A fuzzy multiobjective model for the total cost of logistics in the supplier selection problem in a supply chain

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Abstract

supplier selection problem has a multicriteria nature. Sometimes several conflicting criteria with different importance must be considered in this decision, moreover, uncertainty is not avoidable in real cases. In this paper, a fuzzy non-linear multiobjective model is formulated in such a way to handle multicriteria and uncertain nature of supplier selection problem. The model consists of three objective functions that should be solved while satisfying the constraints of supplier's capacity and buyer's demand: 1) minimizing the total cost of logistics (including the net price, storage, transportation, and ordering costs), 2) maximizing the quality and 3) maximizing the service level. In order to solve the proposed model, a fuzzy weighted additive and mixed integer non-linear programming is developed. The proposed model enables the decision-maker to assign different fuzzy weight to various criteria. By assuming total cost of logistics as an objective function, multi-period inventory management issues are considered. A numerical example is developed to clarify how the model can be applied. Finally, the contributions and recommendations for future researches are discussed.

Keywords
Supplier selection; Fuzzy MCDM; Total cost of logistics; weighted additive; Fuzzy AHP

1. Introduction

Supplier selection is one of the critical activities of purchasing departments. Because of the multicriteria nature of the supplier selection problem, different criteria which are sometimes conflict with each other, must be considered in this decision making process. In the real world decisions, the importance of various criteria are different depending on the purchasing strategies and most of the input information are uncertain. In these cases, fuzzy sets theory is a suitable tool for handling uncertainty. Fuzzy set theories are employed due to the presence of vagueness and imprecision of information in the supplier selection problem. Inventory lot-sizing and supplier choice are closely interrelated (Aissaoui et al. 2007). In this paper a fuzzy non-linear multiobjective model is developed for the supplier selection problem. Through this model, purchase managers can assign different fuzzy weights to the objectives to improve quality, service and reduce total cost of logistics to make improvement in the supply chain performance. By assuming total cost of logistics as a objective function, multi-period inventory management issues are considered, through this costs may be significantly reduced over the planning horizon.

The paper has the following structure. Section 2 presents a brief literature review of the quantitative approaches related to a supplier selection problem. Section 3 presents the suggested supplier selection model. Section 4 provides the necessary background of fuzzy multiobjective programming and weighted additive operator. Section 5 presents one of the FAHP methods for determining the criteria weights. Section 6 presents a general algorithm for solving the model introduced in section 3. Section 7 gives a numerical example and explains the results. Finally, section 8 is devoted to conclusions and recommendations.

2. Literature review

The Quantitative approaches can be divided into two groups: 1) Single objective and 2) multiple objectives. (Aissaoui et al. 2007) . In single objective models, only one criterion is considered as objective function and others are modeled as constraints so they will have the same weight which is an unrealistic assumption in many real world cases. The literature of supplier selection with multiple objective models is as follows:

Buffa and Jackson (1983) proposed a goal programming model for purchase planning. Sharma et al.(1989) proposed a goal programming formulation for attaining goals pertaining to price, quality and lead-time under demand and budget constraints. Weber and current (1993), introduced a multiobjective programming model for supplier selection, affirming that this alternative has several advantages over single objective analysis. Ghodsypour and O'Brien (1998) developed a decision support system by integrating the analytical hierarchy process (AHP) with linear programming. Karpak et al. (1999) used a goal programming model which minimizes the costs and maximizes the delivery reliability and the product quality in supplier selection by assigning the order quantities to each supplier. Ghodsypour and O'Brien

(2001) developed a non-linear multiobjective model for this problem which is minimizing the total cost of logistics (consisting of net the price, storage, ordering costs and transportation) and at the same time maximizing the product quality and on-time delivery. Cebi and Bayraktar (2003) proposed an integrated lexicographic goal programming and AHP model, including both qualitative and quantitative conflicting factors. Sanayei et al.(2008) proposed an integrated group decision-making process for supplier selection and order allocation using multi-attribute utility theory and linear programming. Demirtas et al.(2008) proposed an integrated approach of analytic network process (ANP) and multi-objective mixed integer linear programming (MOMILP) to consider both tangible and intangible factors in choosing the best suppliers and define the optimum quantities among selected suppliers. All the above models are deterministic and, therefore, unsuitable to obtain an effective solution for supplier selection problem in real cases due to the vagueness of the information available for the parameters.

There are some researches to handle uncertainty in the supplier selection models. Morlacchi (1997) developed a model based on this logic that combines fuzzy set theory with AHP to evaluate small suppliers in the engineering and machine sectors. In addition, Holt (1998) and Erol et al. (2003) discussed the application of fuzzy set theory in finding the best overall rated supplier among the existing set of suppliers. These studies deal with a single sourcing supplier selection problem in which one supplier can satisfy all the buyers' needs. Kumar et al (2004) proposed a fuzzy goal programming model for multiple sourcing supplier selection problems with three primary goals of minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries subject to the realistic constraints in buyer's demand and vendor's capacity. They used Zimmerman's weightless technique (1978) in their model which assumes that there is no difference between objective functions. Amid et al. (2006) proposed a fuzzy multiobjective linear model for supplier selection. In this model, they apply an asymmetric fuzzy decision making technique in order to enable the DM to assign different weights to various criteria. Chen et al. (2006) proposed a systematic approach to extend the TOPSIS to solve the supplier selection problem in a fuzzy environment. Amid et al. (2007) proposed a fuzzy multiobjective and mixed-integer linear programming model which takes the price breaks into account. Wang et al. (2009) proposed a fuzzy model for supplier selection in quantity discount environments . Boran et al.(2009) proposed TOPSIS method combined with intuitionistic fuzzy set to select appropriate supplier in group decision making environment.

