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An Exact Solution for Classic Coupled Thermoporoelasticity in Cylindrical Coordinates

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ABSTRACT

In this paper the classic coupled thermoporoelasticity model of hollow and solid cylinders under radial symmetric loading condition (*r*, *t*) is considered. A full analytical method is used and an exact unique solution of the classic coupled equations is presented. The thermal and pressure boundary conditions, the body force, the heat source, and the injected volume rate per unit volume of a distribute water source are considered in the most general forms, and no limiting assumption is used. This generality allows simulation of various applicable problems.

Keywords: Coupled thermoporoelasticity; Hollow cylinder; Exact solution

1 INTRODUCTION

COUPLED thermal and poro-mechanical processes play an important role in a number of problems of interest in the geomechanics such as stability of boreholes and permeability enhancement in geothermal reservoirs. A thermoporoelastic approach combines the theory of heat conduction with poroelastic constitutive equations and coupling the temperature field with the stresses and pore pressure.

There are a limited numbers of papers that present the closed-form or analytical solution for the coupled thermoporoelasticity problems. Bai [1] invastigated the response of saturated porous media subjected to local thermal loading on the surface of semi-infinite space. He used the numerical integral methods for calculating the unsteady temparature, pore pressure and displacement fields. This author also studied the fluctuation responses of saturated porous media subjected to cyclic thermal loading [2]. In the mentioned paper, an analytical solution was proposed by using the Laplace transform and the Gauss-Legendre method and Laplace transform inversion. Droujinine [3] invatastigated dispersion and attenuation of body waves in a wide range of materials representing realistic rock structures. He used the time-domain asymptotic ray theory to a new generalized coordinate-free wave equation with an arbitrary tensor relaxation function. Bai and Li [4] found a solution for cylinderical cavety in saturated thermoporoelastic medium by using Laplace transform and numerical Laplace transform inversion.

The number of papers that present the closed-form or analytical solution for the coupled thermoelasticity problems is also limited. Hetnarski [5] found the solution of the coupled thermoelasticity in the form of a series function. Hetnarski and Ignaczak presented a study of the one-dimensional thermoelastic waves produced by an instantaneous plane source of heat in homogeneous isotropic infinite and semi-infinite bodies of the Green-Lindsay theory type [6]. Also, these authors presented an analysis for laser-induced waves propagating in an absorbing thermoelastic semi-space of the Green-Lindsay theory [7]. Georgiadis and Lykotrafitis obtained a three-dimensional transient thermoelastic solution for Rayleigh-type disturbances propagating on the surface of a half-space [8]. Wagner [9] presented the fundamental matrix of a system of partial differential operators that governs the diffusion of heat and the strains in elastic media. This method can be used to predict the temperature distribution and the strains by an instantaneous point heat, point source of heat, or by a suddenly applied delta force.

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In the present work a full analytical method is used to obtain the response of the governing equations, therefore an exact solution is presented. The method of solution is based on the Fourier's expansion and eigenfunction methods, which are traditional and routine methods in solving the partial differential equations. Since the coefficients of equations are not functions of the time variable (t), an exponential form is considered for the general solution matched with the physical wave properties of thermal and mechanical waves. For the particular solution, that is the response to mechanical and thermal shocks, the eigenfunction method and Laplace transformation is used.

2 GOVERNING EQUATIONS

A hollow cylinder with inner and outer radius r_i and r_o, respectively, made of isotropic material subjected to radial-symmetric mechanical, thermal, and pressure shocks is considered.

The classic theory of thermoporoelasticity for wave propagation is considered to allow coupling between deformation, thermal energy and pressure fields and to describe the physical behavior of the elastic domain to mechanical, thermal and pressure shock loads. Navier equation in terms of the displacement components is obtained as [4]

$$u_{,rr}+\frac{1}{r}u_{,r}-\frac{1}{r^2}u-\alpha\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}p_{,r}-\beta\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}T_{,r}-\rho\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}\ddot{u}=-\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}F(r,t) \quad (1)$$

Heat conduction equation in radial-symmetric direction with the mechanical coupling term is

$$T_{,rr}+\frac{1}{r}T_{,r}-Z\frac{T_\circ}{K}\dot{T}+Y\frac{T_\circ}{K}\dot{p}-\beta\frac{T_\circ}{K}(\dot{u}_{,r}+\frac{1}{r}\dot{u})=-\frac{1}{K}Q(r,t) \quad (2)$$

According to Darcy's law and continuity condition of seepage, the equation of mass conservation can be written as

$$p_{,rr}+\frac{1}{r}p_{,r}-\alpha_p\frac{\gamma_w}{k}\dot{p}-Y\frac{\gamma_w}{k}\dot{T}-\alpha\frac{\gamma_w}{k}(\dot{u}_{,r}+\frac{1}{r}\dot{u})=-\frac{\gamma_w}{k}W(r,t) \quad (3)$$

