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Effects of Slip and Heat Transfer on MHD Peristaltic Flow in An Inclined Asymmetric Channel

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Abstract. Peristaltic transport of an incompressible electrically conducting viscous fluid in an inclined planar asymmetric channel is studied. The asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitude and phase. The closed form solutions of momentum and energy equation in presence of viscous dissipation term are obtained for long wave length and low Reynolds number approximations. The effects of different parameters entering into the problem are discussed numerically and explained graphically.

Keywords: Peristalsis, Froude number, Brinkman number, Heat transfer coefficient.

2000 Mathematics subject classification: 76A05, 76Z05.

1. Introduction

The term peristalsis is used for the mechanism by which a fluid can be transported through a distensible tube when progressive waves of area contraction and expansion propagate along its length. In physiology, peristalsis is an important mechanism for transport of fluid and is used by the body to propel or mix the contents of a tube as in ureter, gastrointestinal tract, bile duct and other glandular ducts. Some biomedical instruments, like the blood

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pumps in dialysis and the heart lung machine use this principle. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited and transport of a toxic liquid is used in nuclear industry to avoid contamination from the outside environment. The problem of the mechanism of peristaltic transport has attracted the attention of many investigators since the first investigation of Latham [7]. A good number of analytical, numerical and experimental [1-3,5-8,11,13-15,17,21,25] studies has been conducted to understand peristaltic action under different conditions with reference to physiological and mechanical situations. Srinivas and Pushparaj [23] have investigated the peristaltic transport of MHD flow of a viscous incompressible fluid in a two dimensional asymmetric inclined channel. However, the interaction of peristalsis and heat transfer has not received much attention, which may become highly relevant and significant in several industrial processes. Also, thermodynamic aspects of blood may become significant in processes like oxygenation and hemodialysis [9,18,19,22,24] when blood is drawn out of the body. Lately, the combined effects of magnetohydrodynamics and heat transfer on the peristaltic transport of viscous fluid in a channel with compliant walls have been discussed by Mekheimer and Abd elmaboud and co-workers [10,16,20]. Very recently, Hayat et al.[4] have expanded the domain of the problem by considering slip conditions on the boundary of the channel; but they have neglected the viscous dissipation term in the energy equation. In fact, a limited attention has been given to the study of peristaltic flow in an inclined asymmetric channel, which is more practical in real field.

The aim of the present investigation is to highlight the importance of heat transfer analysis of MHD peristaltic flow in an asymmetric inclined channel under the influence of slip conditions. The governing equations of momentum and energy have been simplified using long wave length and low Reynolds number approximations. The exact solutions of momentum and energy equations in presence of viscous dissipation term and external heat addition/absorption have been obtained. The features of flow and heat transfer characteristics are analyzed by plotting graphs. In the present paper the Section 2 deals with the formulation of the problems; Section 3 contains the closed form solutions of stream function, temperature, pressure gradient and heat transfer coefficient; numerical results and discussion are presented in Section 4; the conclusions have been summarized in section 5.

2. Mathematical Formulation of the problem

MHD flow of an electrically conducting incompressible viscous fluid is considered in a two dimensional asymmetric channel in presence of a constant

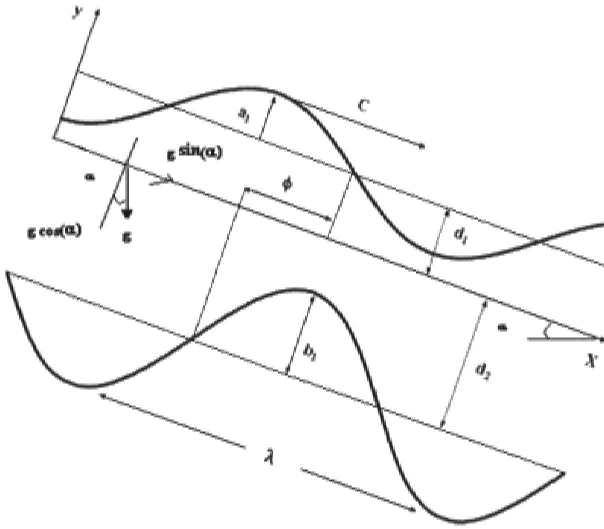

FIGURE 1. Schematic diagram of the physical model

transverse magnetic field B_0 (see Fig.1). The channel flow occurs due to different amplitude and phases of the peristaltic waves with constant speed c along the channel walls

$Y = H_1 = d_1 + a_1 cos\frac{2\pi}{\lambda}(X - ct), upperwall$

$Y = H_2 = -d_2 - b_1 cos(\frac{2\pi}{\lambda}(X - ct) + \phi), lowerwall$ (1)

where a_1 and b_1 are the amplitudes of the waves, λ is the wave length, $d_1 + d_2$ is the width of the channel, t is the time and X is the direction of wave propagation. The phase difference ϕ varies in the range $0 \leq \phi \leq \pi$, in which $\phi = 0$ corresponds to symmetric channel with waves out of phase and $\phi = \pi$ the waves are in phase and a_1, b_1, d_1, d_2 and ϕ satisfies the condition

$$a_1^2 + b_1^2 + 2a_1 b_1 cos\phi \leq (d_1 + d_2)^2 \qquad (2)$$

The lower wall of the channel is maintained at temperature T_1 while the upper wall has temperature T_0

The equations governing the motion for the present problem are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \qquad (3)$$

$$\rho[\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}] = -\frac{\partial P}{\partial X} + \mu(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}) - \sigma B_0^2 U + \rho g sin\alpha, \qquad (4)$$

$$\rho[\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}] = -\frac{\partial P}{\partial Y} + \mu(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}) - \rho g cos\alpha, \qquad (5)$$

