

History and Provenance of the “Chinese” Calendar in the *Zīj-i Īlkhānī*

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Abstract

This article sheds light upon a “Chinese” calendar described in the *Zīj-i Īlkhānī*. In previous studies, some characteristics of the calendar were ascribed to the “Uighurs”. However, I will show that it was not originally associated with the Uighur. This “Chinese” calendar was brought to Iran by the Chinese Taoist Fu Mengchi who accompanied his ruler Hülegü. Fu Mengchi informed Nasīr al-Dīn Tūsī of the Chinese calendrical system, which Tūsī described in his *Zīj-i Īlkhānī*. Soon afterward, the calendar was included in the *zīj* of Muhyī al-Dīn Maghribī, because it was only used among the Mongol ruling class and their Buddhist servants, who were called *Uighur*. Muhyī al-Dīn labeled the “Chinese” calendar the “Chinese-Uighur” calendar, and this title was repeated in subsequent *zīj*es. Therefore, modern scholars have regarded the calendar as a product of the Uighurs. However, the title “Uighur” attached to the calendar in later *zīj*es does not reflect the characteristics of the calendar, but rather the circumstances in which it was utilized.

Key words: *Zīj-i Īlkhānī*, Nasīr al-Dīn Tūsī, Uighur, “Chinese” calendar, Fu Mengchi

1. Introduction

This article sheds light upon the “Chinese” calendrical system described in the *Zīj-i Īlkhānī* by Nasīr al-Dīn Tūsī (1201–1274). The Mongols created a vast transcontinental empire in the 13th century. Under their auspices, various commodities, ideologies, and technologies were disseminated across

Eurasia (Allsen). Therefore, aspects of cross-cultural contact in this period have attracted scholars of various disciplines. Among the elements of this cross-cultural exchange, knowledge of astronomy was one of the first to be held in high esteem by great historians of science such as George Sarton and Joseph Needham. These scholars noted that a Chinese astronomer, who has been called Fu Mengchi, had an academic acquaintance with Tūsī, a polymath representative of the Muslim world in the thirteenth century (Sarton, 1005; Needham with Wang, 218). Although Fu Mengchi has been widely acknowledged, his personage has been cloaked in a dense fog.

His scarce but clear vestige is the “Chinese” calendar in the *Zīj-i Īlkhānī* (compiled ca. 1271), which was subsequently recorded in later *zīj*es (astronomical handbooks) with a few modifications. Analysis of the “Chinese” calendar in various *zīj*es began in the first half of the nineteenth century (Ideler) and has been conducted for a number of astronomical handbooks. For example, it was scrutinized by Itaru Imai in the *Zīj-i Sultānī* compiled ca. 1445, by Edward S. Kennedy in the *Zīj-i Khāqānī* compiled ca. 1420, and by Benno van Dalen *et al.* in the *Zīj-i Īlkhānī*. These studies have already clarified the calendar’s mathematical structure to a great extent.

Despite these great achievements, there remain a few points on which some further elucidations are in order, particularly concerning the characteristics of the calendar and its designation as “Uighur.” According to previous studies, the calendar had some different characteristics from not only the official contemporary Chinese calendar but also from any other calendar adopted by the successive Chinese dynasties. Its peculiarities have been attributed to the influence of the Uighurs (Kennedy, 435; van Dalen 2002, 336; van Dalen *et al.*, 111), who had famously played an important role in the nascent period of the Mongol empire and surely contributed to the Mongol acceptance of the Chinese calendar (Bazin, 402-403). Present recognition of the calendar is well reflected in a statement by Benno van

Dalen, who produced a series of fruitful works on the astronomical contacts between Iran and China in this period (slightly adjusted):

“The Chinese-Uighur calendar, which is of lunisolar type, was a mixture of the official Chinese calendar of the Jin dynasty, which was defeated by the Mongols in 1215, and certain elements from “unofficial” Chinese calendars. One of the latter may have been the calendar used by the Uighurs, who started to serve the Mongol administration around 1210” (van Dalen 2004, 17, n. 2).

In the *Zīj-i Īlkhānī*, however, there is no reference to the Uighurs, even though in later sources the term appears not only in descriptions of the calendrical system, but also in the title *Tarikh-i Khitā wa Uighūr* (Chinese-Uighur Calendar). These indications, as I will show, evidence of Uighur influence cannot be found in the contents of the “Chinese” calendar of the *Zīj-i Īlkhānī*. As previous studies suggest (Imai, *passim*), the calendar is an amalgam of various elements found in several Chinese calendars, not only the official one. The two cores of this amalgam are closely associated with the Jin dynasty (1115–1234), which dominated northern China immediately before the Mongols. Specifically, they are the *Zhong xiu Da ming li*¹ (Revised Great Enlightenment Calendar) and the *Fu tian li* (Heavenly Agreement Calendar) (van Dalen *et al.*, 129).

The personal background of Fu Mengchi is not related to the Uighurs either, but, instead, to northern China. The title given to Fu Mengchi, *sīngsīng*, means “Taoist master,” as is well known among scholars who research Chinese sources of this period. In the early period of the Mongol empire, a Taoist sect known as the *Quan-zhen jiao* (Integral Realization Sect?) established a strong base over northern China, support of the Mongol

1. It is worthwhile remarking on the Chinese word “*li*,” usually translated as “calendar.” *Li* not only stood for a general system by which the beginning, length, and subdivision of a year were fixed, it also indicated an almanac dealing with solstices; the length of days, months, and years; the motion of the sun and moon; planetary revolution periods, and the like (Needham, 9).

court. Also, the Hülegü family had their own fief in Chinese territories, in which a decree was issued concerning Taoists (Takahashi, 33-34).