where (,) denotes partial derivative, u is the displacement component in the radial direction, p is the pore pressure, ρ is bulk mass density, $\alpha=1-\frac{C_s}{C}$ is the Biot's coefficient, $C_s=3(1-2\nu_s)E_s$ is the coefficient of volumetric compression of solid grains, with E_s and ν_s being the elastic modulus and Poisson's ratio of solid grains and $C=3(1-2\nu)E$ is the coefficient of volumetric compression of solid skeleton, with E and ν being the elastic modulus and Poisson's ratio of the solid skeleton, $T_\circ$ is initial reference temperature, $\beta=\frac{3\alpha_s}{C}$ is the thermal expansion factor, α_s is the coefficient of linear thermal expansion of the solid grains, $Y=3(n\alpha_w+(\alpha-n)\alpha_s)$ and $\alpha_p=n(C_w-C_s)+\alpha C_s$ are coupling parameters, α_w and C_w are the coefficients of linear thermal expansion and volumetric compression of pure water, n is the porosity, k is the hydraulic conductivity, γ_w is the unit of pore water and $Z=\frac{((1-n)\rho_s c_s+n\rho_w c_w)}{T_\circ}-3\beta\alpha_s$ is coupling parameter, ρ_w and ρ_s are the densities of pore water and solid grains and c_w and c_s are the heat capacities of pore water and solid grains and K is the coefficient of heat conductivity. Here, $F(r, t)$, $Q(r, t)$ and $W(r, t)$ are the body force, heat generation and the injected volume rate per unit volume of a distribute water source, respectively. The mechanical, thermal, and pressure boundary conditions are

$$
\begin{aligned}
&C_{11}u(r_i,t)+C_{12}u_{,r}(r_i,t)+C_{13}T(r_i,t)+C_{14}p(r_i,t)=f_1(t)\\
&C_{21}u(r_o,t)+C_{22}u_{,r}(r_o,t)+C_{23}T(r_o,t)+C_{24}p(r_o,t)=f_2(t)\\
&C_{31}T(r_i,t)+C_{32}T_{,r}(r_i,t)=f_3(t)\\
&C_{41}T(r_o,t)+C_{42}T_{,r}(r_o,t)=f_4(t)\\
&C_{51}p(r_i,t)=f_5(t)\\
&C_{61}p(r_o,t)=f_6(t)
\end{aligned}
\tag{4}
$$

where C_{ij} are the mechanical, thermal and pressure coefficients, and by assigning different values for them, different types of mechanical, thermal, and pressure boundary conditions may be obtained. These boundary conditions include the displacement, strain, stress (for the first and second boundary conditions), specified temperature, convection, heat flux condition (for the third and forth boundary conditions), and pressure (for the fifth and sixth boundary conditions). $f_1(t)$ to $f_6(t)$ are arbitrary functions which show mechanical, thermal and pressure shocks, respectively. The initial boundary conditions are assumed in the following general form

$$
u(r,0)=f_7(r),\qquad u_{,t}(r,0)=f_8(r),\qquad T(r,0)=f_9(r),\qquad p(r,0)=f_{10}(r)
\tag{5}
$$

where $f_7(t)$ to $f_{10}(t)$ are arbitrary functions which show initial distributions of displacement, temperature and pressure, respectively.

3 SOLUTION

The Eqs. (1) to (3) constitute a system of non-homogeneous partial differential equations with non-constant coefficients (functions of the radius only) has general and particular solutions.

3.1 General solution with homogeneous boundary conditions

Since the coefficients of Eqs. (1) to (3) are independent of the time variable (t), an exponential function form in terms of the time variable may be assumed for the general solution as

$$
\begin{aligned}
&u(r,t)=[U(r)]e^{\lambda t}\\
&T(r,t)=[\theta(r)]e^{\lambda t}\\
&p(r,t)=[P(r)]e^{\lambda t}
\end{aligned}
\tag{6}
$$

Substituting Eq. (6) into homogeneous parts of Eqs. (1) to (3), yields

$$
\begin{aligned}
&U''+\frac{1}{r}U'-\frac{1}{r^2}U+d_1P'+d_2\theta'+d_3\lambda^2U=0\\
&\theta''+\frac{1}{r}\theta'+d_4\lambda\theta+d_5\lambda P+d_6\lambda(U'+\frac{1}{r}U)=0\\
&P''+\frac{1}{r}P'+d_7\lambda P+d_8\lambda\theta+d_9\lambda(U'+\frac{1}{r}U)=0
\end{aligned}
\tag{7}
$$

Eqs. (7) constitute a system of ordinary differential equations, where the prime symbol ($'$) indicates differentiation with respect to the radius variable (r) and d_1 to d_9 are constant parameters given in the Appendix A. The first solutions of U_1 , θ_1 and P_1 are considered as

$$U_1(r)=A_1J_1(\beta r)$$
$$\theta_1(r)=B_1J_0(\beta r) \tag{8}$$
$$P_1(r)=C_1J_0(\beta r)$$

Substituting Eqs. (8) into Eqs. (7) yields

$$\{(-\beta^2+\lambda^2 d_3)A_1-d_2\beta B_1-d_1\beta C_1\}J_1(\beta r)=0$$
$$\{\lambda d_6\beta A_1+\lambda d_4 B_1+(-\beta^2+\lambda d_5)C_1\}J_0(\beta r)=0 \tag{9}$$
$$\{\lambda d_9\beta A_1+(-\beta^2+\lambda d_7)B_1+\lambda d_8 C_1\}J_0(\beta r)=0$$

