Journal of Theoretical and Applied Physics, **4**-3, 15-22 (2010)

B quark gluonic penguin decays

H. Mehrban

Physics Department, Semnan University, Semnan, Iran

Received: 12 August 2010/Accepted: 10 October 2010/ Published: 20 December 2010

Abstract

Two aspects of b-quark decays are investigated. First, according to the structure of penguin decays and second, according to the Effective Hamiltonian theory. In this work, the gluonic penguin term is calculated according to the structure of penguin for various b and $\bar{b}$ quark decays. The total gluonic penguin amplitude to lowest order in α_s is used for the decay process, $b \to q_k g \to q_k q' \bar{q}' (q_k q_i \bar{q}_i)_{i=j}$. The magnetic dipole gluonic penguin term for various b quark decays is calculated. The tree-level and gluonic penguin terms of b-quark are also added to calculated the decay rates of b and $\bar{b}$ quark. It is shown that the gluonic penguin term is small for some decay. The decay rates of processes like $b \to cd\bar{c}(\bar{b} \to \bar{c}\bar{d}c), b \to cs\bar{c}(\bar{b} \to \bar{c}\bar{s}c), b \to ud\bar{u}(\bar{b} \to \bar{u}\bar{d}u)$ and $b \to us\bar{u}(\bar{b} \to \bar{u}\bar{s}u)$ is obtained.

***PACs*:** 25.75.Nq; 12.38.Mh; 14.65.Fy; 14.65.Bt
Keyword: *Gluonic Penguin; Effective Hamiltonian; b Quark; $\bar{b}$ Quark; Magnetic Dipole; Tree Level; Decay Rate.*

1. Introduction

In the Standard Model (SM) [1], flavor-changing neutral currents are forbidden, for example, there is no direct coupling between the b quark and the s or d quarks. Effective flavor-changing neutral currents are induced by one-loop, or penguin diagrams, where a quark emits and reabsorbs a W thus changing flavors twice, as in the $b \to t \to s$ transition depicted in Fig.1. Penguin decays have become increasingly appreciated in recent years [2-3]. These loop diagrams with their interesting combination of CKM matrix elements give insight into the Standard Models [1]. In addition, they are quite sensitive to new physics. The weak coupling of quarks are given by the CKM matrix. For the Standard Model with three generations, the CKM matrix can be described completely by three Euler-type angles, and a complex phase.

Various types of the penguin processes are [5]: electromagnetic, electroweak, and gluonic.

In electromagnetic penguin decays such as $b \to s\gamma$, a charged particle emits an external real photon. The hard photon emitted in these decays is an excellent experimental signature. The inclusive rate is dominated by short distance (perturbative) interactions and can be reliably predicted. The QCD corrections enhance the rate and have been calculated precisely. The electromagnetic penguin decay $b \to d\gamma$ is further suppressed by $|V_{td}|^2/|V_{ts}|^2$ and gives an alternative to $B^0 - \bar{B}^0$ mixing for extraction $|V_{td}|$ [6,7]. Experimentally, inclusive $b \to d\gamma$ has large backgrounds from the dominate $b \to s\gamma$ decays which must be rejected using good particle identification or kinematic separation. The decay $b \to s\ell^+\ell^-$ can proceed via an electroweak penguin diagram where an emitted virtual photon or Z^0 produces a pair of leptons. This decay can also proceed via a box diagram [8]. The Standard Model prediction for the $b \to s\ell^+\ell^-$ decay rate is two orders of magnitude smaller than the $b \to s\gamma$ rate [9,10]. The rate for $b \to s\nu\bar{\nu}$ is enhanced relative to $b \to s\ell^+\ell^-$ primarily due to summing the three neutrino flavors. These decays are expected to be dominated by the weak penguin, since neutrinos do not couple to photons. The predicted rate is only a factor of 10 lower than for $b \to s\gamma$ [11]. Unfortunately, the neutrinos escape detection, making this mode difficult to observe.

Another category of penguin is so-called vertical or annihilation penguin where the penguin loop connects the two quarks in the B meson. These rates are expected to be highly suppressed in the Standard Model since they involve a $b \to d$ transition and are suppressed by $(f_B/m_B)^2 \approx 2 \times 10^{-3}$ [12], where f_B is the B-meson decay constant which parameterizes the probability that the two quarks in the B meson will "find each other", and m_B is the B meson mass. The $B \to \gamma\gamma$ decay is suppressed [13] relative to $b \to s\gamma$ by an additional α_{QCD}. The $B \to \ell^+\ell^-$ decays are helicity-suppressed [7,14]. Because these decays are so suppressed in the Standard Model, they provide a good opportunity to look for non SM effects.

An on- or off-shell gluon can also be emitted from the penguin loop. While the on-shell $b \to sg$ rate had been calculated to be $O(0.1\%)$ [15], the inclusive on- plus off-shell $b \to sg^*$ rate includes contribution from $b \to sq\bar{q}$ and b→ sgg which increase the inclusive rate to 0.5-1% [16,17]. The $b \to dg^*$ penguin rate is smaller by $|V_{td}/V_{ts}|^2$.

*Corresponding author: Hossein Mehrban;
E-mail: hmehraban@semnan.ac.ir
Tel: (+98) 21 77871723
Fax: (+98) 0231 3354082

Unfortunately, there are several difficulties associated with gluonic penguins. There is no good signature for the inclusive $b \to sg^*$ decay, unlike the $b \to s\gamma$ case. The branching fraction of individual exclusive gluonic penguin channels is typically quite small and hadronization effects are difficult to calculate [18,19]. In addition, many gluonic penguin final states are accessible via other diagrams, so the gluonic penguin is difficult to assess. Thus the penguin processes such as $B^0 \to \phi K^0$ that have contributions only from gluonic penguins are eagerly sought.

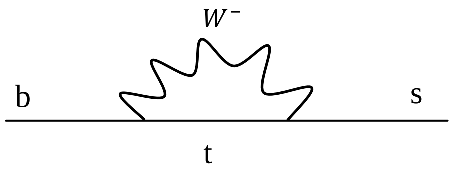

Fig. 1: $b \to s$ loop or penguin diagram.

