Propagation of magnetogasdynamic shock waves in a self-gravitating gas with exponentially varying density

Gorakh Nath[1,4*]
*Corresponding author
Email: gn_chaurasia_univgkp@yahoo.in

Jagdamba Prasad Vishwakarma[2]
Email: jpv_univgkp@yahoo.com

Vinod Kumar Srivastava[3]
Email: vksmaths.saket@gmail.com

Aswani Kumar Sinha[3]
Email: ashwani_gkp@yahoo.com

[1]Department of Mathematics, National Institute of Technology Raipur, G. E. Road, Raipur 492010, India

[2]Department of Mathematics and Statistics, D.D.U. Gorakhpur University, Gorakhpur 273009, India

[3]Department of Mathematics, K. S. Saket P.G. College, Faizabad 224123, India

[4]Present Address: Department of Mathematics, Motilal Nehru National Institute of Technology, Allahabad 211004, India

Abstract

Non-similarity solutions are obtained for one-dimensional adiabatic flow behind a magnetogasdynamic spherical (or cylindrical) shock wave propagating in a self-gravitating perfect gas in the presence of a constant azimuthal magnetic field. The density of the gas is assumed to be varying and obeying an exponential law. The shock wave moves with variable velocity, and the total energy of the wave is non-constant and varies with time. The effects of variation of the Alfven-Mach number and time are obtained. It is investigated that the presence of gravitational field reduces the effects of the magnetic field. Also, the presence of gravitational field increases the compressibility of the medium, due to which it is compressed and therefore the distance between the inner contact surface and the shock surface is reduced. A comparison between the solutions in the cases of the gravitating and the non-gravitating medium with or without magnetic field is made. The solutions are applicable for arbitrary values of time.

Keywords

Shock waves, Magnetic fields, Magnetohydrodynamics, Self-gravitating gas, Blast waves, Electrohydrodynamics

MSC

76L05, 76W05

Introduction

Shock processes can naturally occur in various astrophysical situations, for example, photoionized gas, stellar winds, supernova explosions, collisions between high-velocity clumps of interstellar gas, etc. Shock phenomena, such as a global shock resulting from a stellar pulsation or supernova explosion passing outward through a stellar envelope or perhaps a shock emanating from a point source such as a man-made explosion in the Earth's atmosphere or an impulsive flare in the Sun's atmosphere, have tremendous importance in astrophysics and space sciences. Shock waves are common in the interstellar medium because of a great variety of supersonic motions and energetic events, such as cloud-cloud collision, bipolar outflow from young protostellar objects, powerful mass losses by massive stars in a late stage of their evolution (stellar winds), supernova explosions, central part of star burst galaxies, etc. Shock waves are also associated with spiral density waves, radio galaxies and quasars. Similar phenomena also occur in laboratory situations, for example, when a piston is driven rapidly into a tube of gas (a shock tube), when a projectile or aircraft moves supersonically through the atmosphere, in the blast wave produced by a strong explosion, or when rapidly flowing gas encounters a constriction in a flow channel or runs into a wall. The explanation and analysis for the internal motion in stars is one of the basic problems in astrophysics. According to observational data, the unsteady motion of a large mass of gas followed by sudden release of energy results in flare-ups in novae and supernovae. A qualitative behavior of the gaseous mass may be discussed with the help of the equations of motion and equilibrium taking gravitational forces into account. Numerical solutions for self-similar adiabatic flows in self-gravitating gas were obtained by Sedov [1] and Carrus et al. [2], independently. Purohit [3] and Singh and Vishwakarma [4] have discussed homothermal flows behind spherical shock waves in a self-gravitating gas using similarity method. Shock waves through a variable-density medium have been treated by Sedov [1], Sakurai [5], Nath [6], Rogers [7], Rosenau and Frankenthal [8], Nath et al. [9], Vishwakarma and Yadav [10] and others. Their results are more applicable to the shock formed in the deep interior of stars.

Hayes [11], Laumbach and Probstein [12], Deb Ray [13], Verma and Vishwakarma [14, 15], Vishwakarma [16], Vishwakarma and Nath [17], Nath [18] and Vishwakarma et al. [19] have discussed the propagation of shock waves in a medium where density varies exponentially and obtained similarity and non-similarity solutions. These authors have not taken into account the effects of the self-gravitation of the medium. The shock waves in conducting perfect gas in the presence of a magnetic field can be important for description of shocks in supernova explosions and explosion in the ionosphere. The strong magnetic fields play significant roles in the dynamics of the interstellar medium. Among the industrial applications involving applied external magnetic fields are drag reduction in duct flows, design of efficient coolant blankets in tokamak fusion reactors, control of turbulence of immersed jets in the steel casting process and advanced propulsion and flow control schemes for hypersonic vehicles.

The magnetic fields have important roles in a variety of astrophysical situations. Complex filamentary structures in molecular clouds, shapes and the shaping of planetary nebulae, synchrotron radiation from supernova remnants, magnetized stellar winds, galactic winds, gamma-ray bursts, dynamo effects in stars, galaxies, and galaxy clusters as well as other interesting problems all involve magnetic fields (see [20, 21]).

In the present work, we investigated the effects of the presence of an ambient azimuthal magnetic field and the self-gravitation of the ambient medium on the flow field behind a magnetogasdynamic spherical (or cylindrical) shock wave. Non-similarity solutions for the flow field behind the shock wave are obtained. The density in the medium ahead of the shock is assumed to obey an exponential law. The medium is assumed to be a perfect gas and the initial magnetic field to be constant. The present study can be important to varify the accuracy of the solution obtained by the theory of self-similarity and computational methods such as finite difference scheme, finite element, etc.

Variation of the flow variables behind the shock for different values of the Alfven-Mach number and time is obtained. It is investigated that the presence of gravitational field reduces the effects of the magnetic field. Also, the presence of gravitational field increases the compressibility of the medium, due to which it is compressed and therefore the distance between the inner contact surface and the shock surface is reduced. A comparison between the solutions in the cases of the self-gravitating and the non-gravitating medium is made for both the magnetic and non-magnetic cases.

