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String cloud and domain walls with quark matter for plane symmetric cosmological model in bimetric theory

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Abstract

We have studied the plane symmetric cosmological solutions for quark matter coupled with the string cloud and domain walls in the context of Rosen's (General Relativity and Gravitation 4:435, 1973) bimetric theory. It is observed that, in this theory, string cloud and domain walls do not exist. As we know that, at the early stage of evolution of the universe, domain walls as well as cosmic strings do appear which lead to the formation of galaxies. Thus, it may be said that the bimetric relativity does not help to describe the early era of the universe.

Keywords: Strange quark matter, Domain walls, Cosmic strings, Bimetric theory

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Introduction

Rosen [1] formulated a bimetric theory of gravitation which satisfies the principles of covariance and equivalence. He has modified the formalism of the general relativity theory introducing into it, besides the metric tensor g_{ij} associated with the line element

$$ds^2 = g_{ij}dx^i dx^j \tag{1}$$

a second metric tensor corresponding to flat space-time described by the line element

$$d\sigma^2 = \gamma_{ij}dx^i dx^j \tag{2}$$

at each point of the space-time. The tensor g_{ij} describes gravitation and interacts with matter. The back ground metric γ_{ij} refers to inertial forces. The theory also agrees with present-day observational facts pertaining to general relativity. The field equations of Rosen's bimetric theory of gravitation derived from variational principles are

$$N_j^i - \frac{1}{2}N\delta_j^i = -8\pi\kappa T_j^i, \tag{3}$$

where

$$N_j^i = \frac{1}{2}\gamma_b^a(g^{hi}g_{hj|a})_{|b}$$

and

$$N = N_i^i,\ g = \det(g_{ij}),\ \gamma = \det(\gamma_{ij}),\ \kappa = \sqrt{(\frac{g}{\gamma})}$$

A vertical bar (|) denotes the covariant differentiation with respect to γ_{ij}, and T_j^i is the energy momentum tensor of the matter field. With the advent of new theories of gravitation, it has been a common practice to investigate several aspects of the new theories and then compare them with the results of Einstein's theory of gravitation. With this motivation, a lot of work has been done in Rosen's bimetric theory of gravitation. In particular, Mohanty et al. [2] and Mohanty and Sahoo [3] have shown the non-existence of perfect fluid or mesonic perfect fluid models in bimetric theory, while Reddy [4] has established the non-existence of Bianchi type string cosmologies in this theory. Sahoo [5] and Sahoo et al. [6] have studied Kantowski-Sachs and Bianchi type cosmic strings coupled with Maxwell fields in bimetric theory. Recently, Jain et al. [7] have studied axially symmetric space-time with wet dark fluid in bimetric theory.

The origin of our universe is one of the greatest cosmological mysteries even today. The exact physical situation at early stage of the formation of our universe is still a subject of study. At the very early stages of evolution of the universe, it is generally assumed that during the phase

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transition (as the universe passes through its critical temperature), the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as domain walls, strings, and monopoles [8].

The concept of string theory was developed to describe events of the early stage of the evolution of the universe. The general relativistic treatment of strings was initiated by Satchel [9]. Moreover, they may act as gravitational lenses and may give rise to density fluctuations, leading to the formations of galaxies [10]. Strings posses stress energy and are coupled to the gravitational field. There is even a mode describing the gravitation, which is an important reason why string theory has received so much attention. The gravitational effects of the cosmic strings have been extensively discussed by Vilenkin [10], Gott [11], Letelier [12], and Adhav et al. [13] in general relativity. Also, Tikekar et al. [14] have presented a class of cylindrically symmetric models in string cosmology. Cosmic strings seem to be the best candidate for being observed, and the presence of strings in the early universe do not contradict the present-day observations.

