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The comparison of homotopy perturbation method with finite difference method for determination of maximum beam deflection

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Abstract

This paper deals with the determination of maximum beam deflection using homotopy perturbation method (HPM) and finite difference method (FDM). By providing some examples, we compare the results with exact solutions and conclude that HPM is more accurate, more stable and effective and can therefore be found widely applicable in structure engineering.

Keywords: Nonlinear differential equations, Analytical approximate methods, Numerical methods, Force method, Exact solution, Maximum beam deflection

Introduction

Nonlinear systems have been widely used in many areas of physics and engineering and are of significant importance in mechanical and structural dynamics for the comprehensive understanding and accurate prediction of motion and deformation. In order to develop engineering and applied science, it is necessary to study analytical and numerical methods for solving all available problems. Various methods for solution of such equations have been proposed. Surveys of the literature with numerous references and useful bibliographies have been given by Nayfeh [1], Mickens [2], Jordan and Smith [3], and more recently by He [4].

Most of nonlinear differential equations have no explicit solutions which are expressible in finite terms; even if a solution can be found, it is often too complicated to display clearly the principal features of the solution. Due to such difficulties, one of the most time-consuming and difficult tasks appear among the researchers of nonlinear problems.

With the rapid development of nonlinear science, there appears an ever-increasing interest of scientists in the analytical asymptotic techniques for nonlinear problems, and several analytical approximation methods have been developed to solve linear and nonlinear ordinary and partial differential equations.

Some of these techniques include perturbation method (PM) [5,6], variational iteration method [7,8], homotopy perturbation method (HPM) [9-12], energy balance method [13-17], variational approach method [18-20], parameter-expansion method [21-24], amplitude-frequency formulation [25-27], iteration perturbation method [28,29], etc. Among these methods, the PM and HPM are considered to be two of the powerful methods capable of handling strongly nonlinear behaviors and can converge to an accurate solution for smooth nonlinear systems.

The application of HPM in linear and nonlinear problems has been devoted by scientists and engineers, because this method continuously deforms the under study problem which is difficult into a simple problem which is easy to solve. The HPM was first proposed by Ji-Huan He in 1999 [9] for solving the linear and nonlinear differential and integral equations. The method is a coupling of the traditional perturbation method and homotopy in topology. This method, which does not require a small parameter in an equation, has a significant advantage in providing an analytical approximate solution to a wide range of nonlinear problems in applied sciences. Recent development of the HPM is presented by Ji-Huan He in 2008 [10]. Elementary

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introduction and interpretation of the method are given in the publications [9-12].

One of the responsibilities of the structural design engineer is to devise arrangements and proportions of members that can withstand economically and efficiently for the conditions anticipated during the lifetime of a structure. A central aspect to this structure is the calculation of the beam deformation, which has very wide applications in structural engineering. The nonlinear differential equation of beam deformation under static load is given in the following form [30]:

$$\left(\frac{d^2}{dx^2}y(x)\right) - \left(\frac{M(x)}{EI}\right)\left[1 + \left(\frac{d}{dx}y(x)\right)^2\right]^{\frac{3}{2}} = 0. \qquad (1)$$

In Equation 1, M is the bending moment, E is the elastic modulus, and I is the second moment of area that must be calculated with respect to axis perpendicular to the applied loading. Note that the bending moment changes for different conditions of supporting and loading.

The finite difference method by Taylor [31], which is one of the known numerical methods in civil engineering-structure, and civil engineers frequently may be consulted for analyzing the engineering structures, such as beams, columns, and plates. For analyzing beams using FDM and determination of maximum beam deflection, we need to eliminate the nonlinear part of differential equation and change nonlinear differential equation to linear one. We mention that this change reduces the accuracy of numerical solution results. In small structures this error is negligible, but in large structures the error rate will increase.

In this paper, we introduce HPM to civil engineers for analyzing the engineering structures, such as beams, column, and plates. The main target of this paper is to solve the nonlinear differential equation of beam elastic deformation with different conditions of supporting and loading and then to determine the maximum beam deflection by applying the HPM and to compare the approximate results with FDM and exact solution.

