

Learning from Ant Society in Optimizing Concrete Retaining Walls

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Abstract: In the present paper, lessons are learnt from ant society so that humankind can optimize his engineering issues. As an example of such issues, a reinforced concrete retaining wall for which the application of optimization can reduce the costs involved is considered. Traditional design procedure for reinforced concrete retaining walls is unable to design an optimized wall unless a large trial effort is undertaken. This paper introduces a learning procedure from ants, which is a general search technique for the solution of difficult combinatorial problems with its theoretical roots based on the foraging behavior of ants. This methodology arrives at an optimal design for concrete retaining walls due to its capability to explore and exploit the solution space effectively. The basis of analysis in this paper is to determine the minimum weight and costs in the design of concrete retaining walls following a computation of lateral total thrust on the wall due to backfill pressures, bearing capacity consideration, settlement analysis, stability analysis, and application of design of reinforced concrete principles. The results clearly indicate that ant colony can educate engineers comprehensively to reach a minimum cost justified retaining wall through an optimization approach.

Keywords: Ant Colony, Education, Optimization, Concrete Retaining Wall, Swarm Intelligent.

1- Introduction

This paper attempts to illustrate how practicing engineers can learn from ants to follow a procedure to arrive at an optimal design of concrete retaining wall. Ant colony optimization (ACO) is basically a general search technique for the solution of difficult combinatorial problems with its theoretical roots based on the foraging behavior of ants. ACO is based on the indirect communication of a colony of simple agents, called artificial ants, mediated by artificial pheromone trails. The pheromone trails in ACO serve as distributed numerical information, which the ants use to probabilistically construct solutions to the problem being dealt with.

Concrete retaining walls commonly used to support earth, coal, ore piles, and water are most widely structures amongst various categories of retaining walls in civil engineering practice. Optimization of retaining walls massive structures is a popular topic in civil engineering due to the complexity of the problem and its benefits to industry due to economical considerations. Current structural optimization software packages often lack the ability to find optimal designs because of their deterministic nature, while those employing stochastic methods are not tailored specifically for retaining walls and massive concrete structures.

Classic optimization search methods are rudimentarily based on direct search methods. Direct search methods belong to a class of optimization methods that do not compute derivatives. Examples of direct search method are the Nelder Mead Simplex method, Hooke and Jeeves's pattern search, the box method, and Dennis and Torczon's parallel direct search algorithm employing a multi-sided simplex. However, these algorithms suffer from both trapping in local minima and increasing running time.

An optimum design of retaining walls has been the subject of a number of studies: Saribas and Erbatur presented a detailed study on optimum design of reinforced concrete cantilever retaining walls using cost and weight of the walls as objective functions. In this study, they controlled overturning failure, sliding failure, shear and moment capacities of toe slab, heel slab, and stem of wall as constraints [1]. Ceranic and Fryer proposed an optimization algorithm based on Simulated Annealing, which can compute the minimum cost design of reinforced concrete retaining walls [2]. Sivakumar and Munwar introduced a Target Reliability Approach for design optimization of retaining walls [3]. Ahmadi Nedushan and Varae proposed an optimization algorithm based on particle swarm optimization [4]. They claim that this method requires fewer number of function evaluations, while leading to better results in optimization of retaining walls [4].

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2- Learning from Ant Colony for Optimization

From years of study and observation, ethologists have found that ants, although almost completely blind, are able to successfully navigate between their nest and food sources and in the process, discover the shortest path between these points [6]. The ant colony is able to determine the shortest path to food sources using pheromone trails. As an ant moves, it deposits pheromones along its path. A single ant will move essentially at random, however, another ant following behind it will detect the pheromone trail left by the lead ant and will be inclined to follow it. Once an ant selects a path, it lays additional pheromones along the path, reinforcing the increasing pheromone level of the trail and increasing the probability that subsequent ants will follow this path. This type of collective feedback and emerging knowledge in the ant colony is a form of autocatalytic behavior [7].

