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A fuzzy production model with probabilistic resalable returns

A.Nagoorgani[a] [1], P. Palaniammal[b]
[a]PG & Research Department of Mathematics,
Jamal Mohamed College (Autonomous),
Tiruchirappalli-620 020,Tamil Nadu, India.
[b] PG & Research Department of Mathematics,
Seethalakshmi Ramaswami College (Autonomous),
Tiruchirappalli-02,Tamil Nadu, India.

Email: ganijmc@yahoo.co.in
Email: pp_src@yahoo.com

Abstract. In this paper, a fuzzy production inventory model with resalable returns has been analysed in an imprecise and uncertain environment by considering the cost and revenue parameters as trapezoidal fuzzy numbers. The main objective is to determine the optimal fuzzy production lotsize which maximizes the expected profit where the products leftout at the end of the period are salvaged and demands which are not met directly are lost. The modified graded mean integration representation method is used for defuzzification of fuzzy parameters of production lotsize and expected profit. An example is presented to illustrate the method applied in the model.

Keywords : Fuzzy production model, Fuzzy random variable, Modified graded mean integration representation, Returned resalable products, Trapezoidal fuzzy numbers.

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[1]Correspondence author

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1. Introduction

A production inventory model in which the items are produced one and are sold after the completion of production of all items where the sold products may be returned by the customers to the firm within a specified time so that the returned products will be resalable provided they are undamaged.
The model in which the concept of resalable returns was first considered by Vlachos and Dekker [11] with two restrictive assumptions namely

(i) the return of a fixed percentage of the sold products excludes the prospect of variability in the net demand and
(ii) a single reselling process for each returned product will not consider high return rates because it will lead to multiple resale of products. However, these two restrictions are not applied in the analysis of the 'newsboy problem' with resalable returns by Mostard, Koster and Teunter [9].

In [8], the newsboy problem transforms the normally distributed demand of product into a fuzzy random variable with imprecise probabilities, where the probability of a fuzzy event is a fuzzy number [1]. Recently, demand has been considered as fuzzy random variable by Dutta et al in [4]. The probabilities of return by a sold product and those of resale of a returned product have been assumed to be fuzzy numbers.

In this paper, first a result which involves an infinite series of fuzzy numbers has been proposed. Next, a fuzzy random variable and its fuzzy expectation have been defined. Further, a brief outline of a modified graded mean integration representation of a trapezoidal fuzzy number has been developed and discussed. Following this notion, assumptions along with a mathematical model and the formulation of the problem with an example are presented and is finally followed by conclusion.

2. Preliminaries

The concept of infinite series of fuzzy numbers, the notion of fuzzy random variable with its expectation and the representation of a modified graded mean integration of generalized fuzzy numbers are presented below.

2.1. **Infinite series of fuzzy numbers.** An infinite series of fuzzy numbers is obtained by the following preposition:
If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal normal fuzzy number, where a_1, a_2, a_3, a_4 are positive real numbers, then the relation

$$\tilde{1} \oplus \tilde{A} \oplus \tilde{A}^2 \oplus \tilde{A}^3 \oplus \cdots = (1\Theta\tilde{A})^{-1}, |\tilde{A}| < 1$$

holds.

Proof. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number [2, 3]. Then

$$\tilde{A}^2 = (a_1^2, a_2^2, a_3^2, a_4^2); \tilde{A}^3 = (a_1^3, a_2^3, a_3^3, a_4^3); \tilde{A}^4 = (a_1^4, a_2^4, a_3^4, a_4^4)$$

and so on. Therefore,

$$\begin{aligned}\tilde{1} \oplus \tilde{A} \oplus \tilde{A}^2 \oplus \tilde{A}^3 \oplus \cdots = &(1,1,1,1) \oplus (a_1, a_2, a_3, a_4)\\ &\oplus (a_1^2, a_2^2, a_3^2, a_4^2) \oplus (a_1^3, a_2^3, a_3^3, a_4^4)\\ &\oplus (a_1^4, a_2^4, a_3^4, a_4^4)\\ =&((1-a_1)^{-1}, (1-a_2)^{-1}, (1-a_3)^{-1}(1-a_4)^{-1})_{TRFN}\\ =&(\tilde{1}\Theta\tilde{A})^{-1}\end{aligned}$$

where $|a_1| < 1$, $|a_2| < 1$, $|a_3| < 1$, $|a_4| < 1$.
Thus, $\tilde{1} \oplus \tilde{A} \oplus \tilde{A}^2 \oplus \tilde{A}^3 \oplus ... = (\tilde{1}\Theta\tilde{A})^{-1}, where|\tilde{A}| < 1$. □

