\beta)R^2}\left(e^{-Rh}-e^{-RNT}\right) \\
&\quad - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}(h-NT)e^{-RNT} \\
&\quad \left. - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\left(e^{(\theta+\beta)NT-(\theta+\beta+R)h} - e^{-RNT}\right)\right].
\end{aligned} \tag{52}
$$

Present expected value of interest earned in the last cycle, EIEL_{11}, is given by

$$
\begin{aligned}
\mathrm{EIEL}_{11} &= \sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} \mathrm{IEL}_{11}.f(h)\mathrm{d}h \\
&= I_e.m_0.p_0 \left[-\frac{\alpha\theta+P\beta}{(\theta+\beta)R}\left(Te^{-\lambda T}+\frac{e^{-\lambda T}-1}{\lambda}\right)\right. \\
&\quad - \frac{\alpha\theta+P\beta}{(\theta+\beta)R^2}\left\{\frac{\lambda}{R+\lambda}\left(e^{-(R+\lambda)T}-1\right)-\left(e^{-\lambda T}-1\right)\right\} \\
&\quad + \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left(Te^{-\lambda T}+\frac{e^{-\lambda T}-1}{\lambda}\right) \\
&\quad + \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\left\{\frac{\lambda}{\theta+\beta+R+\lambda}\left(e^{-(\theta+\beta+R+\lambda)T}-1\right)\right. \\
&\quad \left.\left. -\left(e^{-\lambda T}-1\right)\right\}\right]\frac{e^{-\gamma}}{1-e^{-(\gamma+\lambda T+RT)}}.
\end{aligned} \tag{53}
$$

Subcase (1b) $(NT+M \le t \le NT+t_1)$

Present value of interest earned in last cycle, IEL_{12}, is given by

$$
\begin{aligned}
\mathrm{IEL}_{12} &= I_e.m_0.p_0 e^{-\gamma(N+1)} \int_{NT}^{NT+M} \left\{\alpha+\beta q(t)\right\} \\
&\quad (NT+M-t)e^{-Rt}\mathrm{d}t = I_e.m_0.p_0 e^{-(\gamma+RT)N}.e^{-\gamma} \\
&\quad \left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M - \frac{\alpha\theta+P\beta}{(\theta+\beta)R^2}\left(1-e^{-RM}\right)\right. \\
&\quad - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}.M + \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2} \\
&\quad \left.\left(1-e^{-(\theta+\beta+R)M}\right)\right].
\end{aligned} \tag{54}
$$

Present expected value of interest earned in the last cycle, EIEL_{12}, is given by

$$
\begin{aligned}
\mathrm{EIEL}_{12} &= \sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} \mathrm{IEL}_{12}.f(h)\mathrm{d}h \\
&= I_e.m_0.p_0 \left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M - \frac{\alpha\theta+P\beta}{(\theta+\beta)R^2}\left(1-e^{-RM}\right)\right. \\
&\quad - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}.M \\
&\quad \left. + \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\left(1-e^{-(\theta+\beta+R)M}\right)\right] \\
&\quad \times \frac{(1-e^{-\lambda T})e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.
\end{aligned} \tag{55}
$$

Present value of interest paid in the last cycle, IPL_{12}, is given by

$$\begin{aligned}\mathrm{IPL}_{12} &= I_c.p_0.e^{-\gamma(N+1)}\int_{NT+M}^{h} q(t).e^{-Rt}\mathrm{d}t\\ &= I_c.p_0.e^{-\gamma(N+1)}.\frac{P-\alpha}{\theta+\beta}\left[\frac{1}{R}\left(e^{-R(NT+M)}-e^{-Rh}\right)\right.\\ &\quad\left.+\frac{1}{\theta+\beta+R}\left(e^{(\theta+\beta)NT-(\theta+\beta+R)h}-e^{-RNT-(\theta+\beta+R)M}\right)\right].\end{aligned} \tag{56}$$

Present expected value of interest paid in the last cycle, EIPL_{12}, is given by

$$\begin{aligned}\mathrm{EIPL}_{12} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\mathrm{IPL}_{12}.f(h)\mathrm{d}h\\ &= I_c.p_0.\frac{P-\alpha}{\theta+\beta}\left[\frac{1}{R}\left\{e^{-RM}\left(1-e^{-\lambda T}\right)\right.\right.\\ &\quad\left.-\frac{\lambda}{R+\lambda}\left(1-e^{-(R+\lambda)T}\right)\right\}\\ &\quad-\frac{1}{\theta+\beta+R}\left\{e^{-(\theta+\beta+R)M}\left(1-e^{-\lambda T}\right)\right.\\ &\quad\left.\left.-\frac{\lambda}{\theta+\beta+R+\lambda}\left(1-e^{-(\theta+\beta+R+\lambda)T}\right)\right\}\right].\\ &\quad\times\frac{e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.\end{aligned} \tag{57}$$

Subcase (1c) $(NT+t_1 \le t \le (N+1)T)$

The expression of the present expected value of interest earned in the last cycle, EIEL_{13}, is same as Eq. (55).

