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String cosmological models with bulk viscosity in Nordtvedt's general scalar-tensor theory of gravitation

Uma Maheswara Rao Velagapudi* and Neelima Davuluri

Abstract

We have obtained and presented spatially homogeneous Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Nordtvedt's (Astrophys. J. 161:1059, 1970) general scalar-tensor theory of gravitation with the help of a special case proposed by Schwinger. It is observed that only the bulk viscous cosmological model exists in the case of Bianchi type IX universe. Some important features of the models, thus obtained, have been discussed.

Keywords: Bulk viscosity; Cosmic strings; Bianchi type II, VIII, and IX metrics; Nordtvedt's general scalar-tensor theory

Introduction

Nordtvedt [1] proposed a general class of scalar-tensor gravitational theories in which the parameter ω of the Brans-Dicke theory is allowed to be an arbitrary (positive definite) function of the scalar field ($\omega \to \omega(\phi)$). The study of scalar field cosmological models in the framework of Nordtvedt's theory with the help of a special case proposed by Schwinger [2] has attracted many research workers. This general class of scalar-tensor gravitational theories includes the Jordan [3] and Brans and Dicke [4] theories as special cases. These general cases of scalar-tensor theories are seen to lead to a super richness (or arbitrariness) of possible theories.

The field equations of the general scalar-tensor theory proposed by Nordtvedt (using geometrized units with $c = 1$, $G = 1$) are

$$R_{ij}-\frac{1}{2}g_{ij}R = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) -\phi^{-1}\left(\phi_{i;j}-g_{ij}\phi^{,k}_{\ ;k}\right) \tag{1}$$

$$\phi^{,k}_{\ ;k} = \frac{8\pi T}{3+2\omega} - \frac{1}{(3+2\omega)}\frac{d\omega}{d\phi}\phi_{,i}\phi^{,i} \tag{2}$$

where R_{ij} is the Ricci tensor, R is the curvature invariant, T_{ij} is the stress energy of the matter, and the comma and semicolon denote partial and covariant differentiation, respectively.

Also, we have the energy conservation equation

$$T^{ij}_{;j} = 0 \tag{3}$$

Several investigations have been made in higher-dimensional cosmology in the framework of different scalar-tensor theories of gravitation. In particular, Barker [5], Ruban and Finkelstein [6], Banerjee and Santos [7,8], and Shanti and Rao [9,10] are some of the authors who have investigated several aspects of Nordtvedt's general scalar-tensor theory in four dimensions. Rao and Sree Devi Kumari [11] have discussed a cosmological model with a negative constant deceleration parameter in a general scalar-tensor theory of gravitation. Rao et al. [12] have obtained Kaluza-Klein radiating model in a general scalar-tensor theory of gravitation.

The study of string theory has received considerable attention in cosmology. Cosmic strings are important in the early stages of evolution of the universe before the particle creation. Cosmic strings are one-dimensional topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe. Reddy and Subba Rao [13] have studied axially symmetric cosmic strings and domain walls in a theory proposed by Sen [14] based on Lyra [15] geometry. Mohanty and Mahanta [16] have studied a five-dimensional axially symmetric string cosmological model in Lyra [15] manifold. Rao and

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Vinutha [17] have studied axially symmetric cosmological models in a theory of gravitation based on Lyra [15] geometry.

In order to study the evolution of the universe, many authors constructed cosmological models containing a viscous fluid. The presence of viscosity in the fluid introduces many interesting features in the dynamics of homogeneous cosmological models. The possibility of bulk viscosity leading to inflationary-like solutions in general relativistic Freidmann-Robertson-Walker (FRW) models has been discussed by several authors [18-22]. Bali and Dave [23], Bali and Pradhan [24], Tripathy et al. [25,26], and Rao et al. [27] have studied various Bianchi-type string cosmological models in the presence of bulk viscosity.