2.2. **Fuzzy random variable and its fuzzy expectation.** The concept of fuzzy random variables was first introduced by Kwakernaak [7]. In later years, the same concept was discussed by Puri, Ralescu [10] and Gil, Lopez-Diaz [6]. Now, the definition in [5] has been considered as below :

Let R^p be the p-dimensional Euclidean space. Let $F(R^p)$ be the class of upper semi continuous functions in $[0,1]$ with compact closure of the support.

Then $F_c(R)$ is the sub-class of convex sets of $F(R)$. Given a probability space (Ω, A, P), a mapping $\chi : \Omega \to F_c(R)$ is said to be a fuzzy random variable if for all $\alpha \in [0,1]$, the two real-valued mappings inf $\chi_\alpha : \Omega \to R$ and sup $\chi_\alpha : \Omega \to R$ defined for all $\omega \in \Omega$ such that $\chi_\alpha(\omega) = [\inf(\chi(\omega))_\alpha, \sup(\chi(\omega))_\alpha]$ are real-valued random variables.

The fuzzy expectation of a random variable is a unique fuzzy number which is defined by $E(\tilde{x}) = \int \tilde{x} dP = \left\{\left(\int x_\alpha^- dP, \int x_\alpha^+ dP\right)/0 < \alpha < 1\right\}$ where the fuzzy random variable is $[X]_\alpha = [x_\alpha^-, x_\alpha^+]$ for $\alpha \in [0,1]$. The a-cut of the fuzzy expectation is defined by $u_\alpha = [E(\tilde{x})]_\alpha = [E(x_\alpha^-), E(x_\alpha^+)]$ for $\alpha \in [0,1]$. In [6], it is also proved that $E(\tilde{x}) \in F$ and $[E(\tilde{x})]_0 = \int_\Omega x_0 dP = [E(x_0^-), E(x_0^+)]$ for $\alpha = 0$.

2.3. **Modified Graded Mean Integration Representation of a Generalised Fuzzy Number.** The method of defuzzification of a generalized trapezoidal fuzzy number by its graded mean integration representation was proposed by Chen and Hsieh [2, 3]. However, the modified h-level graded mean value of the fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)_{TRFN}$ is assumed to be $h\left[\beta L^{-1}(h) + (1-\beta)R^1(h)\right]$, where β is termed as the decision maker's optimism parameter and $0 \leq$ b ≤ 1, and the respective modified graded mean integration representation is given by

$$\text{(1)}\quad \text{MGMIR}(\tilde{A}) = \frac{\int_0^w h\left[\beta L^{-1}(h) + (1-\beta)R^{-1}(h)\right] dh}{\int_0^w hdh}$$

$$= \frac{\int_0^w h\left[\beta(a_1+(a_2-a_1)h/w) + (1-\beta)(a_4-(a_4-a_3)h/w)\right] dh}{\int_0^w hdh}$$

$$\text{(2)}\quad = \frac{\beta a_1 + a_2 + a_3 + (1-\beta)a_4}{3}$$

If equal weightage is given to the left and right points of the membership function then it is reduced graded mean integration representation formula.

3. Methodology

Mostard and Teunter [8] assumed that there are no interdependencies between ordered items and hence analysed a single item purchase inventory model. The same reason has been followed with respect to a single item production inventory model.

It has been assumed that there is a single replenishment opportunity at which some quantity of products is produced before the start of the selling season. The gross demand denoted by Gd is the total number of products produced during the season to fulfil the demand. The following assumptions has been made in this model.

Assumptions.

