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Effect of Through Stationary Edge and Center Cracks on Static Buckling Strength of Thin Plates under Uniform Axial Compression

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ABSTRACT
Thin plate structures are more widely used in many engineering applications as one of the structural members. Generally, buckling strength of thin shell structures is the ultimate load carrying capacity of these structures. The presence of cracks in a thin shell structure can considerably affect its load carrying capacity. Hence, in this work, static buckling strength of a thin square plate with a centre or edge crack under axial compression has been studied using general purpose Finite Element Analysis software ANSYS. Sensitivity of static buckling load of a plate with a centre or a edge crack for crack length variation and its vertical and horizontal orientations have been investigated. Eigen buckling analysis is used to determine the static buckling strength of perfect and cracked thin plates. First, bifurcation buckling loads of a perfect thin plate with its mode shapes from FE eigen buckling analysis are compared with analytical solution for validating the FE models. From the analysis of the cracked thin plates, it is found that vertical cracks are more dominant than horizontal cracks in reducing buckling strength of the thin plates. Further, it is also found that as the crack length increases, buckling strength decreases.

Keywords: Thin plate structures; Buckling strength; Cracks

1 INTRODUCTION

THIN plate structures are one of the most widely used shell structures in civil, mechanical, aerospace, and marine engineering fields. Generally, thin shell structures mainly fail by buckling. Hence, buckling load of the structure is the ultimate load carrying capacity of the structure. Like other types of structures, they are susceptible to various types of defects and damages such as geometrical imperfections, cracking, corrosion, chemical attack, and time-dependant material degradation, which may impair their structural integrity and affect their service life by reducing its buckling strength. In case of aged steel structures, in addition to corrosion, fatigue cracking is another important factor of age related structural degradation, which has been primary source of costly repair work of aged steel structures. Cracking damage has been found in welded joints and local areas of stress concentrations such as at the weld intersections of longitudinals, frames and griders. To assess the reliability of the aged structures, prediction of ultimate strength of structural members taking into account the effect of structural damages are required. Hence, in the present study, FE models are generated for predicting the ultimate strength of the thin steel plate under axial compression, where crack damage is considered as an influencing parameter. Thin plates in between stiffeners of a stiffened plate are integral part of ship structures, offshore oil platforms, lock gates and floating docks as shown in Fig. 1 [1]. These stiffened plates are designed to withstand the axial compression due to sagging and hogging moments. The analysis of typical stiffened plate structure in a ship can be performed at grillage level, stiffened panel level, and bare plate element level between longitudinal and transverse stiffeners.

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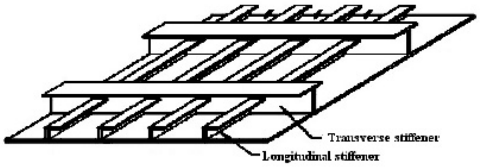

Fig. 1
Typical ship deck in between the bulk heads and deep longitudinal.

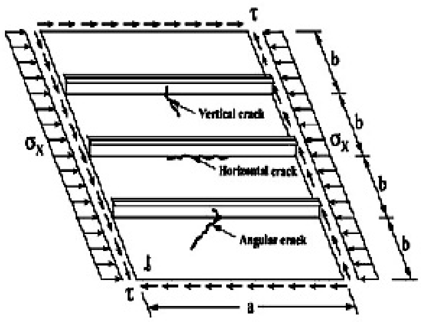

Fig. 2
Stiffened steel panel with three types of crack orientations and under axial load.

Local buckling and collapse of plating between the stiffeners is a basic failure mode and is important to evaluate the exact strength for safe design. The bending rigidities of the boundary edges of plates in between transverse frames and between longitudinal stiffeners are quite high compared to that of the plate itself. The rotational restraints along the plate edges can be considered to be small for plates subjected to axial compression. Hence, the plate elements in the present study are considered as simply supported along all the edges. Different types of crack orientations on stiffened steel plate are shown in Fig. 2.

