

Capacity Region of the Compound Multiple Access Channels with Common Message and in the Presence of Intersymbol Interference

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Abstract—The capacity region of a two-user linear Gaussian compound Multiple Access Channel with common message (CMACC) and intersymbol interference (ISI) under an input power constraint is derived. To obtain the capacity region we first convert the channel to its equivalent memoryless one by defining an n -block memoryless circular Gaussian CMACC model. We then make use of the discrete Fourier transform (DFT) to decompose the n -block circular Gaussian CMACC into a set of independent parallel channels whose individual capacities can be found easily. Finally we derive the capacity of this n -block memoryless circular Gaussian CMACC (n -CGCMACC) based on the DFT decomposition. Since our channel is a special case of a synchronous multi-terminal channel, the capacity region of the Gaussian CMACC with ISI is the same as the capacity region of the n -CGCMACC in the limit of infinite block length. We also investigate the capacity regions for some special cases of the Gaussian CMACC with ISI using the obtained capacity region, and finally, provide some numerical results to show the loss in the rate caused by ISI.

Keywords—Capacity region; compound multiple access channels; Gaussian channels; intersymbol interference (ISI)

I. INTRODUCTION

The two-user compound multiple-access channel (CMAC) is a two-sender two-receiver communication channel in which two senders want to transmit messages to both receivers concurrently and each of the two receivers need to decode all messages sent from both senders [1]–[4]. In the CMAC two basic and important multi-user channels MAC (uplink channel) and broadcast channel (BC) (downlink

channel) can be seen concurrently. In other words, from the receivers standpoint the CMAC can be considered as a combination of two MACs, and from the senders standpoint it can be considered as two BCs. The CMAC was first studied by Maric *et al.* [1] where they established the capacity regions of CMAC with common message (MACC) and of CMAC with conferencing encoders. Simeone *et al.* [2] extended the channel models in [1] to the case with conferencing decoders and studied the CMACC with conferencing

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decoders and CMAC with both conferencing encoders and decoders. More recently, achievable rate regions have been provided for the CMACC with specially correlated channel states [3] and for the CMACC with generalized feedback [4].

One of the main causes of performance degradation in digital communication systems that makes the communication less reliable is intersymbol interference (ISI). ISI is an inevitable and undesirable phenomenon in which symbols interfere with the previous or subsequent transmitted symbols. Two major causes of the ISI are (i) multipath propagation and (ii) the passing of a signal through a bandlimited channel. The capacity characterization of the channel with ISI because of having memory is not so easy. The main idea, which until now has been the basis of all approaches for deriving the capacity regions of the various channels with ISI, is to convert the ISI channel to its equivalent memoryless one using n -block memoryless circular Gaussian channel model and then apply the discrete Fourier transform (DFT) to decompose the n -block channels into independent channels whose capacities can be found easily. This idea was first introduced by Hirt and Massey [5] where they employed this method to derive the capacity of a single-user discrete Gaussian channel with ISI. Cheng and Verdu [6] found the capacity region of a two-user linear Gaussian MAC with ISI. Goldsmith and Effros [7] derived the capacity region of a finite-memory BC with ISI and colored Gaussian noise and showed that this capacity region is equal to the capacity region of an n -circular Gaussian BC as n grows to infinity. They also proved a more general result that states that any synchronous multi-terminal channel and its circular approximation have the same capacity region. Recently, Choudhuri and Mitra [8] derived single-letter expressions for the achievable rates and an upper bound on the capacity of a relay channel with ISI and additive colored Gaussian noise.

In this paper we characterize the capacity region of the two-user Gaussian compound MAC with common message (GCMACC) and in the presence of ISI using the same approach that has been used to obtain the capacity of the single-user and synchronous multi-user channels with ISI [5]–[9]. We first define a similar channel model, n -block memoryless circular Gaussian CMACC. We next make use of the DFT to decomposed our circular Gaussian CMACC into a set of n -parallel, memoryless and independent scalar channels whose individual capacities are given by prior results. Then we derive the capacity of this n -block memoryless circular Gaussian CMACC based on the DFT decomposition. Since in the CMACC both senders are allowed to transmit a common message cooperatively, the channel is block (or frame)-synchronous and hence, we can utilize the obtained result in [7] for our synchronous two-user channel. Finally, we derive the capacity region of the original Gaussian CMACC using this fact that this capacity region equals that of the n -block circular Gaussian CMACC in the limit of infinite block length [5]–[9]. One of the benefits of studying the CMACC with ISI is that we can examine the capacity regions of the strong interference channel with common message (SICC) and ISI, of the MAC with ISI and with/without

common message and of the CMAC with unidirectional cooperation and ISI as special cases of it. The remainder of this paper is organized as follows. In Section II, we define the linear GCMACC with ISI and n -block circular GCMACC. The main result is presented in Section III. In Section IV, we discuss some corollaries of the main result. Some numerical results are provided in Section V to show the loss in the rate caused by ISI. The derived capacity region in Theorem 1 is proved in Section VI. Finally, a conclusion is prepared in Section VII.

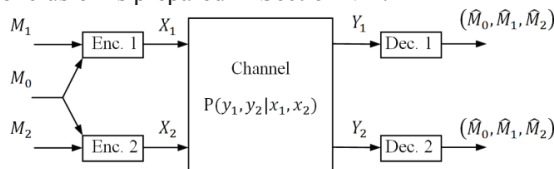


Fig. 1. Two-user compound MACC.

II. DEFINITIONS AND CHANNEL MODEL

We use notations and formulations similar to [7] and [8]. We use $*$ and \otimes to denote the linear and circular convolutions, respectively. $\langle A \rangle_n$ equals A modulo n except when A is zero or an integer multiple of n , in which case $\langle A \rangle_n = n$. We denote sequence (\dots, s_1, s_2, \dots) by $\{s\}$, subsequence (s_a, \dots, s_b) as $\{s_k\}_{k=a}^b$ and vector (s_1, \dots, s_n) by s^n . $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and the conjugate transpose of (\cdot) , respectively, and $\bar{\alpha} = 1 - \alpha$. Also, for a matrix A , $|A|$ denotes the absolute value of the determinant of A .