While the gluonic penguin gives rise only to hadronic final states, several other processes can contribute to the same final states. One important contribution is from the tree-level $b \to u$ decay. For example the $b \to us\bar{u}$ transition and the $b \to sg^*$ penguin transition both contribute to $B^0 \to K^+\pi^-$. However, the $b \to us\bar{u}$ transition is Cabibbo-Suppressed, so the penguin process is expected to dominate [20-23]. On the other hand, in $B \to \pi^+\pi^-$ for example, the small $b \to dg^*$ contribution is expected to be dominant by the non-cabibbo-suppressed tree-level $b \to ud\bar{u}$ transition. In general, most decays to hadronic final state with ϕ mesons or non-zero net strangeness are expected to be dominate by gluonic penguin and hadronic final states with zero net strangeness are expected to be dominated by tree-level $b \to u$.

Electroweak penguin also contributes to hadronic final states. Every gluonic penguin can be converted to an electroweak penguin by replacing the gluon with a Z^0 or γ. Electroweak penguin with internal Z^0 or γ emission are suppressed relative to the corresponding strong gluonic penguin. In the hairpin process the gluon, Z^0, or γ is emitted externally and subsequently forms a meson. The vertical electroweak penguin diagram, with the lepton pair replaced by a de-quark pair, is highly suppressed and is important only for decays such as $B^0 \to \phi\phi$, where there is no other diagrams contribution [24,25].

In the annihilation diagram the b and $\bar{u}$ quarks in a B^- meson annihilate form a virtual W^-. The annihilation diagram is suppressed by $|V_{ub}|$ and f_B/m_B and is expected to be mostly negligible. In the exchange diagram, a $b \to u$ transition and a $\bar{d} \to \bar{u}$ transition occur simultaneously via the exchange of a W between the b and $\bar{d}$ quarks in a $\bar{B}^0$ meson. The exchange process is also suppressed by $|V_{ub}|$ and f_B/m_B, and is also expected to be negligible, except in decays such as $B^0 \to K^+K^-$ where there is no favored diagrams contribution [26-28].

Although $s \to u$ loop diagrams are important in K decays, those decays are typically dominated by large non-perturbative effects. A notable exception is $K^+ \to \pi^+ \nu\bar{\nu}$. This decay is expected to be dominated by electroweak penguins and could eventually provide a measurement of $|V_{td}|$. Penguin processes are also possible in c and t decays, but these particles have the CKM-favoured decays $c \to s$ and $t \to b$ accessible to them. Since the b quark has no kinematically-allowed CKM-favoured decay, the relative importance of the penguin decay is greater. The mass of the top quark the main contributor to the loop, is large, and the coupling of the b quark, the t quark, $|V_{tb}|$, is very close to unity, both strengthening the effect of the penguin. The $b \to s(b \to d)$ penguin transition is sensitive to $|V_{ts}|/|V_{td}|$ which will be extraordinarily difficult to measure in top decay. Information from the penguin decay will complement information on $|V_{ts}|$ and $|V_{td}|$ from $B_s - \bar{B}_s$ and $B^0 - \bar{B}^0$ mixing [29].

Since the Standard Model loops involves the heaviest known particles (t, W ,Z), rates for these processes are very sensitive to non-SM extension with heavy charged Higge or supersymmetric particle. Therefore, measurement of loop processes constitutes the most sensitive low energy probes for such extensions to the Standard Model.

2. Gluonic Penguin

Conservation of the gluonic current requires the $b \to q_k g$ vertex to have the structure [30,31]:

$$\Gamma_\mu^a(q^2) = (ig_s/4\pi^2)\, \bar{u}_k\,(p_k) T^a V_\mu(q^2)\, u_b(p_b), \tag{1}$$

where

$$V_\mu(q^2) = \left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)\gamma^\nu[F_1^L(q^2)P_L + F_1^R(q^2)P_R] + i\sigma_{\mu\nu}q^\nu[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \tag{2}$$

Here F_1 and F_2 are the electric (monopole) and magnetic (dipole) form factors, $q = q_g = p_b - p_k$ is the gluon four momentum, $P_{L(R)} \equiv (1 \mp \gamma_5)/2$ are the chirality projection operators and $T^a (a = 1, \cdots, 8)$ are the $SU(3)_c$ generators normalized to $Tr(T^a T^b) = \delta^{ab}/2$. The $\bar{b} \to \bar{q}_k g$ vertex is:

$$\Gamma_\mu^a(q^2) = -(ig_s/4\pi^2)\, \bar{v}_b\,(p_b) T^a \bar{V}_\mu(q^2)\, v_k(p_k), \tag{3}$$

here $\bar{V}_\mu$ has the form (2) with the form factors $F_{1,2}^{L,R}(q^2)$ replaced by $\bar{F}_{1,2}^{L,R}(q^2)$. To lowest order in α_s the penguin amplitude for the decay process $b \to q_k g \to q_k q' \bar{q}' \left(q_k q_i \bar{q}_j\right)_{i=j}$ is:

$$M^{peng} = -i(\alpha_s/\pi)\left[\bar{u}_k(p_k) T^a \Lambda_\mu u_b(p_b)\right]\left[\bar{u}_{q'}(p_{q'})\gamma^\mu T^a v_{\bar{q}'}\left(p_{\bar{q}'}\right)\right], \tag{4}$$

where $\alpha_s = g_s^2/4\pi$ and:

$$\Lambda_\mu \equiv \gamma_\mu\left[F_1^L(q^2)P_L + F_1^R(q^2)\ P_R\right] + \left(i\sigma_{\mu\nu}q^\nu/q^2\right)\left[F_2^L(q^2)P_L + F_2^R(q^2)\ P_R\right]. \quad (5)$$