Whereas there is no evidence for Uighur influence, we thus see that strong connections to northern China can be found in Fu Mengchi's personal background and in his "Chinese" calendar. The available clues lead to the assumption that he was more affiliated with northern China than with the regions further west, the land of Uighurs.

In scrutinizing Fu Mengchi and his calendar, it will be necessary to make a slight adjustment in our understanding of the cross-cultural contacts of the Mongol period. Before going into details, it is useful to remark on calendars and astronomical activities in the Mongol period in general?

2. Historical Background

By 1206, Temüjin completed the task of forcefully unifying the tribes of Mongolia. In that year, following a decision of the *quriltai* (the council of tribal chiefs), he was acknowledged as khan of the consolidated Turko-Mongol tribes¹ and took the new title Chinggis Khan (Morgan, 1986, 63). Within a short period after that, he and his successors would create a huge empire across Eurasia.

Since ancient times, the Turko-Mongols had appreciated solar motion not directly, by astronomical observations or calculations, but indirectly, through its effects on vegetation, in a way suitable to their pastoral economy (Bazin, 119). In the official Chinese history of the sixth-century dynasty it is stated that

“ [Turks] do not know the succession of years, and only count it based on the grass turning green” (Ling & De, 910; Bazin, 118).

The Mongols seem to have adopted the Chinese calendric system through contacts with the Jin dynasty around 1201 (Bazin, 402). Despite the

1. The term “Turko-Mongols” is used because, concerning the tribes of Mongolia in the twelfth century, it is by no means clear in all cases which were Turkic and which Mongol (Morgan, 1986, 56).

disharmony between the astronomically-determined beginning of the Chinese year and the Turko-Mongolian nomadic tradition, according to which the year starts at the beginning of spring, the Mongols accepted the Chinese calendar (Melville, 84).

Therefore, at first, the Mongols continued to use the Revised Great Enlightenment Calendar of the Jin dynasty. Xu Ting, who was the Sung ambassador to the Mongolian court in 1237, reported that he encountered the Chinese calendar being used in the Mongol court. Upon inquiry, the calendar was identified as that made by Yelü Chucai (1190-1244), who accompanied Chinggis Khan on several campaigns and was famous as the chief “Chinese” advisor of the early Mongol rulers (Allsen, 165-166). In Chinggis’s military expedition to the west, Yelü Chucai compiled a calendar which took into account the difference in geographical longitude between Samarqand and mainland China. This was the *Xi zheng geng-wu yuan li* (Western Expedition Calendar with Epoch Year Geng-Wu [i.e., 1210]). With the exception of the small correction due to the difference in geographical longitude, it is identical to the Revised Great Enlightenment Calendar. He also wrote the *Ma da ba li*¹, which was based on the “methods of western regions”. After the eastern Muslim lands had been annexed to the Mongol empire and Chinese territories had been put under direct Mongol rule, Muslim astronomers came to these territories. In 1267, the most famous of these astronomers, Jamāl al-Dīn (fl. 1267-91)², compiled and submitted the *Wan nian li* (Myriad Years Calendar) to Qubilai, the de facto first emperor of the Yuan dynasty (1271–1368), which constituted the eastern part of the Mongol empire. Qubilai promulgated it to a limited

1. The meaning of this term is yet to be determined (Allsen, 165).

2. Without doubt, Jamāl al-Dīn initiated the most significant phase in the history of West Asian astronomy in Yuan China. His activities in China are crucial to any consideration of astronomical exchange in this period (Allsen, 166; van Dalen, 2002, 336, 340-341).

extent. Unfortunately, the contents of both the *Ma da ba li* and the Myriad Years Calendar are currently unknown (Yabuuchi, 1997, 11-12).

Although, in this way, the project for calendrical reform had been ongoing since the time of Chinggis Khan, it made rapid progress after Lin-an, the capital of the Southern Song dynasty, was occupied in 1276. That is because afterward, it was possible to make use of not only northern Chinese astronomical achievements -from the Jin to the Yuan dynasties- but also southern ones, and to carry forward the reform on this integrated astronomical accumulation. In 1280, a year after Qubilai finally defeated the Southern Song dynasty and reunified China under one rule, the new *Shou shi li* [be consistent in rendering Chinese words in pinyin; until now you had not yet used any hyphens, from here on you start doing it in some cases but not all] (Season-Granting Calendar) was compiled, and almanacs based on it were widely distributed beginning the following year (Yamada, 179; Yabuuchi, 1997, 13). It was by far the most accurate calendar in the tradition of Chinese mathematical astronomy and continued to be used for almost 400 years¹; although calendar reform was also conducted in the following Ming dynasty (1368–1644), that dynasty's calendar, the *Da tong li* (Great Concordance Calendar), was very similar to the Season-Granting Calendar (van Dalen, 2002, 340).

Meanwhile, in the 1250s, a brother of Qubilai, Hülegü, led extensive military campaigns in the Middle East. By taking Baghdad in 1258, he ended the Abbasid caliphate. He reigned over Iran and Iraq from 1256 to 1265 and founded the Ilkhanid dynasty (1256–1336; van Dalen, 2002, 329). The first aim of Hülegü's campaign was to exterminate the Ismā'īlīs, a goal he achieved along with "releasing" Tūsī, who resided in their fortress. Under the Ilkhanid patronage, the Maragha observatory was established, and

1. For the structure of the Season-Granting Calendar and the astronomical reforms that underlie its compilation, we can refer to Nathan Sivin's work (Sivin).