(1) Demand is always less than the production capacity of the system.
(2) Items are produced completely once before the start of sales.
(3) Items sold are to be returned with a fixed time limit usually 10 to 30 days.
(4) An undamaged returned product is collected, tested and then ready for resale.
(5) The demand has been distributed normally.

4. Mathematical formulation of the model

The symbols involved in defining the relevant cost and revenue parameters are described for the formulation of the model under study.

$\tilde{P}_c$ - production cost per unit.
$\tilde{S}_p$ - selling price per unit.
$\tilde{S}_c$ - salvage cost per unit.
$\tilde{C}_c$ - collection cost per unit.
$\tilde{P}_{sr}$ - probability that a sold product is returned.
$\tilde{P}_{rr}$ - probability that a returned product is resold.

Let $\tilde{E}^R_{G_d}$ be the expected revenue per unit of satisfying gross demand which includes the salvage revenue provided that the sold product is returned but not resalable. Then

$$\begin{aligned}\tilde{E}^R_{G_d} &= \text{Total Selling Price } \Theta \text{ Total Collection Cost} \oplus \text{Total Salvage Cost}\\ &= \left(1\Theta\tilde{P}_{sr}\right) \otimes \tilde{S}_p\Theta\tilde{P}_{sr}\otimes\tilde{C}_c\oplus\tilde{P}_{sr}\otimes\left(1\Theta\tilde{P}_{rr}\right)\otimes\tilde{S}_c\end{aligned}$$

Let $\tilde{E}^R_{N_d}$ be the expected revenue per unit of satisfying net demand which includes the repeated selling of a product until it is either not returned (or) returned but not resalable. If a product is sold, returned and resold again and again, then

$$\begin{aligned}\tilde{E}^R_{N_d} &= [\tilde{1}\oplus\tilde{P}_{sr}\otimes\tilde{P}_{rr}\oplus\tilde{P}_{sr}\otimes\tilde{P}_{rr}\otimes\tilde{P}_{sr}\otimes\tilde{P}_{rr}\\ &\quad\oplus\tilde{P}_{sr}\otimes\tilde{P}_{rr}\otimes\tilde{P}_{sr}\otimes\tilde{P}_{rr}\otimes\tilde{P}_{sr}\otimes\tilde{P}_{rr}+\cdots]\otimes\tilde{E}^R_{G_d}\\ &= \left[\tilde{1}\oplus\tilde{P}_{sr}\otimes\tilde{P}_{rr}\oplus\left(\tilde{P}_{sr}\otimes\tilde{P}_{rr}\right)^2\oplus\left(\tilde{P}_{sr}\otimes\tilde{P}_{rr}\right)^3\oplus\cdots\right]\otimes\tilde{E}^R_{G_d}\\ &= \left(\tilde{1}\Theta\tilde{P}_{sr}\otimes\tilde{P}_{rr}\right)^{-1}\otimes\tilde{E}^R_{G_d}\\ &= \tilde{E}^R_{G_d}\emptyset\left(\tilde{1}\Theta\tilde{P}_{sr}\otimes\tilde{P}_{rr}\right)\text{ by using the result (2.1).}\end{aligned}$$

Assume that the net demand $\tilde{N}_d$is a fuzzy random variable with the given set of data $\{(\tilde{y}_1,\tilde{p}_1),(\tilde{y}_2,\tilde{p}_2),(\tilde{y}_3,\tilde{p}_3),\cdots,(\tilde{y}_n,\tilde{p}_n)\}$ where $\tilde{y}_i$,$\tilde{p}_i$ denote the observations and the respective probabilities of the fuzzy random variable $\tilde{N}_d$ for $i=1$ to n. As the data being imprecise with fuzzy probability, the data set and its corresponding probabilities have been assumed to be trapezoidal fuzzy numbers. Thus, for i = 1 to n, $\tilde{y}_i$ and $\tilde{p}_i$ have been represented respectively as $\tilde{y}_i=(y_{i_1},y_{i_2},y_{i_3},y_{i_4})$ and $\tilde{p}_i=(p_{i_1},p_{i_2},p_{i_3},p_{i_4})$.

