Implementation of Traditional (S-R)-Based PM Method with Bayesian Inference

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KEYWORDS

ABSTRACT

In order to perform Preventive Maintenance (PM), two approaches have evolved in the literature. The traditional approach is based on the use of statistical and reliability analysis of equipment failure. Under statistical-reliability (S-R)-based PM, the objective of achieving the minimum total cost is pursued by establishing fixed PM intervals, which are statistically optimal, at which a decision to replace or overhaul equipments or components is made. The second approach involves the use of sensor-based monitoring of equipment condition in order to predict occurrence of machine failure. Under condition-based (C-B) PM, intervals between PM works are no longer fixed, but are performed only "when needed". It is obvious that Condition Based Maintenance (CBM) needs an on-line inspection and monitoring system that causes CBM to be expensive. Whenever this cost is infeasible, we can develop other methods to improve the performance of traditional (S-R)-based PM method. In this research, the concept of Bayesian inference was used. The time between machine failures was observed, and Bayesian inference is employed in (S-R)-based PM, it is tried to determine the optimal checkpoints.

1. Introduction

Preventive maintenance (PM) considers repair, replacement decisions of equipment in order to avoid unexpected failure during production. The objective of any PM program is to minimize the total cost of inspection, repair, and equipment downtime measured in terms of increased production costs (Man et al., 1995).

In order to do PM, two approaches were developed. The traditional approach is based on the use of statistical analysis of time to failure. Under statistical-reliability (S-R)-based PM, the objective is to ascertain the minimum total cost by determining fixed PM intervals, which are statistically optimal, at which a decision to replace or overhaul equipments or components is made. The second approach involves the use of sensor-based monitoring of equipment condition in order to estimate when equipment failure will occur. Under condition-based (C-B) PM, intervals between PM actions are not fixed (Man et al., 1995).

The primary disadvantage of (S-R)-based PM is that the results of the calculations are based on the expected mean of variables. If the standard deviations of these means are large, thus the probability of ascertaining the correct maintenance interval is small. Other disadvantages include more emergency maintenance, more overtime, and less equipment utilization (Man et al., 1995).

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Paper first received April 10, 2012, and in accepted form Dec. 22, 2012.

Condition-Based Maintenance (CBM) is done in response to significant deterioration in equipment's condition as indicated by a shift in a monitored parameter. Predictive maintenance allows the machine to be taken off-line at a predetermined time, which allows total cost to be minimized (Saranga, 2002).

It has been proven that CBPM is an effective way to minimize maintenance costs, improve safety and reduce the frequency and severity of machine failures (Zhou et al., 2006; Mobley, 1989). CBM has been widely accepted in practice in the past few years since it considers maintenance decisions to be made based on the current state of the equipments. Thus it avoids unnecessary maintenance (Jardine et al., 1997). Many researches have been done in the field of maintenance.

Niaki and Fallahnezhad (2007) employed Bayesian inference and stochastic dynamic programming to design a decision-making framework in production environment. Further, Fallahnezhad et al. (2007) determined the optimal policy for two-machine replacement problem using Bayesian inference in the context of the finite mixture model. They discussed the analysis of time-to-failure data and proposed an optimal decision-making procedure for machine replacement strategy. Moreover, Fallahnezhad and Niaki (2010) proposed a dynamic programming model for two series machine replacement problem. Also Fallahnezhad and Niaki (2011) proposed a Markov model for two series machine replacement problem. Fallahnezhad et al. (2012) proposed a single machine replacement policy in a finite horizon model based on the quality of items produced. Also Fallahnezhad et al. (2014) proposed a single machine replacement policy in a finite horizon model based on hazard rate of the machine. Iravani and Duenyas (2002) considered integrated decisions of maintenance and production in a production system with a deteriorating machine.

Ivy and Nembhard (2005) integrated Statistical Quality Control (SQC) and partially observable Markov Decision Processes (POMDP) for maintenance decision making of deteriorating systems. Merrick et al (2003) presented a Bayesian semiparametric proportional hazards model to describe the failure behavior of machine tools. The semi parametric setup is introduced using a mixture of Dirichlet process prior. Giardoni and Colosimo (2007) considered a repairable system that operates under a maintenance strategy that calls for complete preventive repair actions at prescheduled times. Kuo and Yang (2000) presented A Bayesian approach that uses Gibbs sampling. Nair et al. (2001) presented some applications of Bayesian inference in reliability by Gibbs sampling.