Present value of interest paid in the last cycle, IPL_{13}, is given by

$$\begin{aligned}\mathrm{IPL}_{13} &= I_c.p_0.e^{-\gamma(N+1)}\int_{NT+M}^{h} q(t).e^{-Rt}\mathrm{d}t\\ &= I_c.p_0.e^{-\gamma(N+1)}\left[\int_{NT+M}^{NT+t_1} q(t).e^{-Rt}\mathrm{d}t\right.\\ &\quad\left.+\int_{NT+t_1}^{h} q(t).e^{-Rt}\mathrm{d}t\right] = I_c p_0 e^{-\gamma(N+1)}\\ &\quad\left[e^{-RNT}\frac{P-\alpha}{\theta+\beta}\left\{\frac{1}{R}(e^{-RM}-e^{-Rt_1})\right.\right.\\ &\quad\left.-\frac{1}{\theta+\beta+R}\left(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1}\right)\right\}\\ &\quad+\frac{\alpha}{\theta+\beta}\left\{\frac{e^{(\theta+\beta)T}}{\theta+\beta+R}\right.\\ &\quad\times\left(e^{-(\theta+\beta+R)t_1-RNT}-e^{-(\theta+\beta+R)h+(\theta+\beta)NT}\right)\\ &\quad\left.\left.-\frac{1}{R}\left(e^{-Rt_1-RNT}-e^{-Rh}\right)\right\}\right].\end{aligned} \tag{58}$$

Present expected value of interest paid in the last cycle, EIPL_{13}, is given by

$$\begin{aligned}\mathrm{EIPL}_{13} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\mathrm{IPL}_{13}.f(h)\mathrm{d}h\\ &= I_c p_0\left[\frac{P-\alpha}{\theta+\beta}\left\{\frac{1}{R}(e^{-RM}-e^{-Rt_1})-\frac{1}{\theta+\beta+R}\right.\right.\\ &\left.\left(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1}\right)\right\}(1-e^{-\lambda T})\\ &+\frac{\alpha}{\theta+\beta}\left\{\frac{1}{\theta+\beta+R}\left(e^{(\theta+\beta)T-(\theta+\beta+R)t_1}(1-e^{-\lambda T})\right.\right.\\ &\left.-\frac{\lambda}{\theta+\beta+R+\lambda}(e^{(\theta+\beta)T}-e^{-(R+\lambda)T})\right)\\ &\left.\left.-\frac{1}{R}\left(e^{-Rt_1}(1-e^{-\lambda T})-\frac{\lambda}{R+\lambda}(1-e^{-(R+\lambda)T})\right)\right\}\right]\\ &\times\frac{e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.\end{aligned} \tag{59}$$

Total expected interest earned for the last cycle, EI_1E_L, is given by

$$EI_1E_L = \mathrm{EIEL}_{11}+\mathrm{EIEL}_{12}+\mathrm{EIEL}_{13}. \tag{60}$$

Total expected interest paid for the last cycle, EI_1P_L, is given by

$$EI_1P_L = \mathrm{EIPL}_{12}+\mathrm{EIPL}_{13}. \tag{61}$$

Case 2 $(NT+t_1\le NT+M\le (N+1)T)$

Subcase (2a) $(NT\le t\le NT+t_1)$

The expression of the present expected value of interest earned in the last cycle, EIEL_{21}, is same as Eq. (53).

