

Iranian Journal of Mathematical Sciences and Informatics  
Vol. 11, No. 2 (2016), pp 131-137  
DOI: 10.7508/ijmsi.2016.02.010

## Additive Maps Preserving Idempotency of Products or Jordan Products of Operators

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ABSTRACT. Let  $\mathcal{H}$  and  $\mathcal{K}$  be infinite dimensional Hilbert spaces, while  $\mathcal{B}(\mathcal{H})$  and  $\mathcal{B}(\mathcal{K})$  denote the algebras of all linear bounded operators on  $\mathcal{H}$  and  $\mathcal{K}$ , respectively. We characterize the forms of additive mappings from  $\mathcal{B}(\mathcal{H})$  into  $\mathcal{B}(\mathcal{K})$  that preserve the nonzero idempotency of either Jordan products of operators or usual products of operators in both directions.

**Keywords:** Operator algebra, Jordan product, Idempotent.

**2000 Mathematics subject classification:** 46J10, 47B48.

### 1. INTRODUCTION

The study of maps on operator algebras preserving certain properties or subsets is a topic which attracts much attention of many authors. See the references.

Some problems are concerned with preserving a certain property of usual product or other products of operators. For example see [4, 6 – 10, 13, 15, 16].

Let  $\mathcal{R}$  and  $\mathcal{R}'$  be two rings and  $\phi : \mathcal{R} \rightarrow \mathcal{R}'$  be a map. Denote by  $P_{\mathcal{R}}$  and  $P_{\mathcal{R}'}$  the set of all idempotent elements of  $\mathcal{R}$  and  $\mathcal{R}'$ , respectively. The triple Jordan product and the Jordan product of two elements  $A$  and  $B$  are defined as

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$ABA$  and  $\frac{1}{2}(AB + BA)$ , respectively. We say that  $\phi$  preserves the idempotency of product of two elements, the idempotency of triple Jordan product of two elements and the idempotency of Jordan product of two elements, whenever we have

$$AB \in P_{\mathcal{R}} \Rightarrow \phi(A)\phi(B) \in P_{\mathcal{R}'},$$

$$ABA \in P_{\mathcal{R}} \Rightarrow \phi(A)\phi(B)\phi(A) \in P_{\mathcal{R}'}$$

and

$$\frac{1}{2}(AB + BA) \in P_{\mathcal{R}} \Rightarrow \frac{1}{2}(\phi(A)\phi(B) + \phi(B)\phi(A)) \in P_{\mathcal{R}'},$$

respectively. Let  $\mathcal{H}$  and  $\mathcal{K}$  be infinite dimensional Hilbert spaces, while  $\mathcal{B}(\mathcal{H})$  and  $\mathcal{B}(\mathcal{K})$  denote the algebras of all linear bounded operators on  $\mathcal{H}$  and  $\mathcal{K}$ , respectively. In [8], authors characterized some forms of unital surjective maps on  $B(X)$  preserving the nonzero idempotency of product of operators in both directions. Also in [15], authors characterized some forms of linear surjective maps on  $B(X)$  preserving the nonzero idempotency of either products of operators or triple Jordan products of operators.

In this paper, we determine a form of additive mapping  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$  such that the range of  $\phi$  contains all minimal idempotents and  $I$  and also  $\phi$  preserves the nonzero idempotency of Jordan products of operators in both directions. Moreover, we determine a form of surjective additive mapping  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$  that preserves the nonzero idempotency of usual products of operators in both directions. Our main result are as follows.

**Theorem 1.1.** *Let  $\mathcal{H}$  and  $\mathcal{K}$  be two infinite dimensional real or complex Hilbert spaces and  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$  be an additive map such that the range of  $\phi$  contains all minimal idempotents and  $I$ . If  $\phi$  preserves the nonzero idempotency of Jordan products of operators in both directions, then  $\phi$  either annihilates minimal idempotents or there exists a bounded linear or conjugate linear bijection  $A : \mathcal{H} \rightarrow \mathcal{K}$  such that  $\phi(T) = \xi ATA^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$  or  $\phi(T) = \xi AT^t A^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$ , where  $\xi = \pm 1$  ( in the case that  $\mathcal{H}$  and  $\mathcal{K}$  are real,  $A$  is linear).*