In this paper, a fuzzy non-linear multiobjective model has been developed for the supplier selection problem dealing with unstructured relevant information and imprecise input data as well as considering different weights for the evaluation criteria and the correct structure of the purchasing costs (including the net price, storage, transportation, and ordering costs) at the same time. This fuzzy model enables the purchasing managers not only to resolve the imprecision of information and consider the correct structure of the purchasing costs but also to take the limitations of buyers and suppliers into account in order to calculate the order quantity assigned to each supplier.

3. A non-linear multiobjective supplier selection model

A general multiobjective model for the supplier selection problem can be formulated as follows (Amid et al., 2006):

$$Min \quad f_k(x_1, x_2, ..., x_n) \qquad k = 1, ..., a \qquad (1)$$

$$Max \quad f_l(x_1, x_2, ..., x_n) \qquad l = a+1, ..., q \qquad (2)$$

$$S.t:$$

$$x \in X \qquad (3)$$

$$X = \left\{ x \middle| g_i(x) = \sum_{i=1}^{n} a_{ri} x_i \le b_r, r = 1,2,...,m, x \ge 0 \right\}$$

Where $f_1, f_2, ..., f_a$ are the negative objectives or criteria-like cost, etc. and $f_{a+1}, f_{a+2}, ..., f_q$ are the positive objectives or criteria such as quality, on-time delivery, etc. X is the set of feasible solutions which satisfy the constraints such as buyer's demand, supplier's capacity, etc.

In this paper, a non-linear multiobjective model proposed by Ghodsypour and O'Brien (2001) is used as the foundation and then an algorithm is proposed to solve that model in the fuzzy environment. The main assumption is that the buyer would like to choose the best suppliers among the n vendors whose capacities are limited. It is also assumed that after the lot of the ith supplier is finished, the $i+1$th supplier's lot will be received. The inventory level of the problem with one supplier is shown in Figure1.

Figure 1 : Inventory level of the problem with one supplier

The model proposed by Ghodsypour and O'Brien (2001) consists of three goals which are minimization of the total cost of logistics (including the net price, storage, transportation and ordering costs), maximization of the quality, and maximization of the service level. These goals are subject to two sets of constraints which are supplier's capacity and buyer's demand.

Before describing the model the following notations are defined:

D: annual demand

Q: ordered quantity to all suppliers in each period

Q_i: ordered quantity to ith supplier in each period

T: length of each period

T_i: part of period in which the lot of ith supplier (Q_i) is used

r: inventory holding cost rate

X_i : percent of Q assigned to ith supplier

n : number of suppliers

A_i : ordering cost of ith supplier

P_i : price of ith supplier

C_i : annual capacity of ith supplier

q_i : perfect rate of ith supplier

q_a : minimum accepted perfect rate of incoming parts.

$$Y_i = \begin{cases} 0 & if \quad X_i = 0 \\ 1 & if \quad X_i \succ 0 \end{cases}$$

Based on this notation, the multiobjective model is as follows:

$$Min(Z_1) = (\sum_{i=1}^{n} A_i Y_i)\frac{D}{Q} + \frac{rQ}{2}(\sum_{i=1}^{n} X_i^2 P_i) + \sum_{i=1}^{n} P_i X_i D \quad (4)$$

$$Max(Z_2) = \sum_{i=1}^{n} q_i X_i \quad (5)$$

$$Max(Z_3) = \sum_{i=1}^{n} S_i X_i \quad (6)$$

$S.T$:

$$\sum_{i=1}^{n} q_i X_i \geq q_a \quad (7)$$

$$X_i D \leq C_i \quad (8)$$

$$X_i \leq Y_i \quad (9)$$

$$X_i \geq \varepsilon Y_i \quad (10)$$

$$\sum_{i=1}^{n} X_i = 1 \quad (11)$$

$$Q \succ 0 \quad (12)$$

$$X_i \geq 0, Y_i = 0,1, \quad i = 1,2,\ldots,n \quad (13)$$

After solving the problem, the optimum value for the Q, T, Q_i *and* T_i *will be calculated according to the following equations:*

$$T = \frac{Q}{D}, \quad Q_i = X_i Q, \quad T_i = X_i T \quad (14)$$

Ghodsypour and O'Brien (2001), in their model, assumed that Q *(the optimum order quantity) can be calculated by using the derivative of total cost's function according to the following equation:*

$$Q = \sqrt{\frac{2D\sum_{i=1}^{n} A_i Y_i}{r(\sum_{i=1}^{n} x_i^2 P_i)}} \quad (15)$$

Since the above model is a Multi-Objective Non-Linear Programming (MONLP), it is an incorrect assumption.