In the past few years, ant colony optimization (ACO) algorithms have undergone many changes throughout their development, but each different system retains the fundamental ant behavioral mechanisms. The fundamental theory in an ACO algorithm is the simulation of the autocatalytic, positive feedback process exhibited by a colony of ants. This process is modeled by utilizing a virtual substance called "trail" that is analogous to pheromones used by real ants. Each ACO algorithm follows a basic computational structure outlined by the pseudocode in Fig. 1. An ant begins at a randomly selected point and must decide which of available paths to travel. This decision is based upon the intensity of the trails present upon each path leading to the adjacent points. The path with the most trails has a higher probability of being selected. If no trail is present upon a path, there is zero probability that the ant will choose that path. If all paths have an equal amount of trail (intensity of pheromones, the use of trail is a bit awkward unless it is a word used in this way in your specific literature, the literature of engineering and/or biology), then the ant has an equal probability of choosing each path, and its decision is random.

An ant chooses a path using a decision mechanism and travels along it to another point. Some ACO algorithms now apply a local update to the trail (see Fig. 1). This process reduces the intensity of the trail on the path chosen by the ant. The idea is that when subsequent ants arrive at this point, they will have a slightly smaller probability of choosing the same path as other ants before them. This mechanism is intended to promote exploration among the ants, and helps prevent early stagnation of the search and premature convergence of the solution. The amount of this reduction is not great enough to prevent overall solution convergence. The ant continues to

choose paths to travel between points, visiting each point, until all points have been visited and it arrives back at its point of origin. When it returns to its starting point, the ant has completed a tour (Fig. 1).

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Initialize Trail
Do While (Stopping Criteria Not Satisfied) - Cycle Loop
    Do Until (Each Ant Completes a Tour) - Tour Loop
        Ant Decision Mechanism
        Local Trail Update
    End Do
    Global Trail Update
End Do
    
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Fig 1 Ant colony optimization algorithm in pseudocode

The combination of paths which an ant chooses to complete a tour is a solution to the problem and is analyzed to determine how well it solves the problem. The intensity of the trail upon each path in the tour is then adjusted through a global update process. The magnitude of the trail adjustment reflects how well the solution produced by an ant's tour solves the problem. The paths that make up the tours that best solve the problem receive more trail than those paths that make up poor solutions. In this way, when the ant begins the next tour, there is a greater probability that an ant will choose a path that was part of a tour performed well in the past. When all the ants have completed a tour, all tours have been analyzed and the trail levels on the paths have been updated, an ACO cycle is complete [10]. A new cycle now begins and the entire process is repeated. Eventually, almost all ants will make the same tour on every cycle and converge to a solution. Stopping criteria is typically based on comparing the best solution from the last cycle to the best global solution. If this comparison shows that the algorithm is no longer improving the solution, then the criteria are reached [9].

The first ant algorithm was developed by Dorigo, referred to as ant system (AS) [8]. AS improves on ACO by changing the transition probability, P_{ij}^k , to include heuristic information, and by adding a memory capability by the inclusion of a tabu list. In AS, the probability of moving from node i to node j is given as:

$$P_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^a(t)}{\sum_{j \in N_i^K} \tau_{ij}^a(t)} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases} \quad (1)$$

where τ_{ij}^a represents the *a posteriori* effectiveness of the move from node i to node j , as expressed in

the pheromone intensity of the corresponding link, (i, j) ; η_{ij} represents the *a priori* effectiveness of the move from i to j (i.e. the attractiveness, or desirability, of the move), computed using some heuristic. The pheromone concentrations, τ_{ij} , indicate how profitable it has been in the past to make a move from i to j , serving as a memory of previous best moves [8]. From the above explanation, it is concluded that an ant colony can educate us to optimize our structures including reinforced concrete retaining walls.

3- Concrete Retaining Wall Design

Consider a concrete retaining wall as shown in Fig. 2 with a height of H . Expressions for factors of safety against overturning failure, sliding failure, eccentricity failure and bearing capacity failure will be given subsequently.

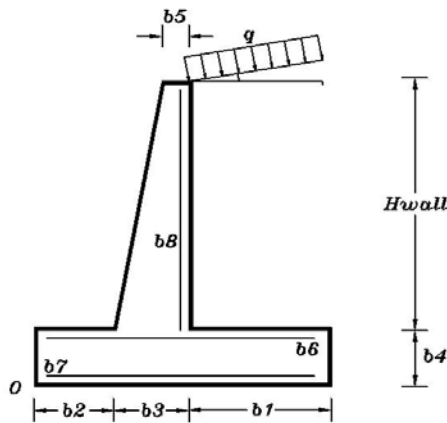


Fig 2 Concrete retaining wall section

Rankine's earth-pressure theory corresponds to the stress and deformation conditions for the states of plastic equilibrium. The resultant active pressure on a vertical plane of height H through a semi-infinite mass of soil whose surface is inclined at an angle β to the horizontal is:

$$P_a = \frac{1}{2} \gamma H^2 \left(\cos \beta \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \right) \quad (2) !$$

where ϕ is the backfill friction angle, γ is the backfill unit weight, and β is the backfill surface angle with the horizontal direction.