$C_p[\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}]=\frac{\kappa}{\rho}(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2})+Q_0+\nu[2(\frac{\partial U}{\partial X})^2+2(\frac{\partial V}{\partial Y})^2+(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X})^2]$ (6)

where U, V are the velocities in the X and Y directions in the fixed frame, α is inclination of the channel with the horizontal, P is the pressure, ρ is the density, μ is the coefficient of viscosity of fluid, ν is the kinematic coefficient of viscosity, σ is the electrical conductivity of the fluid, κ is the thermal conductivity, C_p is the specific heat at constant pressure, Q_0 is the constant heat addition/absorption, g is the acceleration due to gravity and T is the temperature of the fluid.

A wave frame (x,y) is now introduced, moving with velocity c away from the fixed frame (X, Y) by the transformation

$x=X-ct, y=Y, u=U-c, v=V, p(x)=P(X,t)$ (7)

where u, v are the velocities in the x and y directions in the wave frame and p is the pressure in wave frame.

Introducing the following non-dimensional quantities :

$\bar{x}=\frac{x}{\lambda};\bar{y}=\frac{y}{d_1};\bar{u}=\frac{u}{c};\bar{v}=\frac{v}{c\delta};\delta=\frac{d_1}{\lambda};\bar{p}=\frac{d_1^2 p}{c\mu\lambda};h_1=\frac{H_1}{d_1};h_2=\frac{H_2}{d_1};d=\frac{d_1}{d_2};$
$a=\frac{a_1}{d_1};b=\frac{b_1}{d_1};R=\frac{cd_1}{\nu};\theta=\frac{T-T_0}{T_1-T_0};P_r=\frac{\rho\nu C_p}{\kappa};\bar{t}=\frac{ct}{\lambda};M=\sqrt{\frac{\sigma}{\mu}}B_0 d_1;$
$\beta=\frac{Q_0 d_1^2}{(T_1-T_0)\nu C_p};E_c=\frac{c^2}{C_p(T_1-T_0)};F_r=\frac{c^2}{gd_1}$ (8)

in (3)-(6), we finally get (after dropping bars)

$\delta\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$ (9)

$\delta R[(u+1)\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}]=-\frac{\partial p}{\partial x}+(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-M^2(u+1)+\frac{R}{F_r}sin\alpha,$ (10)

$\delta^3 R[(u+1)\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}]=-\frac{\partial p}{\partial y}+\delta^2(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-\delta\frac{R}{F_r}cos\alpha,$ (11)

$\delta R[(u+1)\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}]=\frac{1}{P_r}(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2})+\beta+E_c[2\delta^2(\frac{\partial u}{\partial x})^2+2\delta^2(\frac{\partial v}{\partial y})^2+(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})^2]$ (12)

where R is the Reynolds number, δ is the wave number, M is the Hartmann number, P_r is the Prandtl number, E_c is the Eckert number, F_r is the Froude number and β is the non-dimensional heat source/sink parameter.

Introducing stream function ψ such that $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$, applying long wave length approximation and neglecting the wave number along with low-Reynolds numbers, one can find from (9)-(12) that

$0=-\frac{\partial p}{\partial x}+\frac{\partial^3\psi}{\partial y^3}-M^2(\frac{\partial\psi}{\partial y}+1)+\frac{R}{F_r}sin\alpha,$ (13)

$0=-\frac{\partial p}{\partial y},$ (14)

$$0 = \frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2} + \beta + E_c\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \qquad (15)$$

Considering slip on the wall corresponding boundary conditions for the present problem in the wave frame can be written as

$$\psi = \frac{F}{2}, \frac{\partial\psi}{\partial y} + \beta_1\frac{\partial^2\psi}{\partial y^2} = -1, at \quad y = h_1 = 1 + acos2\pi x,$$

$$\psi = -\frac{F}{2}, \frac{\partial\psi}{\partial y} - \beta_1\frac{\partial^2\psi}{\partial y^2} = -1, at \quad y = h_2 = -d - bcos(2\pi x + \phi), \qquad (16)$$

$$\theta + \gamma\frac{\partial\theta}{\partial y} = 0, at \quad y = h_1,$$

$$\theta - \gamma\frac{\partial\theta}{\partial y} = 1, at \quad y = h_2 \qquad (17)$$

where β_1 is the non-dimensional slip velocity parameter(Knudsen number) and γ is the non-dimensional thermal slip parameter, $F = \int\limits_{h_2}^{h_1}\frac{\partial\psi}{\partial y}dy$ is the flux in the wave frame and a, b, ϕ and d satisfy the relation

$$a^2 + b^2 + 2abcos\phi \leq (1 + d)^2 \qquad (18)$$

3. Solution of the problem

Eq.(14) shows that p is not a function of y. On differentiating Eq.(13) with respect to y, the compatibility equation is as follows

$$\frac{\partial^4\psi}{\partial y^4} - M^2\frac{\partial^2\psi}{\partial y^2} = 0 \qquad (19)$$