The description of homotopy perturbation method

To illustrate the basic ideas of this method, we consider the following equation [9],

$$A(u) - f(r) = 0, r\epsilon\Omega \qquad (2)$$

with the following boundary condition:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r\epsilon\Omega \qquad (3)$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function, and Γ is the boundary of the domain Ω. A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Hence we can write Equation 2 in following form:

$$L(u) - N(u) - f(r) = 0. \qquad (4)$$

The homotopy perturbation structure is given as follows:

$$\begin{aligned} H(v,p) &= (1-p)[L(v) - L(u_0)] \\ &\quad + p[A(v) - f(r)] \\ &= 0, \end{aligned} \qquad (5)$$

where

$$v(r,p) : \Omega \times [0,1] \rightarrow R \qquad (6)$$

In Equation 5, $p\epsilon$ [0, 1] is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Equation 5 can be written as a power series in p, in the following form:

$$v = v_0 + pv_1 + p^2v_2 + p^3v_3 + \ldots \qquad (7)$$

And the best approximation for solution is

$$v = \lim_{p\rightarrow 1} v = v_0 + v_1 + v_2 + v_3 + \ldots \qquad (8)$$

The application of homotopy perturbation method for determination of maximum beam deflection

Beam with two fixed ends, under concentrated load at the middle of span

The nonlinear differential equation of beam deformation with two fixed ends, under concentrated load at the middle of span is in the following form (Figure 1) [30]:

$$\left(\frac{d^2}{dx^2}y(x)\right) - \left(\frac{F(4x-L)}{8EI}\right)\left[1 + \left(\frac{d}{dx}y(x)\right)^2\right]^{\frac{3}{2}} = 0$$

with the following boundary conditions:

$$y(0) = 0, \frac{d}{dx}y(0) = 0. \qquad (10)$$

To solve Equation 9 by HPM, first we change the Equation 9 to the following form:

$$\left(\frac{d^2}{dx^2}y(x)\right) - \left(\frac{F(4x-L)}{8EI}\right)\left[1 + \frac{3}{2}\left(\frac{d}{dx}y(x)\right)^2\right] = 0, \qquad (11)$$

To solve Equation 11 by HPM, we consider the following process after separating the linear and nonlinear parts of the equation. A homotopy can be constructed in the following form:

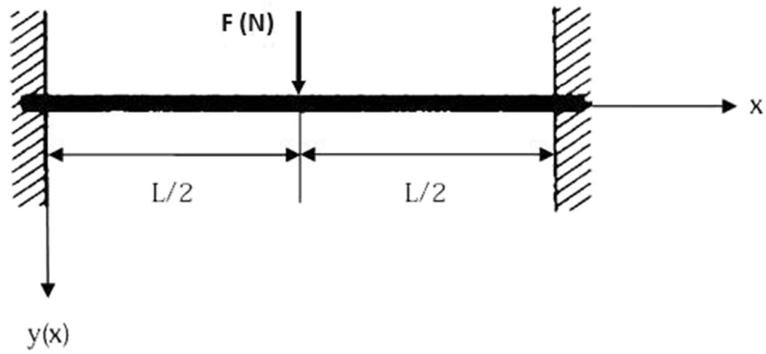

Figure 1 Beam with two fixed ends, under concentrated load at the middle of span.

$$H(y,p) = (1-p)\left[\frac{d^2}{dx^2}y(x) - \frac{F(4x-L)}{8EI}\right] + p\left\{\frac{d^2}{dx^2}y(x) - \frac{F(4x-L)}{8EI}\cdot \left[1+\frac{3}{2}\left(\frac{d}{dx}y(x)\right)^2\right]\right\} = 0. \tag{12}$$