The weighting method for obtaining Pareto optimal solutions is to solve the following weighting problem formulated by taking the weighted sum of all the objective functions of the original MONLP (Sakawa, 1993).

$$Min \quad wf(x) \overset{\Delta}{=} \sum_{j=1}^{q} w_i f_i(x) \tag{16}$$

$$S.t: \quad x \in X \tag{17}$$

Where $w = (w_1, \ldots, w_q)$ *is the vector of weighting coefficients assigned to the objective functions, and assumed to be:*

$$w = (w_1, \ldots, w_q) \geq 0 \tag{18}$$

Each x^* *is a global optimum of Non-Linear Programming (NLP) if and only if satisfies Kuhn-Tucker necessary and sufficient conditions (according to the theorems 1,2).*

Theorem 1 (Kuhn-Tucker necessary conditions)

Let x^* *be a regular point of the constraints of NLP and assume all the functions* $f(x)$ *and* $g_j(x)$ *of the NLP are differentiable. If* x^* *is a local optimum of the NLP, then there exist Lagrange multipliers* $\lambda_i, j = 1, \ldots, r$ *such that:*

$$\nabla_x L(x, \lambda) = \nabla f(x) + \sum_{j=1}^{r} \lambda_j \nabla g_j(x) = 0, \tag{19}$$

$$\lambda_j g_j(x) = 0, \quad j = 1, \ldots, r, \tag{20}$$

$$\lambda_j \geq 0, \quad j = 1, \ldots, r, \tag{21}$$

Theorem 2 (Kuhn-Tucker sufficient conditions)

Let all the $f(x)$ *and* $g_j(x)$ *of the NLP be convex and differentiable. Suppose* x^* *satisfies Kuhn-Tucker conditions. Then* x^* *is a global optimum of NLP.*

In the other hand, simple derivative is not a correct way to calculate the optimum value of Q. *Here we use Kuhn-Tucker conditions to prove that Ghodsypour et al., (2001)'s assumption can be used under some considerations.*

By branching the integer variables (Y_i *which is a binary variable) and substituting their values in the programming, all the objectives become differentiable (In the model solution algorithm,* Y_i *will be omitted from the model). Based on the Kuhn-tucker necessary conditions (Eq(19),(20)), the following equalities will be correct:*

$$\sum_{j=1}^{3} w_j \frac{\partial Z_j}{\partial Q} + \sum_{i=1}^{n+5} \lambda_i \frac{\partial g_j}{\partial Q} = 0 \qquad \textit{According to the Eq}(19) \tag{22}$$

$$\lambda_{n+5} \times Q = 0 \qquad \textit{According to the Eq}(20) \tag{23}$$

In the above equalities, $w = (w_1, w_2, w_3)$ *is the vector of weighting coefficient assigned to the objective functions and assumed to be non-negative. The equality (22) is equivalent to the following one:*

$$w_1(\frac{r}{2}(\sum_{i=1}^{n} x_i^2 P_i) - \frac{D}{Q^2}(\sum_{i=1}^{n} A_i Y_i)) + \lambda_{n+5} = 0 \quad (24)$$

Inequality in constraint (12) guarantees that:

$$\lambda_{n+5} = 0 \quad (25)$$

In a real situation, supplier selection criteria will be determined after vast investigations. By assigning zero weight to any of the criteria, that criteria will be ignored and all the efforts made by DMs have gone to waste. So, assigning zero weight to any of the criteria is not reasonable and by assuming $w \succ 0$ *no practical pareto optimal solution will be omitted.*

Considering (24) and (25), it is evident that equation (15) is true.

In other words, by substituting for Q*, an efficient cut will be applied to the model without missing any practical pareto optimal solution. Considering equation (15) the model is converted to the following:*

$$Min\ (Z_1) = \sqrt{2Dr(\sum_{i=1}^{n} A_i Y_i)(\sum_{i=1}^{n} X_i^2 P_i)} + \sum_{i=1}^{n} P_i X_i D \quad (26)$$

$$Max\ (Z_2) = \sum_{i=1}^{n} q_i X_i \quad (27)$$

$$Max\ (Z_3) = \sum_{i=1}^{n} S_i X_i \quad (28)$$

$S.T:$

$$\sum_{i=1}^{n} q_i X_i \geq q_a \quad (29)$$

$$X_i D \leq C_i \quad (30)$$

$$X_i \leq Y_i \quad (31)$$

$$X_i \geq \varepsilon Y_i \quad (32)$$

$$\sum_{i=1}^{n} X_i = 1 \quad (33)$$

$$X_i \geq 0, Y_i = 0,1, \quad i = 1,2,...,n \quad (34)$$

After solving the problem, the optimum value for the Q, T, Q_i *and* T_i *will be calculated according to the following equations:*

$$T = \frac{Q}{D}, \quad Q = \sum_{i=1}^{n} Q_i, \quad Q_i = X_i Q, \quad Q = \sqrt{(2D\sum_{i=1}^{n} A_i Y_i)/r(\sum_{i=1}^{n} x_i^2 P_i)}, \quad T_i = X_i T \quad (35)$$

