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Some result on simple hyper K-algebras

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Abstract. A simple method is described, to prove some theorems for simple hyper K-algebras and to study positive implicative hyper K-ideals, weak (implicative) hyper K-ideals in simple hyper K-algebras . Beside, some results on positive implicative and (weak) implicative simple hyper K-algebras are presented. Finally classification of simple hyper K-algebras of order 4, which are satisfied in conditions of Theorem 3.29, is going to be calculated .

Keywords: hyper K-algebra, simple hyper K-algebra, (weak) hyper K-ideal, positive implicative hyper K-ideal, (weak) implicative hyper K-ideal.

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1. Introduction

The study of BCK-algebra was initiated by K.Iseki [4] in 1966 as a generalization of concept of set-theoretic difference and propositional calculus. Since then many researches have worked in this area. The hyper structure theory

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was introduced in 1934 by F. Marty [6] at the 8^{th} congress of Scandinavian Mathematicians. Recently R. A. Borzooei, M. M. Zahedi and Y. B. Jun [3,10] have introduced the notions of hyper K-algebra , (weak) hyper K-ideals and defined simple hyper K-algebra of order 3. T. Roudbari and M. M. Zahedi [9] defined 27 different types of positive implicative hyper K-ideals. T. Roudbari , L.Torkzadeh and M. M. Zahedi [8] have defined simple hyper K-algebra in general. In this note the researchers are going to we follow [8] and study simple hyper K-algebras deeply and obtain some results as mentioned in the abstract.

2. Preliminaries

Definition 2.1. [3] Let H be a nonempty set and " $\circ$ " be a *hyper operation* on H, that is " $\circ$ " is a function from $H \times H$ to $P^*(H) = P(H)\backslash\{\phi\}$. Then H is called a hyper K- algebra if it contains a constant "0" and satisfies the following axioms:

(HK1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(HK3) $x < x$,
(HK4) $x < y$, $y < x \Rightarrow x = y$,
(HK5) $0 < x$.
for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$, $\exists b \in B$ such that $a < b$. Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup\limits_{\substack{a \in A \\ b \in B}} a \circ b$ of H.

From now on $(H, \circ, 0)$ is a hyper K-algebra.

Theorem 2.2. [3] For all $x, y, z \in H$ and for all non-empty subsets A and B of H the following statements hold:
(i) $x \circ y < x$, (ii) $A \circ B < A$,
(iii) $A \circ A < A$, (iv) $0 \in x \circ (x \circ 0)$,
(v) $x < x \circ 0$, (vi) $A < A \circ 0$,
(vii) $A < A \circ B$, if $0 \in B$.

Lemma 2.3. [2] For all $x, y, z \in H$ the following statements hold:
(i) $x \circ y < z \Leftrightarrow x \circ z < y$, (ii) $x \in x \circ 0$.

Proof. (i) Let $x, y, z \in H$ be such that $x \circ y < z$. Then there exists $t \in x \circ y$ such that $t < z$. Thus $0 \in t \circ z \subseteq (x \circ y) \circ z = (x \circ z) \circ y$, and hence there exists $w \in x \circ z$ such that $0 \in w \circ y$, i.e., $w < y$. Therefore $x \circ z < y$. The proof of the converse is similar.

(ii) By Theorem 2.2.(i) we have $x \circ 0 < x$. So there exists $t \in x \circ 0$ such that $t < x$. Since $t \in x \circ 0$, then $x \circ 0 < t$ and so by (i) , $x \circ t < 0$. Thus there is $h \in x \circ t$ such that $h < 0$. By (HK5) and (HK4) we get that $h = 0$. So $0 \in x \circ t$, that is $x < t$. Since $x < t$ and $t < x$, then by (HK4), $x = t$. Therefore $x \in x \circ 0$.

Definition 2.4. [8] A hyper K-algebra $(H, \circ, 0)$ is called simple if for all distinct elements $a, b \in H - \{0\}$, $a \not< b$ and $b \not< a$.

Theorem 2.5. [8]The following statements in a simple hyper K-algebra $(H, \circ, 0)$ hold.
(i) $a \circ 0 = \{a\}$, $\forall a \in H - \{0\}$,
(ii) $a \in a \circ b$, for all distinct elements $a, b \in H$,
(iii) $H - \{a\} \subseteq H \circ a$, $\forall a \in H$,
(iv) $a \in b \circ c \Longleftrightarrow c \in b \circ a$, for all distinct elements $a, c \in H$ and $b \in H - \{0\}$,
(v) $x < x \circ a \Longleftrightarrow x \in x \circ a$, $\forall a, x \in H$,
(vi) $A < A \circ b \Longleftrightarrow A \cap (A \circ b) \neq \emptyset$, $\forall b \in H$ and $\emptyset \neq A \subseteq H$,
(vii) $(x \circ y) \circ z < x \circ (y \circ z)$, $\forall x, y, z \in H$,
(viii) If $0 \in I \subseteq H$, then $A \circ B < I \Longleftrightarrow (A \circ B) \cap I \neq \emptyset$, for all non-empty subsets A and B of H.

Definition 2.6. [3] Let I be a nonempty subset of $(H, \circ, 0)$ and $0 \in I$. Then,
(i) I is called a weak hyper K-ideal of H if $x \circ y \subseteq I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$,
(ii) I is called a hyper K-ideal of H if $x \circ y < I$ and $y \in I$ imply that $x \in I$, for all $x, y \in H$.

Definition 2.7. [9] Let I be a nonempty subset of H such that $0 \in I$. Then I is called a positive implicative hyper K-ideal of
(i) type 1, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $(y \circ z) \subseteq I$ imply that $(x \circ z) \subseteq I$,

(ii) type 2, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $(y \circ z) \subseteq I$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(iii) type 3, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $(y \circ z) \subseteq I$ imply that $x \circ z < I$,

(iv) type 4, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $(x \circ z) \subseteq I$,

(v) type 5, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $(x \circ z) \bigcap I \neq \emptyset$

(vi) type 6, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $x \circ z < I$,

(vii) type 7, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $y \circ z < I$ imply that $x \circ z < I$,

(viii) type 8, if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $y \circ z < I$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(ix) type 9 , if for all $x, y, z \in H$, $((x \circ y) \circ z) \subseteq I$ and $y \circ z < I$ imply that $(x \circ z) \subseteq I$,

(x) type 10, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $(y \circ z) \subseteq I$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(xi) type 11, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $(y \circ z) \subseteq I$ imply that $(x \circ z) \subseteq I$,

(xii) type 12, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $(y \circ z) \subseteq I$ imply that $xz < I$,