Definition 1: A two-user discrete memoryless CMACC (shown in Fig. 1) denoted by $\{\mathcal{X}_1 \times \mathcal{X}_2, P(y_1, y_2 | x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2\}$ consists of two input alphabets $\mathcal{X}_1, \mathcal{X}_2$, two output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$, and a transition probability $P(y_1, y_2 | x_1, x_2)$ which is a conditional channel probability of $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ given $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$. The channel is memoryless in the sense that

$$P(y_1^n, y_2^n | x_1^n, x_2^n) = \prod_{k=1}^n P(y_{1,k}, y_{2,k} | x_{1,k}, x_{2,k})$$

In this channel, encoder $i, i = 1, 2$, wants to send a private message $M_i \in \mathcal{M}_i = \{1, \dots, 2^{nR_i}\}$ along with a common message $M_0 \in \mathcal{M}_0 = \{1, \dots, 2^{nR_0}\}$ to both receivers. The common message M_0 and the private messages M_1, M_2 are assumed to be independently and uniformly generated over $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2$, respectively. Encoder $i, i = 1, 2$, maps the messages M_0 and M_i using encoding function $f_i(\cdot)$ into a codeword x_i^n as

$$f_i: \mathcal{M}_0 \times \mathcal{M}_i \rightarrow \mathcal{X}_i^n, \quad x_i^n(M_0, M_i) = (x_{i,1}, \dots, x_{i,n}).$$

Decoder $i, i = 1, 2$, using decoding function $g_i(\cdot)$

$$g_i: \mathcal{Y}_i^n \rightarrow \mathcal{M}_0 \times \mathcal{M}_1 \times \mathcal{M}_2,$$

estimates M_0, M_1 and M_2 based on received sequence y_i^n as $(\hat{M}_0, \hat{M}_1, \hat{M}_2) = g_i(y_i^n)$.

A $(2^{nR_0}, 2^{nR_1}, 2^{nR_2}, n, \epsilon)$ code for the CMACC consists of three message sets $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2$, two encoding functions $f_1(\cdot), f_2(\cdot)$, and two decoding functions $g_1(\cdot), g_2(\cdot)$, such that $\max(P_{e1}^{(n)}, P_{e2}^{(n)}) \leq \epsilon$,



where $P_{ei}^{(n)}$, $i = 1, 2$, denotes the average error probability at the decoder i and is defined as

$$P_{ei}^{(n)} = \frac{1}{2^{n(R_0+R_1+R_2)}} \times \sum_{M_0, M_1, M_2} p(g_i(y_i^n) \neq (M_0, M_1, M_2) | (M_0, M_1, M_2) \text{ sent})$$

A triple (R_0, R_1, R_2) of nonnegative real values is called an achievable rate for the CMACC if for any given $0 < \epsilon < 1$, and for any sufficiently large n , there exists a $(2^{nR_0}, 2^{nR_1}, 2^{nR_2}, n, \epsilon)$ code. The capacity region of the CMACC is the closure of all achievable rate triples (R_0, R_1, R_2) .

The capacity region of the CMACC, determined by Maric *et al.* [1], is

$$\begin{aligned} & \bigcup \{ (R_0, R_1, R_2) : R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \\ & R_1 \leq \min\{I(X_1; Y_1 | X_2, U), I(X_1; Y_2 | X_2, U)\} \\ & R_2 \leq \min\{I(X_2; Y_1 | X_1, U), I(X_2; Y_2 | X_1, U)\} \\ & R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1 | U), I(X_1, X_2; Y_2 | U)\} \\ & R_0 + R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1), I(X_1, X_2; Y_2)\} \} \end{aligned} \quad (1)$$

where the union is over all joint distributions that factor as

$$P(u, x_1, x_2, y_1, y_2) = P(u)P(x_1|u)P(x_2|u)P(y_1, y_2|x_1, x_2) \quad (2)$$

In this paper, we consider the discrete-time linear Gaussian CMACC with ISI shown in Fig. 2, where there are four different sets of ISI coefficients $\{h_{pq,t}\}_{t=0}^m$, $p, q \in \{1, 2\}$, one for each link. In this channel, the input sequences transmitted by user 1 and user 2 are given by $\{x_{1,k}\}$ and $\{x_{2,k}\}$, respectively. The output sequences produced by these input sequences are $\{y_{1,k}\}$ (at the first receiver) and $\{y_{2,k}\}$ (at the second receiver), with $(-\infty < k < \infty)$

$$\begin{aligned} y_{1,k} &= \sum_{t=0}^m (h_{11,t}x_{1,k-t} + h_{21,t}x_{2,k-t}) + z_{1,k} \\ &= h_{11,k} * x_{1,k} + h_{21,k} * x_{2,k} + z_{1,k} \end{aligned} \quad (3)$$

$$\begin{aligned} y_{2,k} &= \sum_{t=0}^m (h_{12,t}x_{1,k-t} + h_{22,t}x_{2,k-t}) + z_{2,k} \\ &= h_{12,k} * x_{1,k} + h_{22,k} * x_{2,k} + z_{2,k} \end{aligned} \quad (4)$$

where $\{z_{1,k}\}$ and $\{z_{2,k}\}$ are zero-mean stationary Gaussian noise processes with autocorrelation functions $R_1[t]$ and $R_2[t]$, respectively. These autocorrelation functions are assumed to have a common finite support t_{max} , i.e., $R_1[t] = R_2[t] = 0$ for $|t| \geq t_{max}$. We also assume that all channel impulse responses $\{h_{pq,t}\}_{t=0}^m$, $p, q \in \{1, 2\}$ have common memory length m . We only consider the case that $m \geq t_{max}$. For the case that $m < t_{max}$, the channel impulse responses can be zero padded to make them equal. Since the outputs are linear combinations of the inputs, for a given m , this channel is called the linear Gaussian CMACC (LGCMAcc) with finite memory m . Moreover, since the channel outputs at a time instance depend on the input symbols

of that time and also previous input symbols, the channels have ISI.