Subcase (2b) $(NT+t_1\le t\le NT+M)$

Present value of interest earned in last cycle, IEL_{22}, is given by

$$\begin{aligned}\mathrm{IEL}_{22} &= I_e.m_0.p_0e^{-\gamma(N+1)}\int_{NT}^{h}\left\{\alpha+\beta q(t)\right\}(h-t)e^{-Rt}\mathrm{d}t\\ &= I_e.m_0.p_0e^{-\gamma(N+1)}\left[\int_{NT}^{NT+t_1}\left\{\alpha+\beta q(t)\right\}(h-t)e^{-Rt}\mathrm{d}t+\int_{NT+t_1}^{h}\left\{\alpha+\beta q(t)\right\}(h-t)e^{-Rt}\mathrm{d}t\right]\\ &= I_e.m_0.p_0e^{-\gamma(N+1)}\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}e^{-RNT}\left\{t_1e^{-Rt_1}-\left(h-NT-\frac{1}{R}\right)(e^{-Rt_1}-1)\right\}\right.\\ &-\frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}e^{-RNT}\left\{t_1e^{-(\theta+\beta+R)t_1}-\left(h-NT-\frac{1}{\theta+\beta+R}\right)\left(e^{-(\theta+\beta+R)t_1}-1\right)\right\}\\ &+\frac{\alpha\theta}{(\theta+\beta)R}\left\{\left(h-NT-t_1-\frac{1}{R}\right)e^{-R(NT+t_1)}+\frac{1}{R}e^{-Rh}\right\}\\ &\left.+\frac{\beta\alpha.e^{(\theta+\beta)(N+1)T}}{(\theta+\beta)(\theta+\beta+R)}\left\{\left(h-NT-t_1-\frac{1}{\theta+\beta+R}\right)e^{-(\theta+\beta+R)(NT+t_1)}+\frac{e^{-Rh}}{\theta+\beta+R}\right\}\right].\end{aligned} \tag{62}$$

Present expected value of interest earned in the last cycle, EIEL_{22}, is given by

$$\begin{aligned}
\mathrm{EIEL}_{22} &= \sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} \mathrm{IEL}_{22}.f(h)\mathrm{d}h \\
&= I_e.m_0.p_0\left[\frac{\alpha\theta + P\beta}{(\theta+\beta)R}\left\{(1-e^{-\lambda T})t_1e^{-Rt_1} + (e^{-Rt_1}-1)\left(Te^{-\lambda T} + (1-e^{-\lambda T})(\frac{1}{R}-\frac{1}{\lambda})\right)\right\}\right. \\
&\quad - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left\{(1-e^{-\lambda T})t_1e^{-(\theta+\beta+R)t_1}\right. \\
&\quad \left. + \left(e^{-(\theta+\beta+R)t_1}-1\right)\left(Te^{-\lambda T} + (1-e^{-\lambda T})(\frac{1}{\theta+\beta+R}-\frac{1}{\lambda})\right)\right\} \\
&\quad - \frac{\alpha\theta}{(\theta+\beta)R}\left\{e^{-Rt_1}\left(Te^{-\lambda T} + (1-e^{-\lambda T})(t_1+\frac{1}{R}-\frac{1}{\lambda})\right) + \frac{\lambda}{R(R+\lambda)}\left(e^{-(R+\lambda)T}-1\right)\right\} \\
&\quad - \frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)}\left\{e^{(\theta+\beta)T-(\theta+\beta+R)t_1}\left(Te^{-\lambda T} + (1-e^{-\lambda T})(t_1+\frac{1}{\theta+\beta+R}-\frac{1}{\lambda})\right)\right. \\
&\quad \left.\left. + \frac{\lambda}{(\theta+\beta+R)(R+\lambda)}\left(e^{-(R+\lambda)T}-1\right)\right\}\right]\frac{e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.
\end{aligned} \tag{63}$$

Subcase (2c) $(NT+M \le t \le (N+1)T)$

Present value of interest earned in last cycle, IEL_{23}, is given by

$$\begin{aligned}
\mathrm{IEL}_{23} &= I_e.m_0.p_0e^{-\gamma(N+1)}\int_{NT}^{NT+M}\left\{\alpha+\beta q(t)\right\}(NT+M-t)e^{-Rt}\mathrm{d}t \\
&= I_e.m_0.p_0e^{-\gamma(N+1)}\left[\int_{NT}^{NT+t_1}\left\{\alpha+\beta q(t)\right\}(NT+M-t)e^{-Rt}\mathrm{d}t\right. \\
&\quad \left. + \int_{NT+t_1}^{(N+1)T}\left\{\alpha+\beta q(t)\right\}(NT+M-t)e^{-Rt}\mathrm{d}t\right] \\
&= I_e.m_0.p_0.e^{-\gamma(N+1)-RNT}\left[\frac{\alpha\theta+P\beta}{(\theta+\beta)R}.M - \frac{P\beta}{(\theta+\beta)R}e^{-Rt_1}(M-t_1)\right. \\
&\quad - \frac{\alpha\theta}{(\theta+\beta)R^2}(1-e^{-RM}) - \frac{P\beta}{(\theta+\beta)R^2}(1-e^{-Rt_1}) \\
&\quad + \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left((M-t_1)e^{-(\theta+\beta+R)t_1}-M\right) \\
&\quad + \frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)}e^{(\theta+\beta)T-(\theta+\beta+R)t_1}(M-t_1) - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\left(e^{-(\theta+\beta+R)t_1}-1\right) \\
&\quad \left. + \frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)^2}e^{(\theta+\beta)T}\left(e^{-(\theta+\beta+R)M}-e^{-(\theta+\beta+R)t_1}\right)\right].
\end{aligned} \tag{64}$$