Fundamental equations and boundary conditions

The fundamental equations governing the unsteady adiabatic spherically (or cylindrically) symmetric flow of an electrically conducting and self-gravitating gas, in the presence of an azimuthal magnetic field may, in Eulerian coordinates can be expressed as [22–24]

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{i\rho u}{r} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left(\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right) + \frac{mG}{r^i} = 0, \tag{2.2}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + (i-1)\frac{hu}{r} = 0, \tag{2.3}$$

$$\frac{\partial m}{\partial r} = 2\pi i\rho r^i, \tag{2.4}$$

$$\frac{\partial U}{\partial t} + u\frac{\partial U}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0, \tag{2.5}$$

where r and t are the independent space and time coordinates, respectively, u is the fluid velocity, ρ is the density, p is the pressure, h is the azimuthal magnetic field, U is the internal energy per unit mass, μ is the magnetic permeability, m is the mass contained in a unit cylinder of radius r or in a sphere of radius r and the dimension of m is taken as $[m] = ML^{i-2}$ where i takes the values 2 and 1 for the respective cases of spherical and cylindrical symmetry, and G is the gravitational constant. In the non-gravitating case, Equation 2.4 and the term $\frac{mG}{r^i}$ in Equation 2.2 do not occur. The electrical conductivity of the gas is assumed to be infinite, and the effects of viscosity and heat conduction are not considered.

The magnetic field equation (2.3) contains all the relevant information needed from Maxwell's equations and Ohm's law; the diffusion term is omitted from it by virtue of the assumed infinite electrical conductivity. The assumption of infinite electrical conductivity of the gas is physically realistic in the case of astrophysical phenomena where the magnetic Reynolds number is very high (or infinite) due to astrophysical scale. The magnetic Reynolds number is a dimensionless parameter defined by $R_{\mathrm{m}} = \frac{\overline{U}\,\overline{L}}{\eta_{\mathrm{m}}}$, where $\overline{U}$ and $\overline{L}$ are the characteristic velocity and characteristic length of the flow field, respectively, and $\eta_{\mathrm{m}} = \frac{1}{\mu\sigma}$ is the magnetic diffusivity (magnetic viscosity), σ being the electrical conductivity of the medium (see p. 169 in [25]). In this case, the Reynolds number R_e of the flow is also very high, as R_e is defined by $R_e = \frac{\overline{U}\,\overline{L}}{\nu}$, where ν is the kinematic coefficient of viscosity. It is well known that the effects of viscosity and heat conduction are negligibly small for the high Reynolds number flow except in the boundary layer region near the solid boundary or in any other region of large variations in velocity and temperature such as inside of a shock wave (see p. 139 in [26]). Therefore, omission of the effects of viscosity and heat conduction on the flow field may be justified in the present study.

The above system of equations should be supplemented with an equation of state. A perfect gas behaviour of the medium is assumed, so that

$$p = \Gamma \rho T, \qquad U = \frac{p}{\rho(\gamma - 1)}, \tag{2.6}$$

where Γ is the gas constant and γ is the ratio of specific heats at constant pressure and volume.

The ambient density of the medium is assumed to obey the exponential law, namely,

$$\rho_a = \rho_0 e^{\delta R}, \tag{2.7}$$

where R is the shock radius and ρ_0 and δ are suitable constants.

We assume that a spherical (or cylindrical) shock is propagating outwardly in the undisturbed ideal gas with infinite electrical conductivity and variable density in the presence of a constant azimuthal magnetic field. The jump conditions at the shock wave are given by the principles of conservation of mass, momentum, magnetic field and energy across the shock [18, 22–24], namely,

$$\begin{aligned} \rho_a V &= \rho_n (V - u_n), \\ h_a V &= h_n (V - u_n), \\ p_a + \frac{1}{2}\mu h_a^2 + \rho_a V^2 &= p_n + \frac{1}{2}\mu h_n^2 + \rho_n (V - u_n)^2, \\ U_a + \frac{p_a}{\rho_a} + \frac{\mu h_a^2}{\rho_a} + \frac{1}{2}V^2 &= U_n + \frac{p_n}{\rho_n} + \frac{\mu h_n^2}{\rho_n} + \frac{1}{2}(V - u_n)^2, \\ m_a &= m_n, \end{aligned} \tag{2.8}$$

where the subscripts 'a' and 'n' denote the conditions immediately ahead and behind of the shock front, respectively, and $V\left(= \frac{dR}{dt}\right)$ denotes the velocity of the shock front.

If the shock is a strong one, then the jump conditions (2.8) become

$$\begin{aligned} u_n &= (1 - \beta)V, \\ \rho_n &= \frac{\rho_a}{\beta}, \\ h_n &= \frac{h_a}{\beta}, \\ p_n &= \left[(1 - \beta) + \frac{1}{2M_A^2}\left(1 - \frac{1}{\beta^2}\right)\right]\rho_a V^2, \\ m_n &= m_a, \end{aligned} \tag{2.9}$$

where $M_A = \left(\frac{\rho_a V^2}{\mu h_a^2}\right)^{1/2}$ is the Alfven-Mach number. The quantity $\beta (0 < \beta < 1)$ is obtained by the relation

$$\beta^2 - \beta\left(\frac{\gamma(M_A^{-2} + 1) - 1}{(\gamma + 1)}\right) + \frac{(\gamma - 2)M_A^{-2}}{(\gamma + 1)} = 0. \tag{2.10}$$

Let the solution of Equations 2.1 to 2.5 be of the form [14, 16, 18]

$$
\begin{aligned}
u &= \frac{1}{t}U(\eta), \\
\rho &= t^{\Omega}D(\eta), \\
p &= t^{\Omega-2}P(\eta), \\
m &= t^{\Omega}K(\eta), \\
\sqrt{\mu}h &= t^{(\Omega-2)/2}H(\eta),
\end{aligned}
\tag{2.11}
$$

where

$$
\eta = te^{\lambda r} \tag{2.12}
$$

while Ω and λ are constants. The variable η assumes a constant value η_0 at the shock surface. Hence,

$$
V = -\frac{1}{\lambda t}, \tag{2.13}
$$

which represents an outgoing shock surface, if $\lambda < 0$.

The solutions of Equations 2.1 to 2.5 in the form (2.11) to (2.13) are compatible with the shock conditions, if

$$
\Omega = 2, \qquad \lambda = -\frac{\delta}{2}. \tag{2.14}
$$

Since necessarily $\lambda < 0$, relation (2.14) shows that $\delta > 0$, thereby meaning that the shock surface expands outwardly in an exponentially increasing medium.

From Equations 2.13 and 2.14, we obtain

$$
R = \frac{2}{\delta}\log\left(\frac{t}{t_0}\right), \tag{2.15}
$$

where t_0 is the duration of the almost instantaneous explosion.