In particular, the domain walls have become important in recent years from cosmological standpoint when a new scenario of galaxy formation has been proposed by Hill et al. [15]. According to them, the formation of galaxies are due to domain walls produced during the phase transitions after the time of recombinations of matter and radiation. Vilenkin [16], Isper and Sikivie [17], Widraw [18], Goetz [19], Mukherji [20], Wang [21], Rahaman et al. [22], Reddy and Subba Rao [23], and Adhav et al. [24] are some of the authors who have investigated several aspects of the domain walls in general relativity. Rahaman [25] and Rahaman, and Mukherji [26] have discussed plane symmetric domain walls in Lyra [27] geometry.

In this study, we will attach strange quark matter to the string cloud and domain walls. It is plausible to attach strange quark matter to the string cloud because one of such transitions during the phase transitions of the universe is quark gluon plasma (QGP)→ hadron gas (called quark-hadron phase transition) when cosmic temperature was $T \sim 200$ MeV. The possibility of the existence of quark matter dates back to early seventies. Itoh [28], Bodmer [29], and Witten [30] proposed two ways of formation of quark matter: the quark-hadron phase transition in the early universe and the conversion of neutron stars into strange one at ultrahigh densities.

Strange quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to volume. In this model, quarks are thought as degenerate Fermi gases, which exist only in a region of space endowed with a vacuum energy density B_C (called as the bag constant). In the framework of this model, the quark matter is composed of massless u, d quarks, massive s quarks, and electrons. In the simplified version of the bag model, assuming quarks are massless and non-interacting, we then have quark pressure $p_q = \rho_q/3$ (ρ_q is the quark energy density); the total energy density is

$$\rho_m = \rho_{\mathrm{q}} + B_C \tag{4}$$

but the total pressure is

$$p_m = p_{\mathrm{q}} - B_C \tag{5}$$

Yavuz et al. [31] and Yilmaz [32,33] have studied 5-D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain walls. Adhav et al. [34,35] have discussed string cloud and domain walls with quark matter in n-dimensional Kaluza-Klein cosmological model in general relativity and strange quark matter attached to string cloud in Bianchi type-III space time in general relativity. Khadekar et al. [36] have confirmed their work to the quark matters which attached to the topological defects in general relativity. Katore and Shaikh [37] have obtained cosmological model with strange quark matter attached to cosmic string for axially symmetric space-time in general relativity. Mahanta et al. [38] have discussed string cloud with quark matter in self-creation cosmology.

The purpose of the present work is to study the plane symmetric cosmological models in bimetric theory of gravitation with quark matter coupled to the string cloud and domain walls. The paper is outlined as follows. In section 'Field equations and their solutions for cosmic strings with quark matter', the field equations and solutions are obtained for dust quark matter coupled with the string cloud in plane symmetric space-time. In the 'Field equations and their solutions for domain walls with quark matter' section, solutions of the bimetric field equations are obtained for strange quark matter coupled to the domain walls in plane symmetric metric. In the 'Conclusion ' section, concluding remarks are given.

Field equations and their solutions for cosmic strings with quark matter

We consider the plane symmetric metric in the form

$$ds^2 = -e^{2\alpha}dx^2 - e^{2\beta}(dy^2 + dz^2) + e^{2\gamma}dt^2, \tag{6}$$

where α, β and γ are functions of z and t.

The background flat space-time corresponding to the metric (6) is

$$d\sigma^2 = -dx^2 - dy^2 - dz^2 + dt^2 \tag{7}$$

The energy momentum tensor for cosmic cloud of strings is given by

$$T_j^i = \rho u^i u_j - \lambda x^i x_j, \tag{8}$$

where ρ is the rest energy density for a cloud with particle attached to them, and λ is the tension density of the strings; they are related by

$$\rho = \rho_p + \lambda \tag{9}$$

Here, ρ_p is the particle energy density. The string is free to vibrate, and different vibrational modes are seen as different masses or spins. Therefore, we will consider quarks instead of particles in the cloud of strings. In this case, from Equation (9), we get

$$\rho = \rho_q + \lambda + B_C. \tag{10}$$

From Equations (8) and (10), we can write the energy momentum tensor for strange quark matter attached to the string cloud as [31]

$$T_j^i = (\rho_q + \lambda + B_C)u^i u_j - \lambda x^i x_j, \tag{11}$$

where u^i is the four-velocity of the string, and x^i represents the direction vector of anisotropy. The string source is along the z-axis which is the axis of symmetry.