Eq. (9) shows that U_1 , θ_1 and P_1 can be the solutions of Eqs. (7), if and only if

$$\begin{bmatrix}-\beta^2+\lambda^2 d_3 & -d_2\beta & -d_1\beta\\ \lambda d_6\beta & \lambda d_4 & -\beta^2+\lambda d_5\\ \lambda d_9\beta & -\beta^2+\lambda d_7 & \lambda d_8\end{bmatrix}\begin{Bmatrix}A_1\\B_1\\C_1\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{10}$$

The non-trivial solution of Eqs. (10) is obtained by equating the determinant of this equation to zero as

$$-\beta^2\lambda^2 d_4 d_7+\beta^6-\beta^4\lambda d_7-\beta^4\lambda d_5+\beta^2\lambda^2 d_5 d_7+\lambda^4 d_3 d_4 d_7-\lambda^2 d_3\beta^4+\lambda^3 d_3\beta^2 d_7+\lambda^3 d_3 d_5\beta^2$$
$$-\lambda^4 d_3 d_5 d_7+\lambda^2 d_6\beta^2 d_2 d_7+\lambda d_6\beta^4 d_1-\lambda^2 d_6\beta^2 d_1 d_7+\lambda d_9\beta^4 d_2-\lambda^2 d_9\beta^2 d_2 d_5+\lambda^2 d_9\beta^2 d_1 d_4=0 \tag{11}$$

Eq. (11) is the first characteristic equation. Thus, it is concluded that U_1 , θ_1 and P_1 satisfy the system of Eqs. (7) and they are the first solution of the system. The second solutions of U_2 , θ_2 and P_2 are considered as

$$U_2(r)=[A_2J_1(\beta r)+A_3rJ_2(\beta r)]$$
$$\theta_2(r)=[B_2J_0(\beta r)+B_3rJ_1(\beta r)] \tag{12}$$
$$P_2(r)=[C_2J_0(\beta r)+C_3rJ_1(\beta r)]$$

Substituting Eqs. (12) to Eqs. (7) yields

$$\{(-\beta^2+\lambda^2 d_3)A_3-d_2\beta B_3-d_1\beta C_3\}rJ_0(\beta r)+\{(\beta^2-d_3\lambda^2)A_2-\frac{2}{\beta}d_3\lambda^2 A_3+d_2\beta B_2+d_1\beta C_2\}J_1(\beta r)=0$$
$$\{\lambda d_6\beta A_3+\lambda d_4 B_3+(-\beta^2+\lambda d_5)C_3\}rJ_1(\beta r)+\{d_6\lambda\beta A_2+d_4\lambda B_2+(-\beta^2+d_5\lambda)C_2+2\beta C_3\}J_0(\beta r)=0 \tag{13}$$
$$\{\lambda d_9\beta A_3+(-\beta^2+\lambda d_7)B_3+\lambda d_8 C_3\}rJ_1(\beta r)+\{d_9\lambda\beta A_2+(-\beta^2+d_7\lambda)B_2+d_8\lambda C_2+2\beta B_3\}J_0(\beta r)=0$$

The expressions for U_2 , θ_2 and P_2 can be the solutions of Eqs. (7), if and only if

$$\begin{bmatrix}-\beta^2+\lambda^2 d_3 & -d_2\beta & -d_1\beta\\ \lambda d_6\beta & \lambda d_4 & -\beta^2+\lambda d_5\\ \lambda d_9\beta & -\beta^2+\lambda d_7 & \lambda d_8\end{bmatrix}\begin{Bmatrix}A_3\\B_3\\C_3\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{14}$$

$$(\beta^2-d_3\lambda^2)A_2-\frac{2}{\beta}d_3\lambda^2 A_3+d_2\beta B_2+d_1\beta C_2=0 \tag{15}$$

$$d_6\lambda\beta A_2+d_4\lambda B_2+(-\beta^2+d_5\lambda)C_2+2\beta C_3=0 \tag{16}$$

$$d_9\lambda\beta A_2+(-\beta^2+d_7\lambda)B_2+d_8\lambda C_2+2\beta B_3=0 \tag{17}$$

The non-trivial solution of Eqs. (14) is obtained by equating the determinant to zero as

$$-\beta^2\lambda^2 d_4 d_7+\beta^6-\beta^4\lambda d_7-\beta^4\lambda d_5+\beta^2\lambda^2 d_5 d_7+\lambda^4 d_3 d_4 d_7-\lambda^2 d_3\beta^4+\lambda^3 d_3\beta^2 d_7+\lambda^3 d_3 d_5\beta^2$$
$$-\lambda^4 d_3 d_5 d_7+\lambda^2 d_6\beta^2 d_2 d_7+\lambda d_6\beta^4 d_1-\lambda^2 d_6\beta^2 d_1 d_7+\lambda d_9\beta^4 d_2-\lambda^2 d_9\beta^2 d_2 d_5+\lambda^2 d_9\beta^2 d_1 d_4=0 \tag{18}$$

Eqs. (15) to (17) give the relations between A_2, A_3, B_2, B_3, C_2 and C_3. They play as the balancing ratios that make Eq. (12) to be the second solution of the system of Eqs. (7). The third solution of the system of the ordinary differential equations with non-constant coefficients (7) must be considered as

$$\begin{aligned}
U_3(r)&=[A_4J_1(\beta r)+A_5 rJ_2(\beta r)+A_6 r^2 J_3(\beta r)]\\
\theta_3(r)&=[B_4J_0(\beta r)+B_5 rJ_1(\beta r)+B_6 r^2 J_2(\beta r)]\\
P_3(r)&=[C_4J_0(\beta r)+C_5 rJ_1(\beta r)+C_6 r^2 J_2(\beta r)]
\end{aligned} \tag{19}$$