Solution to the equations

The flow variables in the flow field behind the shock front will be obtained by solving Equations 2.1 to 2.5. From Equations 2.11, 2.13, and 2.14, we obtain

$$
\frac{\partial u}{\partial t} = \lambda uV - V\frac{\partial u}{\partial r}, \tag{3.1}
$$

$$
\frac{\partial \rho}{\partial t} = -2\rho\lambda V - V\frac{\partial \rho}{\partial r}, \tag{3.2}
$$

$$
\frac{\partial p}{\partial t} = -V\frac{\partial p}{\partial r}, \tag{3.3}
$$

$$
\frac{\partial h}{\partial t} = -V\frac{\partial h}{\partial r}, \tag{3.4}
$$

$$
\frac{\partial m}{\partial t} = -2\lambda mV - V\frac{\partial m}{\partial r}. \tag{3.5}
$$

Using Equations 3.1 to 3.5 and the transformations

$$r' = \frac{r}{R}, \quad u' = \frac{u}{V}, \quad \rho' = \frac{\rho}{\rho_n}, \quad h' = \frac{h}{h_n}, \quad p' = \frac{p}{p_n}, \quad m' = \frac{m}{m_n}, \tag{3.6}$$

in the fundamental equations (2.1) to (2.5), we obtain for spherical symmetry ($i = 2$)

$$\frac{d\rho'}{dr'} = f_1(r', \rho', p', h', m', u'), \tag{3.7}$$

$$\frac{dp'}{dr'} = f_2(r', \rho', p', h', m', u'), \tag{3.8}$$

$$\frac{dh'}{dr'} = f_3(r', \rho', p', h', m', u'), \tag{3.9}$$

$$\frac{dm'}{dr'} = f_4(r', \rho', p', h', m', u'), \tag{3.10}$$

$$\frac{du'}{dr'} = f_5(r', \rho', p', h', m', u'), \tag{3.11}$$

where

$$f_1(r', \rho', p', h', m', u') = \frac{\rho'}{(1-u')}\Big[f_5 + 2\log\frac{t}{t_0} + \frac{2u'}{r'}\Big],$$

$$f_2(r', \rho', p', h', m', u') = \frac{\rho'}{\Big[(1-\beta)\beta + \frac{M_A^{-2}}{2}\Big(\beta - \frac{1}{\beta}\Big)\Big]} \times$$

$$\Big[\Big\{(1-u') - \frac{M_A^{-2}h'}{\beta\rho'(1-u')}\Big\}f_5 + u'\Big(\log\frac{t}{t_0}\Big)$$

$$-\frac{L^* m'}{r'^2}\{2(\log t/t_0) + (\log t/t_0)^{-1} - 2\}(t/t_0)^4 - \frac{h'u'M_A^{-2}}{r'(1-u')\beta\rho'} - \frac{M_A^{-2}h'^2}{\beta\rho' r'}\Big],$$

$$f_3(r', \rho', p', h', m', u') = \frac{h'}{(1-u')}\left[f_5 + \frac{u'}{r'}\right],$$

$$f_4(r', \rho', p', h', m', u') = \frac{4\rho' r'^2(\log\ t/t_0)^3}{\beta[\,2(\log\ t/t_0)^2 - 2(\log\ t/t_0) + 1]},$$

$$f_5(r', \rho', p', h', m', u') = (1-u') \times$$

$$\frac{\Big[\rho'u'\Big(\log\frac{t}{t_0}\Big) - \frac{M_A^{-2}h'^2}{\beta r'} - \frac{h'u'M_A^{-2}}{r'(1-u')\beta} - \frac{L^*m'}{r'^2}\{2\Big(\log\frac{t}{t_0}\Big) + \Big(\log\frac{t}{t_0}\Big)^{-1} - 2\}(t/t_0)^4\Big]}{\Big[\frac{M_A^{-2}h'}{\beta} - \rho'\,(1-u')^2 + \gamma p'\beta\Big\{(1-\beta) + \frac{M_A^{-2}}{2}\Big(1 - \frac{1}{\beta^2}\Big)\Big\}\Big]},$$

and $L^* = \pi\rho_0 G t_0^2$.

Also, the total energy of the disturbance is given by

$$E = 2\pi\int_{\bar{r}}^{R} \rho\Big[U + \frac{1}{2}u^2 + \frac{\mu h^2}{2\rho} - \frac{mG}{r^{i-1}}\Big]r^i dr, \tag{3.12}$$

where $\bar{r}$ is the position of inner boundary of the disturbance. Using (2.6), (2.9) and (3.6), (3.12) becomes

(for $i = 2$)

$$E = \frac{8\pi\mu}{\delta^2\eta_0^2} R^3 \int_{\bar{r}'}^{1} \Big[\Big\{ (1-\beta) + \frac{M_A^{-2}}{2}\Big(1 - \frac{1}{\beta^2}\Big)\Big\} \frac{p'}{(\gamma-1)} + \frac{\rho' u'^2}{2\beta} + \frac{M_A^{-2} h'^2}{2\beta}$$
$$- 2\pi G\eta_0^2 \frac{m'}{r'(\log t/t_0)} \{2(\log t/t_0)^2 - 2(\log t/t_0) + 1\}(t/t_0)^2 \Big] r'^2 dr'. \quad (3.13)$$

Hence, the total energy of the shock wave is non-constant and varies as R^3. The increase of total energy may be achieved by the pressure exerted on the fluid by the inner expanding surface (a contact surface or a piston). A situation very much of the same kind may prevail during the formation of a cylindrical spark channel from exploding wires. In addition, in the usual cases of spark breakdown, time-dependent energy input is a more realistic assumption than instantaneous energy input [27, 28].

In terms of dimensionless variables r', u', ρ', p', h' and m', the shock conditions (2.10) take the form

$$r' = 1, \quad u' = (1-\beta), \quad \rho' = 1, \quad p' = 1, \quad h' = 1, \quad m' = 1. \quad (3.14)$$

Equations 3.7 to 3.12 along with the boundary conditions (3.14) give the solution of our problem. The solution so obtained is a non-similar one, since the motion behind the shock can be determined only when a definite value for time is prescribed.

Results and discussion

The distribution of the flow variables behind the shock front is obtained by the numerical integration of Equations 3.7 to 3.11 with the boundary conditions (3.14) by the Runge-Kutta method of the fourth order.