Similarly, for $\bar{b} \to \bar{q}_k q'\bar{q}'$, the amplitude is:

$$M^{peng} = -i(\alpha_s/\pi)\left[\bar{u}_k(p_k)T^a\Lambda_\mu u_b(p_b)\right]\left[\bar{u}_{q'}(p_{q'})\gamma^\mu T^a v_{\bar{q}'}(p_{\bar{q}'})\right], \quad (6)$$

where $\overline{\Lambda}_\mu$ is obtained from Eq. 5 by the replacement of all the $F(q^2)$ form factors by $\bar{F}(q^2)$ form factors. The top quark dominates in the sum for F_2, hence at value of q^2 (a good approximation), we have $F_2^L(q^2) \approx F_2^L(0)$ and $F_2^R(q^2) \approx F_2^R(0)$ [32], therefore:

$$F_1^L(q^2) = \left(G_F/\sqrt{2}\right)\sum_{i=u,c,t} V_{ik}^* V_{ib} f_1(x_i, q^2),\ F_1^R(0) = 0, \quad (7)$$

$$F_2^L(0)/m_q = F_2^R(0)/m_b = \left(G_F/\sqrt{2}\right)\sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i), \quad (8)$$

where $x_i \equiv m_i^2/M_W^2\ (i = u, c, t)$ and:

$$f_2(x) = -(x/4\,(1-x)^4)[2 + 3x - 6x^2 + x^3 + 6x\ln x], \quad (9)$$

$$f_1(x) = (1/12\,(1-x)^4)[18x - 29x^2 + 10x^3 + x^4 - (8 - 31x + 18x^2)\ln x], \quad (10)$$

$$f_1(x_i, q^2) = (10/9) - (2/3)\ln x_i + (2/3\,z_i) - (2\,(2z_i+1)/3\,z_i)g(z_i), \quad (11)$$

here $z_i \equiv q^2/4m_i^2$ and [33,34]:

$$g(z) = \begin{cases} \sqrt{\dfrac{1-z}{z}}\arctan\left(\sqrt{\dfrac{z}{1-z}}\right), & z < 1 \\ \dfrac{1}{2}\sqrt{\dfrac{1-z}{z}}\left[\ln\left(\dfrac{\sqrt{z}+\sqrt{z-1}}{\sqrt{z}-\sqrt{z-1}}\right) - i\pi\right]. & z > 1 \end{cases} \quad (12)$$

For the u quark, z_i is large and we use the asymptotic form of (11):

$$f_1(x_u, q^2) = (10/9) - (2/3)\left[\ln\left(q^2/M_W^2\right) - i\pi\right], \quad (13)$$

To obtain $F_1^L \gg F_1^R$ and $F_2^R \gg F_2^L$. For the $b \to dq'\bar{q}'$ amplitude we find that F_1^L is dominant. Processes like $b \to ds\bar{s}$ and $\bar{b} \to \bar{d}s\bar{s}$ are expected to be penguin dominated [35] and F_1^L dominates all the other form factors. In the $b \to dq'\bar{q}'$ transition, we again find that $F_1^L \gg F_1^R$, $F_2^R \gg F_2^L$ and the F_1^L amplitude to be dominant.

3. Penguin Amplitude Decays of $b \to q_k q_i \bar{q}_j$

3.1. First Step (Penguin term):

According to Eq. 4, to lowest order the penguin amplitude for the decay process $b \to q_k q_i \bar{q}_j$ is:

$$M^{peng} = -(\alpha_s/\pi)\left[\bar{u}_k(p_k)T^a\Lambda_\mu u_b(p_b)\right]\left[\bar{u}_i(p_i)\gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})\right], \quad (14)$$

where

$$\Lambda_\mu \equiv \gamma_\mu F_1^L(q^2)P_L, \quad (15)$$

because F_1^L dominates for all of the form factors, $P_L = (1-\gamma_5)/2$ and here we neglect the magnetic dipole terms (see (5)). We will calculate the magnetic dipole terms later. The second term with γ^μ can be written as:

$$\gamma^\mu = \gamma^\mu\,(1-\gamma_5)/2 + \gamma^\mu\,(1-\gamma_5)/2. \quad (16)$$

Hence, the lowest order penguin amplitude becomes:

$$M^{peng} = -i(\alpha_s/\pi)\left[\bar{u}_{qk}(p_{qk})T^a\Lambda_\mu u_b(p_b)\right] \times \left[\bar{u}_{Li}(p_i)\tilde{\sigma}^\mu T^a v_{L\bar{j}}(p_{\bar{j}}) + \bar{u}_{Ri}(p_i)\sigma^\mu T^a v_{R\bar{j}}(p_{\bar{j}})\right] \quad (17)$$

Using Eq. 15, we have:

$$M^{peng} = -i(\alpha_s/\pi)F_1^L(q^2)\left\{\left[\bar{u}_{Lk}(p_k)T^a\tilde{\sigma}_\mu u_{Lb}(p_b)\right]\left[\bar{u}_{Li}(p_i)\tilde{\sigma}^\mu T^a v_{L\bar{j}}(p_{\bar{j}})\right] + \left[\bar{u}_{Lk}(p_k)T^a\tilde{\sigma}_\mu u_{Lb}(p_b)\right]\left[\bar{u}_{Ri}(p_i)\tilde{\sigma}^\mu T^a v_{R\bar{j}}(p_{\bar{j}})\right]\right\}. \quad (18)$$