If $\tilde{y}_k$ denote the number of items produced at the beginning of the selling season, then the profit function is, denoted by $\tilde{P}_{rft}$ (or) $\tilde{P}_{rft}(\tilde{y}_k,\tilde{N}_d)$, described by

$$\begin{aligned}\tilde{P}_{rft} &= \tilde{P}_{rft}(\tilde{y}_k,\tilde{N}_d)\\ &= \tilde{E}^R_{N_d}\otimes\tilde{y}_i\Theta\tilde{P}_c\otimes\tilde{y}_k\oplus\tilde{S}_c\otimes(\tilde{y}_k\Theta\tilde{y}_i)\end{aligned}$$

where $\tilde{y}_i\leq\tilde{y}_k$ and $i=1$ to n.

As the demand is a fuzzy random variable, its profit function is also a fuzzy random variable. Therefore, the total expected profit is a unique fuzzy number defined by

$$E\left(\tilde{P}_{rft}\right)=E((P_1,P_2,P_3,P_4))=(EP_1,EP_2,EP_3,EP_4)$$

Hence the fuzzy total expected profit function is determined by

$$
\begin{aligned}
E\left(\tilde{P}_{rft}\right) &= E\left[\tilde{P}_{rft}(\tilde{y}_k,\tilde{N}_d)\right] \\
&= \sum_{i=1}^{n} \oplus \left[\tilde{P}_{N_d} \otimes \tilde{y}_i \Theta \tilde{P}_c \otimes \tilde{y}_k \oplus \tilde{S}_c \otimes (\tilde{y}_k \Theta \tilde{y}_i)\right] \otimes \tilde{P}_i \text{where} \tilde{y}_i \leq \tilde{y}_k \\
&= \sum_{i=1}^{n} \oplus \left[\left(\tilde{P}_{N_d} \Theta \tilde{S}_c\right) \otimes \tilde{y}_i \Theta \left(\tilde{P}_c \Theta \tilde{S}_c\right) \otimes \tilde{y}_k\right] \otimes \tilde{P}_i \\
&= \sum_{i=1}^{n} \oplus \left[\left(\tilde{P}_{N_d} \Theta \tilde{S}_c\right) \otimes \tilde{y}_i \otimes \tilde{P}_i \Theta \left(\tilde{P}_c \Theta \tilde{S}_c\right) \otimes \tilde{y}_k \otimes \tilde{P}_i\right] \\
&= \sum_{i=1}^{n} \oplus \left[\left(P_{N_{d_1}}, P_{N_{d_2}}, P_{N_{d_3}}, P_{N_{d_4}}\right) \Theta \left(S_{c_1}, S_{c_2}, S_{c_3}, S_{c_4}\right)\right] \\
&\quad \otimes \left[\left(y_{i_1}, y_{i_2}, y_{i_3}, y_{i_4}\right) \otimes \left(p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}\right)\right] \\
&\quad \Theta \left[\left(P_{c_1}, P_{c_2}, P_{c_3}, P_{c_4}\right) \Theta \left(S_{c_1}, S_{c_2}, S_{c_3}, S_{c_4}\right)\right] \\
&\quad \Theta \left[\left(y_{k_1}, y_{k_2}, y_{k_3}, y_{k_4}\right) \otimes \left(p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}\right)\right] \\
&= \sum_{i=1}^{n} \oplus \left[\left(P_{N_{d1}} - S_{c_4}, P_{N_{d2}} - S_{c_3}, P_{N_{d3}} - S_{c_2}, P_{N_{d4}} - S_{c_1}\right) \otimes\right. \\
&\left(y_{i1} p_{i_1}, y_{i_2} p_{i_2}, y_{i_3} p_{i_3}, y_{i_4} p_{i_4}\right)\Big] \Theta \left[\left(P_{c_1} - S_{c_4}, P_{c_2} - S_{c_3}, P_{c_3} - S_{c_2}, P_{c_4} - S_{c_1}\right)\right. \\
&\quad \otimes \left(y_{k_1} p_{i_1}, y_{k_2} p_{i_2}, y_{k_3} p_{i_3}, y_{k_4} p_{i_4}\right) \\
&= \sum_{i=1}^{n} \oplus \left[\left(\left(P_{N_{d1}} - S_{c_4}\right) y_{i_1} p_{i_1}, \left(P_{N_{d_2}} - S_{c_3}\right) y_{i_2} p_{i_2}, \left(P_{N_{d_3}} - S_{c_2}\right) y_{i_3} p_{i_3}\right.\right. \\
&\quad \left.\left(P_{N_{d_4}} - S_{c_1}\right) y_{i_4} p_{i_4}\right) \Theta \left[\left(\left(P_{c_1} - S_{c_4}\right) y_{k_1} p_{i_1}, \left(P_{c_2} - S_{c_3}\right) y_{k_2} p_{i2},\right.\right. \\
&\left.\left.\left(P_{c_3} - S_{c_2}\right) y_{k_3} p_{i_3}, \left(P_{c_4} - S_{c_1}\right) y_{k_4} p_{i_4}\right)\right] \\
&= \sum_{i=1}^{n} \oplus \left[\left(\left(P_{N_{d_1}} - S_{c_4}\right) y_{i_1} p_{i_1} - \left(P_{c_4} - S_{c_1}\right) y_{k_4} p_{i_4}\right),\right. \\
&\quad \left(\left(P_{N_{d_2}} - S_{c_3}\right) y_{i_2} p_{i_2} - \left(P_{c_3} - S_{c_2}\right) y_{k_3} p_{i_3}\right), \left(\left(P_{N_{d_3}} - S_{c_2}\right) y_{i_3} p_{i_3} - \left(P_{c_2} - S_{c_3}\right) y_{k_2} p_{i_2}\right), \\
&\quad \left.\left(\left(P_{N_{d_4}} - S_{c_1}\right) y_{i_4} p_{i_4} - \left(P_{c_1} - S_{c_4}\right) y_{k_1} p_{i_1}\right)\right]
\end{aligned}
$$