Bianchi space-times play a vital role in understanding and describing the early stages of the evolution of the universe. In particular, the study of Bianchi types II, VIII, and IX universes are important because familiar solutions like the FRW universe with positive curvature, the de Sitter universe, and the Taub-Nut solutions correspond to Bianchi types II, VIII, and IX space-times. Reddy et al. [28] studied Bianchi types II, VIII, and IX models in the scale-covariant theory of gravitation. Shanthi and Rao [29] studied Bianchi types VIII and IX models in the Lyttleton-Bondi universe. Also, Rao and Sanyasi Raju [30,31] have studied Bianchi types VIII and IX models in zero mass scalar fields and self-creation theory of gravitation. Rao et al. [32-34] have obtained Bianchi types II, VIII, and IX string cosmological models; perfect fluid cosmological models in the Saez-Ballester theory of gravitation; and string cosmological models in general relativity as well as self-creation theory of gravitation, respectively. Rao and Vijaya Santhi [35,36] have obtained Bianchi types II, VIII, and IX perfect fluid magnetized cosmological models and string cosmological models, respectively, in the Brans and Dicke [4] theory of gravitation. Recently, Rao and Sireesha [37,38] have obtained Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Lyra [15] and Brans and Dicke [4] theory of gravitation, respectively. Sadeghi et al. [39] have discussed the cosmic string in the BTZ black hole background with time-dependant tension. Katore et al. [40], Motevalli et al. [41], Saadat [42], Pourhassan [43], Amani and Pourhassan [44], Saadat and Pourhassan [45,46], Motevalli et al. [47], Saadat [48], and Saadat and Farahani [49] are some more authors who have discussed various aspects of bulk viscosity in different theories of gravitation.

In this paper, we will discuss Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Nordtvedt's [1] general scalar-tensor theory with the help of a special case proposed by Schwinger [2], i.e., $3 + 2\omega(\phi) = \frac{1}{\lambda\phi}$, where λ is a constant.

Metric and energy momentum tensor

We consider a spatially homogeneous Bianchi types II, VIII, and IX metrics of the form

$$ds^2 = dt^2 - R^2\left[d\theta^2 + f^2(\theta)d\varphi^2\right] - S^2[d\psi + h(\theta)d\varphi]^2 \tag{4}$$

where (θ, φ, ψ) are the Eulerian angles, and R and S are functions of t only.

It represents Bianchi type II if $f(\theta) = 1$ and $h(\theta) = \theta$, Bianchi type VIII if $f(\theta) = \cosh\theta$ and $h(\theta) = \sinh\theta$, and Bianchi type IX if $f(\theta) = \sin\theta$ and $h(\theta) = \cos\theta$. The energy momentum tensor for a bulk viscous fluid containing one-dimensional string is

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij} - \lambda_s x_i x_j \tag{5}$$

and

$$\bar{p} = p - 3\xi H, \tag{6}$$

where $p = \in_0\rho (0 \le \in_0 \le 1)$. Here, $\bar{p}$ is the total pressure which includes the proper pressure p, ρ is the rest energy density of the system, λ_s is the tension in the string, $\xi(t)$ is the coefficient of bulk viscosity, $3\xi H$ is usually known as the bulk viscous pressure, H is the Hubble parameter, u_i is the four velocity vector, and x_i is a space-like vector which represents the anisotropic directions of the string.

Here, u^i and x^i satisfy the equations

$$g_{ij}u^i u^j = 1,$$

$$g_{ij}x^i x^j = -1,$$

and

$$u^i x_i = 0. \tag{7}$$

We assume the string to be lying along the z-axis. The one-dimensional strings are assumed to be loaded with particles, and the particle energy density is $\rho_p = \rho - \lambda_s$.

In a commoving coordinate system, we get

$$T_1^1 = T_2^2 = -\bar{p}, T_3^3 = \lambda_s - \bar{p}, T_4^4 = \rho, \tag{8}$$

where $\rho, \lambda_s, \bar{p}$ are functions of time t only.