The input sequences are subjected to the following average power constraints for all n :

$$\frac{1}{n} \sum_{k=1}^n E[x_{q,k}^2] \leq P_q, \quad q \in \{1, 2\}. \quad (5)$$

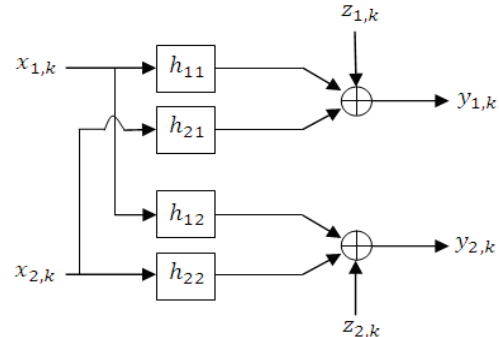


Fig. 2. Two-user linear Gaussian CMACC with ISI.

The noise power spectral densities of the channels are

$$N_q(\omega) = \sum_{t=-(m-1)}^{m-1} R_q[t]e^{-j\omega t}, \quad q \in \{1, 2\}. \quad (6)$$

The transfer functions of the channel links (the DFTs of channel impulse responses) are

$$H_{pq} = H_{pq}(\omega) = \sum_{t=0}^m h_{pq,t}e^{-j\omega t}, \quad (7)$$

which are periodic in ω with period 2π .

Block (or frame)-synchronism refers to the ability of the senders to transmit their codewords simultaneously. Since in the LGCMAcc both transmitters are allowed to send a common message cooperatively, the channel is block (or frame)-synchronous. Therefore, to compute the capacity region of the LGCMAcc with ISI, we modify our channel and define an equivalent n -block memoryless circular channel model called n -block circular Gaussian CMACC (n -CGCMAcc) for $n > m$. Thus, the capacity region of the LGCMAcc with ISI can be computed as the limit of the n -CGCMAcc, as n grows to infinity. Note that a CMACC is n -block memoryless if for any integer K

$$\begin{aligned} & P(y_1^{Kn}, y_2^{Kn} | x_1^{Kn}, x_2^{Kn}) = \\ & \prod_{k=1}^K P(y_{1,(k-1)n+1}^{kn}, y_{2,(k-1)n+1}^{kn} | x_{1,(k-1)n+1}^{kn}, x_{2,(k-1)n+1}^{kn}) \end{aligned} \quad (8)$$

In other words, in the n -block memoryless CMACC the outputs over any n -block are independent of channel inputs and noise samples of other n -blocks.

The n -CGCMAcc over every n -block has input vectors $\{x_{pq,k}\}_{k=1}^n$, $p, q \in \{1, 2\}$ which generate output vectors $\{\tilde{y}_{1,k}\}_{k=1}^n$ and $\{\tilde{y}_{2,k}\}_{k=1}^n$ at the first and second receivers, respectively, so that for $1 \leq k \leq n$

$$\begin{aligned} \tilde{y}_{1,k} &= \sum_{t=0}^{n-1} (\tilde{h}_{11,t} x_{1,(k-t)_n} + \tilde{h}_{21,t} x_{2,(k-t)_n}) + \tilde{z}_{1,k} \\ &= \tilde{h}_{11,k} \circledast x_{1,k} + \tilde{h}_{21,k} \circledast x_{2,k} + \tilde{z}_{1,k} \end{aligned} \quad (9)$$

$$\begin{aligned} \tilde{y}_{2,k} &= \sum_{t=0}^{n-1} (\tilde{h}_{12,t} x_{1,(k-t)_n} + \tilde{h}_{22,t} x_{2,(k-t)_n}) + \tilde{z}_{2,k} \\ &= \tilde{h}_{12,k} \circledast x_{1,k} + \tilde{h}_{22,k} \circledast x_{2,k} + \tilde{z}_{2,k} \end{aligned} \quad (10)$$

where $\{\tilde{h}_{pq,k}\}_{k=0}^{n-1} \triangleq (h_{pq,0}, h_{pq,1}, \dots, h_{pq,m}, 0, \dots, 0)$ for $p, q \in \{1,2\}$, i.e., $\{\tilde{h}_{pq,k}\}_{k=0}^{n-1}$ is an extended version of $\{h_{pq,k}\}_{k=0}^m$ which is extended with $(n-m-1)$ zeros. We can also rewrite (9) and (10) in the following vector forms:

$$\tilde{y}_1^n = \tilde{h}_{11}^n \mathbf{X}_1^c + \tilde{h}_{21}^n \mathbf{X}_2^c + \tilde{z}_1^n \quad (11)$$

$$\tilde{y}_2^n = \tilde{h}_{12}^n \mathbf{X}_1^c + \tilde{h}_{22}^n \mathbf{X}_2^c + \tilde{z}_2^n \quad (12)$$

where for $p, q \in \{1,2\}$, $\tilde{h}_{pq}^n = \{\tilde{h}_{pq,k}\}_{k=0}^{n-1}$, $\tilde{y}_q^n = (\tilde{y}_{q,1}, \dots, \tilde{y}_{q,n})$, $\tilde{z}_q^n = (\tilde{z}_{q,1}, \dots, \tilde{z}_{q,n})$ and \mathbf{X}_q^c is the circulant matrix of the input vector x_q^n , whose first row is defined as $\mathbf{X}_q^c(1, :) = [x_{q,1}, \dots, x_{q,n}]$, and each row vector is rotated one element to the right relative to the preceding row vector. Note that in (9)-(12), the channel impulse responses are fixed vectors and input vectors are circular. We can obtain similar results by considering the fixed input vectors and circular channel impulse response vectors as follows.