In real situations, all the objectives of the supplier selection problem might not be achieved simultaneously because of the system constraints. So the DM is better off to define a tolerance limit and membership function, $\mu(f_k(x))$ *for the* kth *fuzzy goal.*

4. A fuzzy multiobjective programming

Zimmerman (1978) developed a linear programming model for problems (1-3) with fuzzy goals and constraints. In this model, the goal is to find a vector $x_T = (x_1, x_2, ..., x_n)$ *which satisfies the following objectives and constraints:*

$$f_k(x_1, x_2, \ldots, x_n) \lesssim f_k^{'} \qquad k = 1, \ldots, a \tag{36}$$

$$f_l(x_1, x_2, \ldots, x_n) \gtrsim f_l^{o} \qquad l = a+1, \ldots, q \tag{37}$$

$S.t:$

$$g_r(x) = \sum_{i=1}^{n} a_{ri} x_i \lesssim b_r, r = 1, \ldots, h \tag{38}$$

$$g_p(x) = \sum_{i=1}^{n} a_{pi} x_i \leq b_p, r = h+1, \ldots, m \tag{39}$$

$$x_i \geq 0, i = 1, 2, \ldots, n \tag{40}$$

In the formulation (36-40), the symbol ~, indicates the fuzzy environment. It represents the linguistic term 'about' so the symbol $\leq\sim$ in the constraints denotes the statement ' about less or equal to' and the symbol $\geq\sim$ denotes the statement ' about more or equal to', and f_k^o, f_l^o are the aspiration levels that the DM wants to reach.

Using Zimmerman's (1978) approach, every objective function f_j is separated into maximum f_j^+ and minimum f_j^- value by solving:

$$f_k^+ = Max \ f_k, \ f_k^- = Min \ f_k \quad , x \in X \tag{41}$$

$$f_l^+ = Max \ f_l, \ f_l^- = Min \ f_l \quad , x \in X \tag{42}$$

where f_j^+, f_j^- are the individual optima of each objective and they are obtained through solving the multiobjective problem as a single objective by considering each time, only one objective and $x \in X$ indicates that solutions must satisfy the constraints.

He assumed that membership functions $\mu_{f_k}(x)$ are changed linearly from f_j^- to f_j^+. By this assumption the membership functions are as shown in Figure 2.

Figure 2: Objective functions as fuzzy number

By the linearity assumption, membership functions for the fuzzy objectives (minimization (f_k) and maximization goals (f_l)) are given as:

$$\mu_{f_k}(x) = \begin{cases} 1 & f_k \leq f_k^- \\ (f_k^+ - f_k(x))/(f_k^+ - f_k^-) & f_k^- \leq f_k(x) \leq f_k^+ \\ 0 & f_k(x) \geq f_k^+ \end{cases} \tag{43}$$

$$\mu_{f_l}(x) = \begin{cases} 1 & f_l \geq f_l^+ \\ (f_l(x) - f_l^-)/(f_l^+ - f_l^-) & f_l^- \leq f_l(x) \leq f_l^+ \\ 0 & f_l(x) \leq f_l^- \end{cases} \tag{44}$$

Also, the linear membership function for the fuzzy constraints is as follows:

$$\mu_{g_r} = \begin{cases} 1 & g_r(x) \le b_r \\ 1-(g_r(x)-b_r)/d_r & b_r \le g_r(x) \le b_r + d_r \\ 0 & g_r(x) \ge b_r + d_r \end{cases} \quad (45)$$

d_r is the subjectively chosen constants expressing the limit of the admissible violation of the rth inequality constraint (tolerance interval).

The main assumption in Zimmerman's model that there is no difference between fuzzy objective functions and fuzzy constraints is not met in the supplier selection problem. In this problem, fuzzy goals and fuzzy constraints have unequal importance to DM. So there is a need to develop an appropriate operator for decision making in fuzzy situations. In this case, a linear weighted utility function is obtained by multiplying each membership function of fuzzy goals by their corresponding weights and then adding the results together.

Tiwari et al. (1987) proposed a weighted additive model which is equivalent to the following convex single objective fuzzy model (Amid et al., 2006):

$$Max \sum_{j=1}^{q} w_j \lambda_j + \sum_{r=1}^{h} \beta_r \gamma_r \quad (46)$$

$$S.t:$$

$$\lambda_j \le \mu_{f_j}(x), \quad j = 1,2,\ldots,q \quad (47)$$

$$\gamma_r \le \mu_{g_r}(x), \quad r = 1,2,\ldots,h \quad (48)$$

$$g_p(x) \le b_p, \quad p = h+1,\ldots,m \quad (49)$$

$$\lambda_j, \gamma_r \in [0,1], \quad \sum_{j=1}^{q} w_j + \sum_{r=1}^{h} \beta_r = 1, \quad w_j, \beta_r \ge 0 \quad (50)$$

Where w_j and β_r are the weighting coefficients showing the relative importance of the fuzzy goals and constraints.