Present expected value of interest earned in last cycle, EIEL_{23}, is given by

$$\begin{aligned}\mathrm{EIEL}_{23} &= \sum_{N=0}^{\infty} \int_{NT}^{(N+1)T} \mathrm{IEL}_{23}.f(h)\mathrm{d}h \\ &= I_e.m_0.p_0\bigg[\frac{\alpha\theta + P\beta}{(\theta+\beta)R}.M - \frac{P\beta}{(\theta+\beta)R}e^{-Rt_1}(M-t_1) \\ &\quad - \frac{\alpha\theta}{(\theta+\beta)R^2}(1-e^{-RM}) - \frac{P\beta}{(\theta+\beta)R^2}(1-e^{-Rt_1}) \\ &\quad + \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)}\left((M-t_1)e^{-(\theta+\beta+R)t_1} - M\right) \\ &\quad + \frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)}e^{(\theta+\beta)T-(\theta+\beta+R)t_1}(M-t_1) - \frac{\beta(P-\alpha)}{(\theta+\beta)(\theta+\beta+R)^2}\left(e^{-(\theta+\beta+R)t_1} - 1\right) \\ &\quad + \frac{\beta\alpha}{(\theta+\beta)(\theta+\beta+R)^2}e^{(\theta+\beta)T}\left(e^{-(\theta+\beta+R)M} - e^{-(\theta+\beta+R)t_1}\right)\bigg]\frac{e^{-\gamma}(1-e^{-\lambda T})}{1-e^{-(\gamma+RT+\lambda T)}}.\end{aligned} \tag{65}$$

Present value of interest paid in the last cycle, IPL_{23}, is given by

$$\begin{aligned}\mathrm{IPL}_{23} &= I_c.p_0.e^{-\gamma(N+1)}\int_{NT+M}^{h} q(t).e^{-Rt}\mathrm{d}t \\ &= I_c.p_0.e^{-\gamma(N+1)}.\frac{\alpha}{\theta+\beta}\bigg[\frac{1}{\theta+\beta+R} \\ &\quad \times \left(e^{(\theta+\beta-RN)T-(\theta+\beta+R)M} - e^{-(\theta+\beta+R)h+(\theta+\beta)(N+1)T}\right) \\ &\quad + \frac{1}{R}\left(e^{-Rh} - e^{-R(NT+M)}\right)\bigg].\end{aligned} \tag{66}$$

Present expected value of interest paid in the last cycle, EIPL_{23}, is given by

$$\begin{aligned}\mathrm{EIPL}_{23} &= \sum_{N=0}^{\infty}\int_{NT}^{(N+1)T}\mathrm{IPL}_{23}.f(h)\mathrm{d}h \\ &= I_c.p_0\frac{\alpha}{\theta+\beta}\bigg[\frac{1}{\theta+\beta+R}\Big\{e^{(\theta+\beta)T-(\theta+\beta+R)M}(1-e^{-\lambda T}) \\ &\quad + \frac{\lambda}{\theta+\beta+R+\lambda}\left(e^{-(R+\lambda)T} - e^{(\theta+\beta)T}\right)\Big\} \\ &\quad + \frac{1}{R}\Big\{\frac{\lambda}{R+\lambda}\left(1-e^{-(R+\lambda)T}\right) + (e^{-RM-\lambda T} - e^{-RM})\Big\}\bigg] \\ &\quad \times \frac{e^{-\gamma}}{1-e^{-(\gamma+RT+\lambda T)}}.\end{aligned} \tag{67}$$

Total expected interest earned for the last cycle, EI_2E_L,, is given by

$$EI_2E_L = \mathrm{EIEL}_{21} + \mathrm{EIEL}_{22} + \mathrm{EIEL}_{23}. \tag{68}$$