$$(\tilde{y}_1^n)^T = \mathbf{H}_{11}^c (x_1^n)^T + \mathbf{H}_{21}^c (x_2^n)^T + (\tilde{z}_1^n)^T \quad (13)$$

$$(\tilde{y}_2^n)^T = \mathbf{H}_{12}^c (x_1^n)^T + \mathbf{H}_{22}^c (x_2^n)^T + (\tilde{z}_2^n)^T \quad (14)$$

where for $p, q \in \{1,2\}$, \tilde{y}_q^n and \tilde{z}_q^n are defined as above, $\tilde{x}_q^n = (\tilde{x}_{q,1}, \dots, \tilde{x}_{q,n})$, and \mathbf{H}_{pq}^c is the circulant channel matrix between the transmitter p and the receiver q , whose first row is defined as $\mathbf{H}_{pq}^c(1, :) = [h_{pq,0}, 0, \dots, 0, h_{pq,m}, \dots, h_{pq,2}, h_{pq,1}]$. In this paper we utilize (9) and (10) to derive the capacity region. The only difference between the circular channel model (9)-(10) and the linear channel model (3)-(4) is that the channel outputs in (9) and (10) are the circular convolutions of the input codewords with the channel impulse responses, whereas the channel outputs in (3) and (4) are the linear ones. Since the n -CGCMACC is an n -block memoryless channel, the noise samples $\tilde{z}_{1,k}$ and $\tilde{z}_{2,k}$ are n -block independent with the same means and variances as $z_{1,k}$ and $z_{2,k}$, respectively, which their autocorrelation functions $\tilde{R}_1[t]$ and $\tilde{R}_2[t]$ are periodic repetitions of $R_1[t]$ and $R_2[t]$, respectively, for noise samples within an n -block. The same average power constraints (5) are assumed for n -CGCMACC. Note that the n -CGCMACC is not l -block memoryless if l is not an integer multiple of n .

We now decompose the n -CGCMACC defined in (9) and (10) into a set of n -parallel, memoryless and independent scalar GCMACC channels in DFT domain which the capacities of these independent channels can be found easily by prior results. It is worth noting that, due to the invertibility of the DFT, applying the DFT to (9) and (10) does not affect the corresponding capacity region. So, after applying the DFT to both sides of (9) and (10) we obtain

$$\tilde{Y}_{1,k} = \tilde{H}_{11,k} X_{1,k} + \tilde{H}_{21,k} X_{2,k} + \tilde{Z}_{1,k} \quad (15)$$

$$\tilde{Y}_{2,k} = \tilde{H}_{12,k} X_{1,k} + \tilde{H}_{22,k} X_{2,k} + \tilde{Z}_{2,k} \quad (16)$$

where for $1 \leq k \leq n$ and $p, q \in \{1,2\}$, $\tilde{Y}_{q,k}$, $\tilde{H}_{pq,k}$, $X_{q,k}$, and $\tilde{Z}_{q,k}$ are the DFTs of $\tilde{y}_{q,k}$, $\tilde{h}_{pq,k}$, $x_{q,k}$, and $\tilde{z}_{q,k}$, respectively. Consequently, the n -CGCMACC is equivalent to a collection of n parallel CMACCs that the k th-component channel is as shown in Fig. 3.

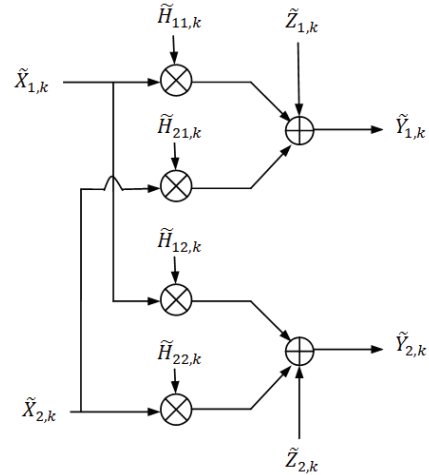


Fig. 3. The k th-component channel.

III. MAIN RESULT: THE CAPACITY REGION OF THE LINEAR GAUSSIAN COMPOUND MAC WITH COMMON MESSAGE AND ISI

In this section, we obtain the capacity region of the n -CGCMACC which is the same as the capacity region of the LGCMACC with ISI in the limit of infinite block length.

Let \mathcal{C} and \mathcal{C}_n denote the capacity regions of the LGCMACC with ISI, and of the n -CGCMACC, respectively. Since our channel is a special case of a synchronous multi-terminal channel, we can utilize the results in [7] and [9] and obtain the capacity region of the LGCMACC with ISI, which is the same as the capacity region of the n -CGCMACC in the limit of infinite block length. Note that by considering the time-sharing principle [5], the capacity region of the block-synchronous CMACC with finite memory is a convex set. Applying the results in [3], [5]-[9] we have:

$$\mathcal{C} = \text{closure} \left(\liminf_{n \rightarrow \infty} \mathcal{C}_n \right) = \text{closure} \left(\limsup_{n \rightarrow \infty} \mathcal{C}_n \right) \quad (17)$$

where $\mathcal{C}_n = \mathcal{C}_n(P_1, P_2)$ is

$$\begin{aligned} & \bigcup \{ (R_0, R_1, R_2) : R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \\ & R_1 \leq \min \frac{1}{n} \{ I(x_1^n; y_1^n | x_2^n, u^n), I(x_1^n; y_2^n | x_2^n, u^n) \} \\ & R_2 \leq \min \frac{1}{n} \{ I(x_2^n; y_1^n | x_1^n, u^n), I(x_2^n; y_2^n | x_1^n, u^n) \} \\ & R_1 + R_2 \leq \min \frac{1}{n} \{ I(x_1^n, x_2^n; y_1^n | u^n), I(x_1^n, x_2^n; y_2^n | u^n) \} \\ & R_0 + R_1 + R_2 \leq \min \frac{1}{n} \{ I(x_1^n, x_2^n; y_1^n), I(x_1^n, x_2^n; y_2^n) \} \} \end{aligned} \quad (18)$$



where the union is over all input vectors x_1^n and x_2^n subjected to the average power constraints (5). Indeed since the n -CGCMACC defined in (9)-(10) is an n -block memoryless CMACC, its capacity region, i.e., (18), follows directly from (1) if we replace (X_1, X_2, U, Y_1, Y_2) by $(x_1^n, x_2^n, u^n, y_1^n, y_2^n)$. The auxiliary random variable U denotes the common message.