The closed form solutions for Eqs.(19) and (15) with boundary conditions (16) and (17) are

$$\psi = A_0 + A_1 y + A_2 coshMy + A_3 sinhMy, \qquad (20)$$

$$\theta = A_4 + A_5 y - \frac{1}{2}\beta P_r y^2 - \frac{1}{8}B_r M^2 \times \left\{2M^2y^2(A_2^2 - A_3^2) + (A_2^2 + A_3^2)cosh2My + 2A_2A_3 sinh2My\right\} \quad (21)$$

where $B_r(= E_c P_r)$ is the Brinkman number,

$$A_0 = -\frac{h_1 + h_2}{2}A_1, \qquad (22)$$

$$A_1 = \frac{MF + (2 + F\beta_1 M^2)tanh\frac{1}{2}M(h_1 - h_2)}{M(h_1 - h_2) - \{2 - \beta_1 M^2(h_1 - h_2)\}tanh\frac{1}{2}M(h_1 - h_2)}, \qquad (23)$$

$$A_2 = \frac{(F+h_1-h_2)sech\frac{1}{2}M(h_1-h_2)sinh\frac{1}{2}M(h_1+h_2)}{M(h_1-h_2)-\{2-\beta_1 M^2(h_1-h_2)\}tanh\frac{1}{2}M(h_1-h_2)}, \quad (24)$$

$$A_3 = -A_2 coth\tfrac{1}{2}M(h_1+h_2) \quad (25)$$

$$A_4 = \tfrac{1}{4}\beta P_r h_1(h_1+2\gamma)\left\{2+B_r M^4(A_2^2-A_3^2)\right\} + \tfrac{1}{8}B_r M^2\left\{(A_2^2+A_3^2)(cosh2Mh_1+2\gamma M sinh2Mh_1)+2A_2A_3(sinh2Mh_1+2\gamma M cosh2Mh_1)\right\} - A_5(h_1+\gamma) \quad (26)$$

$$A_5 = -\frac{1}{h_1-h_2+2\gamma} + \tfrac{1}{4}B_r M^4(A_2^2-A_3^2)(h_1+h_2) + \frac{B_r M^2}{4(h_1-h_2+2\gamma)}\left\{(A_2^2+A_3^2) sinhM(h_1+h_2)+2A_2A_3 coshM(h_1+h_2)\right\}\left\{sinhM(h_1-h_2)+2\gamma M coshM(h_1-h_2)\right\} \quad (27)$$

The pressure gradient is obtained from the dimensionless momentum equation for the axial velocity as

$$\frac{dp}{dx} = \frac{M^3(h_2-h_1-F)\left\{1+M\beta_1 tanh\frac{1}{2}M(h_1-h_2)\right\}}{M(h_1-h_2)-\{2-\beta_1 M^2(h_1-h_2)\}tanh\frac{1}{2}M(h_1-h_2)} + \frac{R}{F_r}sin\alpha \quad (28)$$

The heat transfer coefficient (Z) at the upper wall is given by

$$Z = (h_1)_x\theta_y = (h_1)_x[A_4-\beta P_r y-\tfrac{1}{4}B_r M^3\left\{2My(A_2^2-A_3^2)+(A_2^2+A_3^2)sinh2My+2A_2A_3cosh2My\right\}] \quad (29)$$

Note that if the inclination angle α and the Eckert number E_c are taken equal to zero, the results of the problem reduce exactly to the same as that found by Hayat [25] whose results are in agreement with the previous results obtained Hayat [14] and Srinivas [24].

4. Numerical Results and Discussion

An interesting phenomenon of peristaltic motion in the wave frame is trapping which is basically the formation of an internally circulating bolus of fluid by closed streamlines. This trapped bolus is pushed ahead with the peristaltic wave. The variations of ϕ, M and β_1 on the streamlines are shown in Fig.2-4, respectively. It is observed from Fig.2 that the bolus appears in the center region for ϕ=0 and moves towards right and decreases in size as ϕ increases. Fig.3 depicts that the size and number of trapped bolus decreases by increasing M and slowly disappears for large values of M. The effect of the slip parameter on the trapping is illustrated in Fig.4 and it is observed that the number and size of trapped bolus gradually decreases with increasing slip velocity parameter.

To study the behavior of the distributions of the axial velocity (u), numerical calculations for several values of ϕ, M and β_1 are carried out in Figs.5-7. The influence of ϕ is shown in Fig.5 and it is observed that the axial velocity decreases in magnitude with an increase in the phase angle ϕ. Fig.6 displays that with an increase in M, the magnitude of velocity decreases but it has no effect in the central region of the channel. Fig.7 shows that an increase in β_1 results in increase in velocity distribution.

Figs.8-10 are prepared to discuss the pressure gradient for different values of M, F_r and β_1 . It can be noticed that in the wider portion of the channel $x\epsilon$ [0,0.3] and [0.6,1], the pressure gradient is relatively small , that is, the flow can easily pass without imposition of a large pressure gradient. On the other hand, in a narrow part of the channel $x\epsilon$ [0.3,0.6] a much pressure gradient is required to maintain the same flux to pass it especially near $x = 4.3$. From Figs. 8,9, it may be noted that M increases the maximum amplitude of $\frac{\partial p}{\partial x}$ and it rapidly increases for large magnetic field. But the amplitude decreases on increasing F_r. Fig.10 reveals that the pressure gradient decreases with increase in β_1.

To explicitly see the effects of various parameters, say γ, M and B_r on temperature, Eq.(21)has been numerically evaluated and the results are presented in Figs.11-13. It is observed that the temperature increases with increase of γ, B_r while it decreases with increasing M. Further, it can be noted that the temperature at the lower wall is maximum and it decreases slowly toward the upper wall.