**Theorem 1.2.** *Let  $\mathcal{H}$  and  $\mathcal{K}$  be two infinite dimensional complex Hilbert spaces and  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$  be a surjective additive map. If  $\phi$  preserves the nonzero idempotency of products of operators in both directions, then there exists a bounded linear or conjugate linear bijection  $A : \mathcal{H} \rightarrow \mathcal{K}$  such that  $\phi(T) = \xi ATA^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$  or  $\phi(T) = \xi AT^t A^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$ , where  $\xi = \pm 1$ .*

## 2. PROOFS

In this section we prove our results. First we recall some notations. Let  $X$  and  $Y$  be Banach spaces. Recall that a standard operator algebra on  $X$  is a norm closed subalgebra of  $B(X)$  which contains the identity and all finite rank

operators. Denote the set of all idempotent operators of  $\mathcal{B}(\mathcal{H})$  by  $\mathcal{I}(\mathcal{H})$  and the Jordan product of  $A, B$  by  $A \circ B = \frac{1}{2}(AB + BA)$ . Also denote the dual space  $X$  by  $X^*$ .

For every nonzero  $x \in X$  and  $f \in X^*$ , the symbol  $x \otimes f$  stands for the rank one linear operator on  $X$  defined by

$$(x \otimes f)y = f(y)x. \quad (y \in X)$$

If  $x, y \in \mathcal{H}$ , then  $x \otimes y$  stands for the rank one linear operator on  $\mathcal{H}$  defined by

$$(x \otimes y)z = \langle z, y \rangle x \quad (z \in \mathcal{H})$$

where  $\langle z, y \rangle$  denotes the inner product of  $z$  and  $y$ . We need some lemmas to prove our main result. Throughout this paper,  $\mathcal{A} \subseteq B(X)$  and  $\mathcal{B} \subseteq B(Y)$  are standard operator algebras.

The proof of the following lemma is similar to that of Lemma 2.2 in [15].

**Lemma 2.1.** [15] *Let  $\phi : \mathcal{A} \rightarrow \mathcal{B}$  be an additive map such that preserves the nonzero idempotency of Jordan products of operators. If  $N \in \mathcal{A}$  is a finite rank operator such that  $N^2 = 0$ , then  $\phi(N)^4 = 0$ .*

**Lemma 2.2.** *Let  $\phi : \mathcal{A} \rightarrow \mathcal{B}$  be an additive map. Then the following statements hold.*

(i) *If  $\phi$  preserves the nonzero idempotency of Jordan products of operators, then  $\phi$  is injective.*

(ii) *If  $I \in \text{rng} \phi$  and  $\phi$  preserves the nonzero idempotency of Jordan products of operators in both directions, then  $\phi(I) = I$  or  $\phi(I) = -I$ .*

*Proof.* (i) Assume  $\phi(A) = 0$ . We assert that  $A$  satisfies a quadratic polynomial equation. Otherwise, by the discussion in [11], there exists an  $x \in X$  such that  $x, Ax$  and  $A^2x$  are linear independent. Then there is a linear functional  $f$  such that  $f(x) = f(A^2x) = 0$  and  $f(Ax) = 2$ , because  $\dim X \geq 3$ . Setting  $B = x \otimes f$ , we have  $A \circ B \in P_{\mathcal{A}} \setminus \{0\}$ , implying that

$$\phi(A) \circ \phi(B) \in P_{\mathcal{B}} \setminus \{0\}.$$

This is a contradiction, because  $\phi(A) \circ \phi(B) = 0$ . So by the discussion in [11],  $A$  satisfies a quadratic polynomial equation.

Assume on the contrary that  $A$  is a nonzero operator. For any  $B \in \mathcal{A}$  we have

$$\phi(A) \circ \phi(B) = 0.$$

However, there exists  $B = x \otimes f$  such that  $A \circ B = \frac{1}{2}Ax \otimes f + \frac{1}{2}x \otimes fA$  is a nonzero idempotent, a contradiction. We construct such  $B$ .

