

# Accelerating Bianchi type- $VI_0$ bulk viscous cosmological models in Lyra geometry

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**Abstract** The present study deals with spatially homogeneous and totally anisotropic Bianchi type- $VI_0$  bulk viscous cosmological models in Lyra geometry. The Einstein's field equations have been solved exactly by taking the shear ( $\sigma$ ) in the model proportional to expansion scalar ( $\theta$ ) which leads to  $A = B^n$ , where  $A$  and  $B$  are metric functions and  $n$  is a positive constant ( $n > 1$ ). We also adopt a condition  $\zeta\theta = L$  (constant) where  $\zeta$  is the coefficient of bulk viscosity. It has been found that the displacement vector ( $\beta$ ) is a decreasing function of time and it approaches to a small positive value at late time which is supported by recent observations. It is also found that the distance modulus curve of derived model matches with observations perfectly.

**Keywords** Bianchi type- $VI_0$  universe · Accelerated expansion · Bulk viscosity · Lyra geometry

## Introduction

Several modifications of Riemannian geometry have been proposed so far in an attempt to unify gravitation, electromagnetic field and many other effects in the universe. Weyl [1] tries to unify gravitation and electromagnetism in single space–time geometry. But Weyl's theory was not taken seriously because it was based on the non-integrability of length transfer. Later on, Lyra [2] proposed a

further modification of Riemannian geometry and removed non-integrability of length transfer by introducing a gauge function into the structure-less manifold as a result of which a displacement vector arise naturally. In consecutive investigations Sen [3], Sen and Dunn [4] proposed a new scalar–tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry. Halford [5] has pointed out that the constant vector displacement field  $\phi_i$  in Lyra's geometry plays the role of cosmological constant  $\Lambda$  in the normal general relativistic treatment. It is shown by Halford [6] that the scalar–tensor treatment based on Lyra's geometry predicts the same effects within observational limits as the Einstein's theory.

Cosmological observations on expansion history of the universe indicate that current universe is not only expanding but also accelerating. This late time accelerated expansion of the universe has been confirmed by high redshift supernovae experiments (Riess et al. [7], Perlmutter et al. [8], Bennett et al. [9]). Also, observations such as cosmic background radiation [10, 11] and large-scale structure [12] provide an indirect evidence for late time acceleration.

The simplest model of the observed universe is well represented by Friedmann–Robertson–Walker (FRW) models, which are both spatially homogeneous and isotropic. These models in some sense are good global approximation of the present day universe. But on smaller scales, the universe is neither homogeneous nor isotropic. There are theoretical arguments [13, 14] and recent experimental data regarding cosmic background radiation anisotropies which support the existence of an anisotropic phase that approaches an isotropic one [15]. Bianchi types I–IX cosmological models are important in the sense that these are homogeneous and anisotropic, from which the

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process of isotropization of the universe is studied through the passage of time. Bianchi type-VI<sub>0</sub> space-time is of special interest in anisotropic cosmology. Barrow [16] pointed out that Bianchi type-VI<sub>0</sub> models of the universe give a better explanation of some of the cosmological problems like primordial helium abundance and they also isotropize in a special sense.

Astronomical observations of the large-scale distribution of galaxies in the universe show that the distribution of matter can be satisfactorily described by perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of expanding universe. Various authors [14, 17, 18] have shown that bulk viscosity leads to inflationary-like solution and acts like a negative energy field in an expanding universe. At an early stage of the universe, when neutrino decoupling occurs during radiation era and decoupling of radiation with matter takes place during recombination era, the matter behaves like a viscous fluid. The coefficient of viscosity is known to decrease as the universe expands. Gron [19] has reviewed viscous cosmological models and deduced that viscosity plays an important role in the process of isotropization of the universe. Apart from these qualitative discussions, suitable viscous fluid cosmological models have been discussed in different contexts by several authors such as Pavon et al. [20], Burd and Coley [21], Fabris et al. [22], Johri and Sudharsan [23], Murphy [24], Heller and Klimek [25] and Bali et al. [26, 27].