(xiii) type 13, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $(x \circ z) \subseteq I$,

(xiv) type 14, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(xv) type 15, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that$x \circ z < I$,

(xvi) type 16, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $y \circ z < I$ imply that $x \circ z < I$,

(xvii) type 17, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $y \circ z < I$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(xviii) type 18, if for all $x, y, z \in H$, $((x \circ y) \circ z) \bigcap I \neq \emptyset$ and $y \circ z < I$ imply that $(x \circ z) \subseteq I$,

(xix) type 19, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $x \circ z < I$,

(xx) type 20, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $(x \circ z) \subseteq I$,

(xxi) type 21, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $(y \circ z) \bigcap I \neq \emptyset$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(xxii) type 22, if for all $x, y, z \in H$,$((x \circ y) \circ z) < I$ and $(y \circ z) \subseteq I$ imply that $(x \circ z) \subseteq I$,

(xxiii) type 23, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $(y \circ z) \subseteq I$ imply that $x \circ z < I$,

(xxiv) type 24, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $(y \circ z) \subseteq I$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(xxv) type 25, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $y \circ z < I$ imply that $x \circ z < I$,

(xxvi) type 26, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $y \circ z < I$ imply that $(x \circ z) \bigcap I \neq \emptyset$,

(xxvii) type 27, if for all $x, y, z \in H$, $((x \circ y) \circ z) < I$ and $y \circ z < I$ imply that $(x \circ z) \subseteq I$.

For simplicity of notation we use "PIHKI " instead of " Positive Implicative Hyper K-ideal" .

Definition 2.8. [1] Let I be a nonempty subset of a hyper K-algebra $(H, \circ, 0)$ such that $0 \in I$. Then I is called:

(i) an implicative hyper K-ideal , if for all $x, y, z \in H$, $(x \circ z) \circ (y \circ x) < I$ and $z \in I$ imply that $x \in I$.

(ii) a weak implicative hyper K-ideal , if for all $x, y, z \in H$, $(x \circ z) \circ (y \circ x) \subseteq I$ and $z \in I$ imply that $x \in I$.

Definition 2.9. [2] Let $(H, \circ, 0)$ be a hyper K-algebra. Then $(H, \circ, 0)$ is said to be a positive implicative hyper K-algebra, if $(x \circ z) \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$.

Definition 2.10. [1] Let $(H, \circ, 0)$ be a hyper K-algebra. Then H is called:
(i) a weak implicative , if for all $x, y \in H, x < x \circ (y \circ x)$,

(ii) an implicative , if for all $x, y \in H, x \in x \circ (y \circ x)$,

(iii) a strong implicative , if for all $x, y \in H, x \circ 0 \subseteq x \circ (y \circ x)$.

Theorem 2.11. [8] Let $a \in H - \{0\}$ and $(H, \circ, 0)$ be a simple hyper K-algebra. Then $H - \{a\}$ is a hyper K-ideal of H if and only if $|a \circ x| = 1$, for all $x \in H - \{a\}$.

Definition 2.14. [7] Let $(H, \circ, 0)$ be a hyper K-algebra and S be a nonempty subset of H. Then the sets

$$_{l1}S = \{x \in H \mid \ a < (a \circ x), \forall a \in S\}, \qquad _{l2}S = \{x \in H \mid \ a \in (a \circ x), \forall a \in S\},$$
$$S_{r1} = \{x \in H \mid \ x < (x \circ a), \forall a \in S\} \text{ and } \quad S_{r2} = \{x \in H \mid \ x \in (x \circ a), \forall a \in S\}$$

are called left hyper stabilizer of type 1 of S, left hyper stabilizer of type 2 of S, right hyper stabilizer of type 1 of S and right hyper stabilizer of type 2 of S, respectively.

Theorem 2.13.[8] Every subset of a simple hyper K-algebra $(H, \circ, 0)$ containing 0 is a weak hyper K-ideal.

Definition 2.14. [2] Let $(H_1, \circ_1, 0_1)$ and $(H_2, \circ_2, 0_2)$ be two hyper K-algebras. A mapping $f : H_1 \to H_2$ is said to be a homomorphism if

(i) $f(0) = 0$,

(ii) $f(x \circ y) = f(x) \circ f(y)$, for all $x, y \in H$
If f is 1-1 and onto, we say that f is an isomorphism.

3. Positive implicative hyper K-ideals in simple hyper K-algebras

From now $(H, \circ, 0)$ is a simple hyper K-algebra, unless otherwise is stated.

Remark 3.1. From Definition 2.4 and Theorem 2.5(viii) we get that
(i) I is a $PIHKI$ of type 2 if and only if I is a $PIHKI$ of type 3,

(ii) I is a $PIHKI$ of type 4 if and only if I is a $PIHKI$ of type 9,

(iii) I is a $PIHKI$ of type 5 if and only if I is a $PIHKI$ of type 6(7,8),

(iv) I is a $PIHKI$ of type 11 if and only if I is a $PIHKI$ of type 22,

(v) I is a $PIHKI$ of type 10 if and only if I is a $PIHKI$ of type 12(23,24),

(vi) I is a $PIHKI$ of type 13 if and only if I is a $PIHKI$ of type 18(20,27),

(vii) I is a $PIHKI$ of type 14 if and only if I is a $PIHKI$ of type 15(16,17,19,21,25,26).

Theorem 3.2. Let $a \in H - \{0\}$ and $I = H - \{a\}$. Then I is a $PIHKI$ of type 25(14,15,16,17,19,21,26) if and only if $|a \circ b| = 1$, for all $b \in I$.

Proof. Let I be a $PIHKI$ of type 25. Then we prove that $|a \circ b| = 1$, for all $b \in I$. On the contrary let $|a \circ b| > 1$, for some $b \in I$. By Theorem 2.5(ii) we have $a \in a \circ b$, so there exists $c \in H - \{a\}$ such that $c \in a \circ b$. Thus $(a \circ 0) \circ b = (a \circ b) \circ 0 < I$ and $b \circ 0 < I$ imply that $a = a \circ 0 < I$, which is a contradiction. Therefore $|a \circ b| = 1$, for all $b \in I$.
Conversely, Let $|a \circ b| = 1$, for all $b \in I$. We show that I is a $PIHKI$ of type 25. On the contrary, let $(x \circ y) \circ z < I$ and $y \circ z < I$ but $x \circ z \nless I$, for some $x, y, z \in H$. $x \circ z \nless I$ implies that $x \neq z$ .By Theorem 2.5(ii), $x \in x \circ z$. Thus by hypothesis we get that $x = a$. If $x = y$, then $y \circ z = a \circ z = \{a\} \nless I$, which is a contradiction. If $x \neq y$, then $(x \circ y) \circ z = a \circ z = \{a\} \nless I$, which is also a contradiction. Since I is a $PIHKI$ of type 25.