Total expected interest paid for the last cycle, EI_2P_L, is given by

$$EI_2P_L = \mathrm{EIPL}_{23}. \tag{69}$$

References

Alamri AA, Balkhi ZT (2007) The effects of learning and forgetting on the optimal production lot size for deteriorating items with time varying demand and deterioration rates. Int J Prod Econ 107:125–138

Chang CT, Ouyang LY, Teng JT (2003) An EOQ model for deteriorating items under supplier credits linked to ordering quantity. Appl Math Model 27:983–996

Chiu HN, Chen HM (2005) An optimal algorithm for solving the dynamic lot-sizing model with learning and forgetting in setups and production. Int J Prod Econ 95:179–193

Chung KH, Kim YH (1989) Economic analysis of inventory systems: a rejoinder. Eng Econ 35:75–80

Chung KJ, Liao JJ (2004) Lot-sizing decisions under trade credit depending on the ordering quantity. Comput Oper Res 31:909–928

Das D, Roy A, Kar S (2010a) Improving production policy for a deteriorating item under permissible delay in payments with stock-dependent demand rate. Comput Math Appl 60:1973–1985

Das D, Roy A, Kar S (2010b) A production-inventory model for a deteriorating item incorporating learning effect using genetic algorithm. Adv Oper Res. doi:10.1155/2010/146042

Das D, Kar MB, Roy A, Kar S (2012) Two-warehouse production model for deteriorating inventory items with stock-dependent demand under inflation over a random planning horizon. Cent Eur J Oper Res 20(2):251–280

Datta TK, Pal AK (1992) A note on a replenishment policy for an inventory model with linear trend in demand and shortages. J Oper Res Soc 43:993–1001

Giri BC, Pal SP, Chaudhuri KS (1996) An inventory model for deteriorating items with stock-dependent demand rate. Eur J Oper Res 95:604–610

Goldberg DE (1998) Genetic algorithms in search, optimization and machine learning. Addison Wesley, New York
Goyal SK (1985) Economic order quantity under conditions of permissible delay in payments. J Oper Res Soc 36:335–338
Gurnani C (1985) Economic analysis of inventory systems a reply. Int J Prod Res 23:771–772
Holland HJ (1975) Adaptation in natural and artificial systems. University of Michigan, Michigan
Hou KL (2006) An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. Eur J Oper Res 168:463–474
Kuo WH, Yang DL (2006) Minimizing the total completion time in a single-machine scheduling problem with a time-dependent learning effect. Eur J Oper Res 174:1184–1190
Maiti AK, Maity MK, Maiti M (2006) Two storage inventory model with fuzzy deterioration over a random planning horizon. Appl Math Comput 183:1084–1097
Mandal BN, Phaujdar S (1989a) A note on inventory model with stock-dependent consumption rate. OPSEARCH 26:43–46
Mandal BN, Phaujdar S (1989b) An inventory model for deteriorating items and stock dependent consumption rate. J Oper Res Soc 40:483–488
Michalewicz Z (1992) Genetic algorithms + data structure = evolution programs. Springer, Berlin
Min J, Zhou YW, Liu G-Q, Wang S-D (2012) An EPQ model for deteriorating items with inventory-level-dependent demand and permissible delay in payments. Int J Syst Sci 43(6):1039–1053
Moon I, Yun W (1993) An economic order quantity model with random planning horizon. Eng Econ 39:77–86
Ouyang LY, Chang CT (2013) Quality improvement, setup cost and lead-time reductions in lot size reorder point models with an imperfect production process. Int J Prod Econ 144(2):610–617
Roy A, Maiti MK, Kar S, Maiti M (2007) Two storage inventory model with fuzzy deterioration over a random planning horizon. Math Comput Model 46:1419–1433
Roy A, Pal S, Maiti MK (2009a) A production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon: A fuzzy genetic algorithm with varying population size approach. Comput Ind Eng 57:1324–1335
Roy A, Maiti MK, Kar S, Maiti M (2009b) An inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. Appl Math Model 33(2):744–759
Su SM, Lin SD, Chang LF (2014) The optimal inventory policy for reusable items with random planning horizon considering present value. Appl Math 5:292–299
Teng J-T, Krommyda IP, Skouri K, Lou K-R (2011) A comprehensive extension of optimal ordering policy for stock-dependent demand under progressive payment scheme. Eur J Oper Res 215:97–104