Therefore the defuzzified expected profit function is given by

$$\begin{aligned}\text{MGMIR}\left[E\left(\tilde{P}_{rft}\right)\right] = \sum_{i=1}^{n}\frac{1}{3}\Big[\beta\Big(\left(P_{N_{d_1}}-S_{c_4}\right)y_{i_1}p_{i_1}-\left(P_{c_4}-S_{c_1}\right)y_{k_4}p_{i_4}\Big)\\ +\Big(\left(P_{N_{d_2}}-S_{c_3}\right)y_{i_2}p_{i_2}-\left(P_{c_3}-S_{c_2}\right)y_{k_3}p_{i_3}\Big)\\ +\Big(\left(P_{N_{d_3}}-S_{c_2}\right)y_{i_3}p_{i_3}-\left(P_{c_2}-S_{c_3}\right)y_{k_2}p_{i_2}\Big)\\ +(1-\beta)\Big(\left(P_{N_{d_4}}-S_{c_1}\right)y_{i_4}p_{i_4}-\left(P_{c_1}-S_{c_4}\right)y_{k_1}p_{i_1}\Big)\Big]\end{aligned}$$

Moreover, for the fuzzy optimal production quantity $\tilde{y}_k$, the following condition holds and is given by

$$(3)\quad \text{MGMIR}\left[E\left(\tilde{P}_{rft}\left(\tilde{y}_k,\tilde{N}_d\right)\right)\right]-\text{MGMIR}\left[E\left(\tilde{P}_{rft}\left(\tilde{y}_{k-1},\tilde{N}_d\right)\right)\right]>0. \quad \cdots$$

Thus, the optimal production quantity $\tilde{y}_k$ is found for the production inventory model with resalable returns of items.

5. Example

Consider the production cost per item, $\tilde{P}_c = (12, 12, 12, 12)$ and selling price $\tilde{S}_p = (28.5, 30, 30, 31.5)$. The collection cost, $\tilde{C}_c = (3, 4, 5, 6)$. The salvage cost, $\tilde{S}_c = (4, 5, 6, 7)$. Probability that a sold product being returned, $\tilde{P}_{sr} = (0.42, 0.44, 0.46, 0.48)$. Probability that a returned product being resold, $\tilde{P}_{rr} = (0.42, 0.44, 0.46, 0.48)$. Then, the unit expected revenue for meeting gross demand is calculated as

$$\tilde{E}^R_{G_d} = (11.9736, 13.9880, 15.2056, 17.2788)$$

Therefore, the unit expected revenue for meeting net demand is obtained as

$$\tilde{E}^R_{N_d} = (19.5136897, 23.85402856, 27.23065903, 32.62613293)$$

The optimal fuzzy production quantity along with the respective fuzzy probability information in order to satisfy the demand of the customers is given in the following tabular form in Table 1.