By the above assertion,  $A$  satisfies a quadratic polynomial equation. The spectrum of such  $A$  consists only of eigenvalues. If  $A^2 \neq 0$ , then  $A$  has a nonzero eigenvalue  $\lambda$ , because in this case there exist  $r, s \in \mathbb{C}$  such that  $rs \neq 0$  and  $\lambda$  satisfies a quadratic polynomial equation  $\alpha^2 = r\alpha + s$ . Since  $rs \neq 0$ ,

$\alpha^2 = r\alpha + s$  has a nonzero root. Let  $x$  be its eigenvector. Choose a bounded functional  $f$  with  $f(x) = \frac{1}{\lambda}$  to form  $B = x \otimes f$  with the desired properties.

The remaining case is  $A^2 = 0$ . Since  $A$  is nonzero, we can find a vector  $x$  so that  $Ax \neq 0$  and a functional  $f$  with  $f(x) = 0$  and  $f(Ax) = 1$  to form  $B = x \otimes f$  with the desired properties. The proof is complete.

(ii) Since  $I \in \text{rng}\phi$ , there exists a nonzero operator  $U \in \mathcal{A}$  such that  $\phi(U) = I$ . We show that  $U = I$  or  $-I$ . We have  $\phi(U) \circ \phi(U) \in \mathcal{P}_{\mathcal{B}} \setminus \{0\}$ . Hence we obtain

$$(1) \quad U^2 = U \circ U \in \mathcal{P}_{\mathcal{A}} \setminus \{0\}.$$

This implies that for any  $x \in X$ ,  $x$ ,  $Ux$  and  $U^2x$  are linear dependent. Thus  $U$  satisfies a quadratic polynomial equation, by the discussion in [10]. This together with (1) yields that there exist  $a, b \in \mathbb{C}$  such that we have

$$(2) \quad U^2 = aU + bI.$$

From (1) and (2), we obtain the answers  $(0, 1)$ ,  $(1, 0)$  and  $(-1, 0)$  for  $(a, b)$  which imply that  $U^2 = I$ ,  $U^2 = U$  and  $U^2 = -U$ .

Let  $U^2 = U$ . We assert that  $U = I$ . Assume on the contrary that  $U \neq I$ . From  $U \circ I \in \mathcal{P}_{\mathcal{A}} \setminus \{0\}$ , we obtain

$$(3) \quad \phi(I) = I \circ \phi(I) \in \mathcal{P}_{\mathcal{B}} \setminus \{0\}.$$

On the other hand, there exists an idempotent operator  $T$  such that  $U - T$  is not idempotent. In fact,  $U + S - I$  isn't idempotent, where  $S = I - T$ . Thus

$$(U + S - I) \circ I = U + S - I \notin \mathcal{P}_{\mathcal{A}} \setminus \{0\}$$

which implies that

$$\phi(U + S - I) \circ \phi(I) \notin \mathcal{P}_{\mathcal{B}} \setminus \{0\}$$

This together with (3) yields that

$$\phi(I) \circ \phi(S) \notin \mathcal{P}_{\mathcal{B}} \setminus \{0\}$$

which implies that

$$S = I \circ S \notin \mathcal{P}_{\mathcal{A}} \setminus \{0\}.$$

This is a contradiction, because  $S$  is idempotent. So the proof of assertion is completed.

With a similar proof, the assumption  $U^2 = -U$  yields that  $U = -I$ .

Now let  $U^2 = I$ . We assert that  $U$  is a multiple of  $I$ . Assume on the contrary that  $U$  is a non-scalar operator. Since  $I$  and  $U$  are linear independent, there is a nonzero vector  $x \in X$  such that  $x$  and  $Ux$  are linear independent. Hence there exists  $f \in X^*$  such that  $f(x) = 0$  and  $f(Ux) = 2$ . Setting  $B = x \otimes f$ , we obtain

$$U \circ B \in \mathcal{P}_{\mathcal{A}} \setminus \{0\}$$

which implies that

$$\phi(B) = \phi(U) \circ \phi(B) \in \mathcal{P}_{\mathcal{B}} \setminus \{0\}.$$