For these numerical integrations, we used the 'Mathematica' software in which the number of steps is taken to be 1,000 by default, so that the value of the step size (x) is equal to the distance between a neighbouring point to the piston and the shock front divided by 1,000 (see, for example, for curve 2 with $L^* = 0.001$ in Figure 1, $x = 3.28 \times 10^{-5}$). The Runge-Kutta method of the fourth order gives the value of the interpolating function correct to the first four powers of x and has, therefore, errors of the order of x^5. The Runge-Kutta fourth-order formulae are

$$\rho'(r'+x) = \rho'(r') + \frac{1}{6}(K_{01} + 2K_{11} + 2K_{21} + K_{31}) + O(x^5),$$
$$p'(r'+x) = p'(r') + \frac{1}{6}(K_{02} + 2K_{12} + 2K_{22} + K_{32}) + O(x^5),$$
$$h'(r'+x) = h'(r') + \frac{1}{6}(K_{03} + 2K_{13} + 2K_{23} + K_{33}) + O(x^5),$$
$$m'(r'+x) = m'(r') + \frac{1}{6}(K_{04} + 2K_{14} + 2K_{24} + K_{34}) + O(x^5),$$
$$u'(r'+x) = u'(r') + \frac{1}{6}(K_{05} + 2K_{15} + 2K_{25} + K_{35}) + O(x^5),$$

with

$$K_{0i} = xf_i(r', \rho'(r'), p'(r'), h'(r'), m'(r'), u'(r')),$$
$$K_{1i} = xf_i\left(r' + \frac{x}{2}, \rho'(r') + \frac{K_{01}}{2}, p'(r') + \frac{K_{02}}{2}, h'(r') + \frac{K_{03}}{2}, m'(r') + \frac{K_{04}}{2}, u'(r') + \frac{K_{05}}{2}\right),$$
$$K_{2i} = xf_i\left(r' + \frac{x}{2}, \rho'(r') + \frac{K_{11}}{2}, p'(r') + \frac{K_{12}}{2}, h'(r') + \frac{K_{13}}{2}, m'(r') + \frac{K_{14}}{2}, u'(r') + \frac{K_{15}}{2}\right),$$
$$K_{3i} = xf_i\left(r' + x, \rho'(r') + K_{21}, p'(r') + K_{22}, h'(r') + K_{23}, m'(r') + K_{24}, u'(r') + K_{25}\right),$$

where $i = 1, 2, 3, 4, 5$ and $O(x^5)$, the error terms, are omitted during the numerical calculations.

Figure 1 Variation of reduced flow variables in the region behind the shock front for $\gamma = 5/3$. (**a**) Reduced density ρ', (**b**) reduced pressure p', (**c**) reduced velocity u', (**d**) reduced mass m'. (1) $M_A^{-2} = 0$, $t/t_0 = 3$; (2) $M_A^{-2} = 0.04$, $t/t_0 = 3$; (3) $M_A^{-2} = 0.08$, $t/t_0 = 3$; (4) $M_A^{-2} = 0$, $t/t_0 = 5$; (5) $M_A^{-2} = 0.04$, $t/t_0 = 5$; (6) $M_A^{-2} = 0.08$, $t/t_0 = 5$.

For the purpose of numerical integration, the typical values of physical quantities involved in the computation are taken as [26, 29–32] $\gamma = \frac{5}{3}$; $M_A^{-2} = 0$, 0.04, 0.08; $\frac{t}{t_0} = 3$, 5; and $L^* = 0$, 0.001. For fully ionized gas, $\gamma = \frac{5}{3}$, and therefore, it is applicable to the stellar medium. Rosenau and Frankenthal [8] have shown that the effects of magnetic field on the flow field behind the shock are significant when $M_A^{-2} \geq 0.01$; therefore, the above values of M_A^{-2} are taken for calculation in the present problem. The value $M_A^{-2} = 0$ corresponds to the non-magnetic case. The value $L^* = 0$, $M_A^{-2} = 0$ corresponds to the solution in the non-gravitating and non-magnetic cases, the solution obtained by Vishwakarma [16] in the dust-free case. It should be noted that $0 < \frac{t}{t_0} < 1$ corresponds to the shock free flow and $1 < \frac{t}{t_0} < \infty$ corresponds to the flow under the influence of shock (i.e., shock formation requires that $\frac{t}{t_0} > 1$); therefore, the above values of $\frac{t}{t_0}$ are taken for calculation to know the flow field behind the shock at different times. Starting from the shock front, the numerical integration is carried out until the singularity of the solution

$$\beta\gamma p'\left\{(1-\beta) + M_A^{-2}\left(1 - \frac{1}{\beta^2}\right)\right\} - \frac{M_A^{-2}h'}{\beta} - \rho(1-u')^2 = 0, \qquad (4.1)$$

is reached. This marks the inner boundary of the disturbance, and at this surface, the value of r' ($= \bar{r}'$) remains constant. The inner boundary is the position in the flow field behind the shock front at which the velocity of the inner boundary and the fluid velocity are equal. Also, the velocity of the inner boundary and shock velocity are in a constant ratio. The results are shown in Figures 1a,b,c and 2. These figures show that the self-gravitation of the medium has a significant effect on the flow variables.

Figure 2 Variation of reduced magnetic field h' in the region behind the shock front for $\gamma = 5/3$. (2) $M_A^{-2} = 0.04$, $t/t_0 = 3$; (3) $M_A^{-2} = 0.08$, $t/t_0 = 3$; (5) $M_A^{-2} = 0.04$, $t/t_0 = 5$; (6) $M_A^{-2} = 0.08$, $t/t_0 = 5$.

Table 1 shows the variation of the density ratio $\beta \left(= \frac{\rho_a}{\rho_n}\right)$ across the shock front and the position of the inner expanding surface for different values of M_A^{-2} with $\gamma = \frac{5}{3}$; $L^* = 0, 0.001$; and $\frac{t}{t_0} = 3, 5$ in both the gravitating and non-gravitating cases. The shock strength decreases with an increase in the strength of the magnetic field. Also, Table 1 shows that the distance of the inner expanding surface from the shock front is less in the case of the gravitating medium in comparison with that in the case of the non-gravitating medium. Physically, it means that the gas behind the shock is compressed in the gravitating medium, that is, the shock strength is increased in the gravitating medium.