Orthonormalization of u^i and x^i is given as

$$u^i u_i = 1, \quad u^i x_i = 0, \quad x^i x_i = -1. \tag{12}$$

In co-moving coordinate system, from Equations (11) and (12),

$$T_1^1 = 0 = T_2^2; \quad T_4^4 = \rho; \quad T_3^3 = \lambda; \quad T_j^i = 0, \quad i \neq j. \tag{13}$$

Rosen's bimetric field equations (3) for the plane symmetric metric (6) with the help of Equations (7), (11), (12), and (13) becomes

$$(\alpha_{44} - \alpha_{33}) - 2(\beta_{44} - \beta_{33}) - (\gamma_{44} - \gamma_{33}) = 0 \tag{14}$$

$$(\alpha_{44} - \alpha_{33}) + (\gamma_{44} - \gamma_{33}) = 0 \tag{15}$$

$$(\alpha_{44} - \alpha_{33}) + (\gamma_{44} - \gamma_{33}) = 16\pi\kappa\lambda \tag{16}$$

$$-(\alpha_{44} - \alpha_{33}) - 2(\beta_{44} - \beta_{33}) + (\gamma_{44} - \gamma_{33}) = -16\pi\kappa\rho \tag{17}$$

where the suffixes 3 and 4 hereafter, denote ordinary differentiation with respect to z and t, respectively.

The field equations (14) to (17) give the solution

$$\lambda = 0 \quad \text{and} \quad \rho = 0. \tag{18}$$

Using solution (18) in Equations (14) to (17), we obtain

$$\alpha_{44} - \alpha_{33} = \beta_{44} - \beta_{33} = \gamma_{44} - \gamma_{33} \tag{19}$$

Hence, from Equation (19), the vacuum solutions of the field equations in bimetric theory can be expressed in terms of harmonic functions in $t - z$ and $t + z$;

$$\alpha = f_1(t - z) + g_1(t + z) \tag{20}$$

$$\beta = f_2(t - z) + g_2(t + z) \tag{21}$$

$$\gamma = f_3(t - z) + g_3(t + z), \tag{22}$$

which represent the stationary plane waves moving with unified velocity to positive and negative directions. The spatial volume of the model is given by

$$V^3 = \sqrt{-g} = e^{\alpha + 2\beta + \gamma}. \tag{23}$$

Field equations and their solutions for domain walls with quark matter

A thick domain wall can be viewed as a soliton-like solution of the scalar field equation coupled with gravity. There are two ways of studying the thick domain walls. One way is to solve the gravitational field equations with an energy momentum tensor describing a scalar field ψ with self-interactions contained in a potential $\upsilon(\psi)$ given by

$$\psi_{,i}\psi_{,j} - g_{ij}\left[\frac{1}{2}\psi_{,k}\psi^{,k} - \upsilon(\psi)\right]. \tag{24}$$

The second approach is to assume the energy momentum tensor in the form [39]

$$T_{ij} = \rho(g_{ij} + w_i w_j) + p w_i w_j; \qquad w_i w^j = -1, \tag{25}$$

where ρ is the energy density of the wall, p is the pressure in the direction normal to the plane of the wall, and w_i is a unit space-like vector in the same direction.

This perfect fluid form of the domain walls includes quark matter described by $\rho_m = \rho_q + B_C$ and $p_m = p_q - B_C$, and the domain wall tension σ_w is given by $\rho = \rho_m + \sigma_w$ and $p = p_m - \sigma_w$. Here, we use the second approach to study the thick domain walls with quark matter in bimetric theory of gravitation.

In the co-moving coordinate system, we have from (25)

$$T_1^1 = T_2^2 = T_4^4 = \rho; \quad T_3^3 = -p; \quad T_j^i = 0, \quad i \neq j \tag{26}$$

Here, the pressure is taken in the direction of the z-axis. The quantities ρ and p depend on z and t only.