Table : 1

Fuzzy Production Quantity	**Fuzzy Probability**
$\tilde{y}_k = (y_{k_1}, y_{k_2}, y_{k_3}, y_{k_4})$	$\tilde{p}_k = (p_{k_1}, p_{k_2}, p_{k_3}, p_{k_4})$
(2, 3, 4, 5)	(0.025, 0.03, 0.035, 0.04)
(6, 7, 8, 9)	(0.125, 0.13, 0.135, 0.14)
(10, 11, 12, 13)	(0.225, 0.23, 0.235, 0.24)
(14, 15, 16, 17)	(0.325, 0.33, 0.335, 0.34)
(18, 19, 20, 21)	(0.215, 0.22, 0.225, 0.23)
(22, 23, 24, 25)	(0.155, 0.16, 0.165, 0.17)
(26, 27, 28, 29)	(0.035, 0.04, 0.045, 0.05)

It is important to mention that the results of the calculations, in the example as shown in tables 2 to 4 have been done by taking $\beta = 0.4, 0.5$ and 0.6 such that the successive difference of the values of β is 0.1 and letting

$A = [\beta(P_{N_{d_1}} - S_{C_4}) - (1-\beta)(P_{C_1} - S_{C_4})]y_{k_1}p_{k_1}$

$B = [(P_{N_{d2}} - S_{C_3}) - (P_{C_2} - S_{C_3})]y_{k_2}p_{k_2}$

$C = [(P_{N_{d_3}} - S_{C_2}) - (P_{C_3} - S_{C_2})]y_{k_3}p_{k_3}$

$D = [(1-\beta)(P_{N_{d_4}} - S_{C_1}) - \beta(P_{C_4} - S_{C_1})]y_{k_4}p_{k_4}$

Table : 2($\beta = 0.4$)

$\tilde{y}_k$	A	B	C	D
(2, 3, 4, 5)	0.100273794	1.06686221	2.132292264	2.795135952
(6, 7, 8, 9)	1.50410691	10.78716235	16.44911175	17.6093565
(10, 11, 12, 17)	4.51232073	29.99068214	42.95045846	43.60412085
(14, 15, 16, 17)	9.124915254	58.67742157	81.6363324	80.77942901
(18, 19, 20, 21)	7.761191656	49.54982266	68.53796564	67.50253324
(22, 23, 24, 25)	6.838672751	43.62281038	60.31340976	59.39663898
(26, 27, 28, 29)	1.824983051	12.80234652	19.19063038	20.26473565

Table : 3($\beta = 0.5$)

$\tilde{y}_k$	A	B	C	D
(2, 3, 4, 5)	0.1878422425	1.06686221	2.132292264	2.062613294
(6, 7, 8, 9)	2.817633638	10.78716235	16.44911175	12.99446375
(10, 11, 12, 13)	8.452900913	29.99068214	42.95045846	32.17676739
(14, 15, 16, 17)	17.09364407	58.67742157	81.6363324	59.6095242
(18, 19, 20, 21)	14.53898957	49.54982266	68.33796564	49.81211105
(22, 23, 24, 25)	12.81084094	43.62281038	60.31340976	43.8305325
(26, 27, 28, 29)	3.418728814	12.80234652	19.19063038	14.95394638

Table : 4($\beta = 0.6$)