Theorem 1: The capacity region of the two-user linear Gaussian CMAC with common message and ISI is given by

$$\mathcal{C} = \mathcal{C}(P_1, P_2) = \bigcup_{0 \leq \alpha_q(\omega) \leq 1} \left\{ \begin{array}{l} (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \\ R_1 \leq \min\{\mathbb{I}_1, \mathbb{I}_2\} \\ R_2 \leq \min\{\mathbb{I}_3, \mathbb{I}_4\} \\ R_1 + R_2 \leq \min\{\mathbb{I}_5, \mathbb{I}_6\} \\ R_0 + R_1 + R_2 \leq \min\{\mathbb{I}_7, \mathbb{I}_8\} \end{array} \right\} \quad (19)$$

where the terms $\mathbb{I}_i, i \in \{1, \dots, 8\}$ are defined as (20), at the bottom of the page. The union is over power allocation across all the parallel sub-channels. Moreover, $\Psi(x) = \log(1 + x)$.

Proof: Refer to Section V.

IV. COROLLARIES OF MAIN RESULT

In this section, we discuss some special cases of the derived capacity region to demonstrate the breadth of our main result.

A. Compound MAC with ISI

Corollary 1: It is easy to show that the capacity region of the synchronous CMAC with ISI but without common message is similar to (19) provided that we let $\alpha_1 = \alpha_2 = 0$.

$$\begin{aligned} \mathbb{I}_1 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_1(\omega) P_1(\omega) |\tilde{H}_{11}(\omega)|^2}{\tilde{N}_1(\omega)} \right) d\omega \\ \mathbb{I}_2 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_1(\omega) P_1(\omega) |\tilde{H}_{12}(\omega)|^2}{\tilde{N}_2(\omega)} \right) d\omega \\ \mathbb{I}_3 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_2(\omega) P_2(\omega) |\tilde{H}_{21}(\omega)|^2}{\tilde{N}_1(\omega)} \right) d\omega \\ \mathbb{I}_4 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_2(\omega) P_2(\omega) |\tilde{H}_{22}(\omega)|^2}{\tilde{N}_2(\omega)} \right) d\omega \\ \mathbb{I}_5 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_1(\omega) P_1(\omega) |\tilde{H}_{11}(\omega)|^2 + \bar{\alpha}_2(\omega) P_2(\omega) |\tilde{H}_{21}(\omega)|^2}{\tilde{N}_1(\omega)} \right) d\omega \\ \mathbb{I}_6 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_1(\omega) P_1(\omega) |\tilde{H}_{12}(\omega)|^2 + \bar{\alpha}_2(\omega) P_2(\omega) |\tilde{H}_{22}(\omega)|^2}{\tilde{N}_2(\omega)} \right) d\omega \\ \mathbb{I}_7 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{P_1(\omega) |\tilde{H}_{11}(\omega)|^2 + P_2(\omega) |\tilde{H}_{21}(\omega)|^2 + \sqrt{\alpha_1(\omega) \alpha_2(\omega) P_1(\omega) P_2(\omega)} (2\text{Re}\{\tilde{H}_{11}(\omega) \tilde{H}_{21}^*(\omega)\})}{\tilde{N}_1(\omega)} \right) d\omega \\ \mathbb{I}_8 &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{P_1(\omega) |\tilde{H}_{12}(\omega)|^2 + P_2(\omega) |\tilde{H}_{22}(\omega)|^2 + \sqrt{\alpha_1(\omega) \alpha_2(\omega) P_1(\omega) P_2(\omega)} (2\text{Re}\{\tilde{H}_{12}(\omega) \tilde{H}_{22}^*(\omega)\})}{\tilde{N}_2(\omega)} \right) d\omega \end{aligned} \quad (20)$$

B. MAC with ISI

As we mentioned above, in the CMAC each receiver sees a MAC. Therefore, we can study the MAC by considering one of the receivers.

Corollary 2: Without loss of generality, assume that $Y_2 = \emptyset$. Then similar to Theorem 1 we can show that the capacity region of the two-user linear Gaussian MAC with common message and ISI is given by

$$\mathcal{C} = \mathcal{C}(P_1, P_2) = \bigcup_{0 \leq \alpha_q(\omega) \leq 1} \left\{ \begin{array}{l} (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \\ R_1 \leq \mathbb{I}_1 \\ R_2 \leq \mathbb{I}_3 \\ R_1 + R_2 \leq \mathbb{I}_5 \\ R_0 + R_1 + R_2 \leq \mathbb{I}_7 \end{array} \right\} \quad (21)$$

where the terms $\mathbb{I}_i, i \in \{1, 3, 5, 7\}$ are defined as (20). Moreover, by setting $\alpha_1 = \alpha_2 = 0$ in Corollary 2, the capacity region of the synchronous MAC with ISI but without common message is derived from (21).

C. Compound MAC with Unidirectional Cooperation and ISI

Consider a special case of the CMACC in which one transmitter does not have private message. Without loss of generality, we assume that transmitter 1 only has the common message M_0 to send, while transmitter 2 needs to transmit both the common message M_0 and the private message M_2 . We call this channel model the CMAC with unidirectional cooperation or the CMAC with degraded message sets.

Theorem 2: An achievable rate region for the CMAC with unidirectional cooperation and ISI is given by

$$\mathcal{R}^{uni} = \mathcal{R}^{uni}(P_1, P_2) = \bigcup_{0 \leq \alpha_q(\omega) \leq 1} \left\{ \begin{array}{l} (R_0, R_2): R_0 \geq 0, R_2 \geq 0, \\ R_2 \leq \min\{\mathbb{I}_1^{uni}, \mathbb{I}_2^{uni}\} \\ R_0 + R_2 \leq \min\{\mathbb{I}_3^{uni}, \mathbb{I}_4^{uni}\} \end{array} \right\} \quad (22)$$

$\frac{1}{2\pi} \int_{-\pi}^{\pi} P_q(\omega) d\omega \leq P_q, q=1,2.$

where the terms $\mathbb{I}_i^{uni}, i \in \{1, \dots, 4\}$ are defined as (23), at the top of the next page.