2 LITERATURE REVIEW

The analytical solution to find the bifurcation buckling load for a perfect plate is given in Timoshenko and Gere [2]. The general theory of buckling incorporating imperfections developed by Koiter is given in Brush and Almroth [3]. Bushnell [4] clearly mentions about different buckling behaviours and different types of buckling and different buckling failures of different thin walled structures and it also covers important literature review in this field up to 1985. Estekanchi and Vafai [5] studied about the static buckling strength of cylindrical shells with through cracks in different angle of orientation under axial compressive and tensile loadings using software called CRUX and it is concluded that the cylindrical shells under compression are relatively sensitive to axial cracks. In an article by Satish Kumar and Paik [6], they have discussed about effect of crack size and orientation of crack on buckling strength of thin plate (10mm thickness) under axial compression and also under edge shear loading using linear buckling analysis based on explicitly developed Discrete Kirchhoff-Mindlin Triangular (DKMT) element and they finally concluded that buckling strength of the plate decreases with increase in size and vertical cracks (perpendicular to load direction) are dominant in decreasing the buckling strength of thin plate then horizontal cracks.

Paik et al. [7] have conducted an experimental and numerical study on the ultimate strength of cracked steel plate elements subjected to axial compression or tensile loads. The ultimate strength reduction characteristics of plate elements due to cracking damage were investigated with varying size and location of the cracking damage, both experimentally and numerically. A series of nonlinear finite element analyses for cracked plate elements are performed. Based on the experimental and numerical results obtained from the study, theoretical models for predicting the ultimate strength of cracked plate elements under axial compression or tension were developed. The results of the experiments and numerical computations obtained were documented. Kacianauskas et al. [8] in their work applied adaptive FE mesh technique for crack and crack tip modelling to analyse of elastic-plastic problem of single edge notch bend (SENB) specimen, and concluded that these proposed h-adaptive FE strategy is applicable for solving material nonlinear problems of SENB specimen.

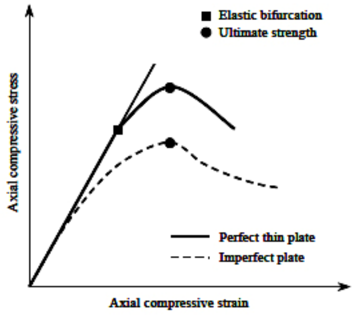

Fig. 3
Load vs. edge displacement curve of perfect and imperfect thin plates.

From the results of the nonlinear solution it was shown that stress and strain fields are obtained with the difference up to 5 % using the adaptive FE meshes. Hence in these works, four types of cracks are considered a) vertical edge crack b) horizontal edge crack c) vertical center crack and d) horizontal center crack. Effect of these orientation of cracks and its size on static buckling strength of thin plate under uniform axial compression are studied using Linear FE analysis based on implicit Shell93 element which is suitable for the plate (5mm) taken for the study.

3 ANALYTICAL SOLUTION

The analytical solution for buckling strength of perfect thin plate (as explained in Fig. 3) was derived by Timoshenko and Gere [1] as explained below. The equilibrium path of a rectangular flat plate is divided into the primary and secondary equilibrium path. The governing non-linear equations for both primary and secondary paths are given by Koiter:

$$\begin{aligned}
&\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
&\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\
&D\nabla^4 w-\left(N_x\frac{\partial^2 w}{\partial x^2}+2N_x\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}\right)=p
\end{aligned}\tag{1}$$

where

N_x=Axial force per unit length in *X*-direction
N_y = Axial force per unit length in *Y*-direction
N_{xy} = Shear stress in *XY*-plane
w = Lateral deflection

The equation (1) is known as Von Karman plate equation. The primary equilibrium path is also known as the pre-buckling path for a perfect plate. It can be seen that it is linear in nature. Hence, omitting the quadratic and cubic terms in the Von Karman plate equation, the linear equilibrium equation for the primary path is given by

$$\begin{aligned}
&\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
&\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\
&D\nabla^4 w=p
\end{aligned}\tag{2}$$

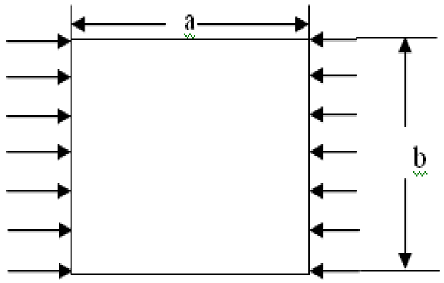

Fig. 4
Rectangular Plate subjected to axial compression with simply supported edges.