The relationship between the optimal solution of the above problem and the pareto optimal concept of the multi-objective non-linear programming (MONLP) be characterized by the following theorem:

Theorem 1:

If x^* is an optimal solution to the problem (46-50) with $0 < \mu_{f_j(x)}, \mu_{g_r(x)} < 1$ holding for all j, r, then x^* is a pareto optimal solution to the MONLP (Sakawa, 1993).

5. Mikhilov (2003)'s approach for fuzzy analytic hierarchy process

In real world decision problems, uncertainty is not avoidable because of the incomplete information or knowledge, inherent complexity and uncertainty within the decision environment, lack of an appropriate measure or scale. So it is sometimes unrealistic or even unfeasible to require the DM to assign exact numerical values to the comparison judgments, using fuzzy judgments instead of precise comparisons is more natural or realistic. In this paper, we use FAHP for deriving priorities from fuzzy pairwise comparison judgments. A number of methods have been developed to deal with fuzzy comparison judgments, here we use Mikhailov's (2003) approach for deriving crisp weights from fuzzy judgments. This approach has the following advantages:

It does not require the construction of fuzzy comparison matrices.

It can derive priorities from an incomplete set of fuzzy judgments.

It can be applied when some of the judgments are presented as intervals or crisp values.

It doesn't require using fuzzy ranking procedures in order to compare the final scores (Different ranking procedures, often give different ranking results). (Mikhailov ,2003).

Consider a prioritization problem with n elements, and suppose that the DM can provide a set $F=\{\tilde{a}_{ij}=(l_{ij},m_{ij},u_{ij})\}$ of $m \le \frac{n(n-1)}{2}$ fuzzy comparison judgments, $i=1,2,\dots,n-1$, $j=2,3,\dots,n$ $j>i$, which are represented as normal convex fuzzy sets or fuzzy numbers. The Mikhailov (2003)'s approach has the following steps:

By applying $\alpha-cuts$ the initial set of fuzzy comparisons $F=\{\tilde{a}_{ij}\}$ will be converted to series of L interval sets $F_l=\{a_{ij}(\alpha_l)=(l_{ij}(\alpha_l),u_{ij}(\alpha_l))\},\ l=1,2,\dots,L.$ where the bounds of $\alpha-cut$ intervals are:

$$l_{ij}(\alpha)=\alpha(m_{ij}-l_{ij})+l_{ij} \quad (51)$$

$$u_{ij}(\alpha)=\alpha(m_{ij}-u_{ij})+u_{ij} \quad (52)$$

By having $m \le \frac{n(n-1)}{2}$ interval pairwise comparison judgments F_l at the level $\alpha=\alpha_l$, and by solving the following linear programming, a sequence of crisp priorities $(w(\alpha_l)=(w_1(\alpha_l),w_2(\alpha_l),\dots,w_n(\alpha_l))^T,\ \ l=1,\dots,L,\ \ 0=\alpha_1 < \alpha_2 < \dots < \alpha_L=1$ will be obtained.

$$Max\ \lambda \quad (53)$$

$$d_k\lambda+(w_i-w_j u_{ij}(\alpha))\le d_k \quad (54)$$

$$d_{k'}\lambda+(-w_i+w_j u_{ij}(\alpha))\le d_{k'} \quad (55)$$

$$\sum_{i=1}^{n} w_i=1,\ w_i > 0,\ \ k=1,2,\dots,m\ \ k'=1,2,\dots,m \quad (56)$$

In the above linear programming d_k is a tolerance parameter, denoting the admissible interval of approximate satisfaction of the crisp equivalent inequalities. The tolerance parameters should be chosen large enough to ensure the non-emptiness of the feasible area and a positive value of λ^*. Mikhailov (2003) showed that values of these parameters greater than or equal to 1 satisfy such requirements. If the fuzzy judgments are symmetrical, it is reasonable for all tolerance parameters to be set equal to 1, since usually the DM has no preference about his individual pair wise comparison judgments. The optimal solution to the above linear program is a vector (w^*,λ^*), whose first component represents the crisp priority vector at the level $\alpha=\alpha_l$ whereas the second component is an indicator for the inconsistency of the DM's judgments. When the interval judgments are consistent, $\lambda^* \geq 1$, for inconsistent judgments the consistency index, λ^*, takes a value between 1 and 0 that depends on the degree of inconsistency and the value of the tolerance parameters $(d_k$.