Theorem 3.3. Let $a \in H - \{0\}$ and $I = H - \{a\}$. If I is a $PIHKI$ of type 27(13,18,20), then
(i) $|a \circ b| = 1$, for all $b \in I$,
(ii) $b \circ c \neq H$, for all $b, c \in H$.

Proof. (i) On the contrary, let $|a \circ b| > 1$, for some $b \in I$. Then there exists $t \in H - \{a\}$ such that $t \in a \circ b$, so $(a \circ t) \circ b < I$ .Thus $(a \circ t) \circ b < I$ and $t \circ b < I$ imply that $a \circ b \subseteq I$, which is a contradiction, because $a \in a \circ b$ and $a \notin I$. Therefore $|a \circ b| = 1$, for all $b \in I$.
(ii) If there exists $b, c \in H$ such that $b \circ c = H$, then $(b \circ 0) \circ c < I$ and $0 \circ c < I$ imply that $H = b \circ c \subseteq I$, which is impossible. Therefore $b \circ c \neq H$, for all $b, c \in H$.

The following example shows that the converse of the above theorem is not true in general.

Example 3.4. The following table shows a simple hyper K-algebra structure on $H = \{0, 1, 2, 3\}$.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,2,3\}$
1	$\{1\}$	$\{0\}$	$\{1,2,3\}$	$\{1,2,3\}$
2	$\{2\}$	$\{2\}$	$\{0,2,3\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,3\}$

Then $3 \circ b = \{3\}$, for all $b \in H - \{3\}$ and $b \circ c \neq H$, for all $b, c \in H$, but $I = H - \{3\}$ is not a $PIHKI$ of type 27, because $(3 \circ 0) \circ 3 = \{0, 3\} < I$ and $0 \circ 3 < I$, while $3 \circ 3 = \{0, 3\} \not\subseteq I$.

Theorem 3.5. Let $a \in H - \{0\}$ and $I = H - \{a\}$. Then I is a $PIHKI$ of type 10(12,23,24) if and only if $|a \circ b| = 1$, for all $b \in I$.

Proof. The proof is similar to the proof of Theorem 3.2 , by imposing some modifications.

Theorem 3.6. Let $a \in H - \{0\}$ and $I = H - \{a\}$. If $|a \circ b| = 1$, for all $b \in I$, then I is a $PIHKI$ of type 6(5,7,8).

Proof. Let $(x \circ y) \circ z \subseteq I$ and $y \circ z < I$, we show that $x \circ z < I$. If $x = z$, it is clear that $x \circ z < I$. Now let $x \neq z$. consider two cases: case (i) $x \neq a$, case(ii) $x = a$.
Case(i). By Theorem 2.5(ii) we get that $x \in x \circ z$ and so $x \circ z < I$.
Case(ii). We consider the following sub-cases and show that $(x \circ y) \circ z \not\subseteq I$ or $y \circ z \not< I$:
Case $(\dot{i})$. $y = x$ implies that $\{a\} = y \circ z = x \circ z = a \circ z \not< I$,
Case $(\dot{i}\dot{i})$. $y \neq x$ implies that $\{a\} = (x \circ y) \circ z \not\subseteq I$. Therefore I is a $PIHKI$ of type 6.

The following table shows that the converse of the above theorem is not true in general.

Example 3.7. The following table shows a simple hyper K-algebra structure on $H = \{0, 1, 2, 3\}$.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0,2\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{1,2\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3,2\}$	$\{0\}$

We see that $I = H - \{1\}$ is a PIHKI of types 5,6,7 and 8, but $|1 \circ 2| \neq 1$.

Theorem 3.8. Let $a \in H - \{0\}$ and $I = H - \{a\}$. Then I is a $PIHKI$ of type
10(12,14,15,16,17,19,21,23,24,25,26) if and only if I is a hyper K-ideal of H.

Proof. The proof follows from Theorems 2.11, 3.2 and 3.5.

Theorem 3.9. In any simple hyper K-algebra $(H, \circ, 0)$, $I = \{0\}$ is a $PIHKI$ of type 3(2,5,6,7).

Proof. Let $(x \circ y) \circ z \subseteq I$ and $y \circ z < I$. Thus $0 \in y \circ z$ and $(x \circ z) \circ y = (x \circ y) \circ z = \{0\}$. Hence by simplicity of H we get that $x \circ z = \{0\}$or$\{y\}$or$\{0, y\}$. If $x \circ z = \{0\}$or$\{0, y\}$, it is clear that $x \circ z < I$. If $x \circ z = \{y\}$, then by Theorem 2.2(i) $x \circ z < \{x\}$ implies that $y = 0$ or $y = x$. If $y = 0$, then $x \circ z < I$, if $y = x$ and $y \neq 0$, then by hypothesis we get that $0 \in x \circ z = \{y\}$, which is not true.

Theorem 3.10. Let H be a positive implicative hyper K-algebra and $\emptyset \neq A \subseteq H$. Then $A_{r1}(A_{r2})$ and ${}_{l1}A({}_{l2}A)$ are PIHKI of types 1,2,...,9.

Proof. By Theorem 2.14, $A_{r1}(A_{r2})$ and ${}_{l1}A({}_{l2}A)$ are weak hyper K-ideals. We show that A_{r1} is a PIHKI of type 6. Let $(x \circ y) \circ z \subseteq A_{r1}$, $y \circ z \bigcap A_{r1} \neq \emptyset$. Since H is a positive implicative hyper K-algebra, then $(x \circ y) \circ z = (x \circ z) \circ (y \circ z) \subseteq A_{r1}$. Also since $y \circ z \bigcap A_{r1} \neq \emptyset$, there exists $t \in y \circ z \bigcap A_{r1}$. We have $(x \circ z) \circ t \subseteq (x \circ z) \circ (y \circ z) \subseteq A_{r1}$ and $t \in A_{r1}$. Hence $x \circ z \subseteq A_{r1}$. Therefore, $x \circ z < A_{r1}$, that is A_{r1} is a PIHKI of type 6. The proofs of the other types are similar to above, by some suitable changes.

Theorem 3.11. Let $a \in H - \{0\}$ and $I = H - \{a\}$. If I is an implicative hyper K-ideal, then
(i) $|a \circ b| = 1$, for all $b \in I$,
(ii) $a \notin a \circ a$,
(iii) $a \notin 0 \circ b$, for all $b \in H - \{a\}$,
(iv) $a \notin b \circ a$, for all $b \in H$.