Tūsī became its director. He compiled an astronomical handbook titled *Zīj-i Īlkhānī* at the observatory, shortly before his death in 1274. Because various human and material resources were concentrated at Maragha, a range of highly significant research was done there, and the achievements of scholars who were active at the Maragha observatory had a lasting effect on the astronomical output of future generations, up until the European Renaissance (Saliba, 2006, 367-368).

3. Historical Sources concerning Fu Mengchi

We have only a faint idea of who Fu Mengchi was. There are four known primary sources concerning him:

1. “Tārīkh-i Chīn” (“History of China”)¹, a section/chapter? in *Jāmi‘ al-Tawārīkh*, by Rashīd al-Dīn Hamadānī
2. *Tārīkh-i Banākatī*, by Dāwūd b. Muhammad Banākatī
3. *Tanksūq-nāma*
4. “Tārīkh-i Khitā,” in *Zīj-i Īlkhānī*, by Nasīr al-Dīn Tūsī.

Among these sources, Rashīd al-Dīn’s “Tārīkh-i Chīn” and the history by Banākatī refer directly to Fu Mengchi by name. The latter is an abridged version of the former; thus, its statements do not provide any new information concerning Fu Mengchi (Banākatī, 338). Furthermore, the description concerning him in the *Tanksūq-nāma*, which is a translation of a Chinese medical text, is also based on Rashīd al-Dīn’s “History of China” (Rashīd al-Dīn, *Tanksūq-nāma*, 16)² Rashīd al-Dīn’s statement in “Tārīkh-i Chīn,” in *Jāmi‘ al-Tawārīkh*, is, therefore, the most direct and original

1. The second part of the *Jāmi‘ al-Tawārīkh* is dedicated to world history, including the history of the biblical prophets, Muhammad, the emergence of Islam, the caliphates and major sultanates, a history of the Mongolian and Turkic peoples, the rise of the Chinggisid dynasty, and separate accounts of the Chinese, Indians, Jews, and Franks. The first part of the history of China in this work provides general information on the population and certain aspects of Chinese culture, such as its calendar (Allsen, 83, 91).

2. Koichi Haneda elucidates the provenance of the work (Haneda).

source concerning the Chinese sage, Fu Mengchi; the following passage represents Rashīd al-Dīn's reference to him in the "History of China".

"In the time when the Qanate, or sovereignty all over the world, came to Mönke Qan', he dispatched his own brother Hülegü Khan, son of Tolui Khan, son of Chinggis Khan, to the land of Iran, and the sovereignty of these regions was established on him [Hülegü Khan]. Chinese philosophers, astrologers, and physicians gathered to his presence. Since he reigned with perfect intelligence, ability, and enthusiasm for all sciences, he ordered our lord, the prominent teacher of mankind, and the most distinguished among contemporaries, Khwāja Nasīr al-Dīn al-Tūsī -may God have mercy upon him- to build an observatory and compile an astronomical handbook after his majestic name".

For the reason that Hülegü Khan had seen their [Chinese] astrologers, known the astrological rules according to their methods, and accustomed himself to them, he [Hülegü Khan] ordered Khwāja Nasīr al-Dīn to introduce their [Chinese] calendar and astrological rules and to incorporate these contents into the astronomical handbook that he would compile in such a way that, in compiling calendars, their dates and calculations of their years could be also added to our calendars according to the methods and technical terms that they had. Then, he [Hülegü Khan] ordered **a Chinese, whose name was Fu Mengchi, known as *sīngsīng* -namely, a sage -to explain whatever points he knew about their calendar and astrology to Khwāja Nasīr al-Dīn and to learn astronomy from Khwāja Nasīr al-Dīn. In two days, Khwāja Nasīr al-Dīn acquired whatever he [the Chinese sage] knew in this field and incorporated it into the *Zīj-i Īlkhānī* that he compiled.**

However, that Chinese scholar could not acquire more scientific profit from Khwāja. That scholar knew some of their methods for calculating the dates relative to astrological choices and orders to some extent, [but] he was not quite familiar with how to apply an astronomical handbook or understand celestial movements and their details. No matter the place or

time, it is rare to find a perfect scholar familiar with these kinds of knowledge. Below, it shall be said what the aforementioned scholar explained and what was mentioned in the *Zij-i Īlkhānī* (Rashid al-Dīn, Wan ed., 83-85; idem, Rawshan ed., 5-6; Jahn, 21-22)¹.

According to Rashīd al-Dīn, the Chinese man who explained the Chinese calendrical and astrological knowledge to Tūsī was named Fu Mengchi (QWMHY)² and held the title *sīngsīng*, which Rashīd al-Dīn explained as “sage” (*‘ārif*; Boyle, 253, n. 4). Even though scholars have not identified this Chinese sage with a historical personage up to now, it has been well known that the title *sīngsīng* is the transliteration of the Chinese *xiansheng*. Whereas Western scholars grasped this word as having a more generalized

1. I would like to thank Mohammad Bagheri for his assistance in translating Persian sources. However, needless to say, I am responsible for all mistakes. The following Persian text from the “History of China” is based on Muhammad Rawshan’s edition.