Substituting Eq. (19) into Eq. (7) yields

$$\begin{aligned}
&\{(-\lambda^2 d_3+\beta^2)A_6+d_2\beta B_6+d_1\beta C_6\}r^2J_1(\beta r)+\{(\beta^2-d_3\lambda^2)A_5+d_2\beta B_5+d_1\beta C_5-\frac{4}{\beta}d_3\lambda^2 A_6\}rJ_0(\beta r)\\
&\quad+\{(d_3\lambda^2-\beta^2)A_4-d_1\beta C_4-d_2\beta B_4+\frac{2}{\beta}d_3\lambda^2 A_5+\frac{8}{\beta^2}d_3\lambda^2 A_6\}J_1(\beta r)=0\\
&\{-\lambda d_6\beta A_6-\lambda d_4 B_6+(\beta^2-\lambda d_5)C_6\}r^2J_0(\beta r)+\{d_6\lambda\beta A_4+d_4\lambda B_4+(-\beta^2+d_5\lambda)C_4+2\beta C_5\}J_0(\beta r)\\
&\quad+\{d_6\lambda\beta A_5+d_4\lambda B_5+(-\beta^2+d_5\lambda)C_5+2d_6\lambda A_6+\frac{2}{\beta}d_4\lambda B_6+(2\beta+\frac{2}{\beta}d_5\lambda)C_6\}rJ_1(\beta r)=0\\
&\{-\lambda d_9\beta A_6+(\beta^2-\lambda d_7)B_6-\lambda d_8 C_6\}r^2J_0(\beta r)+\{d_9\lambda\beta A_4+(-\beta^2+d_7\lambda)B_4+d_8\lambda C_4+2\beta B_5\}J_0(\beta r)\\
&\quad+\{\beta d_9\lambda A_5+(-\beta^2+d_7\lambda)B_5+d_8\lambda C_5+2d_9\lambda A_6+(2\beta+\frac{2}{\beta}d_7\lambda)B_6+\frac{2}{\beta}d_8\lambda C_6\}rJ_1(\beta r)=0
\end{aligned} \tag{20}$$

The expressions for U_3 , θ_3 and P_3 can be the solutions of Eqs. (7), if and only if

$$\begin{bmatrix} -\beta^2+\lambda^2 d_3 & -d_2\beta & -d_1\beta \\ \lambda d_6\beta & \lambda d_4 & -\beta^2+\lambda d_5 \\ \lambda d_9\beta & -\beta^2+\lambda d_7 & \lambda d_8 \end{bmatrix}\begin{Bmatrix} A_6\\ B_6\\ C_6\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{21}$$

The non-trivial solution of Eqs. (21) is obtained by equating the determinant of this equation to zero as

$$\begin{aligned}
&(\beta^2-d_3\lambda^2)A_5+d_2\beta B_5+d_1\beta C_5-\frac{4}{\beta}d_3\lambda^2 A_6=0\\
&(d_3\lambda^2-\beta^2)A_4-d_1\beta C_4-d_2\beta B_4+\frac{2}{\beta}d_3\lambda^2 A_5+\frac{8}{\beta^2}d_3\lambda^2 A_6=0\\
&d_6\lambda\beta A_4+d_4\lambda B_4+(-\beta^2+d_5\lambda)C_4+2\beta C_5=0\\
&d_6\lambda\beta A_5+d_4\lambda B_5+(-\beta^2+d_5\lambda)C_5+2d_6\lambda A_6+\frac{2}{\beta}d_4\lambda B_6+(2\beta+\frac{2}{\beta}d_5\lambda)C_6=0\\
&d_9\lambda\beta A_4+(-\beta^2+d_7\lambda)B_4+d_8\lambda C_4+2\beta B_5=0
\end{aligned} \tag{22}$$

$$\beta d_9\lambda A_5+(-\beta^2+d_7\lambda)B_5+d_8\lambda C_5+2d_9\lambda A_6+(2\beta+\frac{2}{\beta}d_7\lambda)B_6+\frac{2}{\beta}d_8\lambda C_6=0$$

$$-\beta^2\lambda^2 d_4 d_7+\beta^6-\beta^4\lambda d_7-\beta^4\lambda d_5+\beta^2\lambda^2 d_5 d_7+\lambda^4 d_3 d_4 d_7-\lambda^2 d_3\beta^4+\lambda^3 d_3\beta^2 d_7+\lambda^3 d_3 d_5\beta^2$$
$$-\lambda^4 d_3 d_5 d_7+\lambda^2 d_6\beta^2 d_2 d_7+\lambda d_6\beta^4 d_1-\lambda^2 d_6\beta^2 d_1 d_7+\lambda d_9\beta^4 d_2-\lambda^2 d_9\beta^2 d_2 d_5+\lambda^2 d_9\beta^2 d_1 d_4=0 \quad (23)$$