Solution of field equations

Now, with the help of (5) to (8), the field equations (1) for the metric (4) can be written as

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{R}\dot{\phi}}{R\phi} + \frac{\dot{S}\dot{\phi}}{S\phi} = -\frac{8\pi\bar{p}}{\phi} \tag{9}$$

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2+\delta}{R^2}-\frac{3S^2}{4R^4}+\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\ddot{\phi}}{\phi}+2\frac{\dot{R}\dot{\phi}}{R\phi}=\frac{8\pi(\lambda_s-\bar{p})}{\phi} \tag{10}$$

$$2\frac{\dot{R}\dot{S}}{RS}+\frac{\dot{R}^2+\delta}{R^2}-\frac{S^2}{4R^4}-\frac{\omega\dot{\phi}^2}{2\phi^2}+2\frac{\dot{R}\dot{\phi}}{R\phi}+\frac{\dot{S}\dot{\phi}}{S\phi}=\frac{8\pi\rho}{\phi} \tag{11}$$

$$\left(\frac{\dot{S}}{S}-\frac{\dot{R}}{R}\right)\frac{h(\theta)\dot{\phi}}{\phi}=0 \tag{12}$$

$$\ddot{\phi}+\dot{\phi}\left(2\frac{\dot{R}}{R}+\frac{\dot{S}}{S}\right)=\frac{8\pi}{(3+2\omega)}(\rho+\lambda_s-3\bar{p})-\frac{1}{3+2\omega}\frac{d\omega}{d\phi}\dot{\phi}^2 \tag{13}$$

$$\dot{\rho}+(\rho+\bar{p})\left(2\frac{\dot{R}}{R}+\frac{\dot{S}}{S}\right)-\lambda_s\frac{\dot{S}}{S}=0. \tag{14}$$

Here, the overhead dot denotes differentiation with respect to t.

When $\delta = 0, -1$ & $+1$, the field equations (9) to (14) correspond to the Bianchi types II, VIII, and IX universes.

By taking the transformation $dt = R^2S\ dT$, the above field equations (9) to (14) can be written as

$$\frac{R''}{R}+\frac{S''}{S}-2\frac{R'^2}{R^2}-\frac{S'^2}{S^2}-2\frac{R'S'}{RS}+\frac{1}{4}S^4+\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi''}{\phi}-\frac{\phi'R'}{\phi R}=\frac{8\pi\bar{p}}{\phi}\left(R^4S^2\right) \tag{15}$$

$$2\frac{R''}{R}-2\frac{R'S'}{RS}-3\frac{R'^2}{R^2}+\delta\left(R^2S^2\right)-\frac{3}{4}S^4+\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi''}{\phi}-\frac{\phi''S'}{\phi S}=\frac{8\pi}{\phi}(\lambda_s-\bar{p})\left(R^4S^2\right) \tag{16}$$

$$2\frac{R'S'}{RS}+\frac{R'^2}{R^2}+\delta\left(R^2S^2\right)-\frac{1}{4}S^4-\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi'}{\phi}\left(2\frac{R'}{R}+\frac{S'}{S}\right)=\frac{8\pi}{\phi}\rho\ \left(R^4S^2\right) \tag{17}$$

$$\left(\frac{S'}{S}-\frac{R'}{R}\right)\frac{h(\theta)\phi'}{\phi}=0 \tag{18}$$

$$(3+2\omega)\phi''+\frac{d\omega}{d\phi}\phi'^2=8\pi(\rho+\lambda_s-3\bar{p})\left(R^4S^2\right) \tag{19}$$

$$\rho'+(\rho+\bar{p})\left(2\frac{R'}{R}+\frac{S'}{S}\right)-\lambda_s\frac{S'}{S}=0, \tag{20}$$

where the dash denotes differentiation with respect to T, and R, S, and ϕ are functions of T only.