Mazzuchi and Soyer (1996) proposed a Bayesian approach to machine replacement problem. Their objective function is:

$$TC(T) = \frac{C_f \int_0^T m(x)dx + C_P}{T} \qquad (1)$$

where m(x) is the hazard function of the time to failure. Clearly, they tried to minimize this objective function with respect to *T*. They have considered an increasing hazard rate for time between system's failures. Watson and Mason (2006) used this criteria along with Bayesian model for maintenance of water pipe networks. Nenes and Panagiotidou (2011) proposed Bayesian model for the joint optimization of quality and maintenance. In this research first it is considered that the maintenance is perfect. This assumption is reasonable where at each PM check point, a repair action with fixed cost C_P is executed, which instantly returns the system to a like-new condition (perfect PM).

It is obvious that CBM needs an on-line inspection and monitoring system that causes CBM to be expensive. When this cost is infeasible, it is desired to develop other methods to improve the performance of traditional (S-R)-based PM methods. In this research, the Bayesian inference is employed for monitoring of the system. However, observation was the time interval between machine failures. By combining this approach with (S-R)-based PM, it is tried to determine the optimal checkpoints and we show that this approach will have a significant difference with traditional (S-R)-based PM in results even when the large number of data is gathered. The paper aims to present a new method which implements Traditional (S-R)-Based PM with Bayesian Inference. The rest of the paper is organized as follows. Section 2 introduces a thorough review of the statistical-reliability (S-R) approach to PM and Bayesian Inference, we discuss mathematical computation of proposed approach and Bayesian inference. Section 3 presents a numerical example for comparing proposed approach with traditional (S-R)-based PM. Finally, the major conclusions of the study are summarized in Section 4.

2. The Statistical-Reliability (S-R) Approach to PM and Bayesian Inference

An example of a typical S-R PM model is for equipments that must be replaced when they reach a particular age and It is reasonable to do preventive maintenance only if the following conditions are satisfied: 1. new parts are better than old parts, 2. The cost of a preventive repair is less than the cost of a failure and repair cost.

The objective in this model is to minimize the average cost per unit time which corresponds to minimize the ratio of the expected cost per cycle and the expected cycle length (Mann et. al. 1995). The component is replaced or repaired at time T or at failure whichever is earlier, hence following objective function is concluded:

$$TC(T) = \frac{C_F \int_0^T f(t)dt + C_P \int_T^\infty f(t)dt}{\int_0^T tf(t)dt + T\int_T^\infty f(t)dt} \qquad (2)$$

where notations are defined as follows,

C_p: the cost of doing a planned preventive maintenance,

C_f: the cost of recovering from a failure,

$TC=$: the total maintenance cost per unit time,

$f(t)$: the probability density function of the time to failure. In the rest of the paper, two cases have been considered. In the first case, it is assumed that time to failure has a constant hazard rate and in the second case, it is assumed that time to failure has an increasing hazard rate

2-1. Constant Hazard Rate

An application of proposed approach is illustrated by specifying the distribution of the time to first failure as exponential with hazard rate λ.

Let t_i denote the time between $(i\text{-}1)^{\text{th}}$ and i^{th} failure in the cycle and R denotes the number of the failures, to use a non-informative prior by assuming that parameters of Gamma distribution converge to zero, i.e., the prior distribution of λ is *Gamma (0,0)*. Then, using Bayesian inference, the posterior distribution of

λ *is also Gamma* with parameters of R and $\sum_{i=1}^{R} t_i$

(Nair et al. 2001). In other words:

$$g\left(\lambda \mid i : t_i\right) \sim \Gamma(R, \sum_{i=1}^{R} t_i) \tag{3}$$

Then, using Bayesian inference again, the probability distribution function of time between failures, *f(t)*, is determined as follows :

$$f\left(t\right) = \int_0^\infty f\left(t \mid \lambda\right) g\left(\lambda\right) d\lambda =$$

$$\int_0^\infty \frac{\lambda^R e^{\left(-\lambda\left(\sum_{i=1}^{R} t_i + t\right)\right)} \left(\sum_{i=1}^{R} t_i\right)^R}{\Gamma(R)} d\lambda = \frac{R\left(\sum_{i=1}^{R} t_i\right)^R}{\left(\sum_{i=1}^{R} t_i + t\right)^{R+1}} \tag{4}$$

For evaluating the function $TC\left(T\right)$, first it is needed to calculate the integrals $\int_0^T tf\left(t\right) dt$ and $\int_0^T f\left(t\right) dt$.

