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Numerical Solution for Non-Uniform Fractional Advection Dispersion Equation

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Introduction: Solute transport modeling in water bodies and especially rives, provides a useful tools for decision-makers and consumers, to take a timely decisions in order to prevent or avoid any catastrophic consequences. Many factors such as velocity, cross-section area, bedform and etc. affect the transport of any solute particle in the media. The aforementioned factors vary both in width and depth, which cause some of the particles to deviate from the main channel to areas in which the particle stays for a longer time. Those particles are released to the main channel gradually causing a heavy-tailed breakthrough curve in downstream of the injection point. The areas where solute spends more time than the main channel is called deadzones. Any area in the channel that reduce the velocity field to relatively zero could be categorized as deadzones, such as areas behind a boulder, in pocket beside the river, behind or inside a vegetation cluster. In streams containing deadzones the classical adevection-dispersion equation (ADE) cannot meet the requirements. Scholars proposed many different approaches to capture the heavy-tailed behavior. One of the renowned and novel approaches to quantify the role of deadzones in natural streams is using Fractional Calculus. This field of mathematics helps to change the fundamental assumption of ADE, which is laid upon "central limit theorem" and subsequently generate a new equation based on fractional partial derivatives, The Fractional Advection –Dispersion Equation (FADE). The FADE provides a better description of solute movement in natural streams, because of its general form and non-locality.

Materials and Methods: In this research, FADE for a non-uniform but steady condition in natural streams has been investigated. FADE is a product of central limit theorem generalization and hence can capture the anomalous behavior of solute transport in streams containing deadzones. One of the important points that can be inferred from tracer study in such streams is that the concentration of a point is influenced by the point in far upstream; that is either the solute particle comes faster than the main cloud or slower and defy the basic assumption. Since classic ADE just considers the points in the closest proximity of the study point to calculate the bearkthrough curve (BTC), it cannot be used in cases that the deadzones are involved; which calls for a more comprehensive consideration of the stream. Interestingly, factional derivatives are non-local; fractional derivatives consider the state of the function in its whole domain. Hence, FADE can accurately describe and calculate the BTC in streams affected by deadzones. The big difference between FADE and classic ADE is that, the former is based on Levy motion while the later developed and shows Brownian motion. Also, the order of spatial derivative in FADE is a value $1 < \alpha \le 2$, unlike the classic ADE in which the order is fixed to 2. To investigate the FADE in depth standard Grunwald definition of fractional derivatives were employed for discretization, and an algorithm is proposed. The more the order of the fractional partial derivative decreases, the more would be the effect of the deadzones, this means that while the order is equal to 2, the streamflow do not experience any deadzones in its way. In order to fit the data, two variables in the FADE needs to be calibrated; the unknown parameters were estimated using Particle Swarm Optimization (PSO) technique.

Conclusion: The model was validated using a set of data from a small stream. The comparison shows that the model can accurately recreate the BTC and perform dramatically better than classic ADE which even in some stations is incapable of predicting the peak concentration and the arrival time of the particle to the station. Quantitatively, the difference between the FADE and observation represented by RMSE in average is 0.236 while for classic ADE is around 4 times higher. The difference in the first two stations is negligible for the stream does not contain any deadzones. This shows that not only the FADE is capable of considering deadzones, but also if needed it can easily reduce to classic ADE. This flexibility offers vast opportunity for application. In addition, the sensitivity of the model to the calibration parameter was analyzed, the analysis shows that both variables (order of the derivative and skewness parameter) should be treated with great care, since a little

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variation in their value drastically alter the shape of the BTC, and decrease the accuracy of the prediction. In this research, the capability of FADE were investigated and proved that this model performing far better in predicting solute transport in natural streams in which the variability of the morphology may hinder the solutes movement.

Keywords: Deadzones, Fractional advection dispersion equation, Numerical solution, Pollutant transport, Stream