The characteristic Eq. (23) is the same as the characteristic Eqs. (11) and (18). This equality is interesting as it prevents mathematical dilemma and complexity and a single value for the eigenvalue β simultaneously satisfies three characteristic Eqs. (11), (18), and (23). Eq. (22) gives the relation between $A_4, A_5, A_6, B_4, B_5, B_6, C_4, C_5$ and C_6. They play as the balancing ratios that help Eq. (19) to be the third solution of the system of Eq. (7). Therefore, the complete general solutions for the solid cylinder are

$$\begin{aligned}
U^g(r)&=A_1J_1(\beta r)+A_3[\zeta_1J_1(\beta r)+rJ_2(\beta r)]+A_6[\zeta_2J_1(\beta r)+\zeta_3rJ_2(\beta r)+r^2J_3(\beta r)]\\
\theta^g(r)&=A_1\zeta_4J_0(\beta r)+A_3[\zeta_5J_0(\beta r)+\zeta_6rJ_1(\beta r)]+A_6[\zeta_7J_0(\beta r)+\zeta_8rJ_1(\beta r)+\zeta_9r^2J_2(\beta r)]\\
P^g(r)&=A_1\zeta_{10}J_0(\beta r)+A_3[\zeta_{11}J_0(\beta r)+\zeta_{12}rJ_1(\beta r)]+A_6[\zeta_{13}J_0(\beta r)+\zeta_{14}rJ_1(\beta r)+\zeta_{15}r^2J_2(\beta r)]
\end{aligned} \quad (24)$$

and for hollow cylinder are

$$\begin{aligned}
U^g(r)&=A_1J_1(\beta r)+A_3[\zeta_1J_1(\beta r)+rJ_2(\beta r)]+A_6[\zeta_2J_1(\beta r)+\zeta_3rJ_2(\beta r)+r^2J_3(\beta r)]\\
&\quad+\hat{A}_1Y_1(\beta r)+\hat{A}_3[\zeta_1Y_1(\beta r)+rY_2(\beta r)]+\hat{A}_6[\zeta_2Y_1(\beta r)+\zeta_3rY_2(\beta r)+r^2Y_3(\beta r)]\\
\theta^g(r)&=A_1\zeta_4J_0(\beta r)+A_3[\zeta_5J_0(\beta r)+\zeta_6rJ_1(\beta r)]+A_6[\zeta_7J_0(\beta r)+\zeta_8rJ_1(\beta r)+\zeta_9r^2J_2(\beta r)]\\
&\quad+\hat{A}_1\zeta_4Y_0(\beta r)+\hat{A}_3[\zeta_5Y_0(\beta r)+\zeta_6rY_1(\beta r)]+\hat{A}_6[\zeta_7Y_0(\beta r)+\zeta_8rY_1(\beta r)+\zeta_9r^2Y_2(\beta r)]\\
P^g(r)&=A_1\zeta_{10}J_0(\beta r)+A_3[\zeta_{11}J_0(\beta r)+\zeta_{12}rJ_1(\beta r)]+A_6[\zeta_{13}J_0(\beta r)+\zeta_{14}rJ_1(\beta r)+\zeta_{15}r^2J_2(\beta r)]\\
&\quad+\hat{A}_1\zeta_{10}Y_0(\beta r)+\hat{A}_3[\zeta_{11}Y_0(\beta r)+\zeta_{12}rY_1(\beta r)]+\hat{A}_6[\zeta_{13}Y_0(\beta r)+\zeta_{14}rY_1(\beta r)+\zeta_{15}r^2Y_2(\beta r)]
\end{aligned} \quad (25)$$

where ζ_1 to ζ_{15} are ratios obtained from Eqs (21) to (22), (14) to (17) and (10) and are given in the Appendix A. Substituting U^g ,θ^g and P^g in the homogeneous form of the boundary conditions (4), three linear algebraic equations are obtained. They are coefficients depends on λ and β. Setting the determinant of the coefficients equal to zero, the second characteristic equation is obtained. Simultaneous solution of this equation and Eq. (11), results into infinite number of two eigenvalues β_n and λ_n. Therefore, U^g ,θ^g and P^g are rewritten as

$$\begin{aligned}
U^g(r)&=A_1[J_1(\beta r)+\zeta_{16}[\zeta_1J_1(\beta r)+rJ_2(\beta r)]+\zeta_{17}[\zeta_2J_1(\beta r)+\zeta_3rJ_2(\beta r)+r^2J_3(\beta r)]]\\
\theta^g(r)&=A_1[\zeta_4J_0(\beta r)+\zeta_{16}[\zeta_5J_0(\beta r)+\zeta_6rJ_1(\beta r)]+\zeta_{17}[\zeta_7J_0(\beta r)+\zeta_8rJ_1(\beta r)+\zeta_9r^2J_2(\beta r)]]\\
P^g(r)&=A_1[\zeta_{10}J_0(\beta r)+\zeta_{16}[\zeta_{11}J_0(\beta r)+\zeta_{12}rJ_1(\beta r)]+\zeta_{17}[\zeta_{13}J_0(\beta r)+\zeta_{14}rJ_1(\beta r)+\zeta_{15}r^2J_2(\beta r)]]
\end{aligned} \quad (26)$$

where ζ_{16} and ζ_{17} are presented in the appendix. Let us show the functions in the brackets of Eqs. (26) by functions $H_\circ, H_1$ and H_2 as