We can assume that the solution of Equation 12 can be written as a power series in p, in the following form:

$$y(x) = y_0(x) + py_1(x) + p^2y_2(x) + p^3y_3(x) + \ldots. \tag{13}$$

By substituting Equation 13 into Equation 12, we have

$$H(y,p) = (1-p)\left\{\frac{d^2}{dx^2}\left[y_0(x) + py_1(x) + p^2y^2(x) + p^3y_3(x)v\right] - \frac{F(4x-L)}{8EI}\right\} + p\left\langle\frac{d^2}{dx^2}\left[y_0(x) + py_1(x) + p^2y_2(x) + p^3y_3(x)\right] - \frac{F(4x-L)}{8EI}\left\{1+\frac{3}{2}\left[\frac{d}{dx}\left(y_0(x) + py_1(x) + p^2y_2(x) + p^3y_3(x)\right)\right]^2\right\rangle = 0 \tag{14}$$

After expansion and rearranging based on coefficient of p-term, we obtain the following results:

$$p^0 : \left(\frac{d^2}{dx^2}y_0(x)\right) + \left(\frac{Fx}{2EI} + \frac{FL}{8EI}\right) = 0, \quad y_0(0) = \frac{d}{dx}y_0(0) = 0, \tag{15}$$

$$p^1 : \left(\frac{d^2}{dx^2}y_1(x)\right) + \left(-\frac{3Fx}{4EI} + \frac{3FL}{16EI}\right)\left(\frac{d}{dx}y_0(x)\right)^2 = 0, \quad y_1(0) = \frac{d}{dx}y_1(0) = 0, \tag{16}$$

$$p^2 : \left(\frac{d^2}{dx^2}y_2(x)\right) + \left(-\frac{3Fx}{2EI} + \frac{3FL}{8EI}\right)\left(\frac{d}{dx}y_0(x)\right)\left(\frac{d}{dx}y_1(x)\right) = 0, \quad y_2(0) = \frac{d}{dx}y_2(0) = 0, \tag{17}$$

By solving Equations 15, 16, and 17, we have

$$y_0(x) = \left(-\frac{1}{12}\cdot\frac{Fx^3}{EI}\right) + \left(\frac{1}{16}\cdot\frac{FLx^2}{EI}\right) \tag{18}$$

$$y_1(x) = \left(\frac{3}{1,024}\cdot\frac{F^3}{(EI)^3}\right)\left(-\frac{8}{21}x^7 + \frac{2}{3}Lx^6 - \frac{2}{5}L^2x^5 + \frac{1}{12}L^3x^4\right) \tag{19}$$

$$y_2(x) = \left(\frac{3}{65,536}\cdot\frac{F^5}{(EI)^5}\right)\left(-\frac{32}{55}x^{11} + \frac{8}{5}Lx^{10} - \frac{16}{9}L^2x^9 + L^3x^8 - \frac{2}{7}L^4x^7 + \frac{1}{30}L^5x^6\right) \tag{20}$$

According to the HPM, we can conclude that

$$y(x) = \lim_{p\to 1} y(x) = y_0(x) + y_1(x) + y_2(x) + y_3(x) + \ldots. \tag{21}$$

Therefore, substituting the values of $y_0(x)$, $y_1(x)$, and $y_2(x)$ from Equations 18, 19, and 20 into Equation 21, we get

$$y(x) = \left[\left(-\frac{1}{12}\cdot\frac{Fx^3}{EI}\right) + \left(\frac{1}{16}\cdot\frac{FLx^2}{EI}\right)\right] + \left[\left(\frac{3}{1,024}\cdot\frac{F^3}{(EI)^3}\right)\cdot\left(-\frac{8}{21}x^7 + \frac{2}{3}Lx^6 - \frac{2}{5}L^2x^5 + \frac{1}{12}L^3x^4\right)\right] + \left[\left(\frac{3}{65,536}\cdot\frac{F^5}{(EI)^5}\right)\left(-\frac{32}{55}x^{11} + \frac{8}{5}Lx^{10} - \frac{16}{9}L^2x^9 + L^3x^8 - \frac{2}{7}L^4x^7 + \frac{1}{30}L^5x^6\right)\right] \quad (22)$$

In the mechanics of materials for beam with two fixed ends, under concentrated load at the middle of span, the deformation is computed by following formula [30]:

$$y_exact(x) = F/48EI\left(3Lx^2 - 4x^3\cdot\right). \quad (23)$$

Note that the Equation 23 was determined by using force method in the mechanics of materials [30].