Variations of the heat transfer coefficient at the wall have been presented in Figs.14-16 for various values of γ, β and B_r with fixed values of other parameters. One can observe that the heat transfer coefficient decreases with increasing in γ, β. Also, it is noted that there is no quantitative change in the behaviour of the heat transfer coefficient near the center but it increases with increasing B_r in the vicinity of the upper wall.

5. Conclusions

In this work, the combined effects of slip condition and heat transfer on MHD peristaltic flow of a viscous incompressible fluid in an asymmetric inclined channel are derived. The closed form analytical solutions of the problem under long wavelength and low Reynolds number approximations are obtained. Streamlines are plotted to discuss the phenomena of trapping. Some deductions are made and results are found to be in agreement with the earlier works. The main findings can be summarized as :

(i)The number and volume of the trapped bolus decreases by increasing both the slip velocity parameter β_1. Moreover, trapped bolus moves towards right and decreases in size as ϕ increases.
(ii)The magnitude of axial velocity increases with increase in the slip velocity parameter β_1 and decreases by increasing ϕ and M .
(iii)The pressure rise for a magnetohydrodynamic fluid is smaller due to slip condition and is maximum in the absence of the slip flow.
(iv)It is analyzed that with increase in M temperature profile decreases; moreover, it is seen that temperature field increases with increasing in γ and B_r.
(v)The rate of heat transfer drops due to the presence of the thermal slip condition, which is very important from the practical point of view.

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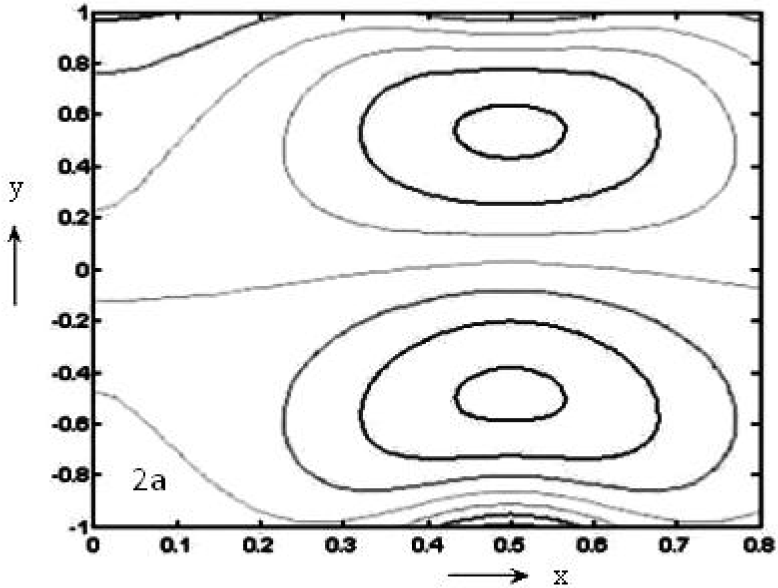

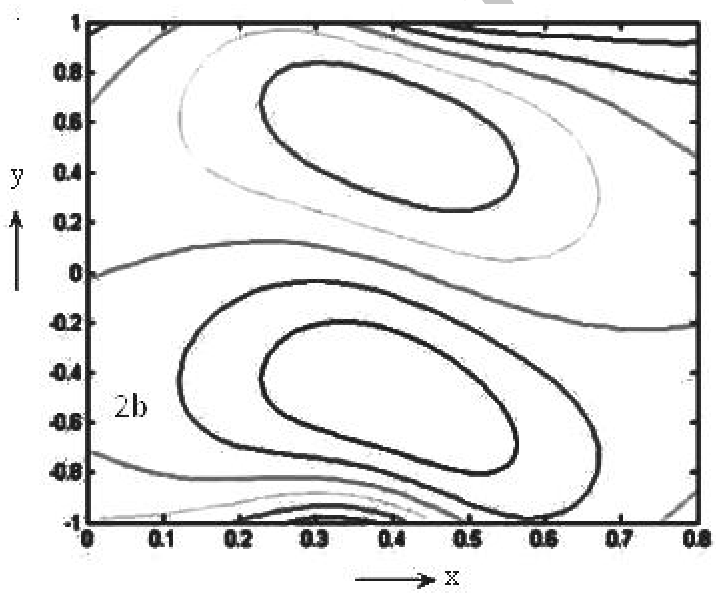

FIGURE 2. Effect of ϕ on streamlines : (a)ϕ=0, (b)ϕ=π/3: a=0.25, b=0.4, d=1.1, M=0.5, $\beta_1 = 0.01$

