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String cloud and domain walls with quark matter in kink cosmological model

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Abstract

We have studied quark matter coupled with string cloud and domain walls in the context of general relativity. For this purpose, we solved Einstein's field equations for quark matter coupled to the string cloud and domain walls in spherical symmetric kink space-time. It is found that cosmic strings and domain walls do not survive in this space-time. Hence, the space-time in both the cases reduces to Minkowskian and the space-time is flat.

Keywords: Quark matter; Domain walls; String cloud; Kink space-time

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Background

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe. At the very early stages of evolution of universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously. The topological stable defects [1] which occur during the phase transition are identified as strings. The other topological defects are monopoles and domain walls. Spontaneous symmetry breaking is an old idea, described within the particle physics in terms of the Higgs field mechanism. The symmetry is called spontaneously broken if the ground state is not invariant under the full symmetry of the Lagrangian density. Thus, the vacuum expectation value of the Higgs field is non-zero. In quantum field theories, broken symmetries are restored at high temperatures.

A star which is smaller than neutron stars, the possibility of a quark star or a compact star, which is supported by degenerate pressure of quark matter, has been pointed out. Such a quark star has been investigated by many authors [1-4]. In their view, it is commonly assumed that, such quark stars contain quark matter in the core region and are surrounded by harmonic matter, although they are in the branch of neutron stars [5]. One of the interesting consequence of the first-order phase transition from quark phase to hadron phase in the early universe is the formation of strange quark matter and it has been attracting much interest [6,7]. It is plausible to attach strange quark matter to the string cosmology. Because string is free to vibrate and different vibration modes of the string represent the different particle types. The different modes are observed as different masses or spins. Satchel [8] and Letelier [9] initiated the relativistic treatment of strings. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin [10], Gott [11] in general relativity. Krori et al. [12], Chatterjee and Bhui [13], Tikekar and Patel [14], and Bhattacharjee and Baruah [15] obtained relativistic string models of Bianchi space time.

In the present paper, we have attached strange quark matter to the string cloud, since one of such transition during the phase transition of the universe could be the quark-glucon plasma (QGP) harden gas when cosmic temperature was $T \approx 200$ MeV. Itoh [4], Bodmar [16], and Witten [6] proposed two ways of formation of strange quark matter: the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities. Alcock et al. [17] and Haensel et al. [18] examined that some neutron stars could actually be strange stars built entirely of strange matters if the hypothesis of the quark matter is true. Cheng et al. [19] have studied strange star properties, while Yavuz et al. [20] studied strange quark matter attached to the string cloud

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in spherical symmetric space-time admitting conformal motion.

Typically, quark matter is modeled with the equation of state (EOS) which is based on the phenomenological bag model of quark matter. In the framework of this model, quarks are thought as degenerate Fermi gas which exists only in the region of space endowed with vacuum energy density B_C. Also in the framework of this model, the quark matter is composed of massless u, d quarks and massive s quarks and electrons. In the simplified bag model, it is assumed that when quarks are massless and non-interacting, we then have quark pressure

$$p_q = \frac{\rho_q}{3}, \tag{1}$$

where ρ_q is the quark energy density.

The total energy density is

$$\rho_m = \rho_q + B_C, \tag{2}$$

but the total pressure is

$$p_m = p_q - B_C. \tag{3}$$

The equation of state for strange quark matter is [21,22]

$$p_m = \frac{1}{3}(\rho_m - 4B_C). \tag{4}$$

Recently, the quark-gluon plasma is created as the perfect liquid in the Brookhaven National Laboratory [23-25]. So, we shall consider the quark-gluon plasma in the form of perfect fluid and also use the following equation of state:

$$p_m = (\gamma - 1)\rho_m, \tag{5}$$

where $1 \leq \gamma \leq 2$ is a constant.