$$\begin{aligned}
H_0&=J_1(\beta r)+\zeta_{16}[\zeta_1J_1(\beta r)+rJ_2(\beta r)]+\zeta_{17}[\zeta_2J_1(\beta r)+\zeta_3rJ_2(\beta r)+r^2J_3(\beta r)\\
H_1&=\zeta_4J_0(\beta r)+\zeta_{16}[\zeta_5J_0(\beta r)+\zeta_6rJ_1(\beta r)]+\zeta_{17}[\zeta_7J_0(\beta r)+\zeta_8rJ_1(\beta r)+\zeta_9r^2J_2(\beta r)\\
H_2&=\zeta_{10}J_0(\beta r)+\zeta_{16}[\zeta_{11}J_0(\beta r)+\zeta_{12}rJ_1(\beta r)]+\zeta_{17}[\zeta_{13}J_0(\beta r)+\zeta_{14}rJ_1(\beta r)+\zeta_{15}r^2J_2(\beta r)
\end{aligned} \quad (27)$$

According to the Sturm-Liouville theorem, these functions are orthogonal with respect to the weight function $p(r)=r$ such as

$$\int_{r_i}^{r_o} H(\beta_n r)H(\beta_m r)r \, \mathrm{d}r = \begin{cases} 0 & n \neq m \\ \|H(\beta_n r)\|^2 & n = m \end{cases} \tag{28}$$

where $\|H(\beta_n r)\|$ is norm of the H function and equals

$$\|H(\beta_n r)\| = \left[\int_{r_i}^{r_o} rH^2(\beta_n r)\mathrm{d}r\right]^{\frac{1}{2}} \tag{29}$$

Due to the orthogonality of function *H*, every piece-wise continuous function, such as *f*(*r*), can be expanded in terms of the function H (either $H_\circ$, H_1 or H_2), and is called the H-Fourier series as

$$f(r) = \sum_{n=1}^{\infty} e_n H(\beta_n r) \tag{30}$$

where e_n equals

$$e_n = \frac{1}{\|H(\beta_n r)\|^2} \int_{r_i}^{r_o} f(r)\, H(r)\, r \, \mathrm{d}r \tag{31}$$

Using Eqs. (6), (26), and (27) the displacement and temperature distributions due to the general solution become

$$\begin{aligned} u^g(r,t) &= \sum_{n=1}^{\infty} \{\sum_{m=1}^{4} a_{nm} e^{\lambda_{nm} t}\} H_0(\beta_n r) \\ T^g(r,t) &= \sum_{n=1}^{\infty} \{\sum_{m=1}^{4} N_{nm} a_{nm} e^{\lambda_{nm} t}\} H_1(\beta_n r) \\ p^g(r,t) &= \sum_{n=1}^{\infty} \{\sum_{m=1}^{4} M_{nm} a_{nm} e^{\lambda_{nm} t}\} H_2(\beta_n r) \end{aligned} \tag{32}$$

where N_{mn} and M_{nm} are ratios obtained by substituting Eqs. (32) to Eq. (1) to (3). Using the initial conditions (5) and with the help of Eqs. (29), (30) and (31), four unknown constants are obtained.

3.2 Particular solution with non-homogeneous boundary conditions

The general solutions may be used as proper functions for guessing the particular solution adopted to the non-homogeneous parts of the Eqs. (1) to (3) and the non-homogeneous boundary conditions (4) as

$$\begin{aligned} u^p(r,t) &= \sum_{n=1}^{\infty} \{\left[G_{1n}(t)J_1(\beta_n r) + G_{2n}(t)rJ_2(\beta_n r) + G_{3n}(t)r^2 J_3(\beta_n r)\right] + r^2 G_{4n}(t)\} \\ T^p(r,t) &= \sum_{n=1}^{\infty} \{\left[G_{5n}(t)J_0(\beta_n r) + G_{6n}(t)rJ_1(\beta_n r) + G_{7n}(t)r^2 J_2(\beta_n r)\right] + r^2 G_{8n}(t)\} \\ p^p(r,t) &= \sum_{n=1}^{\infty} \{\left[G_{9n}(t)J_0(\beta_n r) + G_{10n}(t)rJ_1(\beta_n r) + G_{11n}(t)r^2 J_2(\beta_n r)\right] + r^2 G_{12n}(t)\} \end{aligned} \tag{33}$$

For the solid cylinder, the second type of Bessel function Y is excluded. It is necessary and suitable to expand the body force $F(r, t)$, heat source $Q(r, t)$ and porosity function $W(r, t)$ in H-Fourier expansion form as

$$F(r,t)=\sum_{n=1}^{\infty} F_n(t)H_0(\beta_n r)$$
$$Q(r,t)=\sum_{n=1}^{\infty} Q_n(t)H_1(\beta_n r) \quad (34)$$
$$P(r,t)=\sum_{n=1}^{\infty} P_n(t)H_2(\beta_n r)$$

where $F_n(t)$, $Q_n(t)$ and $P_n(t)$ are

$$F_n(t)=\frac{1}{\left\|H_0(\beta_n r)\right\|^2}\int_{r_i}^{r_o} F(r,t)H_0(\beta_n r)r\,\mathrm{d}r$$
$$Q_n(t)=\frac{1}{\left\|H_1(\beta_n r)\right\|^2}\int_{r_i}^{r_o} Q(r,t)H_1(\beta_n r)r\,\mathrm{d}r \quad (35)$$
$$P_n(t)=\frac{1}{\left\|H_2(\beta_n r)\right\|^2}\int_{r_i}^{r_o} P(r,t)H_2(\beta_n r)r\,\mathrm{d}r$$