Proof: The proof follows immediately by setting $\alpha_1 = 1, \bar{\alpha}_1 = 0$ in Theorem 1 and removing redundant ones from the resulting inequalities.

D. Strong Interference Channel with Common Message and ISI

We now consider the Gaussian strong interference channel with common message (GSICC) [7] and in the presence of ISI. Similar above, we can define linear GSICC (LGSICC) with ISI, and its equivalent n -block memoryless circular channel model called the n -block circular Gaussian SICC (n -CGSICC), for $n > m$. In fact, an n -CGCMACC is an n -CGSICC if

$$I(x_1^n; y_1^n | x_2^n, u^n) \leq I(x_1^n; y_2^n | x_2^n, u^n) \quad (24)$$

$$I(x_2^n; y_2^n | x_1^n, u^n) \leq I(x_2^n; y_1^n | x_1^n, u^n) \quad (25)$$

Theorem 3: The capacity region of the two-user linear Gaussian strong interference channel with common message and ISI is given by (26), where the terms $\mathbb{I}_i, i \in \{1,4,5,6,7,8\}$ are defined as (20), at the bottom of the previous page.

Proof: The proof is the same as the proof of Theorem 1 but with this difference that by applying the conditions (24) and (25) to (18), we have $\min\{\mathbb{I}_1, \mathbb{I}_2\} = \mathbb{I}_1$ and $\min\{\mathbb{I}_3, \mathbb{I}_4\} = \mathbb{I}_4$.

$$\mathcal{C} = \mathcal{C}(P_1, P_2) = \bigcup_{0 \leq \alpha_q(\omega) \leq 1} \left\{ \begin{array}{l} (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \\ R_1 \leq \mathbb{I}_1 \\ R_2 \leq \mathbb{I}_4 \\ R_1 + R_2 \leq \min\{\mathbb{I}_5, \mathbb{I}_6\} \\ R_0 + R_1 + R_2 \leq \min\{\mathbb{I}_7, \mathbb{I}_8\} \end{array} \right\} \quad (26)$$

$\frac{1}{2\pi} \int_{-\pi}^{\pi} P_q(\omega) d\omega \leq P_q, q=1,2.$

V. SIMULATION RESULTS

In this section, we consider the following examples representing different levels of ISI to show the loss in the rate caused by ISI.

- 1) $h_{11} : [1 + 0.2e^{-j\omega}], \quad h_{21} : [1 + 0.3e^{-j\omega}]$
 $h_{12} : [1 + 0.3e^{-j\omega}], \quad h_{22} : [1 + 0.2e^{-j\omega}]$
- 2) $h_{11} : [1 + 0.9e^{-j\omega}], \quad h_{21} : [1 + 0.8e^{-j\omega}]$

$$h_{12} : [1 + 0.8e^{-j\omega}], \quad h_{22} : [1 + 0.9e^{-j\omega}]$$

In these examples, we assume that the transmitters are subjected to the average power constraints $P_1 = 4, P_2 = 5$, the white Gaussian noises have unit variance, and the impulse responses of all channels are normalized to have unit energy, i.e.,

- 1) $H_{11}(\omega) = \frac{1+0.2e^{-j\omega}}{\sqrt{1.04}}, \quad H_{21}(\omega) = \frac{1+0.3e^{-j\omega}}{\sqrt{1.09}}$
 $H_{12}(\omega) = \frac{1+0.3e^{-j\omega}}{\sqrt{1.09}}, \quad H_{22}(\omega) = \frac{1+0.2e^{-j\omega}}{\sqrt{1.04}}$
- 2) $H_{11}(\omega) = \frac{1+0.9e^{-j\omega}}{\sqrt{1.81}}, \quad H_{21}(\omega) = \frac{1+0.8e^{-j\omega}}{\sqrt{1.64}}$
 $H_{12}(\omega) = \frac{1+0.8e^{-j\omega}}{\sqrt{1.64}}, \quad H_{22}(\omega) = \frac{1+0.9e^{-j\omega}}{\sqrt{1.81}}$

Intersymbol interference in example 1 is mild while in example 2 it is stronger. Fig. 4 shows the rate regions of the channels in examples 1 and 2 as well as the rate region of the channel without ISI. It is obvious that with fixed energy for the transfer functions, the rate region decreases as ISI increases as shown in Fig. 4. Note that since the comparison of the regions in a 3-dimensional plot (in terms of (R_0, R_1, R_2)) is difficult to illustrate, we have used Corollary 1 to have a better comparison of the regions in a 2-dimensional plot (in terms of (R_1, R_2)). In fact, we have compared the regions for the Gaussian CMAC with ISI but without common message (i.e., $R_0 = 0$).

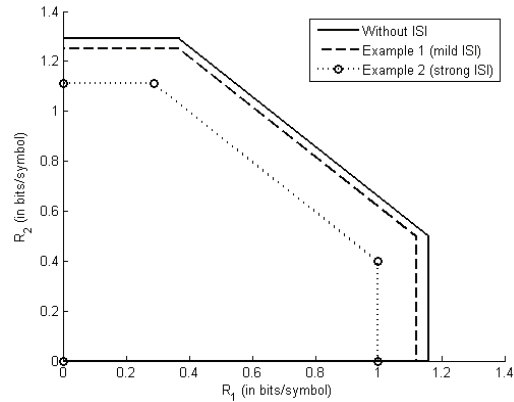


Fig. 4. Comparison of the rate regions of the channels in examples 1 and 2 as well as of the channel without ISI.