Now we can obtain the penguin amplitude for b quark spin projections +1/2 and –1/2:

$$M^{peng} = (-2i/V)(\alpha_s/\pi)F_1^L(q^2)\left\{A_1(\tilde{\sigma}_\mu)(\tilde{\sigma}^\mu) + A_2(\tilde{\sigma}_\mu)(\sigma^\mu)\right\}, \quad (19)$$

here

$$A_1 = \sqrt{(1+v_k)/2}\sqrt{(1+v_i)/2}\sqrt{(1+v_{\bar{j}})/2}\,,$$

$$A_2 = \sqrt{(1-v_k)/2}\sqrt{(1+v_i)/2}\sqrt{(1-v_{\bar{j}})/2}\,. \quad (20)$$

Terms $(\tilde{\sigma}_\mu)(\tilde{\sigma}^\mu)_{LL}$ and $(\tilde{\sigma}_\mu)(\sigma^\mu)_{LR}$ for spin +1/2 and –1/2 obtain by the matrix elements of *L*-*L* handed and *L*-*R* handed for the b quark

$$\langle -_i|_L\tilde{\sigma}^\mu|b_{(1/2)}\rangle_L\langle -_k|_L\tilde{\sigma}_\mu|-_j\rangle_L = \sin\left((\theta_k - \theta_j - \theta_i)/2\right) + \sin\left((\theta_k + \theta_j - \theta_i)/2\right).$$

$$\langle -_i|_L\tilde{\sigma}^\mu|b_{(-1/2)}\rangle_L\langle -_k|_L\tilde{\sigma}_\mu|-_j\rangle_L = \cos\left((\theta_k - \theta_j - \theta_i)/2\right) - \cos\left((\theta_k + \theta_j - \theta_i)/2\right). \quad (21)$$

When dealing with penguin amplitudes we will also need the matrix elements:

$$\langle -_i|_L \tilde{\sigma}^\mu | b_{(1/2)}\rangle_L \langle -_k|_R \sigma_\mu | -_j\rangle_R = \sin((\theta_i - \theta_k - \theta_j)/2) - \sin((\theta_i + \theta_k - \theta_j)/2),$$

$$\langle -_i|_L \tilde{\sigma}^\mu | b_{(-1/2)}\rangle_L \langle -_k|_R \sigma_\mu | -_j\rangle_R = \cos((\theta_i - \theta_k - \theta_j)/2) - \cos((\theta_i + \theta_k - \theta_j)/2). \quad (22)$$

Thus the amplitudes for spin projection +1/2 and –1/2 are given by:

$$M_{1/2}^{peng} = (-i2\sqrt{2}/V)\xi(x)\{A_1[\sin((\theta_i - \theta_j - \theta_k)/2) + \sin((\theta_i + \theta_j - \theta_k)/2)] + A_2[\sin((\theta_k - \theta_i - \theta_j)/2) - \sin((\theta_k + \theta_i - \theta_j)/2)]\}, \quad (23)$$

$$M_{-1/2}^{peng} = (-i2\sqrt{2}/V)\xi(x)\{A_1[\cos((\theta_i - \theta_j - \theta_k)/2) - \cos((\theta_i + \theta_j - \theta_k)/2)] + A_2[\cos((\theta_k - \theta_i - \theta_j)/2) + \cos((\theta_k + \theta_i - \theta_j)/2)]\}. \quad (24)$$

where:

$$\xi(x) = (\alpha_s/\sqrt{2}\pi)(4/3)F_1^L(q^2). \quad (25)$$

Since the colour factor is given by:

$$\sum_{a=1}^{8} T^a T^a = [(1/3) - (1/6) + (1/2)]^2 = 4/9. \quad (26)$$

Also, according to Eq. 7 the $F_1^L(q^2)$ is:

$$F_1^L(q^2) = (G_F/\sqrt{2}) \sum_{i=u,c,t} V_{iq}^* V_{ib} f_1(x_i), = (G_F/\sqrt{2})\{V_{uq}^* V_{ub} f_1(x_u) + V_{cq}^* V_{cb} f_1(x_c) + V_{tq}^* V_{tb} f_1(x_t)\}, \quad (27)$$

here q refers to d and s quarks. We used Eqs.10, 11 and 13, to obtain $f_1(x_u), f_1(x_c)$ and $f_1(x_t)$, respectively.

3.2. Second step (magnetic dipole term)

A charge particle in orbital motion generates a magnetic dipole moment of a magnitude proportional to its orbital angular momentum. Furthermore, a particle with intrinsic angular momentum or spin has an intrinsic magnetic moment. The magnetic dipole term in the penguin amplitude, according to Eq. 5, is:

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu} q^\nu/q^2)[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \quad (28)$$

Also, according to Eq. 8 magnetic (dipole) form factor at $q^2 = 0 (q^2/M_W^2 \ll 1)$ is:

$$\frac{F_2^L(0)}{m_k} = \frac{F_2^R(0)}{m_b} = \frac{g_2^2}{8M_W^2} \sum_i V_{ik}^* V_{ib} f_2(x_i). \quad (29)$$

The top quark is dominant for $F_2^R(0)$, so we can write:

$$F_2^R(0) = m_b (G_F/\sqrt{2})(V_{tk}^* V_{tb}) f_2(x_t). \quad (30)$$

Here $f_2(x_t)$ defined by Eq. 9 and $x_t = m_t^2/M_W^2$, also we saw that $F_2^L(0) \ll F_2^R(0)$, because $m_k \ll m_b$, so the magnetic dipole term becomes:

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu} q^2) F_2^R(0) P_R. \quad (31)$$

Substituting in the penguin amplitude, according to Eq. 4:

$$M^{dip} = \frac{g_s^2}{4\pi^2} [\bar{u}_k(p_k) T^a (i\sigma_{\mu\nu} q^\nu/q^2) F_2^R(0) P_R u_b(p_b)] [\bar{u}_i(p_i) \gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})], \quad (32)$$

where

$$\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu] = (i/2)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu). \quad (33)$$

and

$$\gamma^\mu\gamma^\nu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^\nu \\ \tilde{\sigma}^\nu & 0 \end{pmatrix} = \begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu \end{pmatrix}. \quad (34)$$

Therefore:

$$\sigma^{\mu\nu} = \frac{i}{2} \begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu - \sigma^\nu\tilde{\sigma}^\mu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu - \tilde{\sigma}^\nu\sigma^\mu \end{pmatrix}. \quad (35)$$

The wave function of b and q_k is given by:

$$\bar{u}_k \sigma_{\mu\nu}[(1+\gamma_5)/2] u_b = \frac{i}{2} \begin{pmatrix} \psi_{kL} \\ \psi_{kR} \end{pmatrix}^f \begin{pmatrix} \tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu & 0 \\ 0 & \sigma_\mu\tilde{\sigma}_\nu - \sigma_\nu\tilde{\sigma}_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \psi_{bR} \end{pmatrix} = (i/2)\psi_{kL}(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)\psi_{bR}. \quad (36)$$