Substituting Eq. (33) and (34) into the non-homogeneous form of Eqs. (1) into (3) yield

$$G_2(t)\beta^2-\ddot{G}_2(t)d_3-4\ddot{G}_3(t)\frac{1}{\beta}d_3-d_{13}G_4(t)C_1-4d_{13}G_4(t)C_2\frac{1}{\beta}-d_{16}\ddot{G}_4(t)C_1-4d_{16}\ddot{G}_4(t)C_2\frac{1}{\beta}+G_6(t)d_2\beta$$
$$-d_{15}G_8(t)C_1-4d_{15}G_8(t)C_2\frac{1}{\beta}+G_{10}(t)d_1\beta-d_{14}G_{12}(t)C_1-16d_{14}G_{12}(t)C_2\frac{1}{\beta}-d_{25}d_{10}FC_1-4d_{25}d_{10}FC_2\frac{1}{\beta}=0 \quad (36a)$$

$$-G_1(t)\beta^2+\ddot{G}_1(t)d_3+2\ddot{G}_2(t)\frac{1}{\beta}d_3+8\ddot{G}_3(t)\frac{1}{\beta^2}d_3d_{13}G_4(t)C_0+2d_{13}G_4(t)C_1\frac{1}{\beta}+8d_{13}G_4(t)C_2\frac{1}{\beta^2}+d_{16}\ddot{G}_4(t)C_0$$
$$+2d_{16}\ddot{G}_4(t)C_1\frac{1}{\beta}+8d_{16}\ddot{G}_4(t)C_2\frac{1}{\beta^2}-G_5(t)\beta d_2+d_{15}G_8(t)C_0+2d_{15}G_8(t)C_1\frac{1}{\beta}+8d_{15}G_8(t)C_2\frac{1}{\beta^2}-G_9(t)\beta d_1$$
$$+d_{14}G_{12}(t)C_0+2d_{14}G_{12}(t)C_1\frac{1}{\beta}+8d_{14}G_{12}(t)C_2\frac{1}{\beta^2}+d_{25}d_{10}FC_0+2d_{25}d_{10}FC_1\frac{1}{\beta}+8d_{25}d_{10}FC_2\frac{1}{\beta^2}=0 \quad (36b)$$

$$G_3(t)\beta^2-\ddot{G}_3(t)d_3-d_{13}G_4(t)C_2-d_{16}\ddot{G}_4(t)C_2+G_7(t)\beta d_2-d_{15}G_8(t)C_2+G_{11}(t)\beta d_1-d_{14}G_{12}(t)C_2$$
$$-d_{25}d_{10}FC_2=0 \quad (36c)$$

$$d_6\beta\dot{G}_1(t)+d_{18}E_0\dot{G}_4(t)-\beta^2G_5(t)+d_4\dot{G}_5(t)+2\beta G_6(t)+d_{17}E_0G_8(t)+d_{19}E_0\dot{G}_8(t)+d_5\dot{G}_9(t)+d_{20}E_0\dot{G}_{12}(t)$$
$$+d_{26}d_{11}E_0Q_n(t)=0 \quad (36d)$$

$$-d_6\beta\dot{G}_3(t)-E_2d_{18}\dot{G}_4(t)+\beta^2G_7(t)-d_4\dot{G}_7(t)-d_{17}E_2G_8(t)-d_{19}E_2\dot{G}_8(t)-d_5\dot{G}_{11}(t)-d_{20}E_2\dot{G}_{12}(t)$$
$$-d_{26}E_2Q_n(t)=0 \quad (36e)$$

$$d_6\beta\dot{G}_2(t)+2d_6\dot{G}_3(t)+\left(d_{18}E_1+\frac{2}{\beta}d_{14}E_2\right)\dot{G}_4(t)-\beta^2G_6(t)+d_4\dot{G}_6(t)+2\beta G_7(t)+\frac{2}{\beta}d_4\dot{G}_7(t)$$
$$+\left(d_{17}E_1+\frac{2}{\beta}d_{17}E_2\right)G_8(t)+\left(d_{19}E_1+\frac{2}{\beta}E_2d_{19}\right)\dot{G}_8(t)+\dot{G}_{10}(t)d_5+\frac{2}{\beta}\dot{G}_{11}(t)d_5$$
$$+\left(d_{20}E_1+\frac{2}{\beta}d_{20}E_2\right)\dot{G}_{12}(t)+\left(d_{26}E_1+\frac{2}{\beta}d_{26}E_2\right)Q_n(t)=0 \tag{36f}$$

$$d_9\beta\dot{G}_1(t)+d_{22}D_0\dot{G}_4(t)+d_7\dot{G}_5(t)+d_{23}D_0\dot{G}_8(t)-\beta^2G_9(t)+d_8\dot{G}_9(t)+2G_{10}(t)\beta+d_{21}G_{12}(s)D_0+d_{24}\dot{G}_{12}(t)$$
$$+d_{27}d_{12}D_0W_n(t)=0$$