... و در زمانی که نوبت قاننی و پادشاهی روی زمین به مونککافان رسید، برادر خویش هولاکوخان بن تولوی خان بن چینگیز خان را به ایران زمین فرستاد، پادشاهی این ممالک بر وی مقرر شد، از حکما و منجمان و... اطبای ختای در پندگی وی [جمع] آمده بودند، و چون پادشاهی بغایت با کمال عقل و کفایت بود و مَهْوَس جمله علوم، مَوْلانا سعید استاد البشر افضل المتأخرین خواجه نصیرالدین الطوسی را - رحمة الله علیه - فرمود تا رصد سازد، و زیجی به نام همایون او تألیف کند،

... و به جهت آنکه هولاکو خان منجمان ایشان را دیده بود، و احکام نجومی بر قاعده ایشان دانسته و بدان معتاد شده، فرمود تا خواجه نصیرالدین تاریخ و قواعد نجومی ایشان معلوم گرداند، و آن معنی را در زیجی که می‌سازد بیارد، چنانکه به وقت استخراج تقویم، تاریخ و حساب سالهای ایشان را نیز به موجبی و مصطلحی که دارند به تقویمهای ما اضاف توان کرد. و شخصی ختایی را که نام او قومیحی بود و معروف به سینگسینگ یعنی عارف، فرمود تا از تاریخ و نجومهای ایشان به نکت هرآنچه برآن وقوف دارد با خواجه نصیرالدین تقریر کند و علم نجوم از خواجه نصیرالدین بیاموزد، و هرآنچه او را از آن قسم معلوم بود، خواجه نصیرالدین به مدت دو روز تعلیم کرد و به زیج ایلخانی که ساخت درآورد.

... اما آن حکیم ختایی زیادت بهره ای علمی از خواجه نتوانست یافت. و آن حکیم حساب تاریخ بعضی اختیارات و احکام ایشان می‌دانست، و بر آنچه عمل زیج و ادراک سیر ستارگان باشد، و دقایق آن زیادت وقوفی نداشت. هرچه در هر اقلیمی و هر عهدی حکیمی کامل که بر چنان علوم واقف تواند بود، بنادر یافت شود. آنچه آن حکیم مذکور تقریر کرد، و در زیج ایلخانی مذکور [در آمد] این مقدار است که یاد کرده می‌شود.

2. Although the Chinese sage is provisionally called Fu Mengchi in this article, there is a long history of attempting to identify him that began at the end of the seventeenth century. In 1689, Müller reconstructed this name as FUMNJI from the *Tārīkh-i Banākatī* and vocalized it as *Fau Munjī* (Boyle, 253, n. 4). Later, d’Ohsson represented it as *Fao-moun-dji*, relying on Müller (d’Ohsson, 265). Needham introduced the theory by a modern Chinese scholar that it could be represented *Fu Meng-chi* (Needham with Wang, 375). More recently two Chinese scholars have reconstructed it as *Fu Man-tzu*, with a question mark (Zhou & Gu, 830; cf. Allsen, 162).

meaning, as in, for example, “teacher” or “master” (Jahn, 21-22; Allsen 162), it should be more concretely interpreted as “Taoist master,” in consideration of the contemporary Chinese context. For example, from parts of a Chinese decree issued to a Taoist temple dated to 1238, it is clear that the term was used to represent a Taoist master

“...a decree to instruct any provincial and urban *darughachi* (resident commissioner), governor, and *darughachi* administrating artisans in the lands of the *Han* people. **In [our] words, in the temples of Buddhist monks (*heshang*), the churches of Christian priests, the Taoists’ temples of Taoist masters (*xiansheng*), and the mosques of imams,** representative persons who pray to Heaven shall not be disturbed by laymen or subject to any duties or payments. [Religious] broken or old establishments shall be repaired. They shall pray for [our] long lives and recite scripts in our names. Whosoever should disobey our words, whatever kind of persons are they, shall be slain...” (Takahashi, 20).

In this statement, there is a clear distinction between religious sects, and Taoist masters are expressed as *xianshengs*. Buddhist monks (*heshengs*) also appear in Rashīd al-Dīn’s “History of China” in the form of *khūshāng*

Although the history of Chinese people derives from far ancient times, and the numbers of their years and cycles [are derived] in the way that they already explained to Khwāja Nasīr al-Dīn, there is a historical work in which names of their sovereigns are described in detail and the foundation of accounts is established, and which becomes famous among Chinese people in this time, and is so accurate and certain that all scholars and intellectuals rely on it. It is a work that three reliable scholars compiled in their collaboration, one of whom is named Fūhīn **Khūshāng** -Fūhīn is his proper name, and **Khūshāng represents his attribution, specifically, *bakhshī***- and comes from the city of Tāy‘ānjūy (Tai-an zhou) (Rashid al-Dīn, Wan ed., 86-87; idem, Rawshan ed., 8; Jahn, 24)¹.

هرچند تاریخ اهل ختای عظیم قدیم است و عدد سالها و ادوار ایشان، بموجبی که پیش از این با خواجه نصیرالدین تقریر کرده‌اند، لکن تاریخی که 1. اسامی پادشاهان آنجا در آن مشروح و مفصل است، و بنیاد حکایات بر آن نهاده و درین وقت میان اهل ختای شهرتی دارد، و تاریخی درست و مخفّق

As shall be explained later, *bakhshī* denoted Buddhists in the contemporary Persian sources (Doerfer, 271-277); therefore, Buddhists and Taoists were clearly recognized as distinct in Persian as well as Chinese sources. The above excerpt shows that Fu Mengchi adhered to Taoism.

Thus, we have seen that Fu Mengchi, a Taoist master, came to Iran as part of Hülegü's expedition and informed Tūsī of Chinese astronomical knowledge, which came to be a part of Tūsī's *Zīj-i Īlkhānī*¹. In fact, a section of the *Zīj-i Īlkhānī*'s first chapter, which concerns various calendars and eras in the *zīj*, focuses on the Chinese calendrical system called "Tārīkh-i Khitā"². We will now discuss this section in detail.