Using the probability distribution function, $f\left(t\right)$ that is determine in equation (4), followings are concluded,

$$\int_0^T tf\left(t\right) dt = -T \frac{\left(\sum_{i=1}^{R} t_i\right)^R}{\left(\sum_{i=1}^{R} t_i + x\right)^R} +$$

$$\frac{1}{(1-R)}\left(\frac{\left(\sum_{i=1}^{R} t_i\right)^R}{\left(\sum_{i=1}^{R} t_i + T\right)^{R-1}} - \left(\sum_{i=1}^{R} t_i\right)\right) \tag{5}$$

Also,

$$\int_0^T f\left(t\right) dt = 1 - \frac{\left(\sum_{i=1}^{R} t_i\right)^R}{\left(\sum_{i=1}^{R} t_i + T\right)^R} \tag{6}$$

Now the optimal value of T in function (2) is determined by minimizing the function $TC\left(T\right)$. By differentiating $TC\left(T\right)$ with respect to T and setting it to zero, we get:

$$\frac{\partial TC\left(T\right)}{\partial T} = 0 \tag{7}$$

Assuming W and V is defined as follows,

$$W = \int_0^T tf\left(t\right) dt + T \int_T^\infty f\left(t\right) dt$$

$$V = C_F \int_0^T f\left(t\right) dt + C_P \int_T^\infty f\left(t\right) dt \tag{8}$$

Thus following is concluded,

$$\frac{Wf\left(T^*\right)\left(C_F - C_P\right) - V\left(\int_{T^*}^\infty f\left(t\right) dt\right)}{\left(\int_0^T tf\left(t\right) dt + T \int_T^\infty f\left(t\right) dt\right)^2} = 0 \tag{9}$$

Thus,

$$Wf\left(T^*\right)\left(C_F - C_P\right) = V \int_{T^*}^\infty f\left(t\right) dt \Rightarrow$$

$$\frac{f\left(T^*\right)}{\int_{T^*}^\infty f\left(t\right) dt} = \frac{V}{W\left(C_F - C_P\right)} \tag{10}$$

It is obvious that $\dfrac{f\left(T^*\right)}{\int_{T^*}^\infty f\left(t\right) dt}$ denotes the hazard rate that is equal to $\left(\dfrac{R}{\sum_{i=1}^{R} t_i + T^*}\right)$. Consequently, following is concluded,

$$W \frac{R}{V}\left(C_F - C_P\right) = \left(\sum_{i=1}^{R} t_i + T^*\right) \tag{11}$$

From above equation, T^* is determined.

2-2. Increasing Hazard Rate

Assuming that time to failure has an increasing hazard rate; first following assumptions are made,

1. We illustrate an application of our approach by specifying the distribution of the time to failure as weibull distribution. This means that:

$$F\left(t\left|\alpha,\beta\right.\right)=1-\exp\left(-\alpha t^{\beta}\right),$$
$$m\left(t\left|\alpha,\beta\right.\right)=\alpha\beta t^{\beta-1} \tag{12}$$

2. We select, as an appropriate prior for α, the Gamma distribution given by:

$$h\left(\alpha\right)=\frac{b^{a}}{\Gamma\left(a\right)}\alpha^{a-1}e^{-b\alpha}\approx Gamma\left(a,b\right) \tag{13}$$