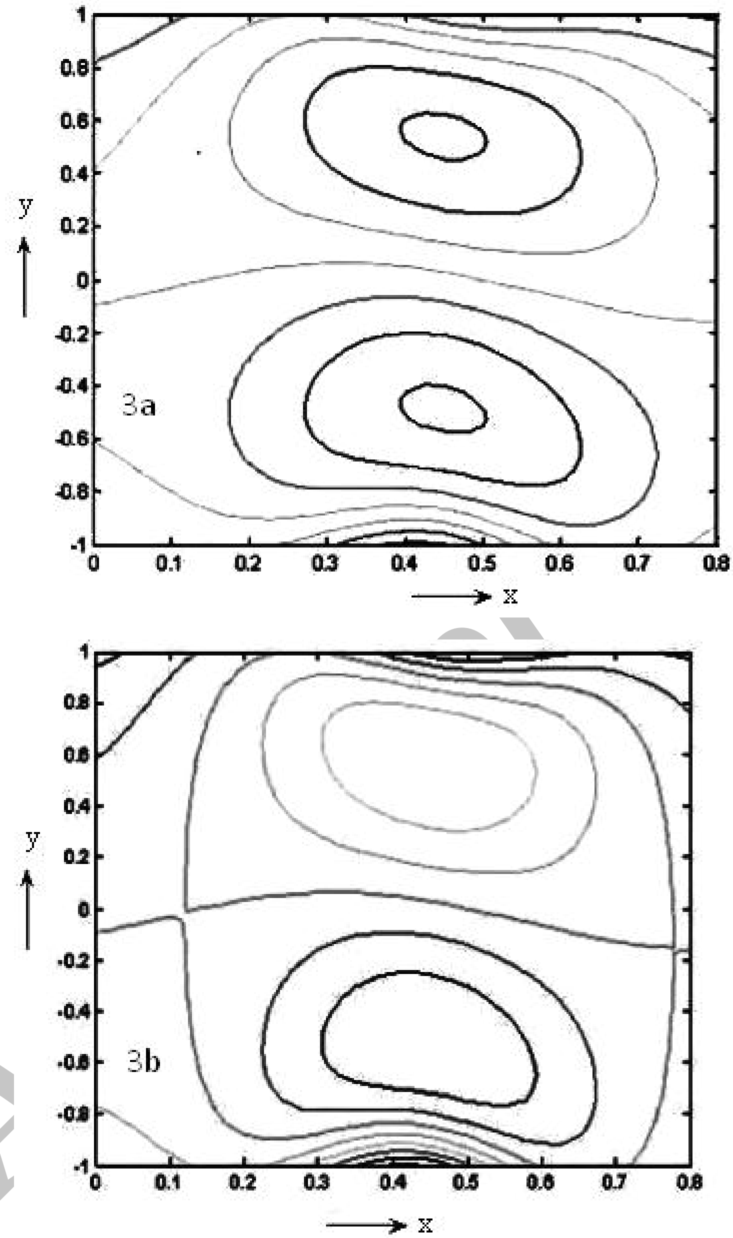

FIGURE 3. Effect of M on streamlines: (a)M=0.05, (b)M=1.5 : a=0.25, b=0.4, d=1.1, $\phi=\pi/6$, β_1=0.01

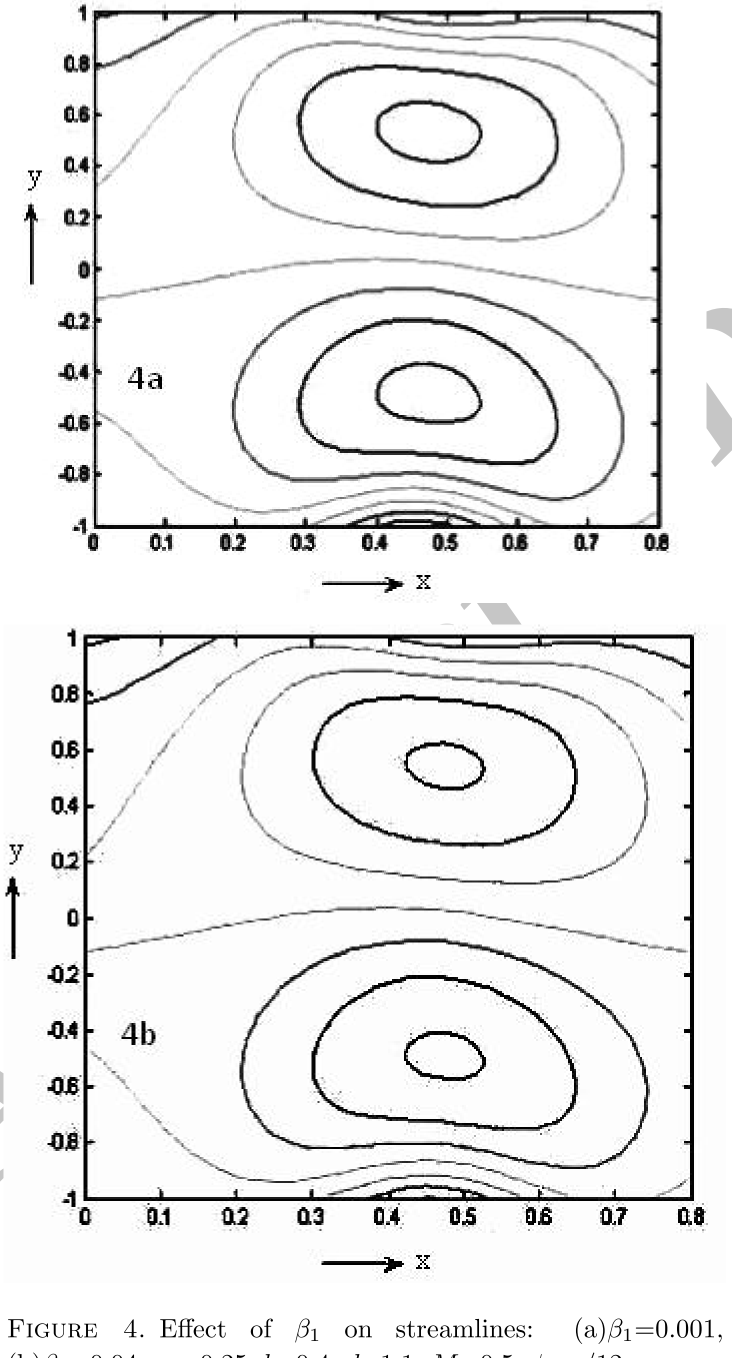

FIGURE 4. Effect of β_1 on streamlines: (a)β_1=0.001, (b)β_1=0.04 : a=0.25, b=0.4, d=1.1, M=0.5, ϕ=$\pi/12$