The secondary equilibrium path is also called as the post-buckling path. It can be seen that it is non-linear in nature. The governing non-linear differential equation in the post-buckling range is given as

$$\begin{aligned} \nabla^4 f &= Eh\left(\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)\right) \\ D\nabla^4 w &= p + \left(\frac{\partial^2 f}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) - 2\left(\frac{\partial^2 f}{\partial x \partial y}\right)\left(\frac{\partial^2 w}{\partial x \partial y}\right) + \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) \end{aligned} \tag{3}$$

where $f = f(x, y)$ denotes the stress-function and

$$\frac{\partial^2 f}{\partial y^2} = N_x, \qquad \frac{\partial^2 f}{\partial x^2} = N_y, \qquad \frac{\partial^2 f}{\partial x \partial y} = N_{xy}$$

The equation (2) is called as the compatibility and equilibrium equation for plates. The buckling load of the perfect flat plate shown in Fig. 4 is given by the formula

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2 \tag{4}$$

where

a = Length of the plate
b = Width of the plate
m = No. of half-lobes formed in x- direction
n = No. of half-lobes formed in y-direction
$D = Eh^3/12\,(1- v^2)$ = flexural rigidity per unit width of the plate
E = Young's modulus of the plate material
h = Thickness of the plate
v = Poisson's ratio

4 LINEAR EIGEN BUCKLING ANALYSIS

In order to obtain the buckling mode shapes to model the geometric imperfections, a linear eigen buckling analysis is first carried out. The basic form of the eigen buckling analysis [9] is given by

$$[K]\{\phi_i\} = \lambda_i [S]\{\phi_i\} \tag{5}$$

where

$[K]$ = Structural stiffness matrix

$\{\phi_i\}$ = Eigen vector
λ_i = Eigen value
$\{S\}$ = Stress stiffness matrix

In eigen buckling analysis, imperfections and non-linearity cannot be included to the structure. Sub-space iteration scheme can be used to extract the load factor or eigen value.

5 PROPERTIES OF THE PLATE CONSIDERED FOR ANALYSIS

5.1 Material properties [10]

Material used: Carbon steel
Young's modulus: 205.8 Gpa
Poisons ratio: 0.3

5.2 Material dimensions [10]
Length: 1m
Breadth: 1m
Thickness: 5mm

6 FINITE ELEMENT MODELING

6.1 FE modelling of perfect plate

FE Analysis were carried out using the general purpose FE package ANSYS 11.0 [11] on a flat plate with simply supported boundary conditions on all the four edges. The axial force is applied on one edge of the plate and the other edge is restrained to move along the load direction. Shell93 (8 node) quadratic element of ANSYS11.0 is used for modeling. The FE model of perfect thin plate is shown in Fig. 5.

6.2 Validation of the result

6.2.1 Mesh Convergence Study

Since the plate taken for analysis is a square plate, equal numbers of elements are taken in both directions. To assess the convergence of the solution, four FE models with 10×10, 20×20, 30×30, 40×40 elements were analyzed. The results are shown in the Fig. 6. For accurate result, the model with 40×40 elements was adopted for all analysis. Buckling loads are obtained for various values of *m* and *n* using analytical Eq. (4) and are compared with first five FE eigen buckling loads shown in Table 1. The deviations in buckling loads are found to be within 0.5%. Fig .7 shows the mode shapes obtained from FE analysis.

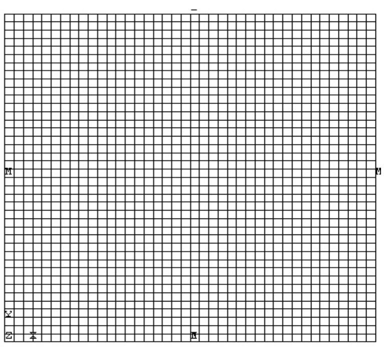

Fig. 5
FE model of thin plate.

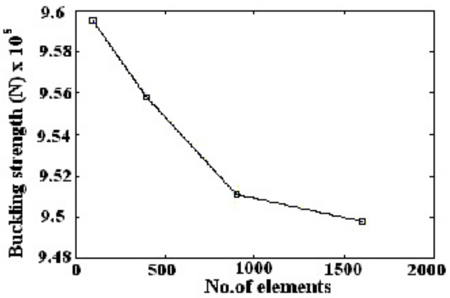

Fig. 6
Mesh convergence.