The results of comparison between HPM with formula in mechanics of materials for beam with two fixed ends and under concentrated load at the middle of span are given in Tables 1 and 2.

Beam with two fixed ends, under linear distributed load

The nonlinear differential equation of beam deformation with two fixed ends, under linear distributed load is in the following form (Figure 2) [30],

$$\left(\frac{d^2}{dx^2}y(x)\right) - \left(\frac{W(6Lx - L^2 - 6x^2)}{12EI}\right)\left[1 + \left(\frac{d}{dx}y(x)\right)^2\right]^{\frac{3}{2}} = 0, \quad (24)$$

Table 1 Comparison of HPM with formula in mechanics of materials for $L = 1.00 \quad m, EI = 500\frac{N}{m^2}, F = 100N$

Displacement from left support x^a (m)	HPM y^b_{HPM} (m)	Formula in mechanics of materials y^c_{exact} (m)
0.10	0.000108	0.000108
0.20	0.000367	0.000367
0.30	0.000675	0.000675
0.40	0.000933	0.000933

[a] $x = L / 2 = 0.50$; [b] $y_{max,\ HPM}$ $(L / 2) = 0.001042$; [c] $y_{max,\ exact}(L / 2) = 0.001042$.

Table 2 Comparison of HPM with formula in mechanics of materials for $L = 3.00 \quad m, EI = 500\frac{N}{m^2}, F = 100N$

Displacement from left support x^a (m)	HPM y^b_{HPM} (m)	Formula in mechanics of materials y^c_{exact} (m)
0.10	0.0004	0.0004
0.20	0.0014	0.0014
0.30	0.0029	0.0029
0.40	0.0049	0.0049
0.50	0.0073	0.0073
1.00	0.0208	0.0208

[a] $x = L / 2 = 1.50$; [b] $y_{max,\ HPM}$ $(L / 2) = 0.0281$; [c] $y_{max,\ exact}$ $(L / 2) = 0.0281$.

with the following boundary conditions:

$$y(0) = 0, \frac{d}{dx}y(0) = 0. \quad (25)$$

To solve Equation 24 by HPM, first we change Equation 24 to the following form:

$$\left(\frac{d^2}{dx^2}y(x)\right) - \left(\frac{W(6Lx - L^2 - 6x^2)}{12EI}\right)\left[1 + \frac{3}{2}\left(\frac{d}{dx}y(x)\right)^2\right] = 0 \quad (26)$$

Now, we use HPM to solve Equation 26. We consider the following process after separating the linear and nonlinear parts of the equation. A homotopy can be constructed in the following form:

$$H(y,p) = (1-p)\left[\frac{d^2}{dx^2}y(x) - \frac{W(6Lx - L^2 - 6x^2)}{12EI}\right] + p\left\{\frac{d^2}{dx^2}y(x) - \frac{W(6Lx - L^2 - 6x^2)}{12EI}\cdot\left[1 + \frac{3}{2}\left(\frac{d}{dx}y(x)\right)^2\right]\right\} = 0 \quad (27)$$

We can assume that the solution of Equation 27 can be written as a power series in p, in the following form:

$$y(x) = y_0(x) + py_1(x) + p^2y_2(x) + p^3y_3(x) + \ddot{A}\!\!\!n \quad (28)$$

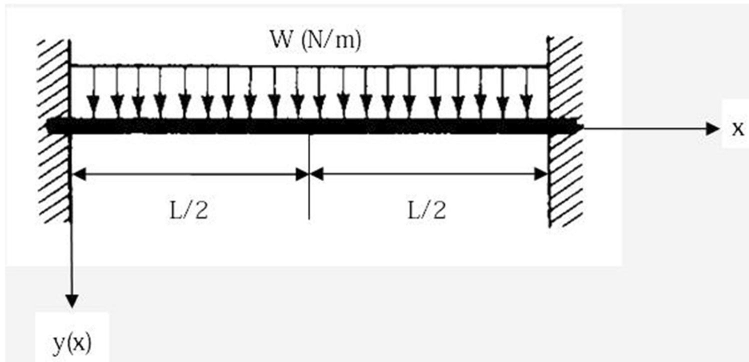

Figure 2 **Beam with two fixed ends, under linear distributed load.**

Substituting Equation 28 into Equation 27 leads to

$$H(y,p) = (1-p)\left[\frac{d^2}{dx^2}(y_0(x) + py_1(x) + p^2y_2(x) + p^3y_3(x)v) - \frac{W(6Lx - L^2 - 6x^2)}{12EI}\right] + \mathrm{p}\left\langle \frac{d^2}{dx^2}\ (y_0(x) + py_1(x) + p^2y_2(x) + p^3y_3(x)) - \frac{W(6Lx - L^2 - 6x^2)}{12EI}\left\{1 + \frac{3}{2}\left[\frac{d}{dx}\left(y_0(x) + py_1(x) + p^2y_2(x) + p^3y_3(x)\right)\right]2\right\}\right\rangle = 0 \tag{29}$$

After expansion and rearranging based on coefficient of p-term, we obtain the following results:

$$p^0 : \left(\frac{d^2}{dx^2}y_0(x)\right) + \left(\frac{wx^2}{2EI} - \frac{wLx}{2EI} + \frac{wL^2}{12EI}\right) = 0, \quad y_0(0) = \frac{d}{dx}y_0(0) = 0, \tag{30}$$

$$p^1 : \left(\frac{d^2}{dx^2}y_1(x)\right) + \left(\frac{3wx^2}{4EI} - \frac{3wLx}{4EI} + \frac{wL^2}{8EI}\right)\left(\frac{d}{dx}y_0(x)\right)^2 = 0, \quad y_1(0) = \frac{d}{dx}y_1(0) = 0, \tag{31}$$

$$p^2 : \left(\frac{d^2}{dx^2}y_2(x)\right) + \left(\frac{3wx^2}{2EI} - \frac{3wLx}{2EI} + \frac{wL^2}{4EI}\right)\left(\frac{d}{dx}y_0(x)\right)\left(\frac{d}{dx}y_1(x)\right) = 0, \quad y_2(0) = \frac{d}{dx}y_2(0) = 0, \tag{32}$$

By solving the Equations 30, 31, and 32, we come to

$$y_0(x) = \left(\frac{w}{12EI}\right)\left(-Lx^3 + \frac{1}{2}L^2x^2 + \frac{1}{2}x^4\right) \tag{33}$$

$$y(x) = \left(\frac{w^3}{1,152(EI)^3}\right)\left(\frac{4}{15}x^{10} - \frac{4}{3}Lx^9 + \frac{11}{4}L^2x^8 - 3L^3x^7 + \frac{11}{6}L^4x^6 - \frac{3}{5}L^5x^5 + \frac{1}{12}L^6x^4\right) \tag{34}$$

$$y_2(x) = \left(\frac{5w^5}{663,552(EI)^5}\right)\left(\frac{2}{5}x^{16} - \frac{16}{5}Lx^{15} + \frac{80}{7}L^2x^{14} - 24L^3x^{13} + \frac{197}{6}L^4x^{12} - \frac{153}{5}L^5x^{11} + \frac{197}{10}L^6x^{10} - \frac{26}{3}L^7x^9 + \frac{5}{2}L^8x^8 - \frac{3}{7}L^9x^7 + \frac{1}{30}L^{10}x^6\right) \tag{35}$$

Table 3 Comparison of HPM with formula in mechanics of materials for $L = 1.00\ \ m, EI = 500\frac{N}{m^2}, w = 100N$

Displacement from left support x^a (m)	HPM y^b_{HPM} (m)	Formula in mechanics of materials y^c_{exact} (m)
0.10	0.000067	0.000067
0.20	0.000213	0.000213
0.30	0.000367	0.000367
0.40	0.000480	0.000480

[a] $x = L / 2 = 0.50$; [b] $y_{max,\ HPM}\ (L / 2) = 0.000521$; [c] $y_{max,\ exact}\ (L / 2) = 0.000521$.