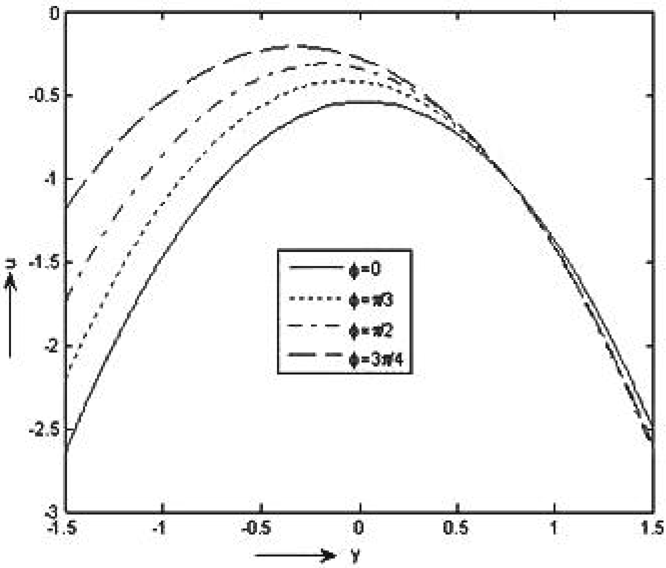

FIGURE 5. Velocity distribution for different values of ϕ: a=0.25,b=0.4,d=1.1,x=0.5

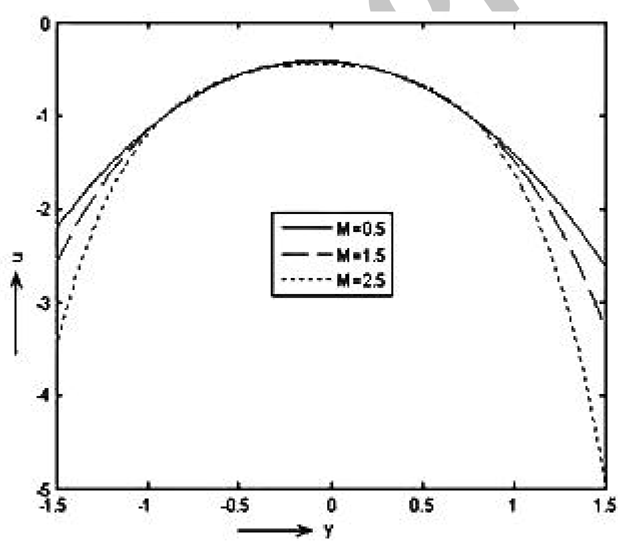

FIGURE 6. Velocity distribution for different values of M: a=0.25,b=0.4,d=1.1,x=0.5

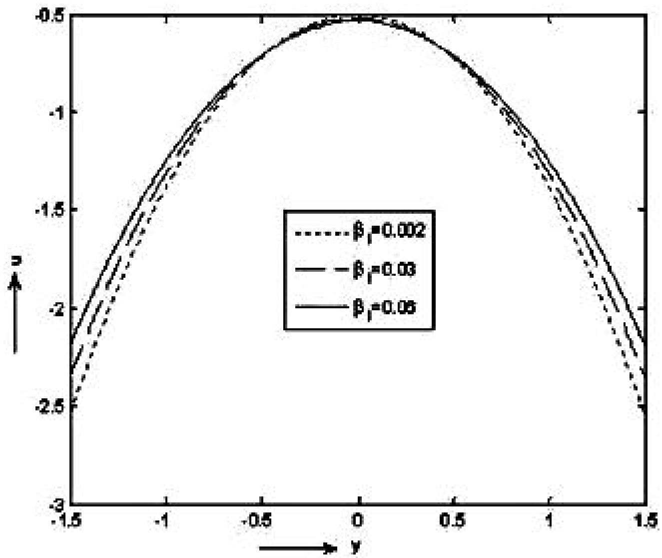

FIGURE 7. Velocity distribution for different values of β_1: a=0.25,b=0.4,d=1.1,x=0.5

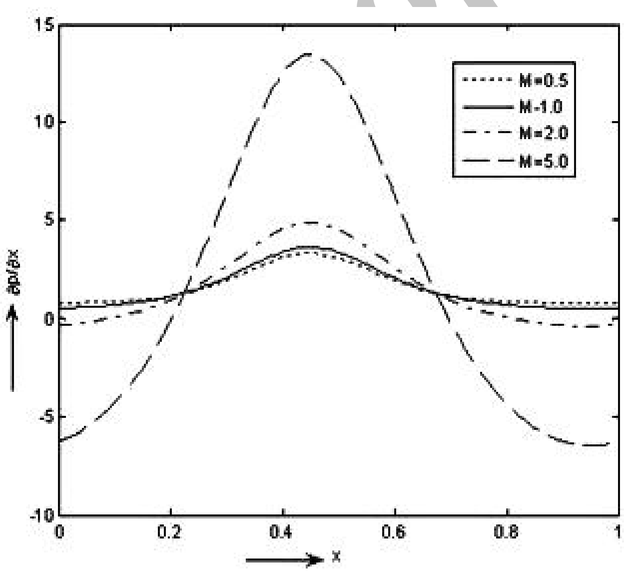

FIGURE 8. Variation of the pressure gradient with x for different values of M: a=0.25,b=0.4,d=1.1