Table 1
Comparison of FE eigen buckling strength with analytical solution

Mode	*m*	*n*	Analytical solution (N)	FE solution (N)	% Deviation
1	1	1	949000	944679.7	0.455
2	2	1	148281	147810.1	0.318
3	3	1	263311	263007.8	0.229
4	2	2	379600	377706.9	0.499
5	4	1	428533	427610.2	0.215

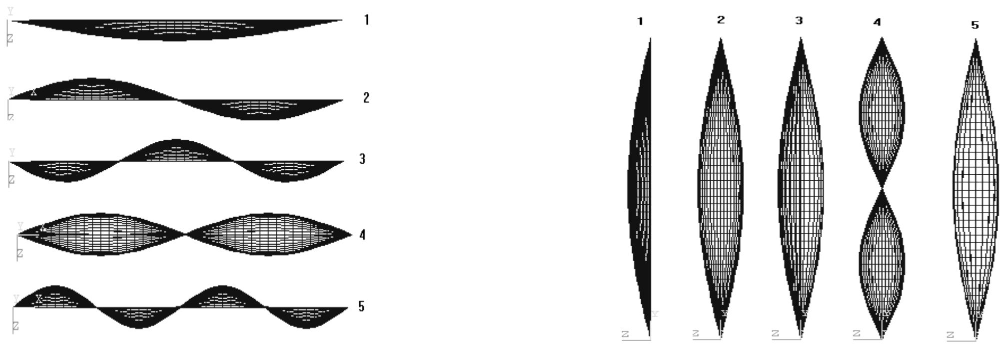

Fig. 7
Eigen mode shapes of perfect plate.

6.3 Modeling of cracked thin plate

Shell93 elements are used for modeling the plate and the crack is modeled as rectangular opening having one end with triangular edge. The length of the crack can be varied. To model the crack, h-type approach is used. In and around the crack tip and along the length of the crack, more number of elements is used. In all the cases crack gap width is maintained as 10mm because the crack face will not come in contact before buckling happens [4]. The models of vertical and horizontal edge crack on thin plates are shown in the Fig. 8 and Fig. 9, respectively. The crack tip angle is maintained 600 at crack tip as given in Ref. [8]. Except crack edge boundary all the other boundary edges are divided into 40 elements. And along the crack edge boundary on both sides of the crack are divided into 20 elements and along the crack edges the element length is maintained as 2mm. Free auto mesh generation scheme of ANSYS is adopted to discretize the model. To capture the high stress gradient along the crack edge and also around the crack tip coarse mesh is refined using auto mesh refinement scheme available in ANSYS to a depth of 3 elements in a similar way they are refined in Ref. [8]. Simply supported boundary condition is applied to all the edges. The load is applied distributed along the vertical boundary edge called load edge and opposite vertical boundary edge called clamped edge is strained to move in plane direction. Similarly plate with center crack is also modeled and models of thin plates with vertical and horizontal center cracks are shown in Fig. 10 and Fig. 11, respectively.

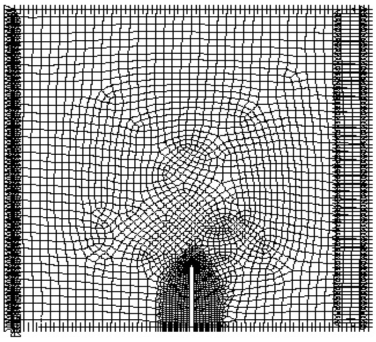

Fig. 8
FE model of thin plate with vertical edge crack.

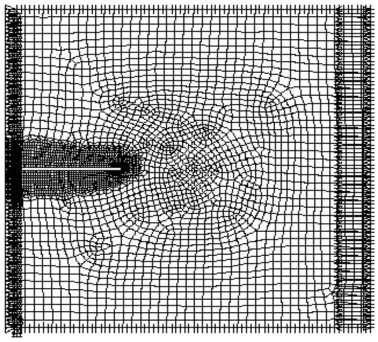

Fig. 9
FE model of thin plate with horizontal edge crack.

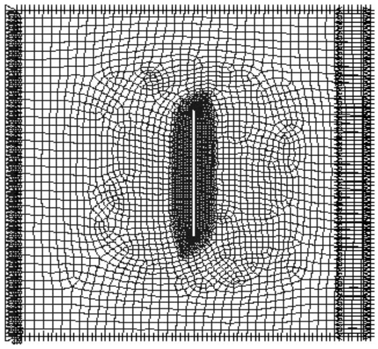

Fig. 10
FE model of thin plate with vertical center crack.

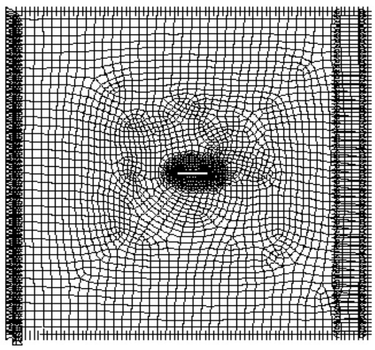

Fig. 11
FE model of thin plate with horizontal center crack.