It is possible to couple quark matter with cosmic strings and domain walls. Because, the strings are free to vibrate and different vibrational modes of the strings represent the different particle types since different modes are seen as different masses or spins. Yilmaz [26,27] have studied 5-D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain walls. Adhav et al. [28,29] have discussed string cloud and domain walls with quark matter in n-dimensional Kaluza-Klein cosmological model in general relativity and strange quark matter attached to string cloud in Bianchi type-III space time in general relativity. Khadekar et al. [30] have confirmed their work to the quark matters which attached to the topological defects in general relativity. Katore and Shaikh [31] have obtained a cosmological model with strange quark matter attached to a cosmic string for axially symmetric space-time in general relativity. Mahanta et al. [32] have discussed string cloud with quark matter in self creation cosmology. Recently, Sahoo and Mishra [33,34] have studied string cloud and domain walls with quark matter for plane symmetric cosmological model and string cloud with strange quark matter in axially symmetric space-time in bimetric theory.

A formulation of the general relativity theory is given in terms of three postulates about a mathematical model for space time. This model is a manifold M with a metric g of the Lorentz signature. The physical significance of the metric (space time) is given by the first two postulates: those of local casualty and of local conservation of energy momentum. These postulates are common to both general and special theory of relativity, and so are supported by the experimental evidence of the latter theory. The third postulate, the field equations for the metric g, is less well experimentally established. However, most of our results will depend only on the property of field equations that gravity is attractive for positive matter densities. This property is common to both general theory and alternative theories of relativity.

The metrical kinks discovered by Finkelstein and Misner [35] have arisen in the context of topology changing space-times [36] and in the study of black holes [37,38]. Although some interesting kink space-times can be found by explicit construction [39,40]; however, in view of the profusion of space-times representing known solutions of the Einstein equations (of varying degrees of physical reasonableness) [41], it is logical to look for kink space-times in this well-established list. Williams [42] has studied the rotating kink space-time in 2+1 Dimensions. The Gödel kink space-time has been identified by Harriott and Williams [43]. A natural generalization of a previously known (2+1)-dimensional kinked perfect fluid space-time [42] was shown to produce a (3+1)-dimensional space-time that does not represent a physically acceptable (perfect or imperfect) fluid but which, nevertheless, can satisfy the weak, strong and dominant energy conditions [44]. Our purpose in this paper is to construct this topic in relation to a broader class of kink space-time. The purpose of the present work is to study spherically symmetric kink cosmological models in general relativity with quark matter coupled to the string cloud and domain walls.

Field equations and their solutions for string cloud with quark matter

Here, we considered the simplest spherically symmetric space-time in the presence of a kink which is given by [35]

$$ds^2 = -\cos 2\alpha dt^2 - 2\sin 2\alpha dr dt + \cos 2\alpha dr^2 + r^2 d\Omega^2, \tag{6}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ and $\alpha = \alpha(r)$.

The energy momentum tensor for string cloud is given by

$$T^i_j = \rho u^i u_j - \lambda x^i x_j, \tag{7}$$

where ρ is the rest energy density for a cloud with particle attached to them and λ is the tension density of the strings; they are related by

$$\rho = \rho_p + \lambda, \tag{8}$$

where ρ_p is the rest energy density of particles.

The string is free to vibrate and different vibrational modes are seen as different masses or spins. Therefore, we will consider quarks instead of particles in the cloud of strings. In this case from (8), we get

$$\rho = \rho_q + \lambda + B_C \tag{9}$$

From (7) and (9), we can write the energy momentum tensor for strange quark matter attached to the string cloud as [20]

$$T^i_j = (\rho_q + \lambda + B_C)u^i u_j - \lambda x^i x_j, \tag{10}$$

where u^i is the four velocity of the string and x^i representing the direction vector of anisotropy. The string source is along the z-axis, which is the axis of symmetry.