4. The "Chinese" Calendar in the *Zīj-i Īlkhānī*

4.1. Chinese Calendars

In Chapter 3, the personage of Fu Mengchi has been clarified as much as possible. Now, after some general remarks on the "Chinese" calendar, the contents of his calendar are scrutinized. Although the "Chinese" calendar was also described in the various Persian and Arabic *zīj*es of the following

[است]. تمامت حکما و دانایان بر آن اعتماد کرده، کتابی است که آن را سه حکیم معتبر به اتفاق ساخته اند. یکی از نام فوهین خوشانگ، فوهین اسم است و خوشانگ صفت یعنی بخشی، و از شهر تائی عان جوی بوده است.

1. Although the accepted view is that Fu Mengchi worked at the Maragha observatory (e.g., Sayılı, 1960, 205-207), this does not have a solid foundation: to my best knowledge, there are no sources that prove the fact. Referring to research by Mamedbeili, John Boyle states that the Chinese astronomer's name is mentioned in a Tehran manuscript of Mu'ayyad al-Dīn 'Urdī's treatise on the instruments of the Maragha observatory, *Risāla fi Kayfiya al-Arsād* (Boyle, 253, n. 4); however, Muhammad Tarbiyat, on whom Mamedbeili relies in this regard (Mamedbeili, 194), does not bring evidence to show the existence of Fu Mengchi at the Maragha observatory, despite referring to the manuscript (Tarbiyat, 377-378). It is possible that Fu Mengchi's acquaintance with Tūsī was a personal one. I would like to thank Benno van Dalen for providing me with the relevant pages of Mamedbeili's monograph.

2. The word *Khitā* originally referred to a nomadic people who dominated a vast area in northern China from the tenth century and established the Liao Dynasty (916-1125). By the Mongol period, the Turko-Mongols and the Western people used *Khitā* to refer to northern China and its inhabitants. The Russian word, Китай, representing China, is derived from it (Isahaya, 2009, 171).

periods, almost all the descriptions were based on that in the *Zīj-i Īlkhānī*. For the following information on the general description of the “Chinese” calendar, I have mainly relied on an article by van Dalen (van Dalen, 2002, 334-335)¹.

The “Chinese” calendar shares basic characteristics with calendars officially adopted in successive Chinese dynasties, which were lunisolar; therefore, lunar months and solar years are compounded to keep pace with lunations and solar motion. In addition, the calendar employed an abstract duodecennial cycle of “twelve branches,” in conjunction with a decennial series of “ten trunks,” to derive a sexagenary cycle for denoting years and days (Melville, 83).

Each month starts with the day of a new moon and consists of 29 or 30 days. To determine the day of the new moon, at first, the time of the *mean* new moon is calculated from the average length of a lunation; in the Chinese calendrical system in the *Zīj-i Īlkhānī*, this length is taken to be 29.5306 days. To obtain the time of the *true* new moon, the time of the mean new moon is corrected by means of the solar and lunar equations, whose maximum values are 0.1840 days and 0.3844 days, respectively. The period of the solar equation corresponds to the solar year, but that of the lunar equation, called the anomalistic month, which is, specifically, 27.5556 days long, is somewhat smaller than a lunar month.

1. More details can be found in an article by van Dalen *et al.* (van Dalen *et al.*, 1997). Note that among many extant manuscripts of the *Zīj-i Īlkhānī*, the following three, all of which were copied shortly after the time of compilation are utilized in this article.

1. British Library Ms. Or. 7464; This manuscript was produced/copied at Maragha in 676 AH/1277–78, only three years after Tūsī’s death, and includes the “longer” introduction, which does not appear in any other manuscripts (Boyle). On the assumption that this manuscript well preserves the form of the original, it is taken as the basic text.
2. Bibliothèque nationale de France, Ms. Ancien fonds persan 163 ; This is also quite an early manuscript and, according to notes on the manuscript (Tūsī, Paris Ms. 2r, 3r), the copyist was a son of the author, Asīl al-Dīn b. Naṣīr al-Dīn (d. 715 AH/1315; Richard, 179).
3. Dār al-Kutub al-Miṣrīya, Ms. Dār al-Kutub Mīqāt Fārsī 1; This manuscript was copied at Maragha in 692 AH/1293 (King, 203).

The solar year in this calendar starts around the time when the sun passes through the halfway point of the zodiacal sign Aquarius; it has a length of 365.2436 days and is divided into twenty-four equal parts, called *qi*. The beginning of the first month is calculated on the basis of the starting point of the second division of the solar year, the *Yu-shui*; practically, the first month immediately precedes the entrance of the sun into the zodiacal sign Pisces. An ordinary year consists of twelve lunar months (354 or 355 days), but, to conform to the solar year, the calendar requires insertion of a leap month every second or third year. The leap month is that which includes only one starting point of a solar division (all other months include two starting points).

Although Fu Mengchi's calendar basically relied on the official Chinese calendrical system, it differed in several respects from the Chinese model.

4.2. *Fu tian li* (Heavenly Agreement Calendar)

As mentioned above, Fu Mengchi's calendrical system has a number of characteristics that are rather atypical for an official Chinese calendar. They are summarized by the following four points.

1. Use of a point different than the winter solstice to prescribe the solar year.
2. Use of a parabolic interpolation scheme for the solar equation.
3. Use of a peculiar value of the anomalistic month (27.5556 days).
4. Use of decimal notation to represent the fraction of a day instead of specific denominators (cf. van Dalen, 2002, 335).