VI. PROOF OF THEOREM 1

Let $W_0, W_1,$ and W_2 be Gaussian random variables distributed according to $\mathcal{CN}(0,1)$. Considering the joint probability distribution (1) we define the following mappings.

$$U_k = (\sqrt{\alpha_1 P_1} + \sqrt{\alpha_2 P_2}) W_0$$

$$X_{1,k} = \sqrt{\alpha_1 P_1} W_0 + \sqrt{\bar{\alpha}_1 P_1} W_1$$

$$X_{2,k} = \sqrt{\alpha_2 P_2} W_0 + \sqrt{\bar{\alpha}_2 P_2} W_2$$

where $\alpha_q \in [0,1]$ and $\bar{\alpha}_q = 1 - \alpha_q$ for $q \in \{1,2\}$.



$$\begin{aligned}
 \mathbb{I}_1^{\text{uni}} &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_2(\omega) P_2(\omega) |\tilde{H}_{21}(\omega)|^2}{\tilde{N}_1(\omega)} \right) d\omega \\
 \mathbb{I}_2^{\text{uni}} &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{\bar{\alpha}_2(\omega) P_2(\omega) |\tilde{H}_{22}(\omega)|^2}{\tilde{N}_2(\omega)} \right) d\omega \\
 \mathbb{I}_3^{\text{uni}} &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{P_1(\omega) |\tilde{H}_{11}(\omega)|^2 + P_2(\omega) |\tilde{H}_{21}(\omega)|^2 + \sqrt{\alpha_2(\omega) P_1(\omega) P_2(\omega)} (2\text{Re}\{\tilde{H}_{11}(\omega) \tilde{H}_{21}^*(\omega)\})}{\tilde{N}_1(\omega)} \right) d\omega \\
 \mathbb{I}_4^{\text{uni}} &= \frac{1}{4\pi} \int_{-\pi}^{\pi} \Psi \left(\frac{P_1(\omega) |\tilde{H}_{12}(\omega)|^2 + P_2(\omega) |\tilde{H}_{22}(\omega)|^2 + \sqrt{\alpha_2(\omega) P_1(\omega) P_2(\omega)} (2\text{Re}\{\tilde{H}_{12}(\omega) \tilde{H}_{22}^*(\omega)\})}{\tilde{N}_2(\omega)} \right) d\omega
 \end{aligned} \tag{23}$$

By using these mappings, and considering the channel model described by (15) and (16), we obtain

$$\begin{aligned}
 \tilde{Y}_{1,k} &= \tilde{H}_{11,k} X_{1,k} + \tilde{H}_{21,k} X_{2,k} + \tilde{Z}_{1,k} \\
 &= \tilde{H}_{11,k} (\sqrt{\alpha_1 P_1} W_0 + \sqrt{\bar{\alpha}_1 P_1} W_1) \\
 &\quad + \tilde{H}_{21,k} (\sqrt{\alpha_2 P_2} W_0 + \sqrt{\bar{\alpha}_2 P_2} W_2) + \tilde{Z}_{1,k}
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 \tilde{Y}_{2,k} &= \tilde{H}_{12,k} X_{1,k} + \tilde{H}_{22,k} X_{2,k} + \tilde{Z}_{2,k} \\
 &= \tilde{H}_{12,k} (\sqrt{\alpha_1 P_1} W_0 + \sqrt{\bar{\alpha}_1 P_1} W_1) \\
 &\quad + \tilde{H}_{22,k} (\sqrt{\alpha_2 P_2} W_0 + \sqrt{\bar{\alpha}_2 P_2} W_2) + \tilde{Z}_{2,k}
 \end{aligned} \tag{28}$$

By the invertibility of the DFT and considering (27) and (28), we evaluate the mutual information terms in (18). Let us choose $I(x_1^n; y_1^n | x_2^n, u^n)$ and $I(x_1^n, x_2^n; y_2^n | u^n)$ for evaluation. The other terms can be evaluated similarly. It is worth noting that for any real sequence d^n , its DFT D^n has the property that $D_k = D_{n-k}^*$, $1 \leq k \leq n$, where D^* denotes the complex conjugate of D . Thus, without losing any information, we can reconstruct the entire sequence d^n using the DFT terms $\{D_1, \dots, D_l\}$, where $l = \lfloor \frac{n}{2} \rfloor$. Therefore, we have:

$$\begin{aligned}
 \blacksquare I(x_1^n; y_1^n | x_2^n, u^n) &= I(X_1^l; \tilde{Y}_1^l | X_2^l, U^l) \\
 &= \sum_{k=1}^l I(X_{1,k}; \tilde{Y}_{1,k} | X_{2,k}, U_k) \\
 &= \sum_{k=1}^l \{h(\tilde{Y}_{1,k} | X_{2,k}, U_k) - h(\tilde{Y}_{1,k} | X_{1,k}, X_{2,k}, U_k)\} \\
 &= \sum_{k=1}^l \frac{1}{2} \log \left(\frac{|\text{cov}(\tilde{H}_{11,k}(\sqrt{\alpha_1 P_1} W_1) + \tilde{Z}_{1,k})|}{|\text{cov}(\tilde{Z}_{1,k})|} \right) \\
 &= \sum_{k=1}^l \frac{1}{2} \log \left(\frac{(\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{11}(\omega_k)|^2 + \tilde{N}_1(\omega_k))}{\tilde{N}_1(\omega_k)} \right) \\
 &= \sum_{k=1}^l \frac{1}{2} \log \left(1 + \frac{\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{11}(\omega_k)|^2}{\tilde{N}_1(\omega_k)} \right) \\
 &= \sum_{k=1}^l \frac{1}{2} \Psi \left(\frac{\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{11}(\omega_k)|^2}{\tilde{N}_1(\omega_k)} \right) \\
 \blacksquare I(x_1^n, x_2^n; y_2^n | u^n) &= I(X_1^l, X_2^l; \tilde{Y}_2^l | U^l) \\
 &= \sum_{k=1}^l I(X_{1,k}, X_{2,k}; \tilde{Y}_{2,k} | U_k)
 \end{aligned}$$

$$\begin{aligned}
 &= \sum_{k=1}^l \{h(\tilde{Y}_{2,k} | U_k) - h(\tilde{Y}_{2,k} | X_{1,k}, X_{2,k}, U_k)\} \\
 &= \sum_{k=1}^l \frac{1}{2} \log \left(\frac{|\text{cov}(\tilde{H}_{12,k}(\sqrt{\alpha_1 P_1} W_1) + \tilde{H}_{22,k}(\sqrt{\alpha_2 P_2} W_2) + \tilde{Z}_{2,k})|}{|\text{cov}(\tilde{Z}_{2,k})|} \right) \\
 &= \sum_{k=1}^l \frac{1}{2} \times \\
 &\Psi \left(\frac{(\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{12}(\omega_k)|^2 + \bar{\alpha}_2(\omega_k) P_2(\omega_k) |\tilde{H}_{22}(\omega_k)|^2)}{\tilde{N}_2(\omega_k)} \right)
 \end{aligned}$$