It is usually required that the factor of safety against overturning be at least 1.5. It is normally in the 1.5-2 range, depending on the importance of the wall. This is commonly determined by taking moments of all forces about the wall toe. The factor of safety is the ratio of the moment of the forces resisting

overturning to the moment of forces tending to cause overturning.

Sum of moments of forces tending to resist overturning about point O (Fig. 2) can be expressed as:

$$\sum M_R = M_c + M_s + M_q + M_v \quad (3)$$

The summation of the moments of forces tending to overturning about point O is expressed as:

$$\sum M_O = P_{ah} \bar{y} \quad (4)$$

where M_c , M_s , and M_v are moments about the toe point O as shown in Fig. 2 due to weight W_c , W_s , and P_{av} , respectively. Here W_c is weight of the concrete; W_s is weight of the soil; P_{av} is the vertical component. M_q is related to surcharge load. Various parameters are defined as: ϕ_1 = friction angle of the back fill soil, δ = wall friction angle normally assumed to be $2\phi_1/3$, γ_1 = unit weight of the backfill soil (kN/m^3) and \bar{y} = moment arm. The factor of safety against overturning failure is expressed as:

$$FS_{\text{overturning}} = \frac{\sum M_R}{\sum M_O} \quad (5)$$

The overall wall stability requires safety against sliding. The sum of the horizontal resisting forces can be written as:

$$\sum F_r = C_a B' + \sum W \tan \delta + P_D \quad (6)$$

The sum of the horizontal driving forces is given by:

$$\sum F_d = P_{ah} \quad (7)$$

where C_a is adhesion coefficient between base slab and base soil, γ is unit weight of soil below the base slab of retaining wall (kN/m^3), ϕ_2 = friction angle of the soil below the base slab of the retaining wall, $\sum W$ = sum of the vertical forces acting on retaining wall, and P_p is passive earth pressure developed by the soil in front of the wall.

The factor of safety against sliding failure can be expressed as:

$$FS_{\text{sliding}} = \frac{\sum F_r}{\sum F_d} \quad (8)$$

The stability number is usually in the range of 1.5-2, depending on the importance of the wall. For stability, the line of action of the resultant force must lie within the middle third of the foundation base. The factor of safety against eccentricity failure is given by:

$$\frac{B}{6} > e \quad (9)$$

where B = base width of the wall and e = eccentricity of the result and force.

In many instances involving the construction of embankments, overpasses or bridge approaches, it is necessary to construct a retaining wall backfilled to a considerable elevation above the existing ground surface. In these circumstances, precaution must be taken to ensure that a base failure beneath the weight of the fill does not occur. If the subsoil consists of coarse material, there is no likelihood of such a failure. However, if the subsoil consists of fine material, it is necessary to check their bearing capacity. The stability of the base against a bearing capacity failure is achieved by using a suitable safety factor with the computed ultimate bearing capacity where the safety factor is usually taken as 2 for granular soil and 3 for cohesive soil. The allowable soil pressure can be computed using the following bearing capacity equation:

$$q_{ult} = CN_c d_c i_c + \bar{q} N_q d_q i_q + 0.5 \gamma B N_\gamma d_\gamma i_\gamma \quad (10)$$

where d = depth factors, i = inclination factors which are based on the load inclination since there exist both vertical and horizontal loads, B = width of the footing (base width), $\bar{q} = \gamma D$, and D = depth of the base. In Eq. (10), N_c , N_q , and N_γ are bearing capacity factors as functions of ϕ [5]. This maximum pressure on the soil at the wall toe is given by:

$$q_{max} = \frac{\sum W}{B'} \quad (11)$$

The factor of safety against bearing capacity failure is defined as:

$$FS_q = \frac{q_{ult}}{q_{max}} \quad (12)$$

4- Concrete Retaining Wall Optimization

An optimal concrete retaining wall design is one with the minimal weight and cost that still allows the

wall to satisfy given constraints. The basic stability requirements for a wall for all conditions of loading are being safe for overturning, sliding, and bearing capacity failure [5].