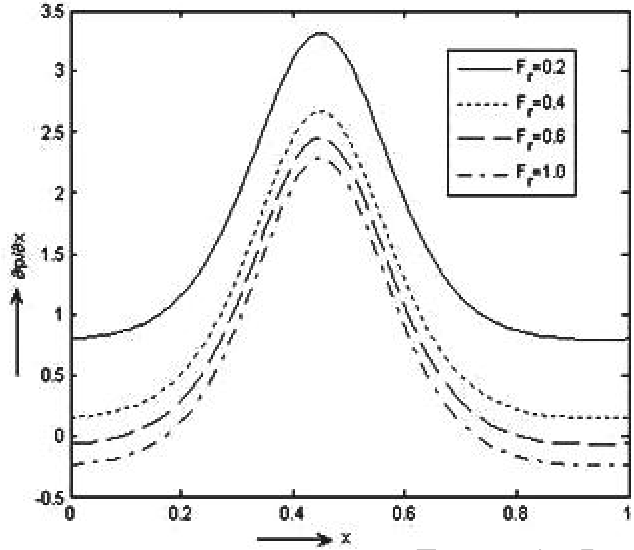

FIGURE 9. Variation of the pressure gradient with x for different values of F_r: a=0.25,b=0.4,d=1.1

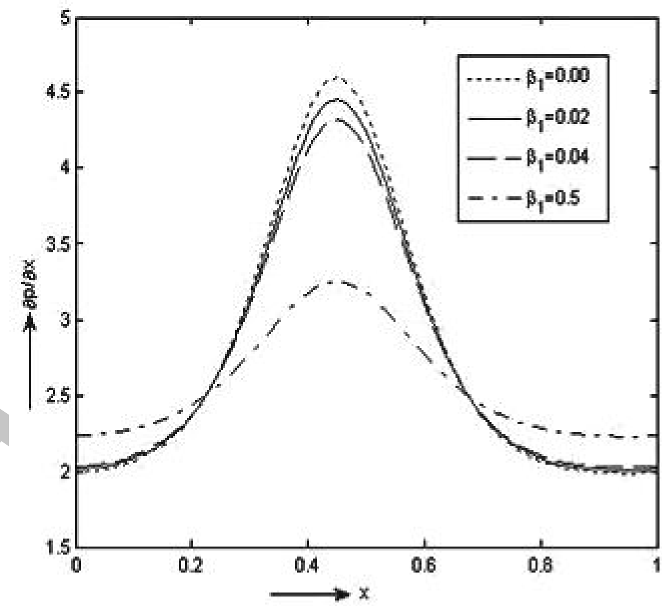

FIGURE 10. Variation of the pressure gradient with x for different values of β_1: a=0.25,b=0.4,d=1.1

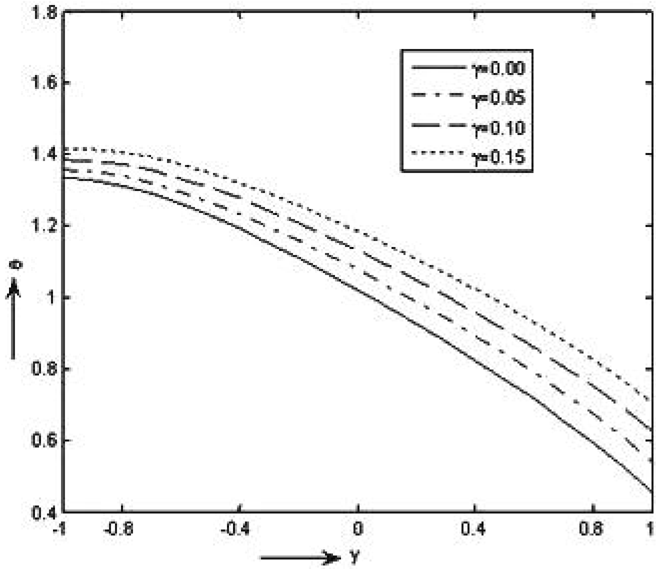

FIGURE 11. Variation of temperature with y for different values of γ: a=0.25, b=0.4, d=1.1, x=0.5

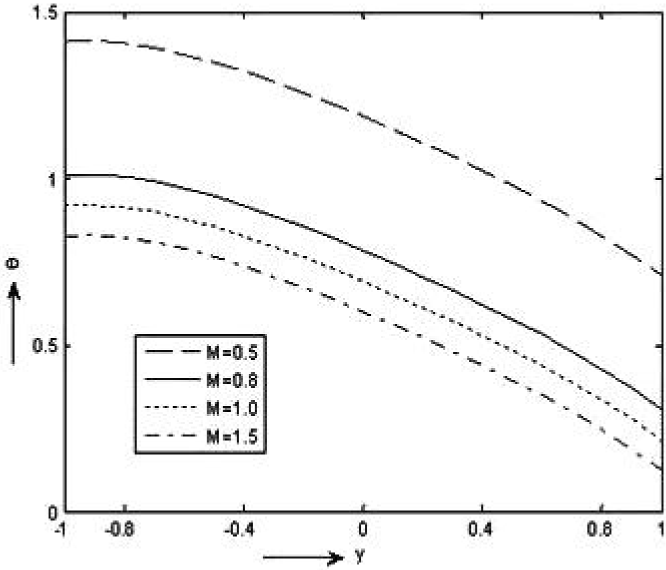

FIGURE 12. Variation of temperature with y for different values of M: a=0.25, b=0.4, d=1.1, x=0.5