The relative importance of all priorities is not the same and depends on the level of α . A small value of α yields a construction of interval judgments, having large spreads, which indicates a high level of uncertainty and correspondingly, less reliable priorities. These considerations suggest that the value of α can be used as a weighting factor of the solutions, so we can obtain aggregated values of the priorities by a weighted sum of the type:

$$W_j = \frac{\sum_{l=1}^{L} \alpha_l w_j(\alpha_l)}{\sum_{l=1}^{L} \alpha_l} \qquad (57)$$

6. Model solution algorithm

The model discussed in this paper is a mixed integer non-linear programming whereas the software packages for solving non-linear programming (Gino, Lindo/Lingo, Microsoft Excel Solver) can solve only pure non-linear programming with continuous variables. Ghodsypour and O' Brien (2001) developed an algorithm for a similar model which is applicable for our model by some changes. The proposed algorithm for the model solution is as follows:

1. Formulate the supplier selection model based on the general model and buyer-suppliers constraints.
2. Find individual optima of each objective (f_j^+, f_j^-), through solving the multiobjective problem as a single objective by considering only one objective, each time.
3. For the fuzzy objective functions and fuzzy constraints determine membership functions according to (43-45).
4. Use Appendix A's questioner to determine fuzzy objectives and constraints relative weights.
5. Derive crisp weights from fuzzy judgments by using Mikhailo's approach.
6. By using weighted additive operator & from results of previous steps ,formulate the equivalent crisp model according to (46-50).
7. Make a list of all the combinations of Y_is (at most 2^n times).
8. Omit the combinations which can not satisfy the demand constraint.
9. Substitute the values of Y_is in the integer programming to change it to PNP. If the set $\{S\}$ is defined as the set of Y_is for which their values are equal to one, the total cost objective function (f_1) will be equivalent to the following:

$$f_1 = \sqrt{2Dr(\sum_{i\in S} A_i)(\sum_{i=1}^{n} P_i X_i^2 D)} + \sum_{I=1}^{n} P_i X_i D) \qquad (58)$$

Use one of the above mentioned software packages to solve the PNPs and find the best solutions for each case. In general, the answer which is found by the software *package can be either the local or global optimum. In order to verify that the optimum solution is a global one, it is necessary to show that the constraints and objective functions are convex. The convexity of the model is discussed by Ghodsypour and O'Brien (2001).*

10. *Choose the best answer from all the feasible cases.*

7. Numerical example

Assume that three suppliers should be managed for one product. The purchasing criteria are total cost of logistics (including: net price, storage, transportation and ordering costs), quality and service. The capacity and demand constraints of suppliers also considered. It is assumed that the input data from supplier's performance on these criteria are not known precisely. The de-fuzzified values of their cost, quality and service and constraints of suppliers are presented in table 1. The minimum accepted perfect rate and the inventory holding cost rate are 0.97 and 0.2 respectively.

The demand is predicted to be 10000 and the demand constraint is a fuzzy one.

Table 1Collected data for numerical example

Supplier	Price	Ordering Cost	Quality	On-time del	Capacity
S1	5	9	0.95	0.94	5000
S2	6	8	1	0.92	6000
S3	2	4	0.98	0.99	4000

For simplicity, the solution will be considered according to the algorithm's steps.

Step 1:

Based on the suppliers information, the fuzzy multiobjective formulation of numerical example is presented as follows:

Find $x^T = (x_1, x_2, x_3)$ to satisfy:

$$\tilde{Z}_1 = \sqrt{4000\ (9Y_1 + 8Y_2 + 4Y_3) \times (5X_1^2 + 6X_2^2 + 2X_3^2)} + 10000 \times (5X_1 + 6X_2 + 2X_3) \tilde{\leq} Z_1^0$$

$$\tilde{Z}_2 = 0.95X_1 + X_2 + 0.98X_3 \tilde{\geq} Z_2^0$$

$$\tilde{Z}_3 = 0.94X_1 + 0.92X_2 + 0.99X_3 \tilde{\geq} Z_3^0$$

$$S.T:$$

$$0.95X_1 + X_2 + 0.98X_3 \geq 0.97$$

$$X_1 \leq 0.5 \quad , \quad \varepsilon Y_1 \leq X_1 \leq Y_1$$

$$X_2 \leq 0.6 \quad , \quad \varepsilon Y_2 \leq X_2 \leq Y_2$$

$$X_3 \leq 0.4 \quad , \quad \varepsilon Y_3 \leq X_3 \leq Y_3$$

$$X_1 + X_2 + X_3 \cong 1$$

$$X_i \geq 0 \quad , Y_i = 0,1$$

The three objective functions z_1, z_2, z_3 *are total cost, quality and service, respectively and* x_i *is percent of order quantity assigned to* i *th supplier.*

Table 2 The data set for the membership functions

	=0μ	=1μ	=0μ
Z_1 (total cost)	-	39948	56468
Z_2 (Quality)	0.96	0.99	-
Z_3 (Service)	0.93	0.96	-
D (Demand)	0.95	1	1.05

Figure 3
Membership functions

Step 2,3:
Individual optima of each objective (f_j^+, f_j^-), through solving the multiobjective problem as a single objective are listed in Table 2. Membership functions of fuzzy objective functions and fuzzy constraints are determined according to the(43-45)
Step 4,5:
Assume that for determining w_j , DM's linguistic comparisons are used. It is not possible to make mathematical operations directly on linguistic values. In the literature about FAHP, one can find a variety of different fuzzy scales. The triangular fuzzy conversion scale in Table 3 is used in this paper.