In regard to the first point, the *Zīj-i Īlkhānī* reads as follows:

"Fifth Section, on knowing the beginnings of the divisions of the solar year which occur in each year

Whenever we want to know the beginning of each of the twenty-four divisions in a given year, we must know which day and double-hour *chāgh* fell, [or will fall,] on the beginning of the *Līchun* (*Li-chun*; the first division)

in the proceeding and following year in a sexagenary cycle. We call it “the starting point of the division of a year,” which is called “*kījū*” (*qi-shou*) in the Chinese language” (Tūsī, London Ms. 5v; idem, Paris Ms. 7r; idem, Cairo Ms. 6r)¹.

This passage shows that the *Li-chun* was regarded as the starting point for computation of the divisions of the solar year, *qi-shou*, in the *Zīj-i Īlkhānī*. Under the Chinese tradition, the *Dong-zhi* (winter solstice) was taken as the starting point. This rule was first changed in the *Fu tian li* (Heavenly Agreement Calendar) compiled by Cao Shi-wei during the Jian-zhong era (780–783) of the Tang dynasty (618–907). In regard to the *Fu tian li*, the *New Standard History of the Five Dynasties* offers the following passage:

During the Jian-zhong era, the mystic Cao Shi-wei first changed the old methods. This calendar, which set the fifth year of the Xian-qing era (600 AD) as the start of the epoch and the *Yu-shui* as the starting point of the year, was named the Heavenly Agreement Calendar. [However,] it was used only among the people under the name of civil calendar (*xiao-li*) (Ou Yang, 670; Yabuuchi, 1982, 2).

As Yabuuchi noted, in the above passage “the starting point of the year” should be corrected to “the starting point of the division of the solar year” (Yabuuchi, 1982, 4). Therefore, the use of a different point from the winter solstice to set the start of the solar year commenced with the Heavenly Agreement Calendar, and the “Chinese” calendar in the *Zīj-i Īlkhānī* followed this precedent.

Next, we address the use of a parabolic interpolation scheme for the solar equation. Although the contents of the Heavenly Agreement Calendar were unknown for a long time, a small fragment entitled “*Futenreki nitten sa rissei*” (the table of the Heavenly Agreement Calendar concerning the solar

فصل پنجم، در معرفت مبادی اقسام سالهای شمسی که واقع باشد در هر سال، هرگاه که خواهیم که مبدأ هر قسمی از اقسام بیست و چهارگانه در . 1. سالی معین بدانیم باید که در یک سال پیش از آن سال یا بعد از آن دانسته باشیم که مبدأ لیچن در کدام روز و جاغ بوده است از دور ستینی و ما آنرا اصل اقسام سال می خوانیم و آنرا بلغت قتیایان کیجو می خوانند.

equation) was found at the Tenri Library, Japan, in March 1963. From this, it was proved that the data were all given by the following formula:

$$y = (182 - x) x / 33^1$$

Then, the parabolic interpolation scheme utilized in the table had the same base as that of the “Chinese” calendar in the *Zīj-i Īlkhānī*:

$$y = (2/9) x (182 - x) \text{ (Kennedy, 438).}$$

In contrast to the former formula, in which the values were denoted in Chinese degrees, the solar equation is here expressed in terms of the fraction of a day of lunar elongation. This method had also never been found in Chinese calendars prior to the Heavenly Agreement Calendar (Nakayama, 451). It subsequently succeeded to the official calendars in the Song period and eventually was used in the Season-Granting Calendar in the Yuan dynasty, extended to third-order formulas (Chen).

In regard to the third point, the value of the anomalistic month (27.5556 days) utilized in the “Chinese” calendar in the *Zīj-i Īlkhānī* is, as Itaru Imai properly stated, quite inaccurate, and this inexact value is rare in Chinese calendars (Imai, 33). Although Edward Kennedy remarked that the approximate value $248/9 \approx 27.5556$ days originated in Babylonia (Kennedy, 441), according to Kiyoshi Yabuuchi it was also adopted in an official Chinese calendar, the *Qin tian li* (Veneration for Heaven Calendar) used from 958 to 963, which imitated the Heavenly Agreement Calendar (Yabuuchi, 1963, 95).

Finally, the adoption of decimal notation to represent the fraction of a day was unusual. The fractions of a day used to mark the constants of Chinese calendars were, in general, denoted with specific denominators; for instance, the lengths of a year and month were represented as $365 \frac{1274}{5230}$ and $29 \frac{2775}{5230}$, respectively, in the Revised Great Enlightenment Calendar. In

1. In this formula, y is the solar equation and x is the mean solar anomaly, both expressed in Chinese degrees. The aforementioned formula reflects revisions made by Ikkei Suzuki to the original version introduced by Shigeru Nakayama (Suzuki, 72-73; Nakayama, 451).

the “Chinese” calendar described in Islamic sources?, decimal notation was substituted for this specific notation, and a common denominator of 10,000 was adopted. According to Yabuuchi, it was another way in which the Heavenly Agreement Calendar “first changed the old methods.” This new method was also adopted in the Season-Granting Calendar (Yabuuchi, 1982, 5).

Based on the evidence of these four points, it is clear that the “Chinese” calendar in the *Zij-i Īlkhānī* had strong similarities with the Heavenly Agreement Calendar. Except for that denoted by the first point, these elements common to both calendars are devices to simplify computation. This is one reason that the Heavenly Agreement Calendar was not officially adopted and was instead called a “civil calendar” -significant digits are sometimes lost when these devices are used. However, the Heavenly Agreement Calendar was adopted as a text for the examination of the Astronomical Bureau in the Jin and Yuan dynasties, even though it was a “civil calendar” (Yamada, 119-125). This may explain why the “Chinese” calendar in the *Zij-i Īlkhānī* also inherited its simplifying elements.”