Motivated by the situation discussed above, in this paper we have obtained accelerating Bianchi type-VI<sub>0</sub> bulk viscous cosmological models with a time-dependent displacement field within the framework of Lyra's geometry. To get the deterministic model of the universe, we have assumed two conditions: (1)  $\zeta\theta = L$  (constant) and (2)  $\sigma \propto \theta$ . The physical and geometrical aspects of the models are also discussed.

### Metric and field equations

We consider the spatially homogeneous and anisotropic Bianchi type-VI<sub>0</sub> space-time in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{-2x} dz^2 \quad (1)$$

where the metric potentials  $A$ ,  $B$  and  $C$  are functions of the cosmic time  $t$ .

Einstein's modified field equation in normal gauge for Lyra's manifold obtained by Sen [3] is given by (in geometrized unit where  $8\pi G = 1$ ,  $c = 1$ )

$$R_i^j - \frac{1}{2} g_i^j R + \frac{3}{2} \phi_i \phi^j - \frac{3}{4} \phi_k \phi^k g_i^j = -T_i^j, \quad (2)$$

where  $\phi_i$  is the displacement vector defined as  $\phi_i = (0, 0, 0, \beta(t))$  and other symbols have their usual

meaning as in Riemannian geometry. We assume the cosmic matter consisting of bulk viscous fluid given by the energy momentum tensor

$$T_i^j = (p + \rho) v_i v^j + p g_i^j - \zeta \theta (g_i^j + v_i v^j), \quad (3)$$

where  $v_i = (0, 0, 0, -1)$ ,  $v^i v_i = -1$ ,  $v_4 = -1$ ,  $v^4 = 1$ ,  $p$  is the isotropic pressure,  $\rho$  the matter density,  $v^i$  the fluid flow vector and  $\beta$  the gauge function.

The equation of state for the fluid is taken as

$$p = \omega \rho, \quad (4)$$

where  $0 \leq \omega \leq 1$  is a constant. For the metric (1), the field Eqs. (2) together with (3) lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -(p - \zeta\theta) - \frac{3}{4} \beta^2, \quad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -(p - \zeta\theta) - \frac{3}{4} \beta^2, \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(p - \zeta\theta) - \frac{3}{4} \beta^2, \quad (7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \rho + \frac{3}{4} \beta^2, \quad (8)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \quad (9)$$

Here and in what follows, an overhead dot denotes ordinary differentiation with respect to  $t$ . The energy conservation equation  $T_{ij}^j = 0$  leads to,

$$\dot{\rho} + (\rho + p - \zeta\theta) \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \quad (10)$$

and conservation of L.H.S of (2) leads to

$$\left( R_i^j - \frac{1}{2} g_i^j R \right)_{;j} + \frac{3}{2} (\phi_i \phi^j)_{;j} - \frac{3}{4} (\phi_k \phi^k g_i^j)_{;j} = 0. \quad (11)$$

Equation (11) leads to

$$\begin{aligned} \frac{3}{2} \phi_i \left[ \frac{\partial \phi^j}{\partial x^j} + \phi^l \Gamma_{lj}^j \right] + \frac{3}{2} \phi^j \left[ \frac{\partial \phi_i}{\partial x^j} - \phi_l \Gamma_{ij}^l \right] \\ - \frac{3}{4} g_i^j \phi_k \left[ \frac{\partial \phi^k}{\partial x^j} + \phi^l \Gamma_{lj}^k \right] - \frac{3}{4} g_i^j \phi^k \left[ \frac{\partial \phi_k}{\partial x^j} - \phi_l \Gamma_{kj}^l \right] = 0. \end{aligned} \quad (12)$$

Equation (12) is automatically satisfied for  $i = 1, 2, 3$ .