The wall optimization problem can be expressed as:

$$g = \text{Min } W(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) \quad (13)$$

$$h = \text{Min } C(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) \quad (14)$$

while considering:

$$FS_q \geq 3 \quad (15)$$

$$FS_{over} \geq (FS_{over})_{all} \quad (16)$$

$$FS_{slid} \geq (FS_{slid})_{all} \quad (17)$$

where g = objective function, W = total weight, h = objective function, C = total price, b_1 = heel projection, b_2 = toe projection, b_3 = stem thickness at bottom, b_4 = thickness of base slab, b_5 = stem thickness at top, b_6 = horizontal steel area of the heel per unit length of the wall, b_7 = horizontal steel area of the toe per unit length of the wall, b_8 = vertical steel area of the stem per unit length of the wall, FS_q = safety against bearing capacity failure, FS_{slid} = safety factor against sliding, FS_{over} = safety factor against overturning, $(FS_{slid})_{all}$, and $(FS_{over})_{all}$ = allowable values for FS_{slid} and FS_{over} , respectively.

Two objective functions attributed to weight and costs have been chosen for flexibility of use and for comparison purposes. In cost minimization, the objective function is defined as:

$$h(x) = C_s W_s + C_c V_c \quad (18)$$

where C_s = unit cost of steel, C_c = unit cost of concrete, W_s = weight of steel per unit length of the wall, and V_c = volume of concrete per unit length of the wall.

For weight optimization the objective function is defined as:

$$g(x) = W_s + 100 V_c \gamma_c \quad (19)$$

where γ_c = unit weight of concrete and 100 is used for consistency of units.

The ACO algorithm adapted for concrete retaining wall optimization is illustrated in the Fig. 3.

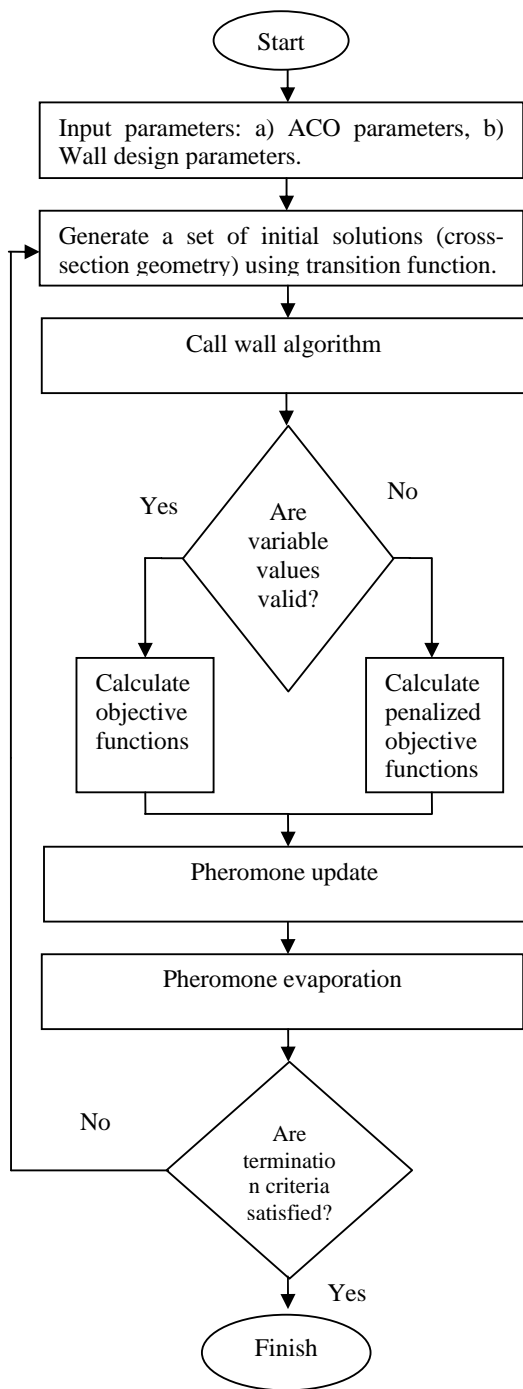


Fig 3 ACO application applied reinforced wall

5- Verification

5.1 Example 1 To check the performance, robustness, and accuracy of the developed algorithm, a retaining wall studied by Saribas and Erbatur [1] is considered. The details of this wall and other necessary input parameters are given in

Table 1. It is noted that all values given in this table are for a unit length of the wall.