Table 4 Comparison of HPM with formula in mechanics of materials for $L = 3.00\ m, EI = 500\frac{N}{m^2}, w = 100N$

Displacement from left support x^a (m)	HPM y^b_{HPM} (m)	Formula in mechanics of materials y^c_{exact}(m)
0.10	0.0007	0.0007
0.20	0.0026	0.0026
0.30	0.0055	0.0055
0.40	0.0090	0.0090
0.50	0.0130	0.0130
1.00	0.0334	0.0334

[a] $x = L / 2 = 1.50$; [b] $y_{\max,\ HPM}\ (L / 2) = 0.0422$; [c] $y_{\max,\ exact}\ (L / 2) = 0.0422$.

According to the HPM, we can conclude that

$$y(x) = \lim_{p\to 1} y(x) \\ = y_0(x) + y_1(x) + y_2(x) + y_3(x) + \dots \qquad (36)$$

Therefore, substituting the values of $y_0(x)$, $y_1(x)$, and $y_2(x)$ from Equations 33, 34, and 35 into Equation 36, we obtain the following result:

$$y(x) = \left[\left(\frac{w}{12EI}\right)\left(-Lx^3 + \frac{1}{2}L^2x^2 + \frac{1}{2}x^4\right)\right] + \left[\left(\frac{w^3}{1,152(EI)^3}\right)\left(\frac{4}{15}x^{10} - \frac{4}{3}Lx^9 + \frac{11}{4}L^2x^8 - 3L^3x^7 + \frac{11}{6}L^4x^6 - \frac{3}{5}L^5x^5 + \frac{1}{12}L^6x^4\right)\right] + \left[\left(\frac{5w^5}{663,552(EI)^5}\right)\left(\frac{2}{5}x^{16} - \frac{16}{5}Lx^{15} + \frac{80}{7}L^2x^{14} - 24L^3x^{13} + \frac{197}{6}L^4x^{12} - \frac{153}{5}L^5x^{11} + \frac{197}{10}L^6x^{10} - \frac{26}{3}L^7x^9 + \frac{5}{2}L^8x^8 - \frac{3}{7}L^9x^7 + \frac{1}{30}L^{10}x^6\right)\right] \qquad (37)$$

In the mechanics of materials for beam with two fixed ends, under linear distributed load, the deformation is computed by following formula [30]:

$$y_{\text{exact}}(x) = \frac{wx^2}{24EI}(L-x)^2 \qquad (38)$$

Once more, we note that the Equation 38 was determined by using force method in the mechanics of materials [30].

The results of comparison between HPM with the formula in mechanics of materials for beam with two fixed ends, under linear distributed load, are given in Tables 3 and 4.

The application of finite difference method for the determination of maximum beam deflection

In this section, we consider beam with two fixed ends, under linear distributed load (Figure 3) [31].

For the determination of maximum beam deflection with the FDM, we consider the linear differential equation in following form [31],

$$\frac{d^4y}{dx^4} = \frac{w}{EI}, \qquad (39)$$

with the following boundary conditions:

$$y(0) = 0, \frac{d}{dx}y(0) = 0. \qquad (40)$$

Now writing Equation 39 in the finite difference form, we obtain [31]:

$$\frac{1}{h^4}\left[-4y_{i+1} + 6y_i - 4y_{i-1} + y_{i-2}\right] = \frac{w}{EI} \qquad (41)$$

where h is the interval. We divided the beam into six equal parts as we illustrated in Figure 3. Applying Equation 41 at each interior point 1 to 5, we get the following:

$$i = 1 \to [y_3 - 4y_2 + 6y_1 - 4y_0 + y_{-1}] = \frac{wh^4}{EI} = k \qquad (42)$$