Proof. (i) Let $|a \circ b| > 1$, for some $b \in I$. Then there exists $c \neq a$ such that $c \in a \circ b$. Thus $(a \circ b) \circ (c \circ a) < I$ and $b \in I$ imply that $a \in I$, which is not true. Therefore $|a \circ b| = 1$, for all $b \in I$.
(ii) If $a \in a \circ a$, then $0 \in a \circ a \subseteq a \circ (a \circ a) = (a \circ 0) \circ (a \circ a)$. Thus $(a \circ 0) \circ (a \circ a) < I$ and $0 \in I$, so by hypothesis we have $a \in I$, which is impossible. Therefore $a \notin a \circ a$.

(iii) Let there exists $b \in H - \{a\}$ such that $a \in 0 \circ b$. Then by (i) we have $a \in 0 \circ b \subseteq (a \circ a) \circ b = (a \circ b) \circ a = a \circ a$, which is a contradiction by(ii).
(iv) On the contrary, let $a \in b \circ a$, for some $b \in H$. Then $(a \circ 0) \circ (b \circ a) < I$ and $0 \in I$ imply that $a \in I$, which is a contradiction. Therefore $a \notin b \circ a$, for all $b \in H$.

Corollary 3.12. Let $a \in H - \{0\}$ and $I = H - \{a\}$. If I is an implicative hyper K-ideal, then I is a hyper K-ideal of H.

Proof. The proof follows from Theorems 2.11 and 3.11.

Corollary 3.13. Let $a \in H - \{0\}$ and $I = H - \{a\}$. If I is an implicative hyper K-ideal, then I is a $PIHKI$ of type 10(12,14,15,16,17,19,21,23,24,25,26).

Proof. The proof follows from Theorems 3.2, 3.5 and 3.11.

The following table shows that the converse of the above theorem is not true in general.

Example 3.14. The following table shows a simple hyper K-algebra structure on $H = \{0, 1, 2, 3\}$.

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0, 2\}$	$\{0\}$
1	$\{1\}$	$\{0\}$	$\{1, 2\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{2, 3\}$	$\{0\}$

We see that $I = H - \{2\}$ is a PIHKI of types 10, 12, 14,15,, 16,17,,19,21,23,24,25 and 26, but I is not an implicative hyper K-ideal, because $(2 \circ 0) \circ (1 \circ 2) = \{0, 2\} < I, 0 \in I$ and $2 \notin I$.

Theorem 3.15. In any simple hyper K-algebra $(H, \circ, 0), I = \{0\}$ is a weak implicative hyper hyper K-ideal.

Proof. Let $(x \circ z) \circ (y \circ x) \subseteq \{0\}$ and $z \in I$. Then $(x \circ 0) \circ (y \circ z) = x \circ (y \circ z) = \{0\}$. We have $x \circ t = \{0\}, \forall t \in y \circ z$. Then $x < t, \forall t \in y \circ x$. By simplicity of H we get that $t = x$. Thus $y \circ x = \{x\}$. By Theorem 2.2 (i) we have $y \circ x = \{x\} < y$. So $x = 0$ or $0 \neq x = y$. If $x = 0$, then we are done. If $0 \neq x = y$, then $x \circ x = \{x\}$, which is impossible , because $0 \in x \circ x$.

Theorem 3.16. Let I be a weak hyper K-ideal of $(H, \circ, 0)$. Then I is an implicative hyper K-ideal if and only if $x \circ (y \circ x) \subseteq I, \forall x, y \in H$ implies that

$x \in I$.

Proof. $\Rightarrow$. Let $(x \circ (y \circ x)) \subseteq I$. Then we consider the following cases:

Case (i). $x = 0$. Since $0 \in I$, then $x \in I$.
Case (ii). $x \neq 0$. Then $x \circ (y \circ x) = (x \circ 0) \circ (y \circ x) \subseteq I$. Since I is a weak implicative hyper K-ideal and $z = 0 \in I$, thus $x \in I$.

$\Leftarrow$. Let $(x \circ z) \circ (y \circ x) \subseteq I$ and $z \in I$. Then $(x \circ (y \circ x)) \circ z \subseteq I$. Since I is a weak hyper K-ideal and $z \in I$, thus $x \circ (y \circ x) \subseteq I$.

Theorem 3.17. Let $(H, \circ, 0)$ be a simple hyper K-algebra and $a \in H$. Then $I = \{0, a\}$ is a weak implicative hyper K-ideal.

Proof. If $a = 0$, then by Theorem 3.15 we are done. If $a \neq 0$, then let there exists $x, y, z \in H$ such that $((x \circ z) \circ (y \circ x)) \subseteq \{0, a\}$ and $z \in \{0, a\}$. Then we show that $x \in I$.
Consider the following cases :
Case (i). $z = 0$. Thus by Theorem 2.5(i) we get that $(x \circ 0) \circ (y \circ x) = x \circ (y \circ x) \subseteq I$. Now if $x = 0$, then $x \in I$.
If $x \neq 0$, then we have $y = 0, y = x$ or $y \neq x$. Thus we have the following sub-cases:
case $(\acute{i})$. y=0. Hence $x \in x \circ 0 \subseteq (x \circ (0 \circ x)) \subseteq I$, so $x \in I$.
case $(\acute{ii})$. y=x. So $x \in x \circ 0 \subseteq (x \circ (y \circ x)) \subseteq I$, thus $x \in I$.
case $(\acute{iii})$. $y \neq x$. Hence $x \in x \circ y \subseteq (x \circ (y \circ x)) \subseteq I$, so $x \in I$.

Case (ii). $z = a$. Therefore we get that $(x \circ a) \circ (y \circ x) \subseteq I$ and $z \in I$. Now if $x = 0$, then we have done. If $x \neq 0$, then we have $x = a$ or $x \neq a$, if $x = a$, that is $x \in I$. If $x \neq a$, then $y = 0, y = x$ or $y \neq x$. Thus we have the following sub cases:
case $(\acute{i})$. $y = 0$. Hence by Theorem 2.5 $x \in x \circ a = ((x \circ a) \circ 0)) \subseteq ((x \circ a) \circ (0 \circ x)) \subseteq I$, so $x \in I$.
case $(\acute{ii})$. $y = x$. Then $x \in x \circ a \subseteq (x \circ a) \circ (x \circ x)) \subseteq I$, thus $x \in I$.
case $(\acute{iii})$. $y \neq x$. Hence $x \in x \circ y \subseteq (x \circ 0) \circ (y \circ x)) \subseteq I$, so $x \in I$.
Therefore I is a weak implicative hyper K-ideal.