Since we are considering the Bianchi types II, VIII, and IX metrics, we have $h(\theta) = \theta, h(\theta) = \sinh\theta$ & $h(\theta) = \cos\theta$, respectively. Therefore, from Equation (18), we will get the following possible cases with $h(\theta) \neq 0$:

(1) $\frac{S'}{S}-\frac{R'}{R}=0\ and\ \phi'\neq 0$
(2) $\frac{S'}{S}-\frac{R'}{R}\neq 0\ and\ \phi'=0$
(3) $\frac{S'}{S}-\frac{R'}{R}=0\ and\ \phi'=0$

From the above three possibilities, we will consider only the first possibility since the other two cases will give us cosmological models in general relativity.

Cosmological models in Nordtvedt's general scalar-tensor theory

We will get cosmological models in a general scalar-tensor theory only in the case of

$$\frac{S'}{S}-\frac{R'}{R}=0\ and\ \phi'\neq 0.$$

If $\frac{S'}{S}-\frac{R'}{R}=0$, we get $S = R + c$.

Without loss of generality by taking the constant of integration $c = 0$, we get

$$S=R. \tag{21}$$

Using (21), the above field equations (15) to (20) can be written as

$$2\frac{R''}{R}-5\frac{R'^2}{R^2}+\frac{1}{4}R^4+\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi''}{\phi}-\frac{\phi'R'}{\phi R}=\frac{-8\pi\bar{p}}{\phi}\left(R^6\right) \tag{22}$$

$$2\frac{R''}{R}-5\frac{R'^2}{R^2}+\delta R^4-\frac{3}{4}R^4+\frac{\omega\phi'^2}{2\phi^2}+\frac{\phi''}{\phi}-\frac{\phi'R'}{\phi R}=\frac{8\pi}{\phi}(\lambda_s-\bar{p})\left(R^6\right) \tag{23}$$

$$3\frac{R'^2}{R^2}+\delta R^4-\frac{1}{4}R^4-\frac{\omega\phi'^2}{2\phi^2}+3\frac{\phi'R'}{\phi R}=\frac{8\pi}{\phi}\rho\ \left(R^6\right) \tag{24}$$

$$(3+2\omega)\phi''+\frac{d\omega}{d\phi}\phi'^2=8\pi(\rho+\lambda_s-3\bar{p})\left(R^6\right) \tag{25}$$

$$\rho'+3(\rho+\bar{p})\frac{R'}{R}-\lambda_s\frac{R'}{R}=0. \tag{26}$$

The field equations (22) to (25) are four independent equations with six unknowns $R, \omega, \phi, \rho, \lambda_s$ and $\bar{p}$. From Equations (22) to (25), we have

$$(3+2\omega)\phi''+\frac{d\omega}{d\phi}\phi'^2=\phi\left[6\frac{R''}{R}-12\frac{R'^2}{R^2}-\frac{1}{2}R^4+2\delta R^4\right]+\omega\frac{\phi'^2}{\phi}+3\phi''. \tag{27}$$

Here, we obtain the string cosmological models with bulk viscosity in Nordtvedt's general scalar-tensor theory with the help of a special case proposed by Schwinger in the form

$$3+2\omega(\phi)=\frac{1}{\lambda\phi},\lambda=\text{constant.} \tag{28}$$

From (27) and (28), we get

$$\frac{1}{\lambda}\left[\frac{\phi''}{\phi}-\frac{\phi'^2}{\phi^2}\right]+\frac{3}{2}\frac{\phi'^2}{\phi}-3\phi'' = \phi\left[6\frac{R''}{R}-12\frac{R'^2}{R^2}-\frac{1}{2}R^4+2\delta R^4\right] \tag{29}$$

From Equation (29), we get

$$\phi=e^{(k_1T+k_2)}, \tag{30}$$

where k_1 and k_2 are arbitrary constants and

$$6\frac{R''}{R}-12\frac{R'^2}{R^2}-\frac{1}{2}R^4+2\delta R^4=-\frac{3}{2}{k_1}^2 \tag{31}$$