$$i = 2 \to [y_4 - 4y_3 + 6y_2 - 4y_1 + y_0] = \frac{wh^4}{EI} = k \qquad (43)$$

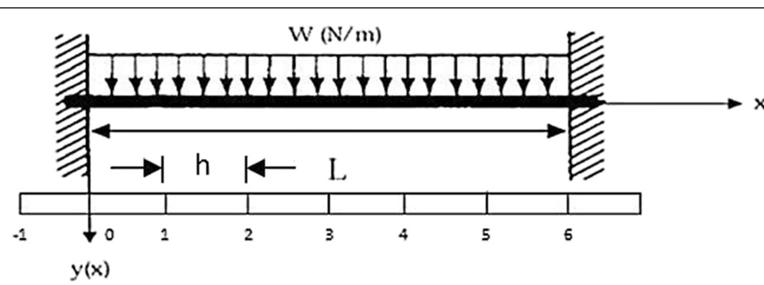

Figure 3 Beam with two fixed ends, under linear distributed load.

Table 5 Results comparison of HPM with FDM and exact solution for the different *L* values

L=3.00\;\text{m}	EI=500N/{{m}^{2}}	w=100NZ	L=3.00\;\text{m}
$L = 1.00\ m, EI = 500\frac{N}{m^2}, w = 100N$	0.00052	0.00051	0.00052
$L = 2.00$ m, $EI = 500N/m^2$, $w = 100N$	0.00833	0.00822	0.00833
$L = 3.00$ m, $EI = 500N/m^2$, $w = 100N$	0.04218	0.04163	0.04218

$$i = 3 \rightarrow [y_5 - 4y_4 + 6y_3 - 4y_2 + y_1] = \frac{wh^4}{EI} = k \quad (44)$$

$$i = 4 \rightarrow [y_6 - 4y_5 + 6y_4 - 4y_3 + y_2] = \frac{wh^4}{EI} = k \quad (45)$$

$$i = 5 \rightarrow [y_7 - 4y_6 + 6y_5 - 4y_4 + y_3] = \frac{wh^4}{EI} = k \quad (46)$$

Note the points labeled -1 and 7 lie outside the beam domain. These are called *imaginary points*. The values at these points can be determined judiciously. Now, applying the difference operator for boundary conditions ($y_0 = y_6 = 0$), we get the following:

$$\frac{dy}{dx} = \frac{1}{2h}[y_{i+1} - y_{i-1}] = 0 \quad (47)$$

$$i = 0 \rightarrow y_1 - y_{-1} = 0 \rightarrow y_{-1} = y_1 \quad (48)$$

$$i = 6 \rightarrow y_7 - y_5 = 0 \rightarrow y_7 = y_5. \quad (49)$$

Also because of symmetry, we have

$$y_1 = y_5, y_2 = y_4. \quad (50)$$

Using Equations 48 to 50 into Equations 42 to 46 in which the latter five equations reduce to the following three meaningful equations in y_1, y_2, and y_3:

$$7y_1 - 4y_2 + y_3 = k \quad (51)$$

$$-7y_1 + 7y_2 - 4y_3 = k \quad (52)$$

$$2y_1 - 8y_2 + 6y_3 = k. \quad (53)$$

Solving this system of equations leads to

$$y_{\max} = y_3 = \frac{80}{24}k = \frac{80}{24} \cdot \frac{wh^4}{EI} \quad (54)$$

But $h = l / 6$, therefore, we get

$$y_{\max} = y_3 = 0.00257\frac{wL^4}{EI} \quad (55)$$

The exact solution for this problem is [31]

$$y_{\max} = \frac{1}{384} \cdot \frac{wL^4}{EI} \quad (56)$$

To illustrate the accuracy of the HPM, we present the comparison results of HPM with FDM and exact solution in Table 5.

Conclusions

In this paper, HPM and FDM have been successfully applied for the determination of maximum beam deflection with specified loading and supporting conditions. Comparison of the results obtained by HPM and FDM with exact solution reveals that the HPM is more accurate and more stable and effective and can therefore be found widely applicable in civil-structure engineering.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

MS carried out the numerical solution of equations and participated in drafted the manuscript. MH conceived of the study and participated in its design and coordination. HEK carried out the software works and the solution of equations using analytical approximate methods. All authors read and approved the final manuscript.

Authors' information

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