For  $i = 4$ , Eq. (12) leads to

$$\begin{aligned} \frac{3}{2} \beta \left[ \frac{\partial}{\partial x^4} (g^{44} \phi_4) + \phi^4 \Gamma_{44}^4 \right] + \frac{3}{2} g^{44} \phi_4 \left[ \frac{\partial \phi_4}{\partial t} - \phi_4 \Gamma_{44}^4 \right] \\ - \frac{3}{4} g^4_4 \phi_4 \left[ \frac{\partial \phi_4}{\partial x^4} + \phi^4 \Gamma_{44}^4 \right] - \frac{3}{4} g^4_4 g^{44} \phi_4 \left[ \frac{\partial \phi_4}{\partial t} - \phi^4 \Gamma_{44}^4 \right] = 0, \end{aligned} \quad (13)$$



which leads to

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \quad (14)$$

Integrating Eq. (9), we obtain

$$B = IC. \quad (15)$$

Here,  $I$  is a constant of integration which can be taken as unity without any loss of generality so that

$$B = C. \quad (16)$$

The average scale factor  $a$  for the metric (1) is defined by

$$a = (AB^2)^{\frac{1}{3}}. \quad (17)$$

The spatial volume  $V$  is given by

$$V = a^3 = AB^2. \quad (18)$$

The generalized mean Hubble parameter  $H$  is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3) \quad (19)$$

where  $H_1 = \frac{\dot{A}}{A}$ ,  $H_2 = H_3 = \frac{\dot{B}}{B}$  are the directional Hubble parameters in the directions of  $x$ ,  $y$  and  $z$  axes, respectively.

The expansion scalar  $\theta$  and shear scalar  $\sigma$  are given by

$$\theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}, \quad (20)$$

$$\sigma^2 = \frac{1}{2}\left[\frac{\dot{A}^2}{A^2} + 2\frac{\dot{B}^2}{B^2}\right] - \frac{1}{6}\theta^2. \quad (21)$$

The mean anisotropy parameter  $A_m$  is given by

$$A_m = \frac{1}{3}\sum_{i=1}^3\left(\frac{\Delta H_i}{H}\right)^2, \quad (22)$$

where  $\Delta H_i = H_i - H$  ( $i = 1, 2, 3$ ). An important observational quantity in cosmology is the deceleration parameter  $q$  which is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right). \quad (23)$$

### Solutions of the Field Equations

To solve the field equations completely, we constrain, the system of equations with proportionality relation of shear scalar ( $\sigma$ ) and expansion scalar ( $\theta$ ) [28]. This condition leads to the following relation between metric potentials

$$A = B^n, \quad (24)$$

where  $n$  is a positive constant ( $n > 1$ ).

The reasons for consideration of Eq. (24) can be explained by the work of Thorne [29]. The observations of the velocity–redshift relation for extragalactic sources suggest that the

Hubble expansion of the universe is isotropic today within approximately 30 % [30, 31]. More precisely, the redshift studies place the limit  $\sigma/H \leq 0.30$  where  $\sigma$  is shear and  $H$  the Hubble constant. Collins et al. [32] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous hypersurface satisfies the condition  $\sigma/\theta$  as constant which leads to the assumption  $A = B^n$ .

We also assume that coefficient of bulk viscosity  $\zeta$  is inversely proportional to the expansion  $\theta$ , i.e.,

$$\zeta\theta = L \text{ (constant)}. \quad (25)$$

The motive behind assuming this condition is explained in Ref. [33–35]. Using Eq. (24) in Eqs. (5) and (6), we obtain

$$2\ddot{B} + 2\alpha\dot{B}^2B^{-1} = 2\gamma B^{1-2n} \quad (26)$$

where  $\alpha = n + 1$ ,  $\gamma = \frac{2}{n-1}$ . Putting  $\dot{B} = \eta(B)$ ,  $\ddot{B} = \eta\frac{d\eta}{dB}$  in Eq. (26) and then integrating, we obtain

$$dt = \left[\frac{\gamma}{2}B^{4-2\alpha} + k_1B^{-2\alpha}\right]^{\frac{1}{2}}dB, \quad (27)$$

where  $k_1$  is a constant of integration.

If we put  $n = 1$ , in Eq. (26) (since  $\gamma = \frac{2}{n-1}$ ) then there arises a singularity. So, we cannot consider  $n = 1$  in the present model to explain the feature of the universe.