Table 1 Variation of β and position of inner expanding surface for different values of M_A^{-2} with $\gamma = 5/3$

M_A^{-2}	β	Position of the inner expanding surface $\bar{r}'$			
		$\frac{t}{t_0} = 3$		$\frac{t}{t_0} = 5$	
		Gravitating case ($L^* = 0.001$)	Non-gravitating case ($L^* = 0$)	Gravitating case ($L^* = 0.001$)	Non-gravitating case ($L^* = 0$)
0	0.2500	0.077	0.033	0.8660	0.040
0.04	0.2921	0.672	0.150	0.8580	0.1634
0.08	0.3302	0.617	0.170	0.8480	0.1870

Figures 1a,b,c and 2 show the variation of the flow variables $\frac{\rho}{\rho_n}$, $\frac{p}{p_n}$, $\frac{u}{V}$ and $\frac{h}{h_n}$, with r' at various values of the parameters M_A^{-2}, L^* and $\frac{t}{t_0}$.

Figures 1a,d and 2 show the distributions of reduced density, reduced mass and reduced azimuthal magnetic field, respectively. These flow variable decrease as we move from the shock front to the inner expanding surface (see Figures 1a,d and 2).

Figure 1b,c shows the distributions of the reduced pressure and the reduced velocity, respectively. The pressure and velocity (except $M_A^{-2} = 0$) both increase from the shock front and approach to maximum near the inner expanding surface (see Figure 1b,c). In the non-magnetic case, i.e., $M_A^{-2} = 0$, the density and velocity decrease as we move from the shock front to the inner expanding surface (see Figure 1b,c).

Conclusions

Non-similarity solutions for propagation of explosion waves in a stellar model, in which density falls exponentially, has been obtained in this paper. The shock wave moves with variable velocity, and the total energy of the wave is not constant and varies with time. It is investigated that the presence of gravitational field reduces the effects of the magnetic field. Also, the presence of gravitational field increases the compressibility of the medium, due to which it is compressed and therefore the distance between the inner contact surface and the shock surface is reduced.

The article concerns with the explosion problem; however, the methodology analysis presented here may be used to describe many other physical systems involving non-linear hyperbolic partial differential equations. The shock waves in conducting perfect gas can be important for description of shocks in supernova explosions and explosion in the ionosphere. Other potential applications of this study include analysis of data from exploding wire experiments and cylindrically symmetric hypersonic flow problems associated with meteors or reentry vehicles (*c.f.* [33]). Also, the present study can be important to varify the accuracy of the solution obtained by the theory of self-similarity and computational methods such as finite difference scheme, finite element, etc.

The following conclusions may be drawn from the finding of the current analysis (see Figures 1a,b,c,d and 2):

1. It is found that the reduced density, reduced pressure and reduced mass increase with the strength of magnetic field, whereas the reduced velocity shows a reverse behaviour in general.
2. By an increase in the strength of magnetic field, the gas behind the shock is more compressed in the gravitating case ($L^* = 0.001$), whereas it is less compressed in the non-gravitating case, i.e., the shock strength is increased in the gravitating case and it is decreased in the non-gravitating case.

3. The reduced velocity, pressure and azimuthal magnetic field increase in the gravitating case and show a reverse behaviour in the non-gravitating case with an increase in time $\left(\frac{t}{t_0}\right)$, whereas the reduced density, reduced mass and shock strength decrease.
4. Due to the presence of gravitation, the pressure, velocity, azimuthal magnetic field and density decrease, in general, as we move inward from the shock.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

GN carried out all the analyses and calculations, conceived and designed the study, and drafted the manuscript. JPV provided guidance at various stages of the study and reviewed the manuscript. VKS and AKS typed the manuscript and drew the graphs. All authors read and approved the final manuscript.

Acknowledgements

The authors are thankful to the referees for giving good comments to improve the manuscript.

References

1. Sedov, LI: Similarity and Dimensional Methods in Mechanics. Mir, Moscow (1982)
2. Carrus, P, Fox, P, Hass, F, Kopal, Z: The propagation of shock waves in a steller model with continuous density distribution. Astrophys. J. **113**, 496–518 (1951)
3. Purohit, SC: Self-similar homothermal flow of self-gravitating gas behind shock wave. J. Phys. Soc. (Japan), **36**, 288–292 (1974)
4. Singh, JB, Vishwakarma, PR: Self-similar solutions in the theory of flare-ups in novae I. Astrophys. Space Sci. **95**, 99–104 (1983)
5. Sakurai, A: Propagation of spherical shock waves in stars. J. Fluid Mech. **1**, 436–453 (1956)
6. Nath, G: Magnetogasdynamic shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of perfect gas with variable density. Adv. Space Res. **47**, 1463–1471 (2011)
7. Rogers, MH: Analytic solutions for blast wave problem with an atmosphere of varying density. Astrophys. J. **125**, 478–493 (1957)
8. Rosenau, P, Frankenthal, S: Equatorial propagation of axisymmetric magnetohydrodynamic shocks I. Phys. Fluids. **19**, 1889–1899 (1976)
9. Nath, O, Ojha, SN, Takhar, HS: Propagation of a shock wave in a rotating interplanetary atmosphere with increasing energy. J. MHD Plasma Res. **8**, 269–282 (1999)
10. Vishwakarma, JP, Yadav, AK : Self-similar analytical solutions for blast waves in inhomogeneous atmospheres with frozen-in-magnetic field. Eur. Phys. J. B. **34**, 247–253 (2003)

11. Hayes, WD: Self-similar strong shocks in an exponential medium. J. Fluid Mech. **32**, 305–315 (1968)

12. Laumbach, DD, Probstein, RF: A point explosion in a cold exponential atmosphere. Part I. J. Fluid Mech. **35**, 53–75 (1968)

13. Deb Ray, G: An exact analytic solution for strong plane shock waves in an exponential medium. Bull. Cal. Math. Soc. **66**, 27–31 (1974)

14. Verma, BG, Vishwakarma, JP: Propagation of magnetogasdynamic plane shock waves in an exponential medium. Nuovo Cimento. **32**, 267–272 (1976)

15. Verma, BG, Vishwakarma, JP: Axially symmetric explosion in magnetogasdynamic. Astrophys. Space Sci. **69**, 177–188 (1980)

16. Vishwakarma, JP : Propagation of shock waves in a dusty gas with exponentially varying density. Eur. Phys. J. B. **16** 369–372 (2000)

17. Vishwakarma, JP, Nath, G: Propagation of shock waves in an exponential medium with heat conduction and radiation heat flux. Modelling Meas. Control B. **77**, 67–84 (2008)

18. Nath, G : Propagation of a strong cylindrical shock wave in a rotational axisymmetric dusty gas with exponentially varying density. Res. Astron. Astrophys. **10**, 445–460 (2010)