Theorem 3.18. Let $(H, \circ, 0)$ be a simple hyper K-algebra, $|H| = n$, $a \in H - \{0\}$ and $I = \{0, a\}$ be a PIHKI of type 27(13, 18, 20). Then
(i) If $0 \in x \circ y$, then $x \circ y = \{0\}$ or $x \circ y = \{0, a\}$,
(ii) $a \circ b = \{a\}$ for $b \neq a$ and $b \in H$.

Proof. (i). On the contrary, let $x \circ y \neq \{0\}$ and $x \circ y \neq \{0, a\}$. Then there exists

$b \neq a$ and $b \neq 0$ such that $b \in x \circ y$. Since $0 \in x \circ y$, then $(x \circ 0) \circ y < I, 0 \circ y < I$, but $x \circ y \nsubseteq I$, which is a contradiction. Therefore $x \circ y = \{0\}$ or $x \circ y = \{0, a\}$
(ii). Let there is $b \in H, b \neq a$ such that $a \circ b \neq \{a\}$. Then by Theorem 2.5 $(a \circ 0) \circ b < I$ and $0 \circ b < I$, but $a \circ b \nsubseteq I$, which is a contradiction. Therefore $a \circ b = \{a\}$ for $b \neq a$ and $b \in H$.

Theorem 3.19. Let $H = \{0, 1, 2, 3\}$ be a simple hyper K-algebra, $3 \circ 1 = 3 \circ 2 = \{3\}$ and $2 \circ 1 = 2 \circ 3 = \{2\}$, then $I = \{0, 1\}$ is a PIHKI of types 25(14, 15,16,17,19,21,26).

Proof. On the contrary , let I does not be a PIHKI of type 25. Then we show that if $(x \circ z) \nless I$, then $((x \circ y) \circ z) \nless I$ or $(y \circ z) \nless I$, for $y \in H$. Now if $(x \circ z) \nless I$, then we have the following possible cases:
Case (i). $x \circ z = \{2\}$,
Case (ii). $x \circ z = \{3\}$,
Case (iii). $x \circ z = \{2, 3\}$.
We consider case (i). By Theorem 2.5 and simplicity of H we get that $x = 2$.
If $z = 0$, then we have the following sub-cases:
Case $(\acute{i})$. $y = 0$, then $(2 \circ 0) \circ 0 = \{2\} \nless I$,
Case $(\acute{ii})$. $y = 1$, then $(2 \circ 0) \circ 1 = \{2\} \nless I$,
Case $(\acute{iii})$. $y = 2$, then $2 \circ 0 = \{2\} \nless I$,
Case $(\acute{iv})$. $y = 3$, then $3 \circ 0 = \{3\} \nless I$.
If $z = 1$, then we have the following sub cases:
Case $(\acute{i})$. $y = 0$, then $(2 \circ 0) \circ 0 = \{2\} \nless I$,
Case $(\acute{ii})$. $y = 1$, then $(2 \circ 1) \circ 1 = \{2\} \nless I$,
Case $(\acute{iii})$. $y = 2$, then $2 \circ 1 = \{2\} \nless I$,
Case $(\acute{iv})$.$y = 3$, then $3 \circ 1 = \{3\} \nless I$.
If $z = 3$, then we have the following sub cases:
Case $(\acute{i})$. $y = 0$, then $(2 \circ 0) \circ 3 = \{2\} \nless I$,
Case $(\acute{ii})$. $y = 1$, then $(2 \circ 1) \circ 3 = \{2\} \nless I$,
Case $(\acute{iii})$. $y = 2$, then $2 \circ 3 = \{2\} \nless I$,
Case $(\acute{iv})$.$y = 3$, then $(2 \circ 3) \circ 3 = \{3\} \nless I$.

Now consider case (ii). The proof is similar to the proof of case (i).
We consider case (iii). So we have the following cases:
Case $(\ddot{i})$. $x = 2, z = 3$ and case $(\ddot{ii})$.$x = 3, z = 2$.
We consider case $(\ddot{i})$. We have the following sub-cases:
Case $(\acute{i})$. $y = 0$, then $(2 \circ 0) \circ 3 = \{2\} \nless I$,
Case $(\acute{ii})$. $y = 1$, then $(2 \circ 1) \circ 3 = \{2\} \nless I$,
Case $(\acute{iii})$. $y = 2$, then $2 \circ 3 = \{2\} \nless I$,
Case $(\acute{iv})$. $y = 3$, then $(2 \circ 3) \circ 3 \nless I$.
We consider case $(\ddot{ii})$. We have the following sub cases:

Case $(\acute{i})$. $y = 0$, then $(3 \circ 0) \circ 2 \not< I$,
Case $(\acute{ii})$. $y = 1$, then $(3 \circ 1) \circ 2 = \{3\} \not< I$,
Case $(\acute{iii})$. $y = 2$, then $(3 \circ 2) \circ 2 \not< I$,
Case $(\acute{iv})$. $y = 3$, then $3 \circ 2 \not< I$.
Therefore $I = \{0, 1\}$ is a PIHKI of types 14(15,16,17,19,21,25,26).

Example 3.20. The following table shows a hyper K-algebra structure on $H = \{0, 1, 2, 3\}$

$\circ$	0	1	2	3
0	$\{0\}$	$\{0\}$	$\{0\}$	$\{0,2,3\}$
1	$\{1\}$	$\{0,1\}$	$\{1,2,3\}$	$\{1,2,3\}$
2	$\{2\}$	$\{2\}$	$\{0,2,3\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,3\}$

We see that H is a simple hyper K-algebra, $3 \circ 1 = 3 \circ 2 = \{3\}$ and $2 \circ 1 = 2 \circ 3 = \{2\}$, then by above theorem $I = \{0, 1\}$ is a PIHKI of types 14(15,16,17,19,21,25,26).

Theorem 3.21. If $I = \{0, 1\}$ is a PIHKI of type14(15,16,17,19,21,25,26) of a simple hyper K-algebra $H = \{0, 1, 2, 3\}$. Then $1 \notin 2 \circ 1, 1 \notin 3 \circ 1, 2 \circ 3 \neq \{2, 3\}$ and $3 \circ 2 \neq \{2, 3\}$.