With the help of Eq. (27), the line element (1) reduces to

$$ds^2 = -\frac{dB^2}{B^{-2\alpha}\left[\frac{\gamma}{2}B^4 + k_1\right]} + B^{2n}dx^2 + B^2(e^{2x}dy^2 + e^{-2x}dz^2). \quad (28)$$

After using the suitable transformation of coordinates  $B = T$ , the above model (28) transforms to

$$ds^2 = -\frac{dT^2}{T^{-2(n+1)}\left[\left(\frac{1}{n-1}\right)T^4 + k_1\right]} + T^{2n}dx^2 + T^2(e^{2x}dy^2 + e^{-2x}dz^2). \quad (29)$$

### Some physical and geometrical features

The displacement vector ( $\beta$ ), energy density ( $\rho$ ) and pressure ( $p$ ) for the model (29) are found to be

$$\beta = k_2\left[\frac{1}{n-1}T^{-4n-2} + k_1T^{-4n-6}\right]^{\frac{1}{2}}, \quad (30)$$

$$\rho = \left(\frac{n+2}{n-1}\right)T^{-2n} + k_1(2n+1)T^{-2n-4} - \frac{3k_2^2}{4(n-1)}T^{-4n-2} - \frac{3}{4}k_1k_2^2T^{-4n-6}, \quad (31)$$

$$p = \omega\left[\left(\frac{n+2}{n-1}\right)T^{-2n} + k_1(2n+1)T^{-2n-4} - \frac{3k_2^2}{4(n-1)}T^{-4n-2} - \frac{3}{4}k_1k_2^2T^{-4n-6}\right], \quad (32)$$

where  $k_2$  is a constant of integration. It is observed that the displacement vector ( $\beta$ ) was large in the beginning, but decreases fast with the evolution of the model analogous to cosmological constant ( $\Lambda$ ). The energy density ( $\rho$ ) and pressure ( $p$ ) are also decreasing function of time  $T$ . The energy density  $\rho \rightarrow \infty$  when  $T \rightarrow 0$  and  $\rho \rightarrow 0$  when  $T \rightarrow \infty$ .

The mean Hubble parameter ( $H$ ), expansion scalar ( $\theta$ ), shear scalar ( $\sigma$ ) are given by

$$H = \left(\frac{n+2}{3}\right) \left[ \left(\frac{1}{n-1}\right) T^{-2n} + k_1 T^{-2n-4} \right]^{\frac{1}{2}}, \quad (33)$$

$$\theta = (n+2) \left[ \left(\frac{1}{n-1}\right) T^{-2n} + k_1 T^{-2n-4} \right]^{\frac{1}{2}}, \quad (34)$$

$$\sigma^2 = \frac{1}{3} (n-1)^2 \left[ \left(\frac{1}{n-1}\right) T^{-2n} + k_1 T^{-2n-4} \right]. \quad (35)$$

The Hubble parameter ( $H$ ), expansion scalar ( $\theta$ ) and shear scalar ( $\sigma$ ) are tend to zero when  $T \rightarrow \infty$  and they become infinite when  $T \rightarrow 0$ .

Average scale factor ( $a$ ) and spatial volume ( $V$ ) are given by

$$a = \left[ T^{-(2n+3)} \sqrt{\left(\frac{1}{n-1}\right) T^4 + k_1} \right]^{\frac{-1}{3}}, \quad (36)$$

$$V = T^{n+2}. \quad (37)$$

The spatial volume  $V \rightarrow 0$  when  $T \rightarrow 0$  and  $V \rightarrow \infty$  when  $T \rightarrow \infty$ .

Also,

$$\lim_{T \rightarrow \infty} \frac{\sigma^2}{\theta^2} = \frac{1}{3} \left( \frac{n-1}{n+2} \right)^2 \neq 0. \quad (38)$$

Therefore, the model does not isotropic for large values of  $T$ .