Table 3Triangular fuzzy conversion scale

Linguistic scale	TFS2[2]	TFRS[1]
Equally important	(1/2,1,3/2)	(2/3,1,2)
Weakly more important	(5/2,3,7/2)	(2/7,1/3,2/5)
Strongly more important	(9/2,5,11/2)	(2/11,1/5,2/9)
Very strongly more important	(13/2,7,15/2)	(2/15,1/7,2/13)
Absolutely more important	(17/2,9,19/2)	(2/19,1/9,2/17)

By using Index A 's questioner (to facilitate comparisons), fuzzy numbers which is equal to DM's fuzzy judgments are as follow:

$a_{23} = (1.5,2,2.5), a_{21} = (2.5,3,3.5), a_{24} = (3.5,4,4.5)$

$a_{31} = (2.5,3,3.5), a_{34} = (2.5,3,3.5), a_{14} = (1.5,2,2.5)$

For deriving the relative weights of the objectives and demand constraints, the following linear program must be solved. The optimal solution of the problem at each level $\alpha = a_l$ is showing in the table 4.

[1] . Triangular Fuzzy Reciprocal Scale
[2] . Triangular Fuzzy Scale

$$Max \quad Z = \lambda$$
$$S.t:$$
$$\lambda + w_2 - w_3(2.5 - 0.5\alpha) \le 1$$
$$\lambda - w_2 + w_3(1.5 + 0.5\alpha) \le 1$$
$$\lambda + w_2 - w_1(3.5 - 0.5\alpha) \le 1$$
$$\lambda - w_2 + w_1(2.5 + 0.5\alpha) \le 1$$
$$\lambda + w_2 - w_4(4.5 - 0.5\alpha) \le 1$$
$$\lambda - w_2 + w_4(3.5 + 0.5\alpha) \le 1$$
$$\lambda + w_3 - w_1(3.5 - 0.5\alpha) \le 1$$
$$\lambda - w_3 + w_1(2.5 + 0.5\alpha) \le 1$$
$$\lambda + w_3 - w_4(3.5 - 0.5\alpha) \le 1$$
$$\lambda - w_3 + w_4(2.5 + 0.5\alpha) \le 1$$
$$\lambda + w_1 - w_4(2.5 - 0.5\alpha) \le 1$$
$$\lambda - w_1 + w_4(1.5 + 0.5\alpha) \le 1$$
$$w_1 + w_2 + w_3 + w_4 = 1$$
$$w_i \succ 0, i = 1,..., 4$$

The relative weights obtained by aggregating the values of priorities for all $\alpha - cuts$ *are as follow:*

$$w_1 = 0.13, w_2 = 0.47, w_3 = 0.29, w_4 = 0.11$$

Table 4Solution of the model for each ?-cut level

?	w1(?)	w2(?)	w3(?)	w4(?)	?
0	0.4561	0.3142	0.1318	0.0980	0.9848
0.1	0.4600	0.3110	0.1306	0.0984	0.9780
0.2	0.4638	0.3078	0.1295	0.0989	0.9713
0.3	0.4668	0.3048	0.1286	0.0998	0.9640
0.4	0.4682	0.3017	0.1283	0.1018	0.9553
0.5	0.4695	0.2988	0.1280	0.1037	0.9466
0.6	0.4709	0.2959	0.1278	0.1054	0.9381
0.7	0.4722	0.2933	0.1276	0.1070	0.9297
0.8	0.4735	0.2906	0.1274	0.1085	0.9213
0.9	0.4749	0.2881	0.1272	0.1098	0.9130
1	0.4762	0.2857	0.1270	0.1111	0.9048

Step 6:

Based on the fuzzy membership functions (according to Fig.3 & Table2) and by using weighted additive operator, the crisp single objective formulation for this example is as follows:

$$Max\ \lambda = 0.13\lambda_1 + 0.47\lambda_2 + 0.29_3\lambda_3 + 0.11_4\gamma_1$$

$S.t:$

$$\lambda_1 \le \frac{56468 - Z_1}{16520}$$

$$\lambda_2 \le \frac{Z_2 - 0.97}{0.02}$$

$$\lambda_3 \le \frac{Z_3 - 0.93}{0.03}$$

$$\gamma_1 \le \frac{105 - 100g_1}{5}$$

$$\gamma_1 \le \frac{100g_1 - 95}{5}$$

$$0.95X_1 + X_2 + 0.98X_3 \ge 0.97$$

$$X_1 \le 0.5 \quad , \quad \varepsilon Y_1 \le x_1 \le Y_1$$

$$X_2 \le 0.6 \quad , \quad \varepsilon Y_2 \le x_2 \le Y_2$$

$$X_3 \le 0.4 \quad , \quad \varepsilon Y_3 \le x_3 \le Y_3$$

$$x_i \ge 0 \quad , Y_i = 0,1$$

$$Z_1 = \sqrt{4000(9Y_1 + 8Y_2 + 4Y_3)\times(5X_1^2 + 6X_2^2 + 2X_3^2)} + 10000 \times (5X_1 + 6X_2 + 2X_3)$$

$$Z_2 = 0.95X_1 + X_2 + 0.98X_3$$

$$Z_3 = 0.94X_1 + 0.92X_2 + 0.99X_3$$

$$g_1 = x_1 + x_2 + x_3$$

Step7. As three suppliers should be evaluated, there are $8(=2^3)$ possibilities of integer programming, which are listed in table 5.