Proof. On the contrary, let $1 \in 2 \circ 1$. Then we get that $(2 \circ 1) \circ 0 < I, 1 \circ 0 < I$, but $2 \circ 1 \not< I$. Thus $1 \notin 2 \circ 1$. Also let $1 \in 3 \circ 1$. Then we get that $(3 \circ 1) \circ 0 < I, 1 \circ 0 < I$ but $3 \circ 1 \not< I$, which is contradiction . So $1 \notin 3 \circ 1$. Now assume that $2 \circ 3 = \{2, 3\}$, so $(2 \circ 3) \circ 3 < I, 3 \circ 3 < I$ but $2 \circ 3 \not< I$. Therefore $2 \circ 3 \neq \{2, 3\}$ and also let $3 \circ 2 = \{2, 3\}$. Then $(3 \circ 2) \circ 2 < I, 2 \circ 2 < I$,but $3 \circ 2 \not< I$. That is $2 \circ 3 \neq \{2, 3\}$.

Theorem 3.22. Let $(H_1, \circ_1, 0_1), (H_2, \circ_2, 0_2)$ be two hyper K-algebras , $H_1 = \{0_1, 1_1, 2_1, 3_1\}, H_2 = \{0_2, 1_2, 2_2, 3_2\}$ and $f : H_1 \to H_2$ is a bijective map such that $f(0_1) = 0_2, f(1_1) = 1_2, f(2_1) = 2_2$ and $f(3_1) = 3_2$. Also suppose that

(i) $|0_1 \circ_1 0_1| \neq |0_2 \circ_2 0_2|$, (ii) $|0_1 \circ_1 1_1| \neq |0_2 \circ_2 1_2|$,

(iii) $|0_1 \circ_1 2_1| \neq |0_2 \circ_2 2_2|$, (iv) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 3_2|$,

(v) $|1_1 \circ_1 0_1| \neq |1_2 \circ_2 0_2|$, (vi) $|1_1 \circ_1 2_1| \neq |1_2 \circ_2 2_2|$,

(vii) $|1_1 \circ_1 1_1| \neq |1_2 \circ_2 1_2|$, (viii) $|1_1 \circ_1 3_1| \neq |1_2 \circ_2 3_2|$,

(ix) $|3_1 \circ_1 3_1| \neq |3_2 \circ_2 3_2|$, (x) $|2_1 \circ_1 0_1| \neq |2_2 \circ_2 0_2|$,

(xi) $|2_1 \circ_1 1_1| \neq |2_2 \circ_2 1_2|$, (xii) $|2_1 \circ_1 2_1| \neq |2_2 \circ_2 2_2|$,

(xiii) $|2_1 \circ_1 3_1| \neq |2_2 \circ_2 3_2|$, (xiv) $|3_1 \circ_1 0_1| \neq |3_2 \circ_2 0_2|$,

(xv) $|3_1 \circ_1 1_1| \neq |3_2 \circ_2 1_2|$ or (xvi) $|3_1 \circ_1 2_1| \neq |3_2 \circ_2 2_2|$.

Then f is not an isomorphism hyper K-algebra.
Proof. We prove theorem for case (i), the proof of the other cases are similar to (i), by imposing some suitable modification. Since f is bijective , we have $|0_1 \circ_1 0_1| = |f(0_1 \circ_1 0_1)|$ and $|0_2 \circ_2 0_2| = |f(0_1) \circ_2 f(0_1)|$. If $|0_1 \circ_1 0_1| \neq |0_2 \circ_2 0_2|$, then $|f(0_1 \circ_1 0_1)| \neq |f(0_1) \circ_2 f(0_1)|$. Therefore f is not isomorphism.

Theorem 3.23. Let $(H_1, \circ_1, 0_1), (H_2, \circ_2, 0_2)$ be two hyper K-algebras , $H_1 = \{0_1, 1_1, 2_1, 3_1\}, H_2 = \{0_2, 1_2, 2_2, 3_2\}$ and $f : H_1 \to H_2$ is a bijective map such that $f(0_1) = 0_2, f(1_1) = 1_2, f(2_1) = 3_2$ and $f(3_1) = 2_2$. Also suppose that

(i) $|0_1 \circ_1 0_1| \neq |0_2 \circ_2 0_2|$, (ii) $|0_1 \circ_1 1_1| \neq |0_2 \circ_2 1_2|$,

(iii) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 2_2|$, (iv) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 2_2|$,

(v) $|1_1 \circ_1 1_1| \neq |1_2 \circ_2 1_2|$, (vi) $|1_1 \circ_1 2_1| \neq |1_2 \circ_2 3_2|$,

(vii) $|1_1 \circ_1 3_1| \neq |1_2 \circ_2 2_2|$, (viii) $|2_1 \circ_1 0_1| \neq |3_2 \circ_2 0_2|$,

(ix) $|2_1 \circ_1 1_1| \neq |3_2 \circ_2 1_2|$, (x) $|3_1 \circ_1 0_1| \neq |2_2 \circ_2 0_2|$,

(xi) $|2_1 \circ_1 2_1| \neq |3_2 \circ_2 3_2|$, (xii) $|2_1 \circ_1 3_1| \neq |3_2 \circ_2 2_2|$,

(xiii) $|3_1 \circ_1 0_1| \neq |2_2 \circ_2 0_2|$, (xiv) $|3_1 \circ_1 1_1| \neq |2_2 \circ_2 1_2|$,

(xv) $|3_1 \circ_1 2_1| \neq |2_2 \circ_2 1_2|$ or (xvi) $|3_1 \circ_1 3_1| \neq |2_2 \circ_2 2_2|$.

Then f is not an isomorphism hyper K-algebra.
Proof. The proof is similar the proof of Theorem 3.22.

Theorem 3.24. Let $(H_1, \circ_1, 0_1), (H_2, \circ_2, 0_2)$ be two hyper K-algebras , $H_1 = \{0_1, 1_1, 2_1, 3_1\}, H_2 = \{0_2, 1_2, 2_2, 3_2\}$ and $f : H_1 \to H_2$ is a bijective map such that $f(0_1) = 0_2, f(1_1) = 3_2, f(2_1) = 2_2$ and $f(3_1) = 1_2$. Also suppose that

(i) $|0_1 \circ_1 0_1| \neq |0_2 \circ_2 0_2|$, (ii) $|0_1 \circ_1 1_1| \neq |0_2 \circ_2 3_2|$,

(iii) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 1_2|$, (iv) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 1_2|$,

(v) $|1_1 \circ_1 1_1| \neq |3_2 \circ_2 3_2|$, (vi) $|1_1 \circ_1 2_1| \neq |3_2 \circ_2 2_2|$,

(vii) $|1_1 \circ_1 3_1| \neq |3_2 \circ_2 1_2|$, (viii) $|2_1 \circ_1 0_1| \neq |2_2 \circ_2 0_2|$,

(ix) $|2_1 \circ_1 1_1| \neq |2_2 \circ_2 3_2|$, (x) $|3_1 \circ_1 0_1| \neq |1_2 \circ_2 0_2|$,

(xi) $|2_1 \circ_1 2_1| \neq |2_2 \circ_2 2_2|$, (xii) $|2_1 \circ_1 3_1| \neq |2_2 \circ_2 1_2|$,

(xiii) $|3_1 \circ_1 0_1| \neq |1_2 \circ_2 0_2|$, (xiv) $|3_1 \circ_1 1_1| \neq |1_2 \circ_2 3_2|$,

(xv) $|3_1 \circ_1 2_1| \neq |1_2 \circ_2 2_2|$ or (xvi) $|3_1 \circ_1 3_1| \neq |1_2 \circ_2 1_2|$.