Bianchi type II (δ = 0) cosmological model

From Equation (31), we get

$$R=\left(\sqrt{3}k_1\operatorname{cosech}(k_1T)\right)^{\frac{1}{2}}. \tag{32}$$

From Equations (22) to (24) we get

$$8\pi\rho=\frac{1}{2\sqrt{3}k_1}\sinh^3(k_1T)\left[e^{(k_1T+k_2)}(1-\coth(k_1T))-\frac{1}{6\lambda}\right] \tag{33}$$

$$8\pi\bar{p}=\frac{1}{6\sqrt{3}k_1}\sinh^3(k_1T)\times\left[e^{(k_1T+k_2)}\left(1-\coth(k_1T)-2\operatorname{cosech}^2(k_1T)\right)-\frac{1}{2\lambda}\right] \tag{34}$$

$$8\pi p=\frac{\varepsilon_0}{2\sqrt{3}k_1}\sinh^3(k_1T)\left[e^{(k_1T+k_2)}(1-\coth(k_1T))-\frac{1}{6\lambda}\right] \tag{35}$$

$$\xi=\frac{1}{12\pi\sqrt{3}k_1^2}\frac{\sinh^4(k_1T)}{\cosh(k_1T)}\times\left[e^{(k_1T+k_2)}\left(\frac{(3\varepsilon_0-1)}{6}\coth(k_1T)-\frac{1}{3}\operatorname{cosech}^2(k_1T)+\frac{(1-3\varepsilon_0)}{6}\right)+\frac{(\varepsilon_0-1)}{12\lambda}\right] \tag{36}$$

$$8\pi\lambda_s=\frac{-1}{\sqrt{3}k_1}e^{(k_1T+k_2)}\sinh(k_1T). \tag{37}$$

The metric (4), in this case, can be written as

$$ds^2=\left(3\sqrt{3}{k_1}^3\operatorname{cosech}^3(k_1T)\right)dT^2-\left(\sqrt{3}k_1\operatorname{cosech}(k_1T)\right)\times\left(d\theta^2+d\varphi^2\right)-\left(\sqrt{3}k_1\operatorname{cosech}(k_1T)\right)(d\psi+\theta d\varphi)^2. \tag{38}$$

Thus, Equation (38) together with (30) and (33) to (37) constitutes a Bianchi type II string cosmological model with bulk viscosity in isotropic form in Nordtvedt's general scalar-tensor theory of gravitation.

Bianchi type VIII (δ = −1) cosmological model

From Equation (31), we get

$$R=\left(\sqrt{\frac{3}{5}}k_1\operatorname{cosech}(k_1T)\right)^{\frac{1}{2}}. \tag{39}$$

From Equations (22) to (24), we get

$$8\pi\rho=\frac{5\sqrt{5}}{2\sqrt{3}k_1}\mathbf{sinh}^3(k_1T)\left[e^{(k_1T+k_2)}(\mathbf{1}\text{-}\mathbf{coth}(k_1T))-\frac{1}{6\lambda}\right] \tag{40}$$

$$8\pi\bar{p}=\frac{5\sqrt{5}}{3\sqrt{3}k_1}\sinh^3(k_1T)\times\left[e^{(k_1T+k_2)}\left(\frac{3}{4}\coth^2(k_1T)-\frac{23}{30}\operatorname{cosech}^2(k_1T)-\frac{1}{2}\coth(k_1T)-\frac{1}{4}\right)-\frac{1}{4\lambda}\right] \tag{41}$$

$$8\pi p=\frac{5\sqrt{5}\varepsilon_0}{2\sqrt{3}k_1}\mathbf{sinh}^3(k_1T)\left[e^{(k_1T+k_2)}(\mathbf{1}\text{-}\mathbf{coth}(k_1T))-\frac{1}{6\lambda}\right] \tag{42}$$