3. For prior distribution of the shape parameter, β, it is convenient to define a discrete distribution by using a discretization of the Beta density on $\left(\beta_{L},\beta_{U}\right)$ since this allows for great flexibility in representing prior uncertainty. The beta density is given by:

$$g\left(\beta\right)=\frac{\Gamma\left(c+d\right)}{\Gamma\left(c\right)\Gamma\left(d\right)}\frac{\left(\beta-\beta_{L}\right)^{c-1}\left(\beta_{U}-\beta\right)^{d-1}}{\left(\beta_{U}-\beta_{L}\right)^{c+d-1}} \tag{14}$$
for $0\le\beta_{L}\le\beta\le\beta_{U}$

where $\beta_{U},\beta_{L},c,d>0$ are specified parameters. Discretized Probability distribution for β is defined as follows:

$$P_{l}=\Pr\left\{\beta=\beta_{l}\right\}=\int_{\beta_{l}-\delta/2}^{\beta_{l}+\delta/2}g\left(\beta\right)d\beta \tag{15}$$

where,

$$\beta_{l}=\beta_{L}+\frac{2l-1}{2}\delta$$
$$\delta=\frac{\beta_{U}-\beta_{L}}{k}\text{ for l=1,2,...,k} \tag{16}$$

4. if t_{i} is the between production of defective products, then with the method of likelihood, we obtain the following posterior results (Mazzuchi, and Soyer 1996)

$$h\left(\overline{\alpha}\left|\beta_{l},t_{1},t_{2},...,t_{n}\right.\right)=Gamma\left(a^{*},b^{*}\right)$$
$$a^{*}=a+n,\quad b^{*}=b+\sum_{i=1}^{n}t_{i}^{\beta_{l}}+\left(t_{B}-\sum_{i=1}^{n}t_{i}\right)^{\beta_{l}}, \tag{17}$$

where t_{B} is the current time, also

$$\Pr\left\{\overline{\beta}_{l}\left|t_{1},t_{2},...,t_{n}\right.\right\}=$$
$$\frac{\beta_{l}^{n}\left\{\prod_{j=1}^{n}t_{j}\right\}^{\beta_{l}-1}\left/\left(b^{*}\right)^{a^{*}}\right.}{\sum_{i=1}^{k}P_{i}\beta_{i}^{n}\left\{\prod_{j=1}^{n}t_{j}\right\}^{\beta_{i}-1}\left/\left(b^{*}\right)^{a^{*}}\right.}P_{l} \tag{18}$$

It is assumed that the posterior state of variable κ is shown with $\overline{\kappa}$.

Like previous model, for evaluating function $TC\left(T\right)$, it is needed to determine the integrals $\int_{0}^{T}tf\left(t\right)dt$ and $\int_{0}^{T}f\left(t\right)dt$.

Thus first $f\left(t\right)$ is determine by conditional probability as follows,

$$f\left(t\right)=\sum_{i=1}^{k}\Pr\left\{\overline{\beta}_{i}\right\}\int_{0}^{\infty}f\left(t\left|\overline{\alpha},\overline{\beta}_{i}\right.\right)h\left(\overline{\alpha}\right)d\alpha dt$$

Thus we have,

$$\int_{0}^{T}tf\left(t\right)dt=$$
$$\sum_{i=1}^{k}\Pr\left\{\overline{\beta}_{i}\right\}\int_{0}^{T^{*}}\int_{0}^{\infty}\left(\alpha\beta_{i}t^{\beta_{i}}e^{-\alpha t^{\beta_{i}}}\frac{b^{*a^{*}}}{\Gamma\left(a^{*}\right)}\alpha^{a^{*}-1}e^{-b^{*}\alpha}\right)d\alpha dt=$$
$$\sum_{i=1}^{k}\Pr\left\{\overline{\beta}_{i}\right\}\int_{0}^{T^{*}}\int_{0}^{\infty}\left(\beta_{i}t^{\beta_{i}}\frac{b^{*a^{*}}}{\Gamma\left(a^{*}\right)}\alpha^{a^{*}}e^{-\left(b^{*}+t^{\beta_{i}}\right)\alpha}\right)d\alpha dt= \tag{20}$$
$$\sum_{i=1}^{k}\left(\Pr\left\{\overline{\beta}_{i}\right\}\int_{0}^{T^{*}}\beta_{i}t^{\beta_{i}}\frac{a^{*}b^{*a^{*}}}{\left(t^{\beta_{i}}+b^{*}\right)^{a^{*}+1}}dt\right)$$