Substituting in the penguin amplitude:

$$M^{dip} = -\frac{g_s^2}{8\pi^2} F_2^R(0)(T^a T^a) \left[\bar{u}_{kL} \frac{q^\nu}{q^2} (\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu) u_{bR}\right] [\bar{u}_i(\tilde{\sigma}^\mu + \sigma^\mu) v_{\bar{j}}]. \quad (37)$$

Substituting $q^\nu = (p_b - p_k)^\nu$ in the above equation, we have:

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[(\bar{u}_{kL}\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)u_{bR}) - (\bar{u}_{kL}(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu u_{bR})] \times [\bar{u}_i\tilde{\sigma}^\mu v_{\bar{j}} + \bar{u}_i\sigma^\mu v_{\bar{j}}], \quad (38)$$

or

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)[(T^aT^a)\frac{1}{q^2}[\langle k_L|\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)|b_R\rangle - \langle k_L|(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu|b_R\rangle] \times [\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle]. \quad (39)$$

We known that:

$$\tilde{\sigma}_\mu\, p_b^\mu|b_L\rangle = m_b|b_R\rangle, \qquad \tilde{\sigma}_\mu\, p_k^\mu|k_L\rangle = m_k|k_R\rangle,$$
$$\sigma_\nu\, p_b^\nu|b_R\rangle = m_b|b_L\rangle, \qquad \sigma_\nu\, p_k^\nu|k_R\rangle = m_k|k_{LR}\rangle,$$
$$\tilde{\sigma}_\mu\,(\sigma_\nu p_b^\nu) = p_{k\mu}\,, \qquad (\tilde{\sigma}_\mu\, p_b^\nu)\sigma_\nu = p_{b\mu}. \quad (40)$$

and

$$(p_b + p_k)_\mu = (2p_b - p_i - p_j)_\mu. \quad (41)$$

Also according to conservation of current

$$(p_i + p_j)_\mu[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle] = m_i[\langle i_R|j_L\rangle + \langle i_L|j_R\rangle] - m_j[\langle i_L|j_R\rangle + \langle i_R|j_L\rangle] = 0. \quad (42)$$

Since in the penguin decays $m_i = m_j$ and $|j\rangle$ is the antiparticle:

$$\tilde{\sigma}_\mu p_{j\mu}|j_L\rangle = -m_j|j_R\rangle. \quad (43)$$

Consequently, the magnetic dipole term of penguin amplitude becomes:

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle - m_k\langle k_R|\sigma_\mu|b_R\rangle - (p_b + p_k)_\mu\langle k_L|b_R\rangle] \times [\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle]. \quad (44)$$

We neglected the term $m_k\langle k_R|\sigma_\mu|b_R\rangle$, because $m_k \ll m_b$:

$$M^{dip} = (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle - (p_b + p_k)_\mu\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - (p_b + p_k)_\mu\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \quad (45)$$

Using Eq. 42 for the second part of Eq.45, results in:

$$M^{dip} = (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle - 2p_{b\mu}\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - 2p_{b\mu}\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle], \quad (46)$$

here b meson is at the rest $\left(p_{b\mu} = (m_b, \vec{0})\right)$ and

$$(T^aT^a) = 4/3,$$
$$d_8 = -\frac{g_s^2}{8\pi^2}F_2^R(0) = -\frac{g_s^2}{8\pi^2}m_b\frac{g_2^2}{8M_W^2}\sum_i V_{ik}^* V_{ib} f_2(x_i), = -(2\sqrt{2}G_F)(m_b/2)(\alpha_s/4\pi)\sum_i V_{ik}^* V_{ib} f_2(x_i). \quad (47)$$

Hence, the magnetic dipole of penguin amplitude is given by:

$$M^{dip} = (4/3)d_8(m_b/q^2)[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle - 2\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - 2\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \quad (48)$$

Now we must calculate each term of Eq. 48 for b spins project -1/2 and +1/2, then squaring these terms and adding all of them and at least averaging.
The b quark is at the rest and to have spin projection –1/2 along angle θ_b, thus the spin projection of b quark of +1/2 is along $\theta_b - \pi(\theta_b \to \theta_b - \pi)$:

$$b\ spin(-1/2)\ and\ angle\ \theta_b \propto (1/\sqrt{2})\begin{pmatrix}-\sin(\theta_b/2)\\ \cos(\theta_b/2)\end{pmatrix},$$

$$b\ spin(+1/2)\ and\ angle\ \theta_b \propto (1/\sqrt{2})\begin{pmatrix}\cos(\theta_b/2)\\ \sin(\theta_b/2)\end{pmatrix}. \quad (49)$$

Substituting the factor of $(1/\sqrt{2})$ in the M^{dip} and negligible terms $\langle k_L|b_R\rangle$, thus the amplitude of magnetic dipole becomes:

$$M^{dip} = A_8 d_8[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle], \quad (50)$$

here

$$A_8 = (1/\sqrt{2})(4/3)(b_b/q^2). \quad (51)$$

The first term of Eq. 50 for b spin project –1/2, according to Fierz transformation:

$$\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle = -\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle, \quad (52)$$

is given by:

$$M^{dip}_{1(-1/2)} \equiv \langle k_L|\tilde{\sigma}^{\mu}|b_L\rangle \ \langle i_L|\tilde{\sigma}_{\mu}|j_L\rangle, = -\cos\big((\theta_b-\theta_i-\theta_k+\theta_j)/2\big) - \cos\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big). \quad (53)$$

and the first term for b spin project +1/2 is given by:

$$M^{dip}_{1(+1/2)} \equiv -\langle k_L|\tilde{\sigma}^{\mu}|b_L\rangle \ \langle i_L|\tilde{\sigma}_{\mu}|j_L\rangle, = -\sin\big((\theta_b+\theta_i-\theta_k-\theta_j)/2\big) + \sin\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big). \quad (54)$$