Coefficient of bulk viscosity ( $\zeta$ ) and mean anisotropy parameter ( $A_m$ ) are found to be

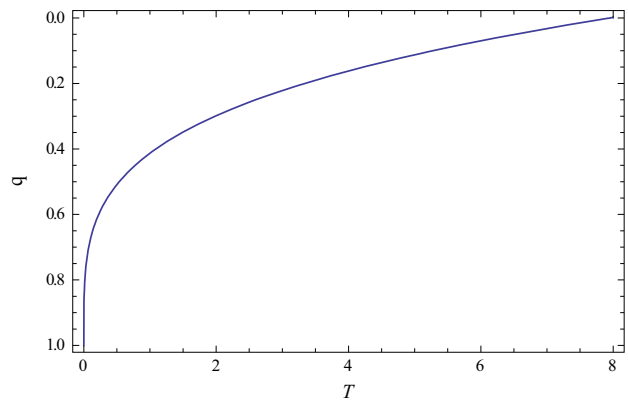
$$\zeta = \left( \frac{L}{n+2} \right) \left[ \frac{1}{n-1} T^{-2n} + k_1 T^{-2n-4} \right]^{\frac{-1}{2}}. \quad (39)$$

$$A_m = \frac{1}{3} \left[ \left( 1 - \frac{3n}{n+2} \right)^2 + 2 \left( 1 - \frac{3}{n+2} \right)^2 \right]. \quad (40)$$

Therefore, the model has constant anisotropy parameter throughout the evolution of the universe except  $n = 1$ .

The deceleration parameter  $q$  is found to be

$$q = -1 + \left( \frac{3}{n+2} \right) \left( \frac{1}{n-1} T^4 + k_1 \right)^{-1} \times \left( (n+2) T^{n+1} \sqrt{\frac{1}{n-1} T^4 + k_1} - \frac{\frac{2}{n-1} T^{n+5}}{\sqrt{\frac{1}{n-1} T^4 + k_1}} \right). \quad (41)$$



**Fig. 1** The variation of  $q$  vs.  $T$  for the model (29) with parameters  $n = 1.25$ ,  $k_1 = 0.001$

Recent observations (Perlmutter et al. [8, 36, 37]; Riess et al. [7, 38]; Tonry et al. [39]; John [40]; Knop et al. [41]) reveal that the value of deceleration parameter  $q$  is confined in the range  $-1 \leq q < 0$  and the present day universe is undergoing an accelerated expansion. We have seen in Fig. 1 that the value of  $q$  lies in the range  $-1 \leq q < 0$  which is consistent with recent observations. The negative value of deceleration parameter implies that our proposed model (29) of the universe is accelerating.

### Distance modulus curve

The distance modulus is given by

$$\mu = 5 \log d_L + 25, \quad (42)$$

where the luminosity distance  $d_L$  is defined as

$$d_L = r_1 (1+z) a_0. \quad (43)$$

Here,  $z$  and  $a_0$  represent redshift parameter and present scale factor, respectively. For the determination  $r_1$ , we assume that photon is emitted by a source with co-ordinate  $(r, t)$  and received at a time  $t_0$  by an observer located at  $r = 0$ . Then, we determine  $r_1$  from

$$r_1 = \int_t^{t_0} \frac{dt}{a}. \quad (44)$$

Using Eqs. (27) and (36) (considering  $k_1 = 0$ ) and solving Eqs. (42)–(44), we can obtain the expression for distance modulus ( $\mu$ ) in terms of redshift parameter ( $z$ ) as