Step 8. By omitting the cases which do not satisfy the demand constraint, only feasible cases must be considered.

Step 9. Substitute the values of Y_is in the integer programming problems and convert them into the PNP models.

Step 10. the optimal answers for all the feasible cases are shown in the Table 6.

Table 5Feasible and unfeasible cases (F=feasible, U=unfeasible)

Cases	Y_1	Y_2	Y_3	Capacities	Situation
1	1	1	1	15000	F
2	1	1	0	11000	F
3	0	1	1	10000	F
4	1	0	1	9000	U
5	0	0	1	4000	U
6	1	0	0	5000	U
7	0	1	0	6000	U
8	0	0	0	0	U

In order to solve these PNP the software package Lindo/Lingo is used. As table 6 shows, the optimum solution has occurred in the case one and it is:

$x_1 = 0.2097, x_2 = 0.399, x_3 = 0.4, q_1 = 141, q_2 = 283,$

$q_3 = 283, \lambda = 0.959, \mu_{z_2} = 1, \mu_{z_3} = 1, \mu_{g_1} = 0.83$

Table 6Optimum solution for the satisfied demand cases

	Case1	Case2	Case3
X1	0.209697	No Feasible Solution	0
X2	0.398788		0.6
X3	0.4		0.4
Z1	42766.38		44345.02
Z2	0.99		0.992
Z3	0.96		0.948
$?_1$	0.829		0.734
$?_2$	1.000		1.000
$?_3$	1.000		0.600
γ_1	0.830		1.000
?	0.959		0.849

In the solution, the level of each objective function 's satisfaction is consistent with the DM's preferences, in other words the below inequalities are correct:

$w_2 \; f w_3 \; f w_1 \quad , \quad \mu_{Z_2} \; f \mu_{Z_3} \; f \mu_{Z_1}$

Based on the DM's preferences, the proposed model has a competence to improve the value of objective functions or performance on the objectives. This model enables the purchasing managers to calculate order quantities to each supplier based on the priority of criteria in a supply chain.

8. Summary and conclusions

Supplier selection is one of the critical duties of purchasing managers. Because of the following reasons, supplier selection problem is one of the complicated problems:

1. *Several conflicting criteria (such as cost, quality, delivery, etc) must be considered.*
2. *The number of supplier selection's criteria and importance of them are under the influence of supply chain strategies.*
3. *Suppliers performance on the various assessing criteria are different.*
4. *May be some limitations are forced to the supply process by suppliers(such as capacity)*
5. *The total cost of purchasing (as opposed to just considering the price), must be considered in the decision making process. Total cost contains transportation, inspection, ordering and storage costs.*
6. *In real cases many of the input data are not known precisely.*

This paper has developed a non-linear multiobjective model to help managers in this decision making. To solve this model, one should run it 2^n times for n suppliers. This process does not take too long because in most practical cases there are usually a maximum of 12 vendors, And also because some cases become omitted, as they can not satisfy the demand constraint (Ghodsypour and O' Brien,2001)

The model not only can handle all the above mentioned aspects of the supplier selection problem but also has a number of advantages:

1. *The model can calculate the economic order quantities (EOQ) for both single and multiple sourcing with and without constraints.*
2. *The model enables the management to reflect the corporate strategies in the purchasing activities.*
3. *A schedule for deliveries can be provided, which tells buyer when and how much should be purchased from each supplier.*
4. *The fuzzy non-linear multiobjective supplier selection problem is transformed into its equivalent crisp non-linear single-objective. This model has less computational complexity, and makes the application of fuzzy methodology more understandable.*
5. *As the model can be solved using only Excel Solver, it is user-friendly and easy to apply by the purchasing management.*

Various types of discount, a case of multi-product, different strategies of ordering are still open for further investigations.

Appendix A.

Questionnaire forms used to facilitate comparisons of objectives and constraints

Questionnaire

Read the following questions and put check marks on the pairwise comparison matrices. If an objective on the left is more important than the one matching on the right, put your check mark to the left of the empty column, under the importance level you prefer. If an objective on the left is less important than the one matching on the right, put your check mark to the right of the empty column under the important level you prefer.

Questions

Q_1 : *How important is cost objective when it is compared with quality objective?*

Q_2 : *How important is cost objective when it is compared with service objective?*

Q_3 : *How important is cost objective when it is compared with demand constraint?*

Q_4 : *How important is quality objective when it is compared with service objective?*

Q_5 : *How important is quality objective when it is compared with demand constraint?*

Q_6 : *How important is service objective when it is compared with demand constraint?*

Importance (or preference) of one main-attribute over another

	Absolutely more important		Very strongly more important		Strongly more important		Weakly more important		Equally important		Equally important		Weakly more important		Strongly more important		Very strongly more important		Absolutely more important	
Cost													ü							Quality
Cost													ü							Service
Cost								ü												Demand
Quality								ü												Service
Quality						ü														Demand
Service							ü													Demand

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