Therefore, $C_n(P_1, P_2)$ can be expressed as

$$\begin{aligned}
 C_n(P_1, P_2) &= \bigcup_{0 \leq \alpha_q(\omega_k) \leq 1} \\
 &\quad \frac{1}{n} \sum_{\substack{P_q(\omega_k) \leq P_q, q=1,2}} \\
 &\left\{ (R_0, R_1, R_2): \begin{aligned} &R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \\ &R_1 \leq \min\{\mathbb{T}_1, \mathbb{T}_2\} \\ &R_2 \leq \min\{\mathbb{T}_3, \mathbb{T}_4\} \\ &R_1 + R_2 \leq \min\{\mathbb{T}_5, \mathbb{T}_6\} \\ &R_0 + R_1 + R_2 \leq \min\{\mathbb{T}_7, \mathbb{T}_8\} \end{aligned} \right\}
 \end{aligned} \tag{29}$$

where the terms $\mathbb{T}_i, i \in \{1, \dots, 8\}$ are defined as (30), at the top of the next page. The $\tilde{H}_{pq}(\omega_k)$, for $p, q \in \{1, 2\}$, is the transfer function of the k th-component channel between the transmitter p and the receiver q ; the $\tilde{N}_q(\omega_k)$ is the noise power spectral density of the k th-component channel of the receiver q . The $P_q(\omega_k)$ is the total power allocated to the k th-component channel by the user q , and $\alpha_q(\omega_k)$ is the fraction of $P_q(\omega_k)$ allocated to the user q on channel k for common message, and $\bar{\alpha}_q(\omega_k)$ is the fraction of $P_q(\omega_k)$ allocated to the user q on channel k for private message. Finally, using properties of Riemann integration, we reach to the desired result in the limit as $n \rightarrow \infty$.

$$\begin{aligned}
 T_1 &= \sum_{k=0}^l \frac{1}{2n} \Psi \left(\frac{\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{11}(\omega_k)|^2}{\tilde{N}_1(\omega_k)} \right) \\
 T_2 &= \sum_{k=0}^l \frac{1}{2n} \Psi \left(\frac{\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{12}(\omega_k)|^2}{\tilde{N}_2(\omega_k)} \right) \\
 T_3 &= \sum_{k=0}^l \frac{1}{2n} \Psi \left(\frac{\bar{\alpha}_2(\omega_k) P_2(\omega_k) |\tilde{H}_{21}(\omega_k)|^2}{\tilde{N}_1(\omega_k)} \right) \\
 T_4 &= \sum_{k=0}^l \frac{1}{2n} \Psi \left(\frac{\bar{\alpha}_2(\omega_k) P_2(\omega_k) |\tilde{H}_{22}(\omega_k)|^2}{\tilde{N}_2(\omega_k)} \right) \\
 T_5 &= \sum_{k=0}^l \frac{1}{2n} \Psi \left(\frac{\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{11}(\omega_k)|^2 + \bar{\alpha}_2(\omega_k) P_2(\omega_k) |\tilde{H}_{21}(\omega_k)|^2}{\tilde{N}_1(\omega_k)} \right) \\
 T_6 &= \sum_{k=0}^l \frac{1}{2n} \Psi \left(\frac{\bar{\alpha}_1(\omega_k) P_1(\omega_k) |\tilde{H}_{12}(\omega_k)|^2 + \bar{\alpha}_2(\omega_k) P_2(\omega_k) |\tilde{H}_{22}(\omega_k)|^2}{\tilde{N}_2(\omega_k)} \right) \\
 T_7 &= \sum_{k=0}^l \frac{1}{2n} \times \\
 &\Psi \left(\frac{P_1(\omega_k) |\tilde{H}_{11}(\omega_k)|^2 + P_2(\omega_k) |\tilde{H}_{21}(\omega_k)|^2 + \sqrt{\alpha_1(\omega_k) \alpha_2(\omega_k) P_1(\omega_k) P_2(\omega_k)} (2\text{Re}\{\tilde{H}_{11}(\omega_k) \tilde{H}_{21}^*(\omega_k)\})}{\tilde{N}_1(\omega_k)} \right) \\
 T_8 &= \sum_{k=0}^l \frac{1}{2n} \times \\
 &\Psi \left(\frac{P_1(\omega_k) |\tilde{H}_{12}(\omega_k)|^2 + P_2(\omega_k) |\tilde{H}_{22}(\omega_k)|^2 + \sqrt{\alpha_1(\omega_k) \alpha_2(\omega_k) P_1(\omega_k) P_2(\omega_k)} (2\text{Re}\{\tilde{H}_{12}(\omega_k) \tilde{H}_{22}^*(\omega_k)\})}{\tilde{N}_2(\omega_k)} \right)
 \end{aligned} \tag{30}$$

VII. CONCLUSION

We derived in this paper the capacity region of a finite-memory compound MAC with common message and ISI under an average input power constraint. Since our channel is a special case of a synchronous multi-terminal channel, this capacity region is equal to the capacity region of an n -circular Gaussian compound MAC with common message as n grows to infinity. We also investigated some special cases of the Gaussian compound MAC with ISI, and by using the obtained capacity region and employing the same method we derived the capacity regions for them. Moreover, some numerical results have been provided to show the loss in the rate caused by ISI.

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