When we consider that a good number of bureaucrats of the Jin dynasty escaped into the Taoist sects, particularly into the Integral Realization, after the fall of the dynasty (Kubo, 167), it is likely that Fu Mengchi, the transmitter of the amalgamated calendar, was one of them. In any case, the “unofficial” elements of his calendar were all derived from the tradition of the Chinese calendars since the Tang period, and there is no evidence of Uighur participation, as suggested by van Dalen, who said, “it is therefore tempting to conjecture that the above-mentioned characteristics of the Chinese-Uighur calendar that stem from unofficial Chinese calendars derive from the original calendar of the Uighurs” (van Dalen, 2002, 336). Although the actual contents of the calendar utilized in the Mongol period by the Uighurs, who usually had their own calendar basically corresponding with the Chinese one, have not become well known, the transliteration into

Persian script of a cycle in the *Zīj-i Īlkhānī* called “the fourth cycle” (*dawr-i chahārum*), which consists of twelve elements for divination, was somewhat different from the transliteration into Uighur script of the same cycle in a document from 1202 (Bazin, 286-288). Also, the dates of an [to be able to write “the” you need to specify it more] Uighurian text written in Brahmi script from 1277 agreed with those of the official Chinese calendar (Bazin, 306-308). Therefore, it is natural to consider that the “Chinese” calendar in the *Zīj-i Īlkhānī* was brought by a Taoist scholar from Chinese territories and had little connection with the contemporary Uighurs.

Incidentally, it has been suggested that Cao Shi-wei, the compiler of the Heavenly Agreement Calendar, came to China from the “western regions,” which included the land of the Uighurs. This is because his family name, Cao, was attached to the people who came from a city of Sogdiana in the Tang period, and it was stated in some Chinese sources that the Heavenly Agreement Calendar was based on Indian methods (Yabuuchi 1982, 3-4). Although Yabuuchi took a prudent attitude to this assumption, there is a strong possibility that Cao Shi-wei came from Sogdiana¹ because it is well known that many Sogdians came to China in the Tang period and propagated Manichaeism, and these Manichaeans, in particular, brought new astrological and calendrical elements into China (Lieu, 232).

In this sense, it is possible that the “Chinese” calendar in the *Zīj-i Īlkhānī* is the epitome of cultural traffic between East and West. Some elements of the calendar were supposedly transmitted from the western regions into China through Sogdian Manichaeans in the eighth century and then, conversely, from China into Iran by the Chinese Taoist (Fu Mengchi) in the thirteenth century.

1. Sogdiana covered territories around Samarkand, Bukhara, Khujand, and Kesh in modern Uzbekistan. The name “Cao” was specifically attached to the people from the city of Kabudan.

5. Turkic Elements in the Calendar

Finally, it is necessary to consider why the calendar in the *Zīj-i Īlkhānī* has been thought to have a relationship with the Uighurs. As mentioned above, the analysis of the “Chinese calendar” described in *zīj*es commenced with that of a *zīj* authored after the *Zīj-i Īlkhānī*, specifically, Ulugh Beg’s *Zīj-i Sultānī* (compiled ca. 1445), in which the Chinese calendrical system derived from Fu Mengchi was given the title of “Chinese-Uighur” calendar (Sédillot, 314). The first *zīj* in which the term Uighur appeared is the *Adwār al-Anwār madā al-Duhūr wa al-Akwār* (ca. 1275) by Muhyī al-Dīn al-Maghribī (Muhyī al-Dīn, 11v)¹, who also worked in the Maragha observatory and who compiled this *zīj* over a period of several years after Tūsī’s *zīj* was completed (Saliba 1983, 391-392). There is no doubt that Muhyī al-Dīn referred to the statements of Tūsī concerning the Chinese calendrical system, since the contents of their *zīj*es scarcely differed despite the difference of language: Muhyī al-Dīn’s *zīj* was in Arabic and Tūsī’s was in Persian. Adding the term “Uighur” was likely connected to observation of which groups utilized the “Chinese” calendar in Iran. It is probable that, in those days, this calendar was properly used only among the ruling class centered upon Mongol royalty and among Buddhist monks protected in the earlier period of the Ilkhanate. This is reflected in the statements of the *Zīj-i Īlkhānī*:

“Exordium

On describing calendars used in this period

The calendar which our sovereigns use is the calendar of the Chinese and Turkic peoples. Those used in our regions are Roman, Arabian, and Persian calendars, and a new calendar was established by Sultan Malikshāh. Astral

1. I express deep appreciation to Benno van Dalen for informing me of this matter.

scholars take these calendars into consideration” (Tūsī, London Ms. 17r; idem, Paris Ms. 5r; idem, Cairo Ms. 3v)¹.

From this passage, it is clear that the “Chinese” calendar was utilized among “our [Mongol] sovereigns and Turks (including a few Chinese),” not by other people in Iran. Furthermore, the following statements appear in the section on the festivals of various calendars.