Table 1 Input parameters

Input parameters	Unit	Symbol	Value
Height of stem	m	H	4.5
Yield strength of reinforcing steel	MPa	f_y	400
Compressive strength of concrete	MPa	f'_c	21
Surcharge load	kPa	q	30
Backfill slope	degree	β	15
Internal friction angle of retained soil	degree	ϕ_1	36
Internal friction angle of base soil	degree	ϕ_2	34
Unit weight of retained soil	kN/m ³	γ_1	17.5
Unit weight of base soil	kN/m ³	γ_2	18.5
Unit weight of concrete	kN/m ³	γ_c	23.5
Cohesion of base soil	kPa	c	100
Depth of soil in front of wall	m	D	0.75
Cost of steel	\$/kg	C_s	0.40
Cost of concrete	kg/m ³	C_c	40
Factor of safety for overturning stability	-	N_o	1.5
Factor of safety against sliding	-	N_s	1.5
Factor of safety for bearing capacity	-	SF_q	3.0

In this case, both weight and cost minimization is considered. Optimum design results are shown in Tables 2 and 3. The optimum values of the design variables are tabulated together with suggested, upper and lower limits for easy interpretation (Table 2). Table 3 shows the results obtained from the present optimization analysis (AS) and those reported by Saribas and Erbatur [1]. As seen, these results are in close agreement. The deviations between two methods are 6.1% and 4.9% for cost and weight optimizations, respectively.

Table 2 Optimum values of design variables

Design variable	Lower bound	Upper bound	Optimum value for minimum cost	Optimum value for minimum weight
b_1 (m)	1.059	1.833	1.385	1.385
b_2 (m)	0.655	1.167	1.143	1.143
b_3 (m)	0.25	0.50	0.251	0.251
b_4 (m)	0.40	0.50	0.40	0.40
b_5 (m)	0.25	0.25	0.25	0.25
b_6 (cm ² /m)	11.059	67.68	14	14
b_7 (cm ² /m)	11.059	67.68	14	14
b_8 (cm ² /m)	5.761	67.68	59	59

Table 3 Optimum values of objective function

Objective function	Unit	Optimum value (Saribas)	Optimum value (AS)
Minimum cost	\$/m	189.546	201.185
Minimum weight	kg/m	5280.96	5540.3

5-2 Example 2 For further validation of the developed optimization method, another example is considered and the results are compared with those given by Saribas and Erbatur, Sivakumar and Munwar, Bowles, and Das [1,3,5,11]. Three walls with heights of 3, 4, and 5 m are considered. Other specifications for the design of these retaining walls are presented in Table 4. To compare the results with Das and Bowles, a value of 0.3 m is assumed for b_5 for all walls.

Tables 5 to 8 compare optimum design results determined from the present method and those given by others as referenced. It is noted that in these tables, some fixed values are considered for b_1 , b_2 , and b_4 . This stems from the fact that Das and Bowles do not optimize these values and they just recommend some experienced-based approximate values which are normally used by engineers in practice [11,5]. As seen in these tables, these values can be easily optimized using the method described in this research or other optimization approaches. As seen in Tables (5-8), the current optimization method gives reasonable results, which may be used in practice as optimized values. It is noted that the values obtained from the present developed optimization method from viewpoints of weight and cost of retaining walls are relatively greater than those given by Saribas and Erbatur [1]. This could be attributed to the fact that the results of Saribas

and Erbatur do not account for uncertainties that exist in the soil, concrete, steel properties, and geometric properties of the wall [1,3].