19. Vishwakarma, JP, Nath, G, Singh, KK: Propagation of shock waves in a dusty gas with heat conduction, radiation heat flux and exponentially varying density. Phys. Scr. **78**, 035402 (2008)

20. Hartmann, L: Accretion Processes in Star Formation. Cambridge University Press, Cambridge (1998)

21. Balick, B, Frank, A: Shapes and shaping of planetary nebulae. Annu. Rev. Astron. Astrophys. **40**, 439–486 (2002)

22. Whitham, GB: On the propagation of shock waves through regions of non-uniform area or flow. J. Fluid Mech. **4**, 337–360 (1958)

23. Vishwakarma, JP, Nath, G: Cylindrical shock wave generated by a piston moving in a non-uniform self-gravitating rotational axisymmetric gas in the presence of conduction and radiation heat-flux. In: Petrova, VM (ed.) Advances in Engineering Research, vol. 2, pp. 537–576. Nova, Hauppauge (2011)

24. Nath, G, Sinha, AK: A self-similar flow behind a magnetogasdynamics shock wave generated by a moving piston in a gravitating gas with variable density : isothermal flow. Phys. Res. Inter. (2011). doi:10.1155/2011/ 782172

25. Liberman, MA, Velikovich, AL: Physics of Shock Waves in Gases and Plasma. Springer, Berlin (1986)

26. Pai, SI: Magnetogasdynamic and Plasma Dynamics. Springer, Wien (1986)

27. Freeman, RA, Craggs, JD: Shock waves from spark discharges. J. Phys. D : Appl. Phys. **2**, 421–427 (1969)

28. Director, MN, Dabora, EK : An experimental investigation of variable energy blast waves. Acta Astronautica **4**, 391–407 (1977)

29. Vishwakarma, JP, Maurya, AK, Singh, KK: Self-similar adiabatic flow headed by a magnetogasdynamics cylindrical shock wave in a rotating non-ideal gas. Geophys. Astrophys. Fluid Dynamics. **101**, 155–167 (2007)

30. Nath, G: Shock waves generated by a piston moving in a non-ideal gas in the presence of a magnetic field: isothermal flow. South East Asian J. Math. Math. Sci. **5**, 69–83 (2007)

31. Deb Ray, G: Similarity solutions for cylindrical blast waves in magnetogasdynamics. Phys. Fluids. **16**, 559–560 (1973)

32. Rosenau, P: Equatorial propagation of axisymmetric magnetohydrodynamic shocks II. Phys. Fluids. **20**, 1097–1103 (1977)

33. Hutchens, GJ: Approximate cylindrical blast theory: near field solutions. J. Appl. Phys. **77**, 2912–2915 (1995)

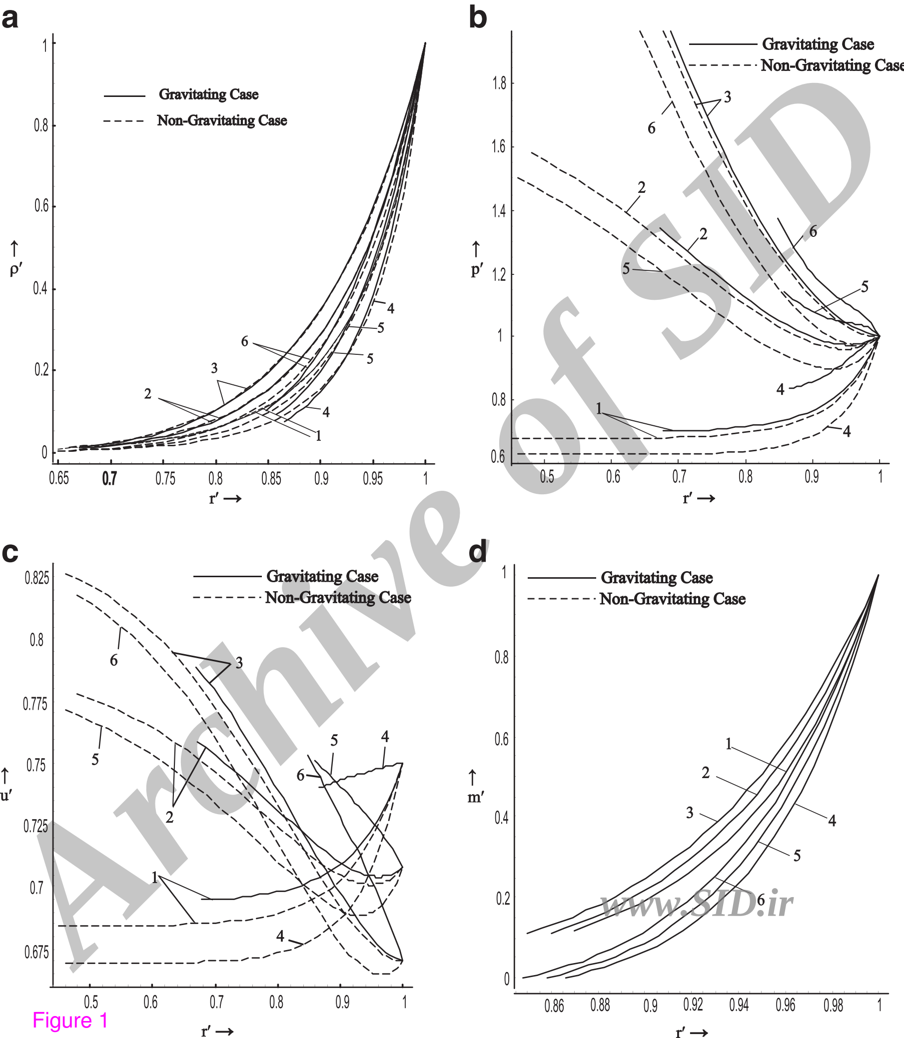
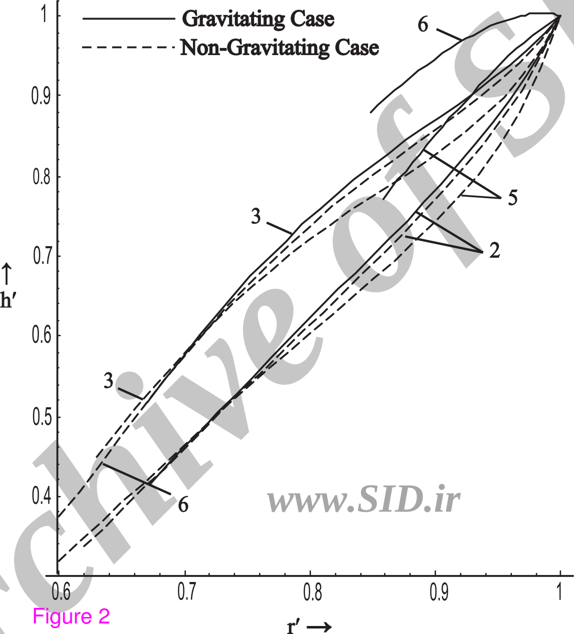

a
Gravitating Case
Non-Gravitating Case
ρ′
r′
b
Gravitating Case
Non-Gravitating Case
p′
r′
c
Gravitating Case
Non-Gravitating Case
u′
r′
d
Gravitating Case
Non-Gravitating Case
m′
r′