Then f is not an isomorphism hyper K-algebra.
Proof. The proof is similar the proof of Theorem 3.22.

Theorem 3.25. Let $(H_1, \circ_1, 0_1), (H_2, \circ_2, 0_2)$ be two hyper K-algebras , $H_1 = \{0_1, 1_1, 2_1, 3_1\}, H_2 = \{0_2, 1_2, 2_2, 3_2\}$ and $f : H_1 \to H_2$ is a bijective map such that $f(0_1) = 0_2, f(1_1) = 2_2, f(2_1) = 1_2$ and $f(3_1) = 3_2$. Also suppose that

(i) $|0_1 \circ_1 0_1| \neq |0_2 \circ_2 0_2|$, (ii) $|0_1 \circ_1 1_1| \neq |0_2 \circ_2 2_2|$,

(iii) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 3_2|$, (iv) $|0_1 \circ_1 2_1| \neq |0_2 \circ_2 1_2|$,

(v) $|1_1 \circ_1 1_1| \neq |2_2 \circ_2 2_2|$, (vi) $|1_1 \circ_1 2_1| \neq |2_2 \circ_2 1_2|$,

(vii) $|1_1 \circ_1 3_1| \neq |2_2 \circ_2 3_2|$, (viii) $|2_1 \circ_1 0_1| \neq |1_2 \circ_2 0_2|$,

(ix) $|2_1 \circ_1 1_1| \neq |1_2 \circ_2 2_2|$, (x) $|3_1 \circ_1 0_1| \neq |3_2 \circ_2 0_2|$,

(xi) $|2_1 \circ_1 2_1| \neq |1_2 \circ_2 1_2|$, (xii) $|2_1 \circ_1 3_1| \neq |1_2 \circ_2 3_2|$,

(xiii) $|3_1 \circ_1 0_1| \neq |3_2 \circ_2 0_2|$, (xiv) $|3_1 \circ_1 1_1| \neq |3_2 \circ_2 2_2|$,

(xv) $|3_1 \circ_1 2_1| \neq |3_2 \circ_2 1_2|$ or (xvi) $|3_1 \circ_1 3_1| \neq |3_2 \circ_2 3_2|$.

Then f is not an isomorphism hyper K-algebra.
Proof. The proof is similar the proof of Theorem 3.22.

Theorem 3.26. Let $(H_1, \circ_1, 0_1), (H_2, \circ_2, 0_2)$ be two hyper K-algebras , $H_1 = \{0_1, 1_1, 2_1, 3_1\}, H_2 = \{0_2, 1_2, 2_2, 3_2\}$ and $f : H_1 \to H_2$ is a bijective map such that $f(0_1) = 0_2, f(1_1) = 2_2, f(2_1) = 3_2$ and $f(3_1) = 1_2$. Also suppose that

(i) $|0_1 \circ_1 0_1| \neq |0_2 \circ_2 0_2|$, (ii) $|0_1 \circ_1 1_1| \neq |0_2 \circ_2 2_2|$,

(iii) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 1_2|$, (iv) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 1_2|$,

(v) $|1_1 \circ_1 1_1| \neq |2_2 \circ_2 2_2|$, (vi) $|1_1 \circ_1 2_1| \neq |2_2 \circ_2 3_2|$,

(vii) $|1_1 \circ_1 3_1| \neq |2_2 \circ_2 1_2|$, (viii) $|2_1 \circ_1 0_1| \neq |3_2 \circ_2 0_2|$,

(ix) $|2_1 \circ_1 1_1| \neq |3_2 \circ_2 2_2|$, (x) $|3_1 \circ_1 0_1| \neq |1_2 \circ_2 0_2|$,

(xi) $|2_1 \circ_1 2_1| \neq |3_2 \circ_2 3_2|$, (xii) $|2_1 \circ_1 3_1| \neq |3_2 \circ_2 1_2|$,

(xiii) $|3_1 \circ_1 0_1| \neq |1_2 \circ_2 0_2|$, (xiv) $|3_1 \circ_1 1_1| \neq |1_2 \circ_2 2_2|$,

(xv) $|3_1 \circ_1 2_1| \neq |1_2 \circ_2 3_2|$ or (xvi) $|3_1 \circ_1 3_1| \neq |1_2 \circ_2 1_2|$.

Then f is not an isomorphism hyper K-algebra.
Proof. The proof is similar the proof of Theorem 3.22.

Theorem 3.27. Let $(H_1, \circ_1, 0_1), (H_2, \circ_2, 0_2)$ be two hyper K-algebras , $H_1 = \{0_1, 1_1, 2_1, 3_1\}, H_2 = \{0_2, 1_2, 2_2, 3_2\}$ and $f : H_1 \to H_2$ is a bijective map such that $f(0_1) = 0_2, f(1_1) = 3_2, f(2_1) = 1_2$ and $f(3_1) = 2_2$. Also suppose that

(i) $|0_1 \circ_1 0_1| \neq |0_2 \circ_2 0_2|$, (ii) $|0_1 \circ_1 1_1| \neq |0_2 \circ_2 3_2|$,

(iii) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 2_2|$, (iv) $|0_1 \circ_1 3_1| \neq |0_2 \circ_2 2_2|$,

(v) $|1_1 \circ_1 1_1| \neq |3_2 \circ_2 3_2|$, (vi) $|1_1 \circ_1 2_1| \neq |3_2 \circ_2 1_2|$,

(vii) $|1_1 \circ_1 3_1| \neq |3_2 \circ_2 2_2|$, (viii) $|2_1 \circ_1 0_1| \neq |1_2 \circ_2 0_2|$,

(ix) $|2_1 \circ_1 1_1| \neq |1_2 \circ_2 3_2|$, (x) $|3_1 \circ_1 0_1| \neq |2_2 \circ_2 0_2|$,

(xi) $|2_1 \circ_1 2_1| \neq |1_2 \circ_2 1_2|$, (xii) $|2_1 \circ_1 3_1| \neq |1_2 \circ_2 2_2|$,

(xiii) $|3_1 \circ_1 0_1| \neq |2_2 \circ_2 0_2|$, (xiv) $|3_1 \circ_1 1_1| \neq |2_2 \circ_2 3_2|$,

(xv) $|3_1 \circ_1 2_1| \neq |2_2 \circ_2 1_2|$ or (xvi) $|3_1 \circ_1 3_1| \neq |2_2 \circ_2 2_2|$.