Table 4 Input parameters [3]

Input parameters	Unit	Symbol	Value
Height of stem	m	H	3, 4, 5
Yield strength of reinforcing steel	MPa	f_y	400
Compressive strength of concrete	MPa	f'_c	21
Surcharge load	kPa	q	25
Backfill slope	degree	β	10
Internal friction angle of retained soil	degree	ϕ_1	36
Internal friction angle of base soil	degree	ϕ_2	0
Unit weight of retained soil	kN/m ³	γ_1	17.5
Unit weight of base soil	kN/m ³	γ_2	18.5
Unit weight of concrete	kN/m ³	γ_c	23.5
Cohesion of base soil	kPa	C	125
Depth of soil in front of wall	m	D	0.75
Cost of steel	\$/kg	C_s	0.40
Cost of concrete	kg/m ³	C_c	40
Factor of safety for overturning stability	-	N_o	1.5
Factor of safety against sliding	-	N_s	1.5
Factor of safety for bearing capacity	-	SF_q	3.0

Table 5 Comparative study for the projection of toe from base of stem, b_2

Height of stem (m)	3	4	5
0.1H (m) (Das, 1999)	!0.3	!0.4	!0.5
0.233H (m) (Bowles, 1996) !	!!0.7	!!0.933	!!1.167
for minimum cost (Saribas and Erbatur, 1996)	!!0.443	!!0.582	!!0.727
for minimum weight (Saribas and Erbatur, 1996)	!!0.436	!!0.603	!!0.789
Sivakumar and Munwar (2008)	!0.72	!0.96	!1.20
for minimum cost (present study)	!0.555	!0.726	!0.939
for minimum weight (present study)	!0.629	!!0.842	!!1.013

Table 6 Comparative study for projection of heel from the base of the stem, b_1 !

Height of stem (m)	! 3.0	! 4.0	! 5.0
0.5H (m) (Das, 1999)	! 1.5	!! 2.0	! 2.5
0.367H (m) (Bowles, 1996)	!! 1.101	!! 1.468	! 1.835
for minimum cost (Saribas and Erbatur, 1996)	!! 0.864	!!! 1.161	!!! 1.411
for minimum weight (Saribas and Erbatur, 1996)	!! 0.873	!!! 1.191	!!! 1.473
Sivakumar and Munwar (2008)	! 0.6	! 0.8	!! 1.0
for minimum cost (present study)	! 1.026	! 1.375	! 1.687
for minimum weight (present study)	! 0.944	! 1.255	!! 1.589

Table 7 Comparative study for the thickness of base slab, b_4

Height of stem (m)	! 3.0	!! 4.0	! 5.0
0.1H (m) (Das, 1999)	! 0.3	! 0.4	! 0.5
0.1H (m) (Bowles, 1996) !	! 0.3	! 0.4	! 0.5
for minimum cost (Saribas and Erbatur, 1996)	!! 0.273	!! 0.364	!! 0.455
for minimum weight (Saribas and Erbatur, 1996)	!! 0.273	!! 0.364	!! 0.455
Sivakumar and Munwar (2008)	! 0.3	! 0.4	! 0.5
for minimum cost (present study)	!! 0.271	!! 0.363	!! 0.450
for minimum weight (present study)	!! 0.270	!! 0.363	!! 0.451

In general, it has been demonstrated that the proposed algorithm method based on ant colony has sufficient capability to educate engineers to optimize reinforced concrete retaining wall design for weight and cost considerations. In fact, design engineers can confidently learn lessons efficiently from ant society on the procedure of their feeding for optimization of reinforced concrete retaining walls.

Table 8 Comparative study for the cross sectional area of the retaining wall (m^2)

Height of stem (m)	!! 3.0	!! 4.0	! 5.0
0.1H (m) (Das, 1999)	!! 1.440	!! 2.380	! 3.550
0.1H (m) (Bowles, 1996) !!	!! 1.440	!! 2.380	!! 3.550
for minimum cost (Saribas and Erbatur, 1996)	!! 1.340	!! 2.071	!! 3.037
for minimum weight (Saribas and Erbatur, 1996)	!! 1.340	!!! 1.962	!! 2.713
Sivakumar and Munwar (2008)	! 1.395	!! 2.080	! 2.875
for minimum cost (present study)	!! 1.407	! 2.073	!! 2.816
for minimum weight (present study)	! 1.405	! 2.070	!! 2.811

6- Conclusions

The present paper has shown how engineers can learn from ant colony for optimization of reinforced concrete retaining walls. By validation of the predicted results on optimizing retaining walls, it has been demonstrated that ant colony can educate efficiently design engineers to find a successful optimization approach, which is a successful random search method. This method educates engineers to find a global minimum in difficult combinational problems, which can hardly be attained by classical optimization methods. It has been demonstrated that the presented algorithm is able to find quickly the geometrical specifications for reinforced concrete retaining walls for which minimum weight and minimum justified costs are involved..

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