Also the second term of Eq. 50 for b spin project –1/2 is given by:

$$M^{dip}_{2(-1/2)} \equiv -\langle k_L|\tilde{\sigma}^{\mu}|b_L\rangle \ \langle i_R|\sigma_{\mu}|j_R\rangle, = -\cos\big((\theta_b+\theta_i-\theta_k-\theta_j)/2\big) - \cos\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big). \quad (55)$$

In addition the second term for b spin project +1/2 is given by:

$$M^{dip}_{1(+1/2)} \equiv -\langle k_L|\tilde{\sigma}^{\mu}|b_L\rangle\langle i_R|\sigma_{\mu}|j_R\rangle, = -\sin\big((\theta_b-\theta_i-\theta_k+\theta_j)/2\big) + \sin\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big). \quad (56)$$

Thus the amplitudes for spin projection +1/2 and –1/2 are given by:

$$M^{dip}_{1(+1/2)} = -A_8 d_8\{[-\sin\big((\theta_b+\theta_i-\theta_k-\theta_j)/2\big) + \sin\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big)] + [-\sin\big((\theta_b-\theta_i-\theta_k+\theta_j)/2\big) + \sin\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big)]\}, \quad (57)$$

$$M^{dip}_{1(-1/2)} = A_8 d_8\{[-\cos\big((\theta_b-\theta_i-\theta_k+\theta_j)/2\big) - \cos\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big)] + [-\cos\big((\theta_b+\theta_i-\theta_k-\theta_j)/2\big) - \cos\big((\theta_b-\theta_i+\theta_k-\theta_j)/2\big)]\}. \quad (58)$$

3.3. Final step (Penguin + Magnetic Dipole)

We want to consider all terms in Eq. 5, by adding the two amplitudes of penguin (see Eqs. 23 and 24) and magnetic dipole (see Eqs. 57 and 58) terms. In this case, the amplitudes for spin projection +1/2 and –1/2 are given by:

$$M^{p+d}_{(+1/2)} = (-i/V)\{[\xi(x)(A_2-A_1) - A_8 d_8(A_1 - A_2)]\sin\big((\theta_k-\theta_i-\theta_j)/2\big) + [-A_1\xi(x) + A_2A_8d_8]\sin\big((\theta_k-\theta_i+\theta_j)/2\big) + [-A_2\xi(x) + A_1A_8d_8]\sin\big((\theta_k+\theta_i-\theta_j)/2\big)\}, \quad (59)$$

$$M^{p+d}_{(-1/2)} = (-i/V)\{[\xi(x)(A_2-A_1) - A_8 d_8(A_1 + A_2)]\cos\big((\theta_k-\theta_i-\theta_j)/2\big) + [A_1\xi(x) - A_2A_8d_8]\cos\big((\theta_k-\theta_i+\theta_j)/2\big) + [A_2\xi(x) - A_1A_8d_8]\cos\big((\theta_k+\theta_i-\theta_j)/2\big)\}. \quad (60)$$

The amplitude of tree level decay rate is:

$$M^{t}_{(+1/2)} = (-i/V)A_1\eta\left[\sin\big((\theta_k-\theta_i-\theta_j)/2\big) + \sin\big((\theta_k-\theta_i+\theta_j)/2\big)\right],$$

$$M^{t}_{(-1/2)} = (-i/V)A_1\eta\left[\cos\big((\theta_k-\theta_i-\theta_j)/2\big) - \cos\big((\theta_k-\theta_i+\theta_j)/2\big)\right], \quad (61)$$

Here:

$$\eta = 2G_F|V_{ib}V^{*}_{jk}|. \quad (62)$$

We added the amplitude of (tree + penguin + magnetic dipole) terms for b spin projection +1/2 and –1/2, so:

$$M^{t+p+d}_{(+1/2)} = (-i/V)\{\lambda_1\sin\big((\theta_k-\theta_i-\theta_j)/2\big) + \lambda_2\sin\big((\theta_k-\theta_i+\theta_j)/2\big) - \lambda_3\sin\big((\theta_k+\theta_i-\theta_j)/2\big)\}, \quad (63)$$

$$M^{t+p+d}_{(-1/2)} = (-i/V)\{\lambda_1\cos\big((\theta_k-\theta_i-\theta_j)/2\big) - \lambda_2\cos\big((\theta_k-\theta_i+\theta_j)/2\big) + \lambda_3\cos\big((\theta_k+\theta_i-\theta_j)/2\big)\}, \quad (64)$$

where:

$$\lambda_1 = A_1(\eta-\xi(x)-A_8d_8) + A_2(\xi(x)-A_8d_8),$$

$$\lambda_2 = A_1\big(\eta-\xi(x)\big) + A_2A_8d_8,$$
$$\lambda_3 = A_2\xi(x) - A_1A_8d_8. \quad (65)$$

Squaring spine average, the amplitude is given by

$$|M^{t+p+d}|^2_{spi-ave} = \frac{1}{2}\left[\left|M^{t+p+d}_{(+1/2)}\right|^2 + \left|M^{t+p+d}_{(-1/2)}\right|^2\right]$$

$$= (-i/V)^2(1/4)(1/8)\{\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 2\lambda_1\lambda_2\cos(\theta_k - \theta_i) + 2\lambda_1\lambda_3\cos(\theta_j - \theta_k) - 2\lambda_2\lambda_3\cos(\theta_j - \theta_i)\}. \quad (66)$$