$$\xi=\frac{5\sqrt{5}}{12\pi\sqrt{3}k_1^2}\frac{\sinh^4(k_1T)}{\cosh(k_1T)}\times\left[e^{(k_1T+k_2)}\left(\frac{1}{4}\coth^2(k_1T)-\frac{23}{60}\operatorname{cosech}^2(k_1T)+\frac{(3\varepsilon_0-1)}{6}\coth(k_1T)+\frac{(1-6\varepsilon_0)}{12}\right)+\frac{(\varepsilon_0-1)}{12\lambda}\right] \tag{43}$$

$$8\pi\lambda_s=\frac{-2\sqrt{5}}{\sqrt{3}k_1}e^{(k_1T+k_2)}\mathbf{sinh}(k_1T) \tag{44}$$

The metric (4), in this case, can be written as

$$\begin{aligned} ds^2 = &\left(\frac{3\sqrt{3}}{5\sqrt{5}}k_1{}^3 \operatorname{cosech}^3(k_1T)\right)dT^2 \\ &-\left(\sqrt{\frac{3}{5}}k_1 \operatorname{cosech}(k_1T)\right)\left(d\theta^2 + \cosh^2\theta\, d\varphi^2\right) \\ &-\left(\sqrt{\frac{3}{5}}k_1 \operatorname{cosech}(k_1T)\right)(d\psi + \sinh\theta\, d\varphi)^2. \end{aligned} \tag{45}$$

Thus, Equation (45) together with (30) and (40) to (44) constitutes a Bianchi type VIII string cosmological model with bulk viscosity in isotropic form in Nordtvedt's general scalar-tensor theory of gravitation.

Bianchi type IX ($\delta = 1$) cosmological model

From Equation (31), we get

$$R = (k_1 \operatorname{sech}(k_1T))^{\frac{1}{2}}. \tag{46}$$

From Equations (22) to (24), we get

$$8\pi\rho = \frac{1}{2k_1}\cosh^3(k_1T)\left[3e^{(k_1T+k_2)}(1-\tanh(k_1T)) - \frac{1}{2\lambda}\right] \tag{47}$$

$$8\pi\bar{p} = \frac{1}{2k_1}\cosh^3(k_1T)\left[e^{(k_1T+k_2)}(1-\tanh(k_1T)) - \frac{1}{2\lambda}\right] \tag{48}$$

$$8\pi p = \frac{\varepsilon_0}{2k_1}\cosh^3(k_1T)\left[3e^{(k_1T+k_2)}(1-\tanh(k_1T)) - \frac{1}{2\lambda}\right] \tag{49}$$

$$\begin{aligned} \xi = &\frac{1}{24\pi k_1^2}\frac{\cosh^4(k_1T)}{\sinh(k_1T)} \\ &\times\left[e^{(k_1T+k_2)}((1-3\varepsilon_0) + (3\varepsilon_0-1)\tanh(k_1T)) + \frac{(\varepsilon_0-1)}{2\lambda}\right] \end{aligned} \tag{50}$$

$$8\pi\lambda_s = 0. \tag{51}$$

The metric (4), in this case, can be written as

$$\begin{aligned} ds^2 = &\left(k_1{}^3 \operatorname{sech}^3(k_1T)\right)dT^2 - (k_1 \operatorname{sech}(k_1T)) \\ &\times\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) - (k_1 \operatorname{sech}(k_1T)) \\ &\times(d\psi + \cos\theta\, d\varphi)^2. \end{aligned} \tag{52}$$

Thus, metric (52) together with (30) and (47) to (51) constitutes a Bianchi type IX bulk viscous cosmological model in isotropic form in Nordtvedt's general scalar-tensor theory of gravitation.