Figure 1



Gravitating Case
Non-Gravitating Case
1
0.9
0.8
0.7
0.6
0.5
0.4
h′ ↑
0.6
0.7
0.8
0.9
1
r′ →
6
5
3
2
3
6

Figure 2

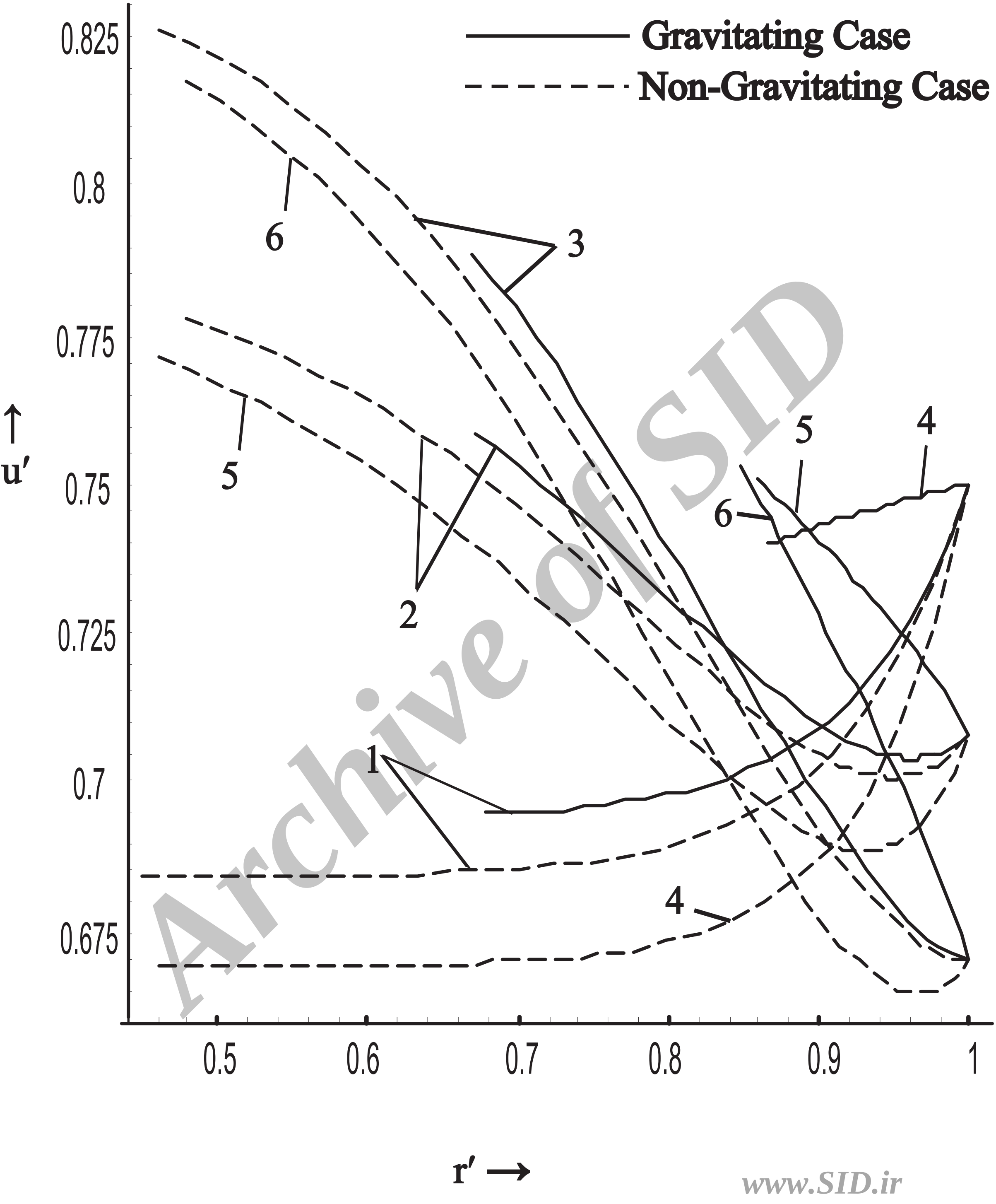


Fig. 1(c)