The field equations (3) of Rosen's bimetric theory of gravity for the line element (6) with (26) take the form

$$(\alpha_{44} - \alpha_{33}) - 2(\beta_{44} - \beta_{33}) - (\gamma_{44} - \gamma_{33}) = -16\pi\kappa\rho \tag{27}$$

$$(\alpha_{44} - \alpha_{33}) + (\gamma_{44} - \gamma_{33}) = 16\pi\kappa\rho \tag{28}$$

$$(\alpha_{44} - \alpha_{33}) + (\gamma_{44} - \gamma_{33}) = -16\pi\kappa p \tag{29}$$

$$-(\alpha_{44} - \alpha_{33}) - 2(\beta_{44} - \beta_{33}) + (\gamma_{44} - \gamma_{33}) = -16\pi\kappa\rho \tag{30}$$

The set of field equations (27) to (30) immediately yields

$$p + \rho = 0. \tag{31}$$

In view of reality conditions $p > 0$ and $\rho > 0$, Equation (31) implies that

$$p = 0 \quad and \quad \rho = 0. \tag{32}$$

When $p = 0 = \rho$ (vacuum), from equations (27) to (30), we obtain which in turn yield the same vacuum solutions given by Equations (20) to (22).

Conclusion

Topological stable objects like domain walls and cosmic strings play a fundamental role in the formation of the universe during the early stage of evolution. It is evident from the literature that Einstein's theory of general relativity has been extensively used to establish the existence of thick domain walls and cosmic strings. Here, it is shown that plane symmetric universe does not accommodate domain walls and cosmic strings coupled with quark matter in Rosen's [1] bimetric theory of relativity. The quark matter is vanish automatically; however, the existence of quark may be possible in this theory. Hence, it is concluded that bimetric theory does not help in any way to study the gravitational effects of thick domain walls and cosmic strings at the early stages of evolution of the universe.

Competing interest
The authors declare that they have no competing interests.

Authors' contributions
All calculations, derivations of the various results and their verifications were carried out by PKS and BM. All authors read and approved the final manuscript.