Orthonormalisation of u^i and x^i is given as

$$u^i u_i = -1, \quad u^i x_i = 0, \quad x^i x_i = 1 \tag{11}$$

In the co-moving coordinate system, from Equations (10) and (11)

$$T^1_1 = 0 = T^2_2; \quad T^4_4 = \rho; \quad T^3_3 = \lambda; \quad T^i_j = 0, \quad i \neq j \tag{12}$$

The Einstein's field equation in general relativity is

$$R^i_j - \frac{1}{2} N \delta^i_j = -8\pi G T^i_j \tag{13}$$

Here, we consider geometrized units so that $8\pi G = c = 1$. The Einstein's field Equation (13) for the line element (6) with the help of Equations (7, 10,11, and 12) becomes

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\alpha_1 \sin 2\alpha = 0 \tag{14}$$

$$\frac{2}{r}\alpha_1 \sin 2\alpha + \alpha_{11}\sin 2\alpha + 2\alpha_1^2 \cos 2\alpha = 0 \tag{15}$$

$$\frac{2}{r}\alpha_1 \sin 2\alpha + \alpha_{11}\sin 2\alpha + 2\alpha_1^2 \cos 2\alpha = \lambda \tag{16}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\alpha_1 \sin 2\alpha = -\rho \tag{17}$$

Here afterwards, the suffix 1 following an unknown function denote ordinary differentiation with respect to r.

The field equations (14 to 17) gives the solution

$$\lambda = 0 \quad and \quad \rho = 0 \tag{18}$$

From (14), we obtain

$$(r \sin^2\alpha)_1 = 0 \tag{19}$$

which on integration yields

$$\alpha = \sin^{-1}\sqrt{\frac{k}{r}}, \tag{20}$$

where k is a constant.

Hence, the simplest spherically symmetric kink space-time takes the form

$$ds^2 = -\left(1 - \frac{2k}{r}\right)dt^2 - 4\sqrt{\frac{k}{r}\left(1 - \frac{2k}{r}\right)}drdt + \left(1 - \frac{2k}{r}\right)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{21}$$

For $r = 2k$, there is no singularity in the space-time.

Field equations and their solutions for domain walls with quark matter

The energy momentum tensor T^D_{ij} of a domain wall in the conventional form [45] is given by

$$T^D_{ij} = \rho U_i U_j + p(g_{ij} + U_i U_j), \tag{22}$$

where U^i is time-like vector such that $U_i U^i = -1$ otherwise 0. Here, we also use the co-moving co-ordinate system. This perfect fluid form of the domain walls include quark matter [26] described by $\rho_m = \rho_q + B_C$ and $p_m = p_q - B_C$ as well as domain walls tension σ_w is given by $\rho = \rho_m + \sigma_w$ and $p = p_m - \sigma_w$ which are related by the bag model equation of state (Equation 4) and equation of state (Equation 5).

Using the line element (6), the field Equation (13) take the form for (22) as

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\alpha_1 \sin 2\alpha = -p \tag{23}$$

$$\frac{2}{r}\alpha_1 \sin 2\alpha + \alpha_{11}\sin 2\alpha + 2\alpha_1^2 \cos 2\alpha = -p \tag{24}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\alpha_1 \sin 2\alpha = \rho \tag{25}$$

The set of field Equations (23 to 25) immediately yields

$$p + \rho = 0 \tag{26}$$

In view of reality conditions $p > 0$, $\rho > 0$, Equation (26) implies that

$$p = 0 \quad and \quad \rho = 0 \tag{27}$$

when $p = 0 = \rho$ (vacuum), from Equations (23-25) which in turn yield the same vacuum solution given by Equation (20).

Conclusion

General relativity provides a rich arena to understand the natural relation between geometry and matter furnished by the Einstein equations. Field equations mean that any two field configurations connected by a diffeomorphism which are empirically indistinguishable and physically identical. Topological stable objects like domain walls and cosmic strings play a fundamental role in the formation of universe during the early stage of evolution.