Also $\int_{0}^{T}f\left(t\right)dt$ is determined as follows,

$$\int_{0}^{T}f\left(t\right)dt=\sum_{i=1}^{k}\Pr\left\{\overline{\beta}_{i}\right\}\left(1-\left(\frac{b^{*}}{T^{*\beta_{i}}+b^{*}}\right)^{a^{*}}\right) \tag{21}$$

Regarding above results, following is concluded,

$$\frac{\partial TC\left(T\right)}{\partial T}=0\Rightarrow$$
$$\frac{W}{V}\sum_{i=1}^{k}\left(\Pr\left\{\overline{\beta}_{i}\right\}\beta_{i}T^{*\beta_{i}-1}\frac{a^{*}b^{*a^{*}}}{\left(T^{*\beta_{i}}+b^{*}\right)^{a^{*}+1}}\right)\left(C_{F}-C_{P}\right) \tag{22}$$
$$=\int_{T^{*}}^{\infty}f\left(t\right)dt=\left(\sum_{i=1}^{k}\Pr\left\{\overline{\beta}_{i}\right\}\left(\left(\frac{b^{*}}{T^{*\beta_{i}}+b^{*}}\right)^{a^{*}}\right)\right)$$

From the above equation, T^* can be determined.

3. Numerical example

In the first numerical example, it is assumed that time to failure has a constant hazard rate. Assume that mean time to failure is equal to 1 for a special machine and time to failure follows exponential distribution. Therefore the probability density function of time to failure is defined as:

$$f(t) = e^{-t} \qquad (23)$$

Assuming C_F=76 and C_P=8, using the traditional approach to minimize the objective function (2), following is concluded:

$$\frac{W}{V} = \frac{1}{68} \Rightarrow T^* = 2.833 \qquad (24)$$

Data has been generated from an exponential distribution with mean 1. The total number of this data is 77.

To find the optimal value of T in objective function (2), a search procedure is applied and it is seen that $T^* = 1.93$ is an optimal solution where the value of objective function is 74.7426 in this case. in the traditional approach (traditional (S-R)-based PM), the value of objective function is 76.90 where $T^* = 2.83$. In the other words, Bayesian approach results in tighter intervals for inspection (0.9 hours difference between the checkpoints of these two approaches) that causes less average cost.

Also, with increasing the number of observations $(77 \rightarrow 139)$, after the search procedure, it is seen that the optimal value of T^* becomes 2.01. since there is still a substantial difference between this approach $(T^* = 2.01)$ and traditional method $(T^* = 2.83)$ (difference between these two checkpoints is 0.82) and also in general, the data of system's failures in the past time is limited, therefore proposed approach is more applied because even when the number of observations is large but there is still a substantial difference between these two approaches. Hence for real situations, when sufficient data is not at hand, this approach can be more effective.

In the second numerical example, it is assumed that time to failure has an increasing hazard rate. It is assumed that time to failure follows a weibull distribution with parameters α and β. The values of β has been generated from the standard beta distribution with parameter *c=2* and *d=1* and the values of α have been generated from an exponential distribution with mean 1.

Also in this case by a search procedure, we could find $T^* = 0.54$ as an optimal solution (the value of objective function is 3.96) but with traditional approach (traditional (S-R)-based PM), the value of objective function is 4.58 where $T^* = 0.95$, in the other words, Bayesian approach causes tighter intervals of inspection (0.41 difference between the checkpoints of these two approaches) that causes less average cost and it means that Bayesian inference makes the decisions more realistic and applied.

4. Conclusion

In this research, Bayesian inference is applied to determine the checkpoint of PM. since the results of the proposed method and traditional (S-R)-based PM are different, hence it is concluded that combining Bayesian approach and traditional (S-R)-based PM can make PM approach more applicable and sensitive. By considering the past performance of machine in the proposed method, decisions will be more confident. Also in numerical example, it has been shown that, however, data comes from an exponential distribution but the results of traditional (S-R)-based PM and proposed approach are substantially different and also when data comes from a weibull distribution with increasing hazard rate , checkpoints of Bayesian inference are substantially different from traditional (S-R)-based PM.

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