This is a contradiction, because  $B$  is a nilpotent such that  $B^2 = 0$  and so by Lemma 2.1,  $\phi(B)$  is a nilpotent operator. So the proof of assertion is completed. By the proved assertion, there exists a nonzero complex number  $\lambda$  such that  $U = \lambda I$ . Since  $U^2 = I$ , we obtain  $\lambda^2 = 1$  and this completes the proof.  $\square$

**Theorem 2.3.** [5] *Let  $\mathcal{H}$  and  $\mathcal{K}$  be two infinite dimensional real or complex Hilbert spaces and  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$  be an additive map preserving idempotents. Suppose that the range of  $\phi$  contains all minimal idempotents. Then  $\phi$  either annihilates minimal idempotents or there exists a bounded linear or conjugate linear bijection  $A : \mathcal{H} \rightarrow \mathcal{K}$  such that  $\phi(T) = ATA^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$  or  $\phi(T) = AT^tA^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$  ( in the case that  $\mathcal{H}$  and  $\mathcal{K}$  are real,  $A$  is linear).*

**Proof of Theorem 1.1.** Since by Lemma 2.2,  $\phi(I) = I$  or  $\phi(I) = -I$ , from  $P = I \circ P \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$  we obtain that  $\phi(P)$  or  $-\phi(P)$  belongs to  $\mathcal{I}(\mathcal{H}) \setminus \{0\}$ . This together with  $\phi(0) = 0$  implies that  $\phi$  or  $-\phi$  preserves the idempotent operators in both directions. Hence the forms of  $\phi$  follows from Theorem 2.3.

**Proposition 2.4.** *Let  $\dim \mathcal{H} \geq 3$ . Let  $A$  be an arbitrary operator of  $\mathcal{B}(\mathcal{H})$  and  $P$  be a rank one idempotent operator. Then  $A \in \mathbb{C}^*P$  if and only if for every  $T \in \mathcal{B}(\mathcal{H})$  such that  $PT \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$  we have  $AT \notin \mathcal{I}(\mathcal{H}) \setminus \{0\}$ , where  $\mathbb{C}^* = \mathbb{C} \setminus \{0, 1\}$ .*

*Proof.* If  $A \in \mathbb{C}^*P$ , then there exists a  $\lambda \in \mathbb{C}^*$  such that  $A = \lambda P$ . hence it is trivial that for every  $T$  such that  $PT \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$  then  $\lambda PT \notin \mathcal{I}(\mathcal{H}) \setminus \{0\}$ .

Conversely, Let  $A \notin \mathbb{C}^*P$ . Since  $P$  is rank one, by [2], there exists either an  $x \in \mathcal{H}$  such that  $Px$  and  $Ax$  are linear independent or an  $x \in \mathcal{H}$  and linear independent vectors  $z_1, z_2 \in \mathcal{H}$  such that  $P = x \otimes z_1$  and  $A = x \otimes z_2$ .

If  $Px$  and  $Ax$  are linear independent, then there exists  $y \in \mathcal{H}$  such that  $\langle Px, y \rangle = \langle Ax, y \rangle = 1$ , because  $\dim \mathcal{H} \geq 3$ . Setting  $T = x \otimes y$  follows that  $PT \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$  and also  $AT \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$ . This is a contradiction.

If  $P = x \otimes z_1$  and  $A = x \otimes z_2$ , then there exist  $y, z_3 \in \mathcal{H}$  such that  $\langle x, z_3 \rangle = \langle y, z_1 \rangle = \langle y, z_2 \rangle = 1$ . Setting  $T = y \otimes z_3$  follows that  $PT \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$  and also  $AT \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$ . This is a contradiction.

These contradictions yields that  $A \in \mathbb{C}^*P$  and this completes the proof.  $\square$

**Lemma 2.5.** *Let  $\phi : \mathcal{A} \rightarrow \mathcal{B}$  be a surjective additive map such that*

$$AB \in \mathcal{P}_{\mathcal{A}} \setminus \{0\} \Leftrightarrow \phi(A)\phi(B) \in \mathcal{P}_{\mathcal{B}} \setminus \{0\}$$

*for every  $A \in \mathcal{A}$  and  $B \in \mathcal{B}$ . Then the following statements hold.*

- (i)  $\phi(I) = I$  or  $\phi(I) = -I$ .