Now, we need to obtain all of the helicity states and then add them together:

$$|M^{t+p+d}|^2_{spi-ave} = (1/V^2)(1/4)\{\alpha_1 + \alpha_2 + \alpha_3(1-v_i^2)^{1/2}(1-v_j^2)^{1/2} - 2v_iv_k(\alpha_4+\alpha_5)\cos(\theta_k-\theta_i) + 2v_jv_k(\alpha_6+\alpha_7)\cos(\theta_j-\theta_k) - 2v_iv_j(\alpha_8+\alpha_9)\cos(\theta_j-\theta_i)\}, \quad (67)$$

here

$$\alpha_1 = (\eta - \xi(x) - A_8d_8)^2 + (\eta + \xi(x))^2 - (A_8d_8)^2,$$

$$\alpha_2 = (\xi(x) - A_8d_8)^2 + \xi(x)^2 + (A_8d_8)^2,$$

$$\alpha_3 = 2[(\eta - \xi(x) - A_8d_8)(\xi(x) - A_8d_8) + (\eta - \xi(x))(A_8d_8) - \xi(x) - A_8d_8],$$

$$\alpha_4 = (\eta - \xi(x) - A_8d_8)(\eta + \xi(x)),$$

$$\alpha_5 = (\xi(x) - A_8d_8)(A_8d_8),$$

$$\alpha_6 = -(\eta - \xi(x) - A_8d_8)(A_8d_8),$$

$$\alpha_7 = (\xi(x) - A_8d_8)\xi(x),$$

$$\alpha_8 = -(\eta - \xi(x)) - A_8d_8$$

$$\alpha_9 = \xi(x) - A_8d_8. \quad (68)$$

After integration in the phase space and change variable x and y, the total decay rate gives:

$$d^2\,\Gamma^{t+p+d}/dxdy = (3/8)(M_b^2/194\pi^3)\,[I_1 + I_2 + I_3]. \quad (69)$$

We assume the momentum of quarks, according to x and y, to be:

$$p_i = x\,M_b/2, \qquad p_k = y\,M_b/2, \qquad p_{\bar{j}} = x\,M_b/2. \quad (70)$$

The phase spaces are

$$I_1 = 6xy\cdot f_{ab}\cdot[\alpha_{10} - 2(\alpha_4+\alpha_5)h_{abc}],$$

$$I_2 = 6xy\cdot f_{ab}\cdot[2(\alpha_6+\alpha_7)h_{bca}],$$

$$I_3 = 6xy\cdot f_{ac}\cdot[-2(\alpha_8+\alpha_9)h_{acb}], \quad (71)$$

where

$$\alpha_{10} = \alpha_1 + \alpha_2 + \alpha_3(1-v_i^2)^{1/2}(1-v_j^2)^{1/2},$$
$$(1-v_i^2)^{1/2} = \sqrt{1-(x^2/(x^2+a^2))},$$
$$(1-v_j^2)^{1/2} = c/[2-\sqrt{x^2+a^2}-\sqrt{y^2+b^2}]. \quad (72)$$

Also:

$$f_{ab} = 2 - \sqrt{x^2+a^2} - \sqrt{y^2+b^2},$$

$$h_{abc} = \frac{(f_{ab})^2 - (c^2+x^2+y^2)}{2\sqrt{x^2+a^2}\sqrt{y^2+b^2}},$$

$$f_{bc} = 2 - \sqrt{x^2+b^2} - \sqrt{y^2+c^2},$$

$$h_{bca} = \frac{(f_{bc})^2 - (a^2+x^2+y^2)}{2\sqrt{x^2+b^2}\sqrt{y^2+c^2}},$$

$$f_{ac} = 2 - \sqrt{x^2+a^2} - \sqrt{y^2+c^2},$$

$$h_{acb} = \frac{(f_{ac})^2 - (b^2+x^2+y^2)}{2\sqrt{x^2+a^2}\sqrt{y^2+c^2}}, \quad (73)$$

here the constants *a*, *b* and *c* are:

$$a = 2\,m_i/M_b, \quad b = 2\,m_k/M_b, \quad c = 2\,m_j/M_b. \quad (74)$$

4. Numerical Results

As an example of the use of the above formalism, we use the standard particle data group parameterization of the CKM matrix with the central values $\theta_{12} = 0.221, \theta_{13} = 0.0035$, $\theta_{23} = 0.041$, and choose the CKM phase δ_{13} to be $\pi/2$. Following Ali and Greub, we treat internal quark masses in tree-level loops with the values (GeV) $m_b = 4.88, m_s = 0.2$, $m_d = 0.01$, $m_u = 0.005, m_c = 1.5$.

The decay rates and branching ratios of tree plus penguin plus magnetic dipole (Eq. 73), for both particles and antiparticles $b \to ud\bar{u}$, $us\bar{u}$, $cd\bar{c}$, $cs\bar{c}$ and $b \to cd\bar{u}$ are shown in Table 1. It can be seen that the decay rate for the antiparticle $\bar{b} \to \bar{u}\bar{d}u$ is greater than the decay rate particle $b \to ud\bar{u}$, and the decay rate antiparticle $\bar{b} \to \bar{c}\bar{d}c$ is less than decay rate particle $b \to cd\bar{c}$, and so one. We consider that, modes $b \to c\bar{u}d$, and $b \to c\bar{c}s$ are dominant. The total decay rates of (tree + penguin + magnetic dipole) is given by:

$$\Gamma^{T+P+MD}_{total,P} = 3.107\times10^{-13}\,GeV,$$

$$\Gamma^{T+P+MD}_{total,AP} = 3.104\times10^{-13}\,GeV.$$

Table 1. Decay rates (DR) and branching ratios (BR) of (tree + penguin + magnetic dipole) of b-quark and b-antiquark $(DR \times 10^{-15}\ GeV)(BR \times 10^{-2})$.