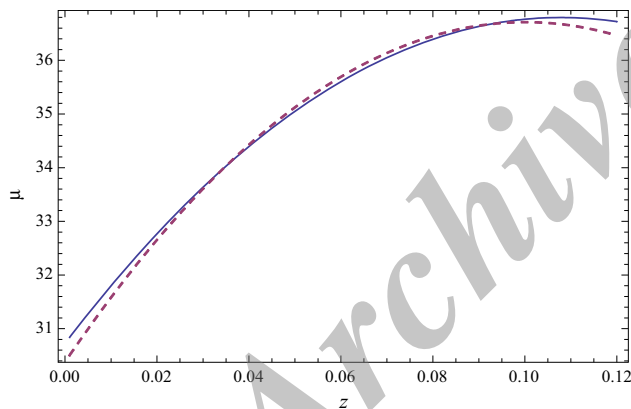
$$\mu = 5 \log \left[ \frac{3}{\sqrt{n-1}} \left( \frac{2n+1}{3H_0} \right)^{\frac{n+2}{3}} (1+z) \left( 1 - (1+z)^{\frac{1-n}{2n+1}} \right) \right] + 25. \quad (45)$$

In this paper, we have analyzed 18 data set out of recently released 38 data set of SN Ia in the range



**Table 1** Comparison of the results of present model with the observational data

Redshift ( $z$ )	Supernovae Ia ( $\mu$ )	Our model ( $\mu$ )
0.0132	$30.35^{+0.26}_{-0.26}$	30.0531
0.0133	$32.54^{+0.22}_{-0.22}$	32.1711
0.0134	$33.10^{+0.22}_{-0.22}$	32.1875
0.0264	$33.93^{+0.2}_{-0.2}$	33.6684
0.0274	$34.33^{+0.1}_{-0.1}$	33.7498
0.0284	$34.61^{+0.23}_{-0.23}$	33.8283
0.0295	$34.82^{+0.17}_{-0.17}$	33.9115
0.0398	$35.18^{+0.17}_{-0.17}$	34.5684
0.0399	$35.91^{+0.17}_{-0.17}$	34.5739
0.0409	$35.84^{+0.16}_{-0.16}$	34.6283
0.0519	$36.09^{+0.16}_{-0.16}$	35.1524
0.0619	$36.35^{+0.17}_{-0.17}$	35.5412
0.0721	$36.33^{+0.17}_{-0.17}$	35.8787
0.0813	$36.75^{+0.16}_{-0.16}$	36.1451
0.0854	$36.71^{+0.16}_{-0.16}$	36.2544
0.0951	$36.86^{+0.16}_{-0.16}$	36.4938
0.0967	$36.76^{+0.16}_{-0.16}$	36.531
0.0979	$37.5^{+0.19}_{-0.19}$	37.54

**Fig. 2** Plot of distance modulus ( $\mu$ ) vs. redshift ( $z$ ) for Supernovae Ia data (solid line) from Amanullah et al. [42] and for our model (dotted line)

$0.0015 \leq z < 0.12$  as reported by Amanullah et al. [42] which are shown in Table 1. The comparison between calculated  $\mu(z)$  and observed  $\mu(z)$  SN Ia data as reported by Amanullah et al. [42] can be seen in Fig. 2. We observed that derived model is fit well with SN Ia observations.

## Conclusion

In this paper, we have obtained accelerating Bianchi type- $VI_0$  bulk viscous cosmological models in the framework of

Lyra's geometry for time-dependent displacement field. To get the deterministic model of the universe, we have assumed two conditions: (1)  $\zeta\theta = L$  (constant) and (2)  $\sigma \propto \theta$ , where  $\zeta$  is the coefficient of bulk viscosity,  $\theta$  the expansion in the model and  $\sigma$  the coefficient of shear viscosity. It is observed that the spatial volume  $V \rightarrow 0$  as  $T \rightarrow 0$  and  $V \rightarrow \infty$  as  $T \rightarrow \infty$ . This shows that the universe starts expanding with zero volume at  $T = 0$  and expands with cosmic time  $T$ . When  $T \rightarrow 0$ , the expansion scalar  $\theta \rightarrow \infty$  and energy density  $\rho \rightarrow \infty$ . Again, when  $T \rightarrow \infty$  then  $\theta \rightarrow 0$  and  $\rho \rightarrow 0$ . Therefore, the model represents expanding, shearing and non-rotating universe with big bang start. Since  $\lim_{T \rightarrow \infty} \frac{\sigma^2}{\theta^2} \neq 0$ , therefore, the model is not isotropic for large values of  $T$ . At  $T = 0$ , Hubble parameter  $H$ , expansion scalar  $\theta$ , shear scalar  $\sigma$ , pressure  $p$  and energy density  $\rho$  are infinite and at late times they become zero. The role of bulk viscosity is to retard expansion in the model. We can see from the above discussion that the bulk viscosity plays a significant role in the evolution of the universe. We also observe that the displacement vector  $\beta$  is large in the beginning of the universe and reduces fast during its evolution so that its behavior coincides with the nature of the cosmological constant  $\Lambda$ . The distance modulus of the derived model fits well with the observational  $\mu(z)$  values (see Fig. 2 and Table 1).