Then f is not an isomorphism hyper K-algebra.

Proof. The proof is similar the proof of Theorem 3.22.

Theorem 3.28. There are 50 non- isomorphism hyper K-algebras of order 4 with simple condition such that they have exactly one PIHKI of order 2 and of type 27(13, 18, 20).

Proof. Let $H = \{0,1,2,3\}$ and assume that the following table shows a probable hyper K-algebra structure on H.

$\circ$	0	1	2	3
0	a_{11}	a_{12}	a_{13}	a_{14}
1	a_{21}	a_{22}	a_{23}	a_{24}
2	a_{31}	a_{32}	a_{33}	a_{34}
3	a_{41}	a_{42}	a_{43}	a_{44}

Let $I = \{0,1\}$ be a PIHKI of type 27. Then by Theorems 2.5 and 3.19 we get that $1 \circ 0 = \{1\}, 2 \circ 0 = \{2\}, 3 \circ 0 = \{3\}, 2 \in a_{31} \bigcap a_{32} \bigcap a_{34}, 3 \in a_{41} \bigcap a_{42} \bigcap a_{43}$ and $1 \circ 2 = 1 \circ 3 = \{1\}$. Since H is a hyper K-algebra, then $0 \in a_{11} \bigcap a_{12} \bigcap a_{13} \bigcap a_{14} \bigcap a_{22} \bigcap a_{33} \bigcap a_{44}$. Hence we get that the only cases for a_{31}, a_{32}, a_{34} are $\{2\}, \{1,2\}, \{2,3\}$ or $\{1,2,3\}$. Also the only cases for a_{41}, a_{42}, a_{43} are $\{3\}, \{1,3\}, \{2,3\}$ or $\{1,2,3\}$. Also by Theorem 3.19(ii), the only cases for $a_{11}, a_{12}, a_{13}, a_{14}, a_{22}, a_{33}$ and a_{44} are $\{0\}$ or $\{0,1\}$. Therefore there are $2^7 \times 4^6$ for H. By considering Definition 2.1, specially HK2 and some manipulation we show that 56 cases are hyper K-algebras. By Theorems 3.22, 3.23 ,3.24 ,3.25, 3.26 and 3.27 we get that there are 50 non- isomorphism hyper K-algebras of order 4 with simple condition such that they have exactly one PIHKI of type 27.

H_1	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{3}	{3}	{0}

H_2	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{3}	{3}	{0}

H_3	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{3}	{3}	{0, 1}

H_4	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{13}	{3}	{0, 1}

H_5	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0}

H_6	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_7	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{13}	{3}	{0, 1}

H_8	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{3}	{3}	{0, 1}

H_9	0	1	2	3
0	{0}	{0}	{0}	{0, 1}
1	{1}	{0}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0}

H_{10}	0	1	2	3
0	{0}	{0}	{0}	{0, 1}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0}

H_{11}	0	1	2	3
0	{0}	{0}	{0}	{0, 1}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{12}	0	1	2	3
0	{0}	{0}	{0}	{0, 1}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0, 1}

H_{13}	0	1	2	3
0	{0}	{0}	{0}	{0, 1}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{14}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{15}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0}

H_{16}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0, 1}

H_{17}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0, 1}

H_{18}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{19}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0}

H_{20}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0}

H_{21}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{22}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{23}	0	1	2	3
0	{0}	{0, 1}	{0}	{0, 1}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0}

H_{24}	0	1	2	3
0	{0}	{0, 1}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{1, 3}	{3}	{0}

H_{25}	0	1	2	3
0	{0}	{0, 1}	{0}	{0, 1}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{0, 1}	{0, 1}

H_{26}	0	1	2	3
0	{0}	{0, 1}	{0}	{0, 1}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{27}	0	1	2	3
0	{0}	{0, 1}	{0, 1}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{28}	0	1	2	3
0	{0}	{0, 1}	{0, 1}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{1, 2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{29}	0	1	2	3
0	{0, 1}	{0}	{0}	{0}
1	{1}	{0, 1}	{1}	{1}
2	{2}	{2}	{0, 1}	{2}
3	{3}	{3}	{3}	{0, 1}

H_{30}	0	1	2	3
0	$\{0,1\}$	$\{0\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{31}	0	1	2	3
0	$\{0,1\}$	$\{0\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{32}	0	1	2	3
0	$\{0,1\}$	$\{0\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{33}	0	1	2	3
0	$\{0,1\}$	$\{0\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{34}	0	1	2	3
0	$\{0,1\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{35}	0	1	2	3
0	$\{0,1\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{36}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{37}	0	1	2	3
0	$\{0,1\}$	$\{0\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{38}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{0\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{39}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{40}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{41}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{42}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{43}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{44}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{45}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{46}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{47}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{48}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{3\}$	$\{0,1\}$

H_{49}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{1,3\}$	$\{3\}$	$\{0,1\}$

H_{50}	0	1	2	3
0	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$	$\{0,1\}$
1	$\{1\}$	$\{0,1\}$	$\{1\}$	$\{1\}$
2	$\{2\}$	$\{1,2\}$	$\{0,1\}$	$\{2\}$
3	$\{3\}$	$\{3\}$	$\{0,1\}$	$\{3\}$

Conclusion. The paper has shown that the nature of PIHKI of type 6 is completely different from type 27 in the simple hyper K-algebras (Theorems 3.3, 3.6 and Examples 3.4, 3.7). The reserchers have introduced a simple and usefule condition to study whether $I = H - \{a\}$, for $a \in H$ is a PIHKI or not. Finally the researchers have proved in a simple positive hyper K-algebra , for all non-empty subsets A of H, $A_{r1}(A_{r2})$ and ${}_{l1}A({}_{l2}A)$ are PIHKI of types 1,2,...,9.

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