Some important features of the models

For Bianchi types II and VIII ($\delta = 0, -1$) cosmological models

The spatial volume for model (38) is given by

$$V = (-g)^{\frac{1}{2}} = \left(\sqrt{3}k_1 \operatorname{cosech}(k_1T)\right)^{\frac{3}{2}} f(\theta). \tag{53}$$

The spatial volume for model (45) is given by

$$V = (-g)^{\frac{1}{2}} = \left(\sqrt{\frac{3}{5}}k_1 \operatorname{cosech}(k_1T)\right)^{\frac{3}{2}} f(\theta). \tag{54}$$

The expression of the expansion scalar θ calculated for the flow vector u^i for both models is given by

$$\theta = u^i{}_{,i} = \frac{-3k_1}{2}\coth(k_1T), \tag{55}$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{5k_1{}^2}{8}\tanh^2(k_1T). \tag{56}$$

The deceleration parameter q is given by

$$\begin{aligned} q &= \left(-3\theta^{-2}\right)\left(\theta_{,i}u^i + \frac{1}{3}\theta^2\right) \\ &= -\left[-2\operatorname{cosech}^2(k_1T) + 1\right]. \end{aligned} \tag{57}$$

Hubble's parameter H is given by

$$H = \frac{-k_1}{2}\tanh(k_1T). \tag{58}$$

The average anisotropy parameter is

$$\begin{aligned} A_m &= \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 0, \text{where}\, \Delta H_i \\ &= H_i - H \ (i = 1, 2, 3). \end{aligned} \tag{59}$$

For Bianchi type IX ($\delta = 1$) cosmological model

The spatial volume for model (52) is given by

$$V = (-g)^{\frac{1}{2}} = (k_1 \operatorname{sech}(k_1T))^{\frac{3}{2}} f(\theta). \tag{60}$$

The expression of the expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i{}_{,i} = \frac{-3k_1}{2}\tanh(k_1T), \tag{61}$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{5{k_1}^2}{8}\tanh^2(k_1 T). \tag{62}$$

The deceleration parameter q is given by

$$\begin{aligned} q &= \left(-3\theta^{-2}\right)\left(\theta_{,i}u^i + \frac{1}{3}\theta^2\right) \\ &= -\left[-2\operatorname{cosech}^2(k_1 T) + 1\right]. \end{aligned} \tag{63}$$

Hubble's parameter H is given by

$$H = \frac{-k_1}{2}\tanh(k_1 T). \tag{64}$$

The average anisotropy A_m parameter is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 0,$$

$$\text{where } \Delta H_i = H_i - H \ (i = 1, 2, 3). \tag{65}$$

Conclusions

In this paper, we have presented Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Nordtvedt's [1] general scalar-tensor theory with the help of a special case proposed by Schwinger [2]. It is observed that all the models are isotropic, and in the case of Bianchi type IX, we will get only the bulk viscous cosmological model. The models presented here are free from singularities, and the spatial volume decreases as time T increases, i.e., all the three models are contracting. We observe that as T approaches to infinity, the expansion scalar θ leads to a constant value. This shows that the universe expands homogeneously. We also observe that the shear scalar σ and the Hubble parameter H are constants as T tends to infinity. The energy density, total pressure, string tension density, and coefficient of bulk viscosity diverge with the increase of time.

For these models, the deceleration parameter q tends to −1 for large values of T. Also, in recent observations of type Ia supernovae, Perlmutter et al. [50] and Riess et al. [51-53] proved that the decelerating parameter of the universe is in the range $-1 \le q \le 0$ and the present-day universe is undergoing an accelerated expansion. Therefore, the present models not only represent accelerating universes but also conform to the well-known fact that the scalar field and the bulk viscosity will play a vital role in getting an accelerated universe. Since $A_m = 0$, the models always represent isotropic universes. These exact models are new and more general and represent the present stage of the universe.

Competing interests
Both authors declare that they have no competing interests.

Authors' contributions
All calculations and derivations of the various results and their verifications were carried out by VUMR and DN. Both authors read and approved the final manuscript.

Authors' information
VUMR is working as a professor, and DN is working as a research scholar in the Department of Applied Mathematics, Andhra University, Visakhapatnam, India.

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