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Application of Archimedean and Extreme values Copula Functions for Multivariate Analysis of Low Flows in Dez Basin

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Introduction: Hydrological phenomena are often multidimensional and very complex. Hence, the joint modeling of two or more random variables is required to investigate the probabilistic behavior of them. To this aim, the copulas can be efficiently utilized to derive multivariate distributions. In addition, the copula functions can quantify the dependence structure between correlated random variables. Estimation of low flow is necessary in different fields of hydrological studies such as water quality management, determination of minimum required flow at downstream for producing electricity and cooling purposes, design of intakes, aquaculture, design of irrigation systems and assessing the effect of long-term droughts on ecosystems. Low flows can be determined based on low flow indices. There are many types of low flow indices which among them the 7-days low flow with different return periods are more popular. Heretofore, numerous studies have been performed in the field of univariate analysis of river low flows, but the low flows of two river branches can be simultaneously analyzed using copula functions. Copula is a flexible approach for constructing joint distribution with different types of marginal distributions. Indeed, the copula is a function which links univariate marginal distributions to construct a bivariate or multivariate distribution function.

Materials and Methods: Hydrological phenomena often have different properties, where for their frequency analysis; they may be examined either individually or concurrently. These variables are not independent, rather they are interconnected and the change in one of them affects the other. Thus, the univariate frequency analysis can bring about some error due to neglecting the interdependence between these random variables. the copula is a function which joint the marginal distribution functions for constructing a bivariate or multivariate function. Development of copula functions is alleged to Sklar (1959) who described how univariate distribution can be jointed to form a multivariate distribution. Generally a copula function is a transfer of a multivariate function from $[-\infty,+\infty]^d$ to $[0,1]^d$. This transfer separate marginal distributions from F function and the copula function, C, is only related to dependency among variables, therefore it present a full description of inner dependency structure. In other words, the Sklar's theorem states that for multivariate distributions, the inner dependency among the variables and univariate marginal distributions is separated and the dependency structure explained by copula function. The copula function divided into many families which among them then the Archimedean copula is widely used in multivariate analysis of hydrological events and also has an explicit formula for its cumulative form which is an important advantage in comparison with elliptical copula functions that have not explicit formula. Application of the copulas can be useful for the accurate multivariate frequency analysis of hydrological phenomena. There are many copula functions and some methods were proposed for estimating the copula parameters. Since the copula functions are mathematically complicated, estimating of the copula parameter is an effortful work. In this study, five different copula functions including, Ali - Mikhail – Haq, Clayton, Frank, Gal ambos and Gumbel-Hougaard were used for multivariate analysis of 7-days low flow in Dez basin.

Results and Discussion: In this study, the low flow of the Dez basin at junction of river branches during 1956-2012 were investigated using copula functions. For this purpose, firstly the 7-days low flow series of considered stations were extracted and then the homogeneity of the series was examined using Mann-Kendall test. The results showed that the 7-days low flow series of Dez basin are homogenous. In the next step, 11 different distribution functions were fitted on low flow series and the Logistic distribution was selected as the

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best fitted marginal distribution for considered stations. After specifying the marginal distributions, the Archimedean and Extreme value families of copula functions were used for multivariate frequency analysis of 7-days low flow. For this study, the best-fitted copula was specified in two ways. For the first specification, the nonparametric empirical copula was computed and compared with the values of the parametric copulas. The parametric copula that was closest to the empirical copula was defined as the most appropriate choice. The second specification was based on the statistical approach. The results indicated that for pair data of Sepid Dasht Sezar and Sepid Dasht Zaz stations, the Gumbel-Hougaard copula had the most accordance with empirical copula. In order to investigate the joint return periods, we used the joint return periods in two cases of AND and OR forms and also conditional joint return period.

Conclusion: Based on the obtained results from joint analysis of the low flow at upstream of the junction of two river branches, it was specified that two river branches of Sepid Dasht Sezar and Sepid Dasht Zaz may experience sever simultaneous drought events every 200 years.

Keywords: Conditional joint return period, Empirical copula, Homogenous, Joint return period, Marginal distribution