“Seventh Division

Concerning famous days in each calendar

The sovereigns of the Mongols celebrate New Year’s Day as well as the first day of each month and the birthday of the sovereign. Buddhist monks practice *bājāq* for three days each month. That is their fasting. They differ in the days, one day earlier or later. On the last days of the months and in the *Jaqshābāt* month (the twelfth month of the “Chinese” calendar), several days are also those of *bājāq*, in which they practice religious observances and eat decided foods” (Tūsī, London Ms. 23v; idem, Paris Ms. 18v; idem, Cairo Ms. 23r)²

This passage is devoted to the explanation of the “famous days” of the “Chinese” calendar. By these statements, it is made clear that, in Iran, people who utilized this calendar were Mongol rulers and the Buddhist monks who served them. In fact, according to Charles Melville, the events recorded in the “Chinese” calendar all concern the activities of the Mongol ruling class or persons closely attached to the ruling elite in the contemporary Persian chronicles (Melville, 85). Moreover, it is worthwhile noting that the term *bakhshiyān*, which denoted Buddhist monks, became synonymous with “Uighur” in the course of Mongol domination in Iran (DeWeese, 82-83, n. 22). Muhyī al-Dīn Maghribī came from the western

مقدمه در ذکر تاریخهای کی درین وقت بکار می‌دارند، تاریخی که پادشاهان ما بکار می‌دارند تاریخ قنایان و ترکانست و آنچ در ولایتها ما بکار می‌دارند تاریخ رومیان و تاریخ عربیان و تاریخ فارسیان باشد و تاریخی محدث که سلطان ملک‌شاه نهاده است و منجمان این تاریخها اعتبار کنند.

فصل هفتم در ایام مشهور از هر تاریخی، پادشاهان مغول اول روز سال نو جشن می‌کنند و اول هر ماهی و روز ولادت پادشاه وقت هم چنین و بخشیان در هر ماه سه روز باجاق می‌کنند و آن روزه ایشانست و در تقدیم و تأخیر بعضی از آن روزها بیک روز میان ایشان خلافت و آخر ماهها و در ماه جقشاباط چند روز هم باجاق باشد و در باجاق عبادت کنند و طعامها معین خورند.

Muslim world and had probably never been farther east than Baghdad and Maragha (Ridawī, 232-237). Therefore, for him, Uighur was not a term representing the people who dwelt in the western region of China, but instead described the Buddhist monks serving the Mongol rulers who utilized their peculiar calendar, which he called the “Chinese-Uighur” calendar in his *zīj*. Originally, the calendar that Fu Mengchi taught to Tūsī had no relationship to the Uighurs, but it eventually became known as the “Chinese-Uighur’s” as a reflection of the political situation in Iran.

There are several Turkic words in the calendar in the *Zīj-i Īlkhānī*; for instance, the names of years and months are described in both Chinese and Turkic, and a Turkic technical term concerning the solar equation also appears (Mercier, 50). However, this fact does not mean that the Uighur—a Turkic people—had anything to do with the creation of the calendar. As Louis Bazin properly stated, from the middle of the thirteenth century, the Turkophonic regions were under Mongol domination. The Mongols, who had been influenced by the Turkic culture, in particular the Uighur culture, since the nascent growth of their empire, mingled further with it as they advanced to the western regions. Eventually, the Mongolian language was only imposed on the eastern regions of the empire that depended directly on the Yuan dynasty, that is, present-day Inner and Outer Mongolia and their border lands. In more western regions, where Mongolic people have remained even up to the present day, the Mongol ruling elites linguistically turkicized themselves through several generations, although preserving their own customs and faith to the blood of Chinggis (Bazin, 403). As a result, the “Chinese” calendar was practiced in Turkic form in Iran,¹ so it was

1. Most Turkic people also utilized the “Chinese” calendar, but in the skeletal and simplified form. In the *Zīj-i Īlkhānī*, it is stated that “Turks curtail the (sexagenary) cycle to a duodecennial one and count it in their language. Their calendrical measure (*qayd-i tārikh-i īshān*) is not known” (Tūsī, London Ms. 5v; idem, Paris Ms. 7r; idem, Cairo Ms. 6r). Finally, this “Turkic” calendar was assimilated into Persian calendrical customs and the year has commenced on Persian New Year, *Nawrūz*, since the Timurid period (Isahaya 2008).

natural to insert Turkic words into the description of the “Chinese” calendar in the *Zīj-i Īlhānī*.



Muhyī al-Dīn, 11v¹

1. It is possible to see Uyghur words on this page, with which the description of the “Chinese” calendar begins.

6. Conclusion

On the basis of a range of evidence, the nature of the “Chinese” calendar has come to light. It was brought to Iran by the Chinese Taoist Fu Mengchi who accompanied his ruler Hülegü. He informed Tūsī of the calendar, which Tūsī described in the *Zīj-i Īlkhānī*. The Chinese calendrical system was then included in the *zīj* of Muhyī al-Dīn Maghribī in the period immediately thereafter, and it was practiced only among the Mongol ruling class and their Buddhist servants, who were called “Uighur.” In reflecting on this social situation, Muhyī al-Dīn labeled the “Chinese” calendar as the “Chinese-Uighur” calendar, and this title succeeded to the later *zīj*es, one of which became the first focus for analysis of the “Chinese” calendar. Thus, the calendar was attributed to the Uighurs, who surely played an important role in the nascent period of the Mongol empire.

It is natural that there are similarities between the Heavenly Agreement Calendar, the “Chinese” calendar in the *Zīj-i Īlkhānī*, and the Season-Granting Calendar, as several previous studies have noted (e.g., Imai, 37; Nakayama, 452), because, as clarified in this article, these calendars were all used or compiled in accordance with the same intellectual foundation, namely, that of China during the late Jin and early Yuan periods.

Studies on the Mongol empire have made remarkable progress (Jackson; Morgan 2004), and historical studies on science in this period have also advanced,¹ but the results of both have not necessarily reflected upon each other. It would give me great pleasure if this work served, even if only in a small way, to bridge the gap between these fields.

1. One of the representative researchers is, of course, Benno van Dalen, on whose works this study greatly relied.

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