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## References

- Weyl, H.: Vorlesungen uker Allgemeine relativitotstheorie. Sber. Preuss. Akad. der Wiss. 465 Sitzunbgsberichte, zu Berlin (1918)
- Lyra, G.: Über eine Modifikation der Riemannschen Geometrie. Math. Z. **54**, 52–64 (1951)
- Sen, D.K.: A static cosmological model. Z. Phys. **149**, 311–323 (1957)
- Sen, D.K., Dunn, K.A.: A scalar-tensor theory of gravitation in a modified Riemannian manifold. J. Math. Phys. **12**, 578–586 (1971)
- Halford, W.D.: Cosmological theory based on Lyra's geometry. Aust. J. Phys. **23**, 863–869 (1970)
- Halford, W.D.: Scalar-tensor theory of gravitation in a Lyra manifold. J. Math. Phys. **13**, 1699–1703 (1972)
- Riess, A.G., et al.: Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron. J. **116**, 1009–1038 (1998)
- Perlmutter, S., et al.: Measurements of  $\Omega$  and  $\Lambda$  from 42 high-redshift supernovae. Astrophys. J. **517**, 565–585 (1999)
- Bennett, C.L., et al.: First-year Wilkinson microwave anisotropy probe (WMAP) observations: preliminary maps and basic results. Astrophys. J. Suppl. Ser. **148**, 1–27 (2003)