Process	DR	BR	Process	DR	BR
$b \to cd\bar{c}$	2.291	0.737	$\bar{b} \to \bar{c}\bar{d}c$	2.224	0.716
$b \to cs\bar{c}$	41.570	13.370	$\bar{b} \to \bar{c}\bar{s}c$	41.630	13.410
$b \to cd\bar{u}$	184.600	59.410	$\bar{b} \to \bar{c}\bar{d}u$	184.600	59.470
$b \to cs\bar{u}$	9.280	2.986	$\bar{b} \to \bar{c}\bar{s}u$	9.280	2.989
$b \to ud\bar{u}$	2.252	0.724	$\bar{b} \to \bar{u}\bar{d}u$	2.684	0.864
$b \to us\bar{u}$	2.236	0.719	$\bar{b} \to \bar{u}\bar{s}u$	2.203	0.709
$b \to ud\bar{c}$	2.368	0.762	$\bar{b} \to \bar{u}\bar{d}c$	2.368	0.762
$b \to us\bar{c}$	1.235	0.397	$\bar{b} \to \bar{u}\bar{s}c$	1.235	0.394

5. Conclusions

The decay rates of the b-quark at the tree-level and the penguin, including the magnetic dipole terms it shown. According to Table 1, the dominant mode in b-quark in the hadronic decays is, $b \to cd\bar{u}$ because the decay rates of $b \to c$ channel are much bigger than $b \to u$, since $V_{cb} \gg V_{ub}$. In addition, the dominant mode in the pure penguin decays is, $b \to s$.

The magnetic dipole terms are small for b-quark decay rates (the magnetic dipole contributions are small) and the decay rate of the tree and the tree plus penguin are also not very different. The decay rates and branching ratios are very similar in all the models but the (tree + penguin + magnetic dipole) total decay rate is about 10% larger than the simple tree or tree plus penguin.

The decay rates of $b-$ and $\bar{b}-$ quark, at the tree-level are exactly the same, but in the pure penguin, tree plus penguin, they are different. For example, $\Gamma_{b\to sd\bar{d}} < \Gamma_{\bar{b}\to\bar{s}\bar{d}d}$, $\Gamma_{b\to ud\bar{u}} < \Gamma_{\bar{b}\to\bar{u}\bar{d}u}$, $\Gamma_{b\to cd\bar{u}} > \Gamma_{\bar{b}\to\bar{c}\bar{d}u}$ and $\Gamma_{b\to cs\bar{c}} \approx \Gamma_{\bar{b}\to\bar{c}\bar{s}u}$, because the total decay rates of $b-$ and $\bar{b}-$ quark must be equal, $\Gamma_b^{total} = \Gamma_{\bar{b}}^{total}$.

Also the decay rates and branching ratios are very similar in all the models. On the other hand, including the penguin induces matter antimatter asymmetries. These are largest in the rare decay $b \to ud\bar{u}$, the decay rate of which, is about 7% smaller than the decay rate $\bar{b} \to \bar{u}\bar{d}u$. Also the rate $b \to su\bar{u}$ is larger than the rate $\bar{b} \to \bar{s}\bar{u}u$.

References

[1] A. Datta, D. London, JHEP. 0404, 72 (2004).
[2] L. Silvestrini, Ann. Rev. Nucl. Part. Sci. **57**, 405 (2007).
[3] G. Eigen, eConf. C **0304052**, WG204 (2003).
[4] A. Pich, arXiv: 0705.4264 (2007).
[5] R. Fleischer, S. Recksiegel, Phys. Rev. D**71**, 051501 (2005).
[6] A. J. Buras, R. Fleischer, S. Recksiegel, F. Schwab, arXiv: 0512059 (2005).
[7] C. S. Kim, S. Oh, C. Yu, Phys. Lett. B **621**, 259 (2005).
[8] A. J. Buras, P. Gambino, U. Haisch, Nucl. Phys. B. **570**, 117 (2000).
[9] P. Koppenburg, Eur. Phys. J. C**33**, 274 (2004).
[10] T. Besmer, A. Steffen, Phys. Rev. D **63**, 055007 (2001).
[11] M. Ciuchini, M. Pierini, L. Silvestrini, Phys. Rev. Lett. **100**, 031802 (2008).
[12] K. C. Yang. Phys. Rev. D **72**, 034009 (2005).
[13] S. Villa, Phys. Rev. D**73**, 05117 (2006).
[14] B. Aubert, Phys. Rev. Lett. **98**, 061803 (2007).
[15] R. Fleischer, S. Recksiegel, arXiv:0511.325 (2005).
[16] Y. Grossman, A. L. Kagan, M. Neubert, JHEP. **9910**, 029 (1999).
[17] M. Beneke, T. Feldmann, D. Seidel, Eur. Phys. J. C**41**,173 (2005).
[18] T. N. Pham, AIP Conf. Proc. **602**, 206 (2001).
[19] R. Fleischer, Eur. Phys. J. C**51**, 849 (2007).
[20] M. Athanas, Phys. Rev.Lett. **80**, 5493 (1998).
[21] A. Deshpande, X. G. He, W. S. Hou, Phys. Rev. Lett. **82**, 2240 (1999).
[22] S. Mishima, T. Yoshikawa, Phys. Rev. D**70**, 094024 (2004).
[23] R. Fleischer, arXiv:0608.010 (2006).
[24] D. S. Du, M. Z. Yang, Phys. Lett. B**358**, 123 (1995).
[25] L. Lesniak, A. Furman, R. Kaminski, B. Loiseau, PoS HEP2005, 249 (2005).
[26] A. Furman, R. Kaminski, L. Lesniak, B. Loiseau, Phys. Lett. B**622**, 207 (2005).
[27] A. Datta, D. London, Phys. Lett. B**595**, 453 (2004).
[28] C. K. Chua, Phys. Lett. B**633**, 70 (2006).
[29] Z. Xiao, C. S. Li, K. T. Chao, Phys. Lett. B**473**, 148 (2000).
[30] [30] M. Beneke, J. Rohrer, D. Yang, Phys.Rev.Lett. **96**, 141801 (2006).
[31] S. A. Abel, W. N. Cottingham, I. B. Wittingham, Phys.Rev.D**58**, 073006 (1998).
[32] Z. Xiao, W. Li, L. Guo, G. Lu," Eur. Phys. J. C**18**, 681 (2001).
[33] W. N. Cottingham, H. Mehrban, I. B. Wittingham, Phys. Rev. D**60**, 114029 (1999).
[34] Particle Data Group, European physical J.C**3,** 1 (2010).
[35] A. Ali, C. Greub, Phys. Rev. D**57**, 2996 (1998).