Figure 3

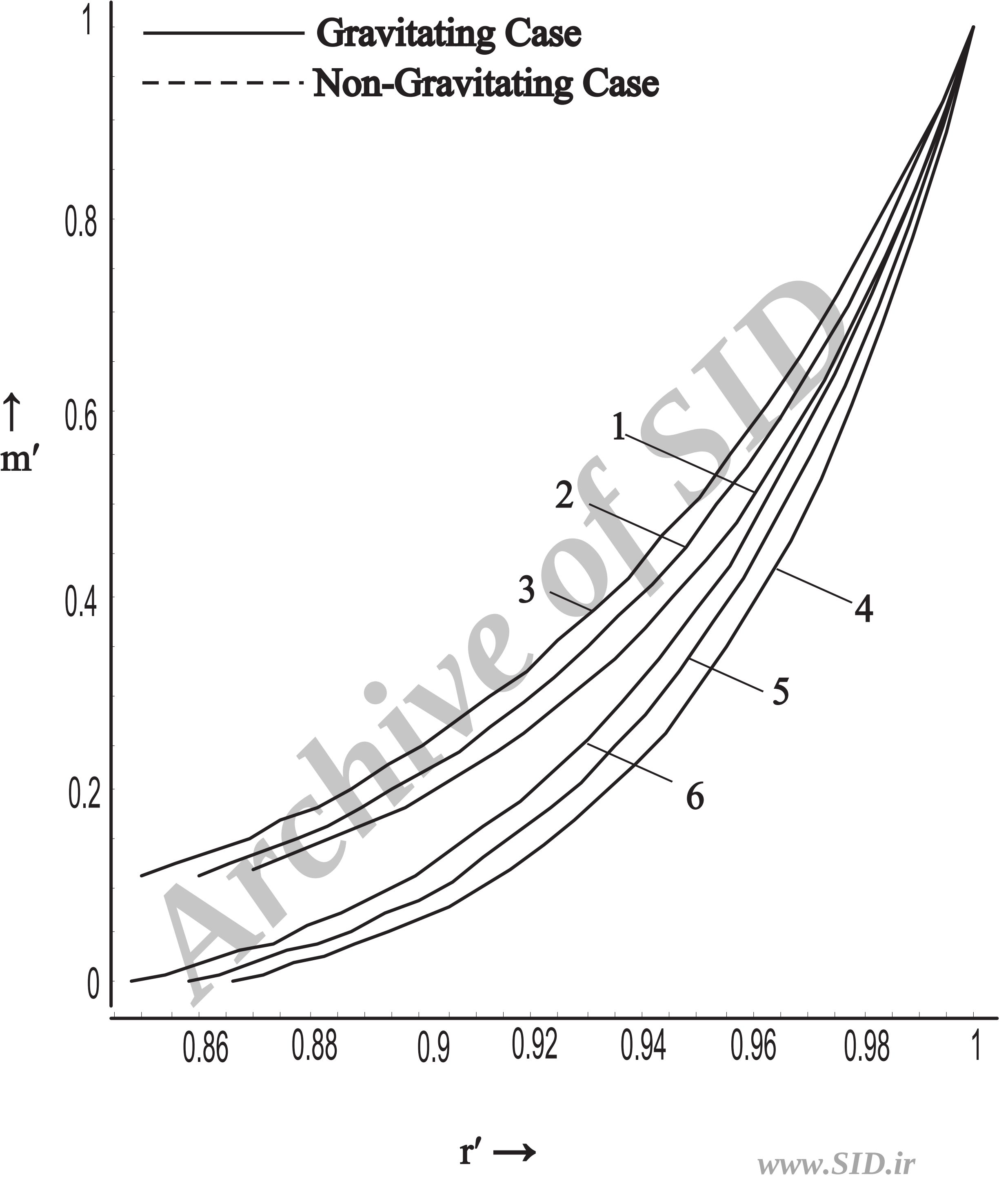


Fig. 1(d)

Figure 4

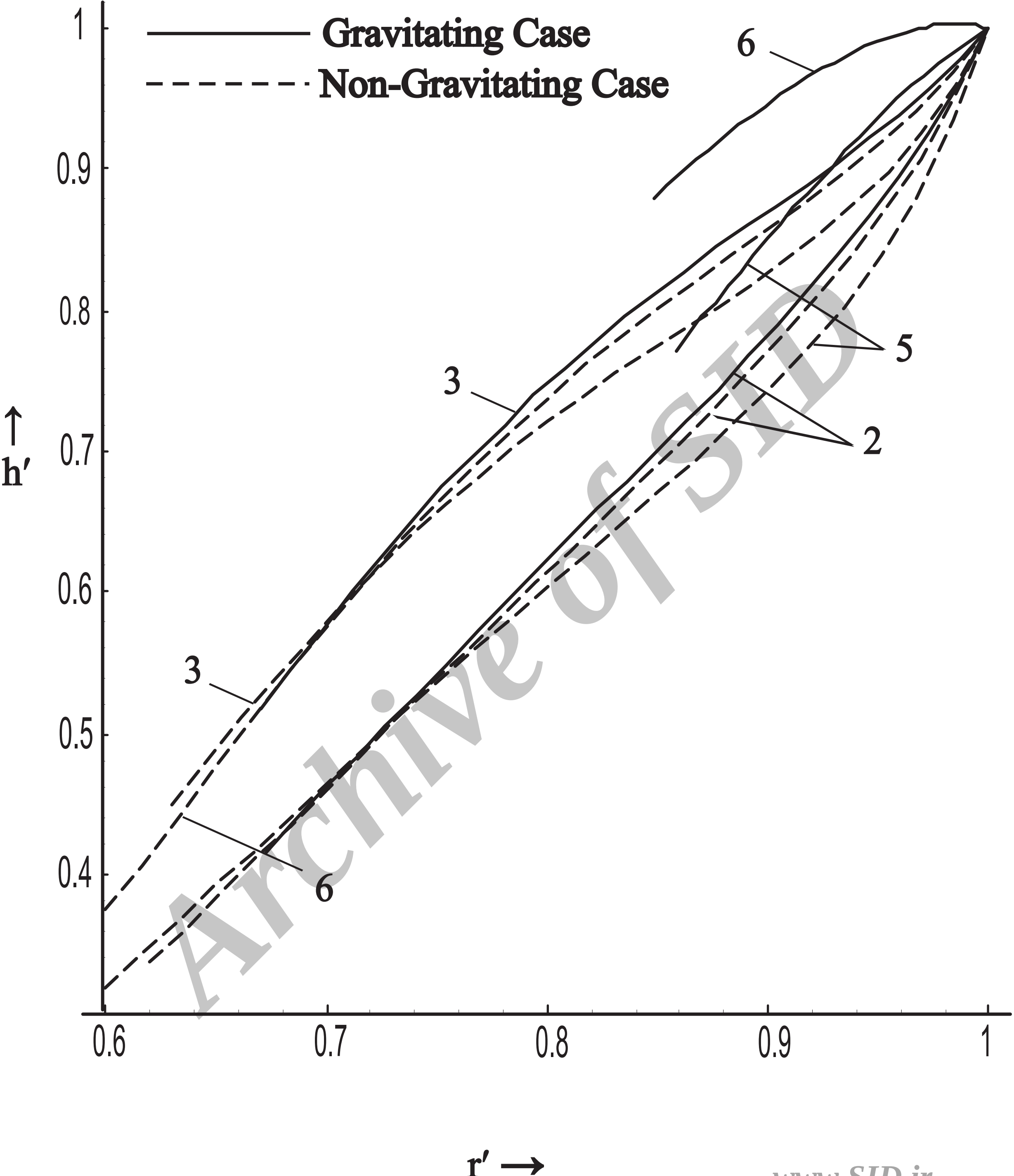


Fig. 2

Figure 5