$\tilde{y}_k$	A	B	C	D
(2, 3, 4, 5)	0.275410691	1.06686221	2.132292264	1.330090634
(6, 7, 8, 9)	4.131160365	10.78716235	16.44911175	8.379570997
(10, 11, 12, 17	12.3934811	29.99068214	42.95045846	20.7497139
(14, 15, 16, 17)	25.06237288	58.67742157	81.6363324	38.43961933
(18, 19, 20, 21)	21.31678748	49.54982266	68.53796564	32.12168882
(22, 23, 24, 25)	18.78300913	43.62281038	60.31340976	28.26442598
(26, 27, 28, 29)	5.012474576	12.80234652	19.19063038	9.643157099

The Optimal Fuzzy Profit $\tilde{P}^*_{rft}$ is given by

$$\tilde{P}^*_{rft} = (10.69728814, 50.85742157, 89.4563324, 142.7090483)$$

Therefore MGMIR $\left(\tilde{P}^*_{rft}\right) = 72.33897406$, where $\beta = 0.5$, which is for high probability of optimal production of items.

From Table : 2, 3 and 4, it is to be noticed that the fuzzy optimal production quantity $(\tilde{y}^*_k)$, the defuzzified optimal production quantity MGMIR$(\tilde{y}^*_k)$, the fuzzy optimal profit $\tilde{P}^*_{rft}$ and the defuzzified optimal profit MGMIR$(\tilde{P}^*_{rft})$ are found and are exhibited respectively by

(i) $(\tilde{y}^*_k) = (14, 15, 16, 17)$,

(ii) MGMIR$(\tilde{y}^*_k) = 15.5$ units,

(iii) $P_{rft}(y^*_k) = (10.69728814, 50.85742157, 89.4563324, 142.7090483)$ and

(iv)MGMIR$(\tilde{P}^*_{rft}) = 76.73936608, 72.33897406$ and 67.93858206 for $\beta = 0.4, 0.5$ and 0.6 respectively.

Conclusion

It is observed that the defuzzified profit decreases with the increase of the parameter β. However, it is found that one can meet the level of optimal profit for $\beta = 0.5$ in accordance with high probability of the optimal production. Moreover, the co-ordination of fuzzy random variable as demand and the consideration of relevant fuzzy costs provide a more realistic model of the production inventory problem under study with resalable returns. This incorporation of the concepts of imprecision and randomness can be extended for better representation of other real life production based inventory problems.

References

[1] D. Chakraborty, Redefining chance-constrained programming in fuzzy environment, *Fuzzy Sets and Systems*, **125**(3) (1994), 327-333.

[2] S.H.Chen and C.H.Hsieh, Graded Mean Integration Representation of Generalized fuzzy number, *Journal of Chinese Fuzzy Systems Association*, **5** (1999), 1-7.

[3] S.J.Chen, S.M.Chen and C.H.Hsieh, Fuzzy Risk Analysis Based on Similarity Measures of Generalised Fuzzy Numbers, *IEEE Transactions on Fuzzy Systems*, **11** (2003), 45-56.

[4] P. Dutta, D. Chakraborty and A.R. Roy, A Single - Period Inventory Model with Fuzzy Random Variable Demand, *Mathematical and Computer Modeling*, **41** (2005), 915-922.

[5] M.A. Gil, M.Lopez - Diaz and D.A. Ralescu, Overview on the Development of Fuzzy Random Variables, *Fuzzy Sets and Systems (Available Online)*, 2006.

[6] M.A. Gil and M.Lopez - Diaz, The λ-average value of the expected value of a fuzzy random variable, *Fuzzy Sets and Systems*, **99** (1998), 347-391.

[7] H. Kwakernaak, Fuzzy Random Variables Definitions and Theorems, *Information Sciences*, **15** (1978), 1-29.

[8] J. Mostard and R. Teunter, The newsboy problems with resalable returns : A single period model and case study, *European Journal of operational Research*, **169** (2006), 81-96.

[9] J. Mostard, R. Koster and R. Teunter, The Distribution - free newsboy problem with resalable returns, *International Journal of Production Economics*, **97** (2006), 81-96.

[10] M.C. Puri and D.A. Ralescu, Fuzzy Random Variables, *Journal of Mathematical Analysis and Applications*, **114** (1986), 409-422.

[11] D. Vlachos and R. Dekker, Return handling options and order quantities for single period products, *European Journal of Operational Research*, **151** (2003), 38-52.