(ii) If  $\mathcal{A} = \mathcal{B}(\mathcal{H})$  with  $\dim \mathcal{H} \geq 3$  and  $\mathcal{B} = \mathcal{B}(\mathcal{K})$ , then  $\phi(\mathbb{C}P) \subseteq \mathbb{C}\phi(P)$ , for every rank one idempotent  $P$ .

*Proof.* (i) It is proved by using Lemma 2.1 and similar to the proof of Lemma 2.2 in [13].

(ii) By (i),  $\phi(I) = I$  or  $\phi(I) = -I$ . Since  $\phi(0) = 0$ , we can conclude from (i) that  $\phi$  or  $-\phi$  preserves the idempotent operators in both directions. By Lemma 2.6 in [14],  $\phi$  or  $-\phi$  preserves the rank one idempotent operators in both directions. If  $A \in \mathbb{C}^*P$ , then by Proposition 2.4, for every  $T \in \mathcal{B}(\mathcal{H})$  such that  $PT \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$  we have  $AT \notin \mathcal{I}(\mathcal{H}) \setminus \{0\}$  which by surjectivity of  $\phi$  imply that for every  $T' \in \mathcal{B}(\mathcal{H})$  such that  $\phi(P)T' \in \mathcal{I}(\mathcal{H}) \setminus \{0\}$  we have  $\phi(A)T' \notin \mathcal{I}(\mathcal{H}) \setminus \{0\}$ . Since  $\phi(P)$  is a rank one idempotent, by Proposition 2.4 we can conclude that  $\phi(A) \in \mathbb{C}^*\phi(P)$ . This together with (i) and  $\phi(0) = 0$  follows that  $\phi(\mathbb{C}P) \subseteq \mathbb{C}\phi(P)$ . This completes the proof.  $\square$

**Proposition 2.6.** *Let  $\mathcal{H}$  and  $\mathcal{K}$  be two infinite dimensional real or complex Hilbert spaces and  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$  be an additive map. If  $\phi$  preserves the idempotent operators, then  $\phi$  preserves the square zero operators.*

*Proof.* Let  $N \in \mathcal{B}(\mathcal{H})$  be a square zero operator. Then we have  $\mathcal{H} = \ker N \oplus M$  for some closed subspace  $M$  of  $\mathcal{H}$ . Thus by this decomposition  $N$  has the following operator matrix

$$N = \begin{pmatrix} 0 & N_1 \\ 0 & 0 \end{pmatrix}.$$

If

$$A = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

then  $A+nN \in \mathcal{I}(\mathcal{H})$  for every natural number  $n$ . It implies that  $\phi(A)+n\phi(N) \in \mathcal{I}(\mathcal{K})$  for every natural number  $n$ . That is,

$$\phi(A) + n\phi(N) = \phi(A)^2 + n(\phi(A)\phi(N) + \phi(N)\phi(A)) + n^2\phi(N)^2$$

for all  $n$ . Setting  $n = 1$  and  $n = 2$  yield

$$\phi(N) = \phi(A)\phi(N) + \phi(N)\phi(A) + \phi(N)^2,$$

$$2\phi(N) = 2(\phi(A)\phi(N) + \phi(N)\phi(A)) + 4\phi(N)^2$$

which imply that  $\phi(N)^2 = 0$  and this completes the proof.  $\square$

**Theorem 2.7.** [1] *Let  $\mathcal{H}$  and  $\mathcal{K}$  be two infinite dimensional complex Hilbert spaces and  $\phi : \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{K})$  be a surjective additive map such that  $\phi(\mathbb{C}P) \subseteq \mathbb{C}\phi(P)$  holds for every rank one operator  $P$ . Then  $\phi$  preserves square zero in both directions if and only if there exists a nonzero scalar  $c$  and a bounded linear or conjugate linear bijection  $A : \mathcal{H} \rightarrow \mathcal{K}$  such that  $\phi(T) = cATA^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$  or  $\phi(T) = cAT^tA^{-1}$  for every  $T \in \mathcal{B}(\mathcal{H})$ .*