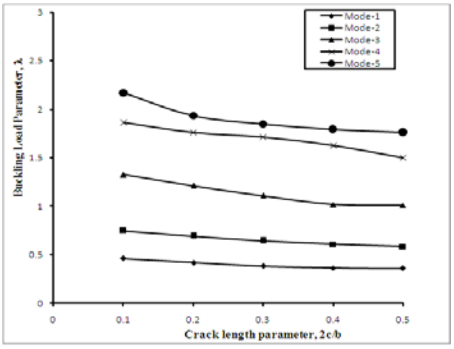

Fig. 12
Variation of buckling parameter of a vertical edge crack under axial compression.

Table 2
Buckling load parameter of a plate with vertical edge crack

Crack Size, $2c/b$	Mode	FE result
	1	0.4609
	2	0.745
0.1	3	1.3239
	4	1.8631
	5	2.1701
	1	0.4172
	2	0.6915
0.2	3	1.2124
	4	1.7607
	5	1.932
	1	0.3839
	2	1.6428
0.3	3	1.1073
	4	1.7122
	5	1.8471
	1	0.3648
	2	0.6076
0.4	3	1.0183
	4	1.6307
	5	1.7932
	1	0.3608
	2	0.5857
0.5	3	1.94
	4	1.498
	5	1.7628

6.3.1 Crack Dimensions:

Length of the crack: 0.1-0.5
Gap of the crack: 10mm
Angle of the crack tip: 600

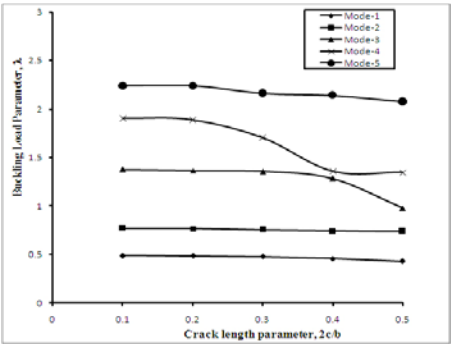

Fig. 13
Variation of buckling parameter of a horizontal edge crack under axial compression.

Table 3
Buckling load parameter of a plate with horizontal edge crack

Crack Size, 2*c*/*b*	Mode	FE result
0.1	1	0.4877
	2	0.7705
	3	1.3754
	4	1.9052
	5	2.2378
	1	0.4854
0.2	2	0.7655
	3	1.3646
	4	1.8887
	5	2.241
	1	0.4767
0.3	2	0.7532
	3	1.355
	4	1.7023
	5	2.1652
	1	0.4584
0.4	2	0.7417
	3	1.2821
	4	1.3549
	5	2.1419
0.5	1	0.4314
	2	0.7381
	3	0.979
	4	1.3463
	5	2.0771

7 RESULTS AND DISCUSSIONS

7.1 Thin plate with vertical edge crack

Fig.12 shows the variation of buckling load parameter λ ($\lambda = N_{xy}b^2/\pi^2 D$) of a plate vs. crack length (2*c*/*b*), where 2*c* is the crack length and *b* is the width of the plate for the vertical crack modeled along the unloaded edge of a plate. Table 2 shows the buckling load parameter values for different crack length. Both Fig. 12 and Table 2 show reduction in elastic buckling load with increase in crack length as expected and at higher modes namely 3, 4, and 5 reduction found to be significant.

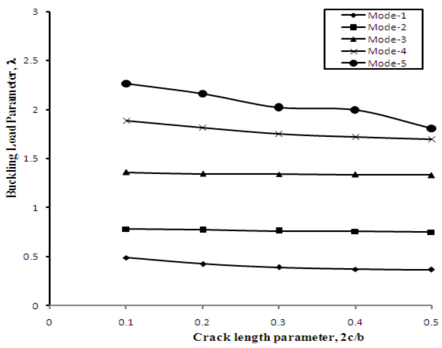

Fig. 14
Variation of buckling parameter of a vertical center crack under axial compression.