It is evident from the literature that Einstein's theory of general relativity has been extensively used to establish the existence of thick domain walls and cosmic strings. Here, it is shown that the spherically symmetric kink universe does not accommodate domain walls and cosmic strings coupled with quark matter in the general theory of relativity. In both the cases, the space-time turns up to be flat. Hence, a vacuum kink model is obtained. It is interesting to note that the vacuum model (21) obtained here has no singularity at $r = 2k$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All calculations, derivations of the various results and their verifications were carried out by PKS and BM. Both authors read and approved the final manuscript.

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References

1. Kibble, TWB: Topology of cosmic domains and strings. J. Phys. A. **9**, 1387 (1976)
2. Gerlach, UH: Equation of state at supranuclear densities and the existence of a third family of superdense stars. Phys. Rev. **172**, 1325 (1968)
3. Ivanenko, D, Kurdgelaidze, DF: Remarks on quark stars. Lett. Nuovo Cimento. **2**, 13 (1969)
4. Itoh, N: Hydrostatic Equilibrium of Hypothetical Quark Stars. Prog. Theor. Phys. **44**(1970), 291
5. Rosenhauer, A, Staubo, EF, Csernai, LP, Φvergard, T, Φstgaard, E: Neutron stars, hybrid stars and the equation of state. Nucl. Phys. **A540**, 630 (1992)
6. Witten, E: Cosmic separation of phases. Phys. Rev. D. **30**, 272 (1984)
7. Farhi, E, Jaffe, RL: Strange matter. Phys. Rev. D. **30**, 2379 (1984)
8. Satchel, J: Thickening the string. I. The string perfect dust. Phys. Rev. D. **21**, 2171 (1980)
9. Letelier, PS: String cosmologies. Phys. Rev. D. **28** (1983)
10. Vilenkin, A: Cosmic strings. Phys. Rev.D. **24**, 2082 (1981)
11. Gott, JR: Gravitational lensing effects of vacuum string: exact results. Astrophys. J. **288**, 422 (1985)
12. Krori, KD, Choudhary, T, Mahanta, CR, Mazumdar, A: Some exact solutions in string cosmology. Gen. Rel. Grav. **22**, 123 (1990)
13. Chatterjee, S, Bhui, B: Homogeneous cosmological model in higher dimension. Mon. Not. R. Astron. Soc. **247**, 57 (1990)
14. Tikekar, R, Patel, LK, Dadhich, N: A class of cylindrically-symmetric models in string cosmology. Gen. Rel. Grav. **26**, 647 (1994)
15. Bhattacharjee, R, Baruah, KK: String cosmologies with a scalar field. Pure Appl. Math. **32**, 47 (2001)
16. Bodmar, AR: Collapsed nuclei. Phys. Rev. D. **4** (1971)
17. Alcock, C, Farhi, E, Olinto, A: Strange stars. Astrophys. J. **310**, 261 (1986)
18. Haensel, P, Zdunik, JL, Sehaeffer, R: Strange quark stars. Astron. Astrophys. **160**, 121 (1986)
19. Cheng, KS, Dai, ZG, Lu, T: Strange stars and related astrophysical phenomena. Int. J. Mod. Phys. **D7**, 139 (1998)
20. Yavuz, I, Yilmaz, I, Baysal, H: Strange quark matter attached to the string cloud in the spherically symmetric space-time admitting conformal motion. Int. J. Mod. Phys. **D14**, 1365 (2005)
21. Kapusta, J: Finite-Temperature Field Theory. Cambridge University Press, Cambridge (1994)
22. Sotani, H, Kohri, K, Harada, T: Restricting quark matter models by gravitational wave observation. Phys. Rev. **D69**, 084008 (2004)
23. Back, B. B, *et al*: The PHOBOS perspective on discoveries at RHIC, PHOBOS Collb. Nucl. Phys. A. **757**, 28 (2005)