**Proof of Theorem 1.2.** By a similar proof to that of Lemma 2.3 in [15], we obtain that  $\phi$  is injective. Since  $\phi(0) = 0$ , we can conclude from part (i) of Lemma 2.5 that  $\phi$  or  $-\phi$  preserves the idempotent operators in both directions. This together with Proposition 2.6 and the injectivity of  $\phi$  implies that  $\phi$  preserves the square zero operators in both directions. Moreover, by part (i) of Lemma 2.5,  $\phi(\mathbb{C}P) \subseteq \mathbb{C}\phi(P)$ , for every rank one idempotent  $P$ . Therefore the forms of  $\phi$  follow from Theorem 2.7. The scalar  $c$  in Theorem 2.7 is the scalar that  $\phi(I) = cI$  (by the proof of this theorem in [2]). This together with the part (i) of Lemma 2.5 completes the proof.

#### ACKNOWLEDGMENTS

This research is partially supported by the Research Center in Algebraic Hyperstructures and Fuzzy Mathematics, University of Mazandaran, Babolsar, Iran.

#### REFERENCES

1. A. Armandnejad, H. R. Afshin, Linear functions preserving multivariate and directional majorization, *Iranian Journal of Mathematical Sciences and Informatics*, **5**(1), (2010), 1-5.
2. Z. Bai, J. Hou, Linear maps and additive maps that preserve operators annihilated by a polynomial, *J. Math. Anal. Appl.*, **271**, (2002), 139154.
3. M. Brešar, P. Šemrl, On locally linearly dependent operators and derivation, *Transactions of the American Mathematical Society*, **351**, (1999), 1257-1275.
4. M.A. Chebotar, W.-F. Ke, P.-H. Lee, N.-C. Wong, Mappings preserving zero products, *Studia Math.*, **155**, (2003), 77-94.
5. J. Cui, Additive drazin inverse preservers, *Linear Algebra Appl.*, **155**, (2007), 77-94.
6. M. Dobovišek, B. Kuzma, G. Lešnjak, C. K. Li, T. Petek, Mappings that preserve pairs of operators with zero triple Jordan Product, *Linear Algebra Appl.*, **426**, (2007), 255-279.
7. L. Fang, G. Ji, Linear maps preserving products of positive or Hermitian matrices, *Linear Algebra Appl.*, **419**, (2006), 601-611.
8. L. Fang, G. Ji, Y. Pang, Maps preserving the idempotency of products of operators, *Linear Algebra Appl.*, **426**, (2007), 40-52.
9. J. Hou, Q. Di, Maps preserving numerical ranges of operator products, *Proc. Amer. Math. Soc.*, **134**, (2006), 1435-1446.
10. J. Hou, L. Zhao, Zero-product preserving additive maps on symmetric operator spaces and self-adjoint operator spaces, *Linear Algebra Appl.*, **399**, (2005), 235-244.
11. I. Kaplansky, *Infinite Abelian Groups*, University of Michigan Press, Ann Arbor, 1954.
12. F. Khalooei, A. Salemi, Linear Preservers of Majorization, *Iranian Journal of Mathematical Sciences and Informatics*, **6**(2), (2011), 43-50.
13. C. K. Li, P. Šemrl, N. S. Sze, Maps preserving the nilpotency of products of operators, *Linear Algebra Appl.*, **424**, (2007), 222-239.
14. A. Taghavi, R. Hosseinzadeh, Linear maps preserving idempotent operators, *Bulletin of the Korean Mathematical Society*, **47**, (2010), 787-792.
15. M. Wang, L. Fang, G. Ji, Y. Pang, Linear maps preserving idempotency of products or triple Jordan products of operators, *Linear Algebra Appl.*, **429**, (2008), 181-189.
16. L. Zhao, J. Hou, Jordan zero-product preserving additive maps on operator algebras, *J. Math. Anal. Appl.*, **314**, (2006), 689-700.

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فرض کنید  $B(K)$  و  $B(H)$  به ترتیب جبر تمام عملگرهای خطی کران دار روی فضاهای هیلبرت  $K$  و  $H$  باشند. در این مقاله فرم نگاشت‌های جمعی از  $B(H)$  به  $B(K)$  که خودتوانی ناصفر ضرب جردن یا ضرب معمولی عملگرها را از دو طرف حفظ می‌کنند، به دست می‌آید.

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