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References

1. Rosen, N: A bi-metric theory of gravitation. Gen. Rel. Grav. **4**, 435 (1973)
2. Mohanty, G, Sahoo, PK, Mishra, B: On Bianchi type-I mesonic cosmological model in bimetric theory. Astrophys. Space. Sci. **281**, 609 (2002)
3. Mohanty, G, Sahoo, PK: Some Bianchi type cosmological models in bimetric theory. Czech. J. Phys. **52**, 1041 (2002)
4. Reddy, DRK: Non-existence of cosmic strings in bimetric theory of gravitation. Astrophys. Space Sci. **286**, 397 (2003)
5. Sahoo, PK: On Kantowski-Sachs cosmic strings coupled with Maxwell fields in bimetric relativity. Int. J. Theor. Phys. **49**, 25 (2010)
6. Sahoo, PK, Mishra, B, Ramu, A: Bianchi types V and VI 0 cosmic strings coupled with Maxwell fields in bimetric theory. Int. J. Theor. Phys. **50**, 349 (2011)
7. Jain, P, Sahoo, PK, Mishra, B: Axially symmetric cosmological model with wet dark fluid in bimetric theory of gravitation. Int. J. Theor. Phys. **51**, 2546 (2012)
8. Kibble, TWB: Topology of cosmic domains and strings. J. Phys. A. **9**, 1387 (1976)
9. Satchel, J: Thickening the string. I. The string perfect dust. Phys. Rev. D. **21**, 2171 (1980)
10. Vilenkin, A: Cosmic strings. Phys. Rev. D. **24**, 2082 (1981)
11. Gott, JR: Gravitational lensing effects of vacuum string: exact results. Astrophys. J. **288**, 422 (1985)
12. Letelier, PS: String cosmologies. Phys. Rev. D. **28**, 2414 (1983)
13. Adhav, KS, Nimkar, AS, Naidu, RL: Axially symmetric non-static domain walls in scalar-tensor theories of gravitation. Astrophys. Space Sci. **312**, 165 (2007)
14. Tikekar, R, Patel, LK, Dadhich, N: A class of cylindrically-symmetric models in string cosmology. Gen. Rel. Grav. **26**, 647 (1994)
15. Hill, CT, Schramm, DN, Fry, JN: Cosmological structure formation from soft topological defects. Comments Nucl. Part Phys. **19**, 25 (1989)
16. Vilenkin, A: Phys. Lett. B. **133**, 177 (1983)
17. Isper, J, Sikivie, P: Gravitationally repulsive domain wall. Phys. Rev. D. **30**, 712 (1984)
18. Widraw, LM: General-relativistic domain walls. Phys. Rev. D. **39**, 3571 (1989)
19. Goetz, G: The gravitational field of plane symmetric thick domain walls. Math. Phys. **31**, 2683 (1990)
20. Mukherji, M: Gravitational fields of cosmic membranes. Class Quantum Gravity. **10**, 131 (1993)
21. Wang, A: Gravitational collapse of thick domain walls: an analytic model. Mod. Phys. Lett. A. **9**, 3605 (1994)
22. Rahaman, F, Chakraborti, S, Kalam, M: Domain wall in Lyra geometry. Int. J. Mod. Phys. D. **10**, 735 (2001)
23. Reddy, DRK, Subba Rao, MV: Axially symmetric cosmic strings and domain walls in Lyra geometry. Astrophys. Space Sci. **302**, 157 (2006)
24. Adhav, KS, Raut, VB, Dawande, MV: Bianchi type-III cosmic strings and domain walls cosmological model in general relativity. Int. J. Theo. Phys. **48**, 1019 (2009)
25. Rahaman, F: Higher dimensional domain wall in Lyra geometry. Astrophys. Space Sci. **282**, 625 (2002)
26. Rahaman, F, Mukherji, R: Domain walls in Lyra geometry. Astrophys. Space Sci. **288**, 389 (2003)
27. Lyra, G: Über eine modifikation der Riemannschen geometrie. Math. Zeitschrift. **54**, 52 (1951)
28. Itoh, N: Prog. Theor. Phys. **44**, 291 (1970)
29. Bodmer, AR: Collapsed nuclei. Phys. Rev. D. **4**, 1601 (1971)
30. Witten, E: Cosmic separation of phases. Phys. Rev. D. **30**, 272 (1984)
31. Yavuz, I, Yilmaz, I, Baysal, H: Strange quark matter attached to the string cloud in the spherical symmetric space-time admitting conformal motion. Int. J. Mod. Phys. D. **14**, 1365 (2005)
32. Yilmaz, I: Domain wall solutions in the nonstatic and stationary Gödel universes with a cosmological constant. Phys. Rev. D. **71**, 103503 (2005)
33. Yilmaz, I: String cloud and domain walls with quark matter in 5-D Kaluza-Klein cosmological model. Gen. Rel. Grav. **38**, 1397 (2006)
34. Adhav, KS, Nimkar, AS, Dawande, MV: String cloud and domain walls with quark matter in n-dimensional Kaluza-Klein cosmological model. Int. J. Theo. Phys. **47**, 2002 (2008)
35. Adhav, KS, Nimkar, AS, Raut, VB, Thakare, RS: Strange quark matter attached to string cloud in Bianchi type-III space time. Astrophys. Space Sci. **319**, 81 (2009)
36. Khadekar, GS, Wanjari, R, Ozel, C: Domain wall with strange quark matter in Kaluza-Klein type cosmological model. Int. J. Theor. Phys. **48**, 2550 (2009)

37. Katore, SD, Shaikh, AY: Cosmological model with strange quark matter attached to cosmic string for axially symmetric space-time. Int. J. Theor. Phys. **51**, 1881 (2012)
38. Mahanta, KL, Biswal, SK, Sahoo, PK, Adhikary, MC: String cloud with quark matter in self-creation cosmology. Int. J. Theor. Phys. **51**, 1538 (2012)
39. Press, WH, Ryden, BS, Spergel, DN: Dynamical evolution of domain walls in an expanding universe. ApJ. **347**, 590 (1989)

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