24. Adams, J, *et al*: Experimental and theoretical challenges in the search for the quarkgluon plasma: The STAR Collaboration's critical assessment of the evidence from RHIC collisions. STAR Collaboration. Nucl. Phys. A. **757**, 102 (2005)
25. Adcox, K, *et al*: Formation of dense partonic matter in relativistic nucleus collisions at RHIC:, PHENIX Collaboration. Nucl. Phys. A. **757**, 184 (2005)
26. Yilmaz, I: Domain wall solutions in the nonstatic and stationary Godel universes with a cosmological constant. Phys. Rev. D. **71**, 103503 (2005)
27. Yilmaz, I: String cloud and domain walls with quark matter in 5-D Kaluza Klein cosmological model. Gen. Rel. Grav. **38**, 1397 (2006)
28. Adhav, KS, Nimkar, AS, Dawande, MV: String cloud and domain walls with quark matter in dimensional Kaluza-Klein cosmological model. Int. J. Theo. Phys. **47**, 2002 (2008)
29. Adhav, KS, Nimkar, AS, Raut, VB, Thakare, RS: Strange quark matter attached to string cloud in Bianchi type-III space time. Astrophys. Space Sci. **319**, 81 (2009)
30. Khadekar, GS, Wanjari, R, Ozel, C: Domain wall with strange quark matter in Kaluza-Klein type cosmological model. Int. J. Theor. Phys. **48**, 2550 (2009)
31. Katore, SD, Shaikh, AY: Cosmological model with strange quark matter attached to cosmic string for axially symmetric space-time. Int. J. Theor. Phys. **51**, 1881 (2012)
32. Mahanta, KL, Biswal, SK, Sahoo, PK, Adhikary, MC: String cloud with quark matter in self-creation cosmology. Int. J. Theor. Phys. **51**, 1538 (2012)
33. Sahoo, PK, Mishra, B: String cloud and domain walls with quark matter for plane symmetric cosmological model in bimetric theory. J. Theor. Appl. Phys. **7**, 12 (2013)
34. Sahoo, PK, Mishra, B: Axially symmetric space-time with strange quark matter attached to string cloud in bimetric theory. Int. J. of Pure Appl. Math 82. **87**(1) (2013)
35. Finkelstein, D, Misner, CW: Some new conservation laws. Ann. Phys. **6**, 230 (1959)
36. Gibbons, GW, Hawking, SW: Kinks and topology change. Phys. Rev. Lett. **69**, 1719 (1992)
37. Finkelstein, D, McCollum, G: Kinks and extensions. J. Math. Phys. **16**, 2250 (1975)
38. Finkelstein, D: Kinks and black holes. In Directions in General Relativity. In: Misner, BL, Hu, MP, Ryan, Jr, Vishveshwara, CV (eds.) *Proceedings of the 1993 International Symposium in Maryland, Volume 1: Papers in honor of Charles*, pp. 99–103. Cambridge University Press, Cambridge, (1993)
39. Dunn, KA, Harriott, TA, Williams, JG: Kinks and geodesic incompleteness. J. Math. Phys. **37**, 5637 (1996)
40. González-Díaz, PF: On the black-hole kink. Int. J. Mod. Phys. **D6**, 57 (1997)
41. Kramer, D, Stephani, H, Herlt, E, MacCallum, M: Exact solutions of Einstein's field equations. Cambridge University Press, Cambridge (1980)
42. Williams, JG: Rotating kink spacetime in 2+1 dimensions. Gen. Relativity Gravit. **30**(1), 27 (1998)
43. Harriott, TA, Williams, JG: Gödel kink spacetime. Gen. Relativity Gravit. **33**(10) (2001)
44. Harriott, TA, Williams, JG: Rotating kink spacetime. Gen. Relativity Gravit. **35**(3), 341 (2003)
45. Okuyama, N, Maeda, K: Domain wall dynamics in brane world and nonsingular cosmological models. Phys. Rev. D. **70**, 064030 (2004)

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