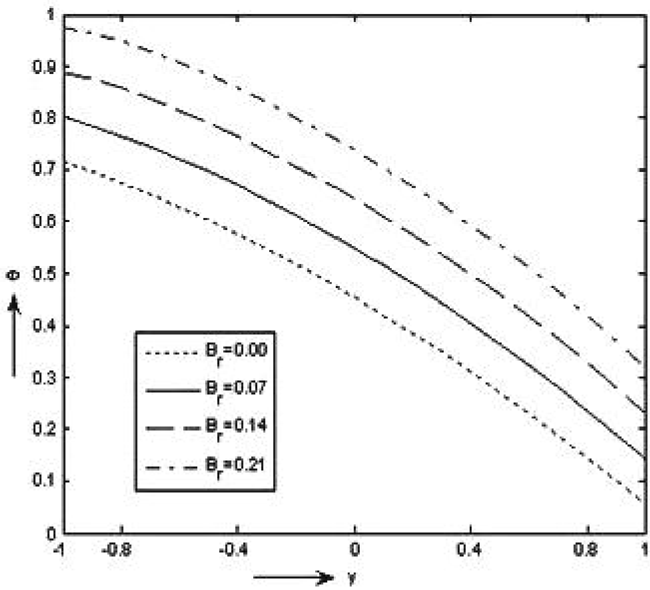

FIGURE 13. Variation of temperature with y for different values of B_r: a=0.25, b=0.4, d=1.1, x=0.5

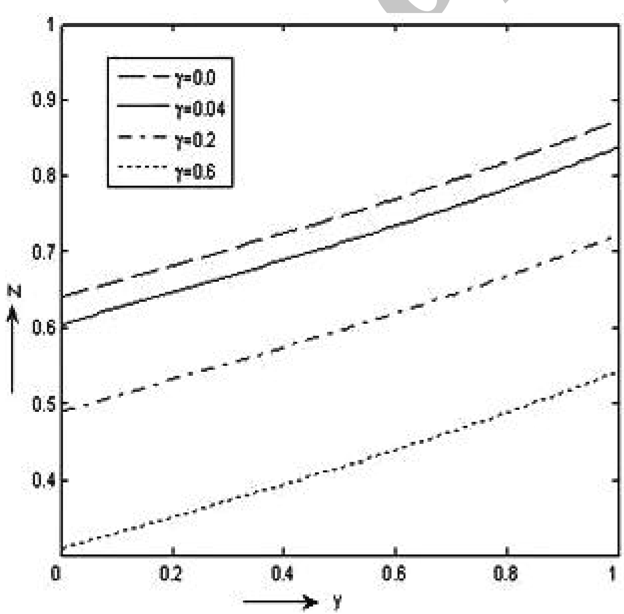

FIGURE 14. Effect of γ on heat transfer coefficient

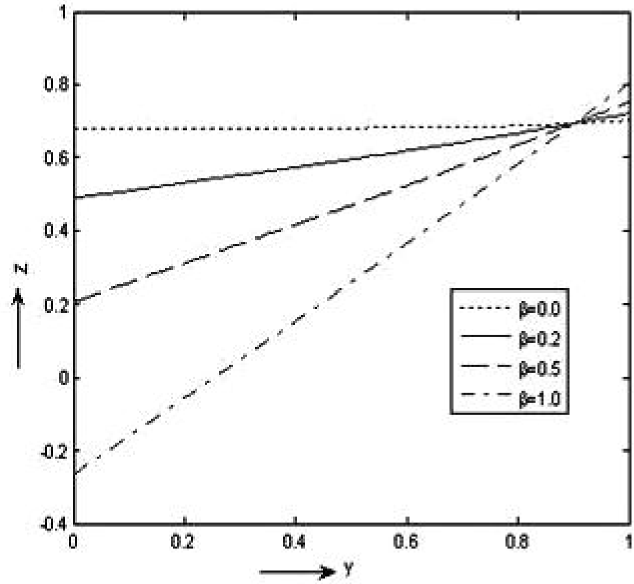

FIGURE 15. Effect of β on heat transfer coefficient

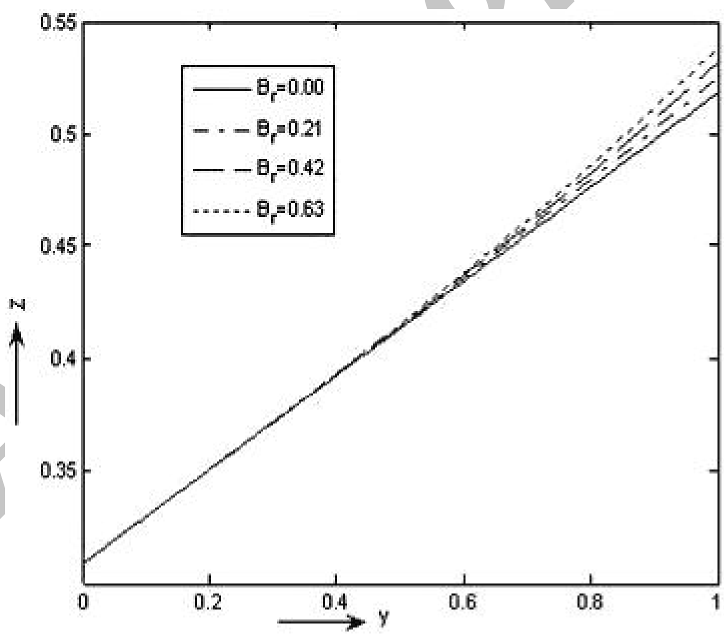

FIGURE 16. Effect of B_r on heat transfer coefficient