10. Spergel, D.N., et al.: First-year Wilkinson microwave anisotropy probe (WMAP) observations: determination of cosmological parameters. *Astrophys. J. Suppl. Ser.* **148**, 175–194 (2003)
11. Oli, S.: Interacting and non-interacting two fluid cosmological models in a bianchi type-VI<sub>0</sub> space times. *Ind. J. Phys.* **86**, 755–761 (2012)
12. Tegmark, M., et al.: Cosmological parameters from SDSS and WMAP. *Phys. Rev. D* **69**, 103501–103526 (2004)
13. Chimento, L.P.: Extended tachyon field, chaplygin gas and solvable k-essence cosmologies. *Phys. Rev. D* **69**, 123517–10 (2004)
14. Misner, C.W.: The isotropy of the universe. *Astrophys. J.* **151**, 431–457 (1968)
15. Land, K., Magueijo, J.: Examination of evidence for a preferred axis in the cosmic radiation anisotropy. *Phys. Rev. Lett.* **95**, 071301–071304 (2005)
16. Barrow, J.D.: Helium formation in cosmologies with anisotropic curvature. *Mon. Not. R. Astr. Soc.* **211**, 221–227 (1984)
17. Padmanabhan, T., Chitre, S.M.: Viscous universes. *Phys. Lett. A* **120**, 433–436 (1987)
18. Johri, V.B., Sudarshan, R.: FRW cosmological models with bulk viscosity and entropy production. *Proceedings of International Conference on Mathematical Modeling in Science and Technology*. vol. 2, Tata McGraw-Hill (1988)
19. Gron, O.: Viscous inflationary universe models. *Astrophys. Space Sci.* **173**, 191–225 (1990)
20. Pavon, D., Bafaluy, J., Jou, D.: Causal Friedmann–Robertson–Walker cosmology. *Class. Quantum Grav.* **8**, 347–360 (1991)
21. Burd, A., Coley, A.: Viscous fluid cosmology. *Class. Quantum Grav.* **11**, 83–105 (1994)
22. Fabris, J.C., Goncalves, S.V.B., Ribeiro, R.: Bulk viscosity driving acceleration of the universe. *Gen. Relativ. Gravit.* **38**, 495–506 (2006)
23. Johri, V.B., Sudharsan, R.: Friedmann universes with bulk viscosity. *Phys. Lett. A* **132**, 316–320 (1988)
24. Murphy, G.L.: Big-bang model without singularities. *Phys. Rev. D* **8**, 4231–4233 (1973)
25. Heller, M., Klimek, Z.: Viscous universes without initial singularity. *Astrophys. Space Sci.* **33**, L37–L39 (1975)
26. Bali, R., Pradhan, A., Amirhashchi, H.: Bianchi type-VI<sub>0</sub> magnetized barotropic bulk viscous fluid massive string universe in general relativity. *Int. J. Theor. Phys.* **47**, 2594–2604 (2008)
27. Bali, R., Banerjee, R., Banerjee, S.K.: Bianchi type-VI<sub>0</sub> magnetized bulk viscous massive string cosmological model in general relativity. *Astrophys. Space Sci.* **317**, 21–26 (2008)
28. Tiwari, R.K., Dwivedi, U.: Kantowski-Sachs cosmological models with time-varying  $G$  and  $\Lambda$ . *FIZIKA B (Zagreb)*. **19**, 1–8 (2010)
29. Thorne, K.S.: Primordial element formation, primordial magnetic fields and the isotropy of the universe. *Astrophys. J.* **148**, 51–68 (1967)
30. Kantowski, R., Sachs, R.K.: Some spatially homogeneous anisotropic relativistic cosmological models. *J. Math. Phys.* **7**, 443–446 (1966)
31. Kristian, J., Sachs, R.K.: Observations in cosmology. *Astrophys. J.* **143**, 379–399 (1966)
32. Collins, C.B., Glass, E.N., Wilkinson, D.A.: Exact spatially homogeneous cosmologies. *Gen. Rel. Grav.* **12**, 805–823 (1980)
33. Bali, R., Pradhan, A.: Bianchi type-III string cosmological models with time dependent bulk viscosity. *Chin. Phys. Lett.* **24**, 585–588 (2007)
34. Saha, B.: Bianchi type-I universe with viscous fluid. *Mod. Phys. Lett. A* **20**, 2127–2143 (2005)
35. Yadav, V.K., Yadav, L.: Some bulk viscous magnetized LRS bianchi type-I string cosmological models in Lyra's geometry. *Rom. J. Phys.* **58**, 64–74 (2013)
36. Perlmutter, S., et al.: Discovery of a supernova explosion at half the age of the universe. *Nature* **391**, 51–54 (1998)
37. Perlmutter, S., et al.: Measurements of the cosmological parameters  $\Omega$  and  $\Lambda$  from the first seven supernova at  $z \geq 0.35$ . *Astrophys. J.* **483**, 565–581 (1997)
38. Riess, A.G., et al.: Type Ia supernova discoveries at  $z > 1$  from the Hubble space telescope: evidence for past deceleration and constraints on dark energy evolution. *Astrophys. J.* **607**, 665–687 (2004)
39. Tonry, J.L., et al.: Cosmological results from high- $z$  supernovae. *Astrophys. J.* **594**, 1–24 (2003)
40. John, M.V.: Cosmographic evaluation of the deceleration parameter using type Ia supernova data. *Astrophys. J.* **614**, 1–5 (2004)
41. Knop, R.A., et al.: New constraints on  $\Omega_M$ ,  $\Omega_\Lambda$  and  $\omega$  from an independent set of 11 high-redshift supernovae observed with the Hubble space telescope. *Astrophys. J.* **598**, 102–137 (2003)
42. Amanullah, R., et al.: Spectra and Hubble space telescope light curves of six type Ia supernovae at  $0.511 < z < 1.12$  and the union 2 compilation. *Astrophys. J.* **716**, 712–738 (2010)