Table 4
Buckling load parameter of a plate with vertical center crack

Crack Size, $2c/b$	Mode	FE result
0.1	1	0.4894
	2	0.7832
	3	1.3615
	4	1.8856
	5	2.2644
	1	0.4258
0.2	2	0.7725
	3	1.3458
	4	1.819
	5	2.1623
	1	0.392
0.3	2	0.7605
	3	1.3407
	4	1.7497
	5	2.0231
	1	0.3726
0.4	2	0.7582
	3	1.3387
	4	1.719
	5	1.9943
0.5	1	0.3672
	2	0.7502
	3	1.332
	4	1.7
	5	1.8075

As inferred by Satish Kumar and Paik [6], here also it is found that dominant reduction in buckling strength of perfect thin plate is noticed due the presence of vertical edge crack. Based on first eigen mode FE, results reduction in buckling strength of the perfect thin plate due to vertical edge crack (10 mm length) is found to be 5.46%.

7.2 Thin plate with horizontal edge crack

Fig. 13 and Table 3 show the variation of buckling load parameter λ of a plate for different length of the horizontal crack modeled along the clamped edge of a plate. The reduction in elastic buckling load is noticed with increase in crack length as expected and at higher modes namely 3, 4, and 5 reduction found to be significant. Based on first eigen mode FE results, reduction in buckling strength of the perfect thin plate due to horizontal edge crack (10 mm

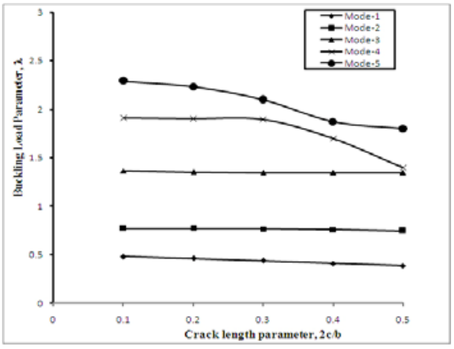

Fig. 15
Variation of buckling parameter of a horizontal center crack under axial compression.

Table 5
Buckling load parameter of a plate with horizontal center crack

Crack Size, 2*c*/*b*	Mode	FE result
	1	0.48
	2	0.7712
0.1	3	1.3662
	4	1.9099
	5	2.2918
	1	0.4619
	2	0.7705
0.2	3	1.3534
	4	1.9046
	5	2.2325
	1	0.4376
	2	0.7665
0.3	3	1.346
	4	1.8953
	5	2.1011
	1	0.4114
	2	0.7577
0.4	3	1.345
	4	1.7001
	5	1.8709
	1	0.3869
	2	0.7444
0.5	3	1.3444
	4	1.3949
	5	1.798

length) is found to be 0.03%. Comparing vertical edge crack and horizontal edge crack effect on the buckling strength it is found that vertical edge is more dominant than horizontal edge crack by 5.4% based on first eigen mode FE results.

7.3 Thin plate with vertical center crack

Fig. 14 and Table 4 show the variation of buckling load parameter λ of a plate for different length of the vertical center crack. The reduction in elastic buckling load is noticed with increase in crack length as expected and at higher modes namely 4 and 5 reduction found to be significant and at mode 3 there is not much significant variation of elastic buckling load and at lower modes namely 1 and 2 the elastic buckling loads are found to be increased with increase in crack length. As found in [2], here also it is found that dominant reduction in buckling strength of perfect thin plate is noticed due to the presence of vertical center crack. Based on first eigen mode FE results, reduction in buckling strength of the perfect thin plate due to vertical center crack (10 mm crack length) is found to be 0.38%.

7.4 Thin plate with horizontal center crack

Fig. 15 and Table 5 show the variation of buckling load parameter λ of a plate for different crack length of horizontal center crack. The reduction in elastic buckling load is noticed with increase in crack length as expected and at modes namely1, 2, 4, and 5 reductions found to be significant and at mode 3 there is not much significant variation of elastic buckling load. Based on first eigen mode FE results reduction in buckling strength of the perfect thin plate due to horizontal center crack (10 mm length) is found to be 1.55%. Comparing vertical center crack and horizontal vertical crack effect on the buckling strength it is found that vertical center is more dominant than horizontal center crack by 0.19% based on first eigen mode FE results.

8 CONCLUSION

From the present study for the thin plate taken for study, the following conclusions are derived:

(i). It is evident that crack damage reduces the ultimate strength of the plate significantly.

(ii). In case of edge crack, vertical edge crack reduces the buckling strength of thin plate considerably than horizontal edge crack. In a similar way, vertical center crack reduces buckling strength than horizontal centre crack but by meager amount.

(iii). From this buckling analysis, it is found that variation of buckling strength for modes 1 and 2 is not significant and it is significant for modes 4 and 5.

Generally edge cracks are more dominant than center cracks in reducing buckling strength.

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