$$-d_9\beta\dot{G}_3(t)-d_{22}D_2\dot{G}_4(t)-d_7\dot{G}_7(t)-d_{23}D_2\dot{G}_8(t)+\beta^2G_{11}(t)-d_8\dot{G}_{11}(t)-d_{21}D_2G_{12}(t)-d_{24}\dot{G}_{12}(t)D_2$$
$$-d_{27}d_{12}E_2W_n(t)=0 \tag{36g}$$

$$d_9\dot{G}_2(t)\beta+d_9\,2\dot{G}_3(t)+\left(d_{22}D_1+\frac{2}{\beta}d_{22}D_2\right)\dot{G}_4(t)+d_7\dot{G}_6(t)+\frac{2}{\beta^2}d_7\dot{G}_7(t)+\left(d_{23}D_1+\frac{2}{\beta}d_{23}D_2\right)\dot{G}_8(t)$$
$$-\beta^2G_{10}(t)+d_8\dot{G}_{10}(t)+2\beta G_{11}(t)+\frac{2}{\beta}d_8\dot{G}_{11}(t)+\left(d_{21}D_1+\frac{2}{\beta}d_{21}D_2\right)G_{12}(t)+\left(d_{24}D_1+\frac{2}{\beta}d_{24}D_2\right)\dot{G}_{12}(t)=0 \tag{36h}$$

where d_{10} to d_{27} are the coefficients of the H-expansion and constant parameters presented in the appendix. By taking Laplace transform of Eq. (36) and using three boundary conditions of Eq. (4) (for solid cylinder only second, fourth and sixth boundary condones are applicable), a system of algebraic equations is obtained and solved by Carmer's methods in the Laplace domain. By the inverse Laplace transform, the functions are transformed into the real time domain and finally $G_{1n}(t)$ to $G_{12n}(t)$ are calculated. In this process, it is necessary to consider the following points:

1. The initial conditions (5) are considered only for the general solutions, the initial conditions of $G_{1n}(t)$ to $G_{12n}(t)$ for the particular solutions are considered equal to zero.
2. Laplace transform of Eq. (36) is in terms of polynomial functions of the Laplace parameter *s* (not the Bessel functions form of *s*). Therefore, the exact inverse Laplace transform is possible and somehow simple.
3. For the hollow cylinder it is enough to include the second type of Bessel function $Y(r)$ in a sequence of the particular solution as

$$u^p(r,t)=\sum_{n=1}^{\infty}\{\left[G_{1n}(t)J_1(\beta_n r)+G_{2n}(t)rJ_2(\beta_n r)+G_{3n}(t)r^2J_3(\beta_n r)\right]$$
$$+\left[G_{4n}(t)Y_1(\beta_n r)+G_{5n}(t)rY_2(\beta_n r)+G_{6n}(t)r^2Y_3(\beta_n r)\right]+rG_{7n}(t)+r^2G_{8n}(t)\}$$
$$T^p(r,t)=\sum_{n=1}^{\infty}\{\left[G_{9n}(t)J_0(\beta_n r)+G_{10n}(t)rJ_1(\beta_n r)+G_{11n}(t)r^2J_2(\beta_n r)\right]$$
$$+\left[G_{12n}(t)Y_0(\beta_n r)+G_{13n}(t)rY_1(\beta_n r)+G_{14n}(t)r^2Y_2(\beta_n r)\right]+rG_{15n}(t)+r^2G_{16n}(t)\}$$
$$p^p(r,t)=\sum_{n=1}^{\infty}\{\left[G_{17n}(t)J_0(\beta_n r)+G_{18n}(t)rJ_1(\beta_n r)+G_{19n}(t)r^2J_2(\beta_n r)\right]$$
$$+\left[G_{20n}(t)Y_0(\beta_n r)+G_{21n}(t)rY_1(\beta_n r)+G_{22n}(t)r^2Y_2(\beta_n r)\right]+rG_{23n}(t)+r^2G_{24n}(t)\} \tag{37}$$

By substituting Eqs. (37) in Eqs. (1) to (3), eighteen equations are obtained, where using the six boundary conditions (4) twenty four functions $G_{1n}(t)$ to $G_{24n}(t)$ are obtained for the hollow cylinder.

4 RESULTS AND DISCUSSIONS

As an example, a solid cylinder with $r_i = 0$, $r_o = 1\,\text{m}$ is considered. The material properties are listed in Table 1. The initial temperature $T_\circ$ is considered to be 293°K. Now, an instantaneous hot spot $T(1,t) = 10^{-3} T_\circ \delta(t)$, where $\delta(t)$ is unit dirac function, is considered and the outside radius of the cylinder is assumed to be fixed (u(1, t) = 0). For plotting the graphs a nondimensional time $\hat{t} = \frac{vt}{r_\circ}$ is considered where $v = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$ is the dilatational -wave velocity. Figs. 1-3 show the wave-front for the displacement, temperature, and pressure. As a second example mechanical shock wave is applied to the outside surface of the cylinder given as $u(1,t) = 10^{-12} u_\circ \delta(t)$, and the surface is assumed to be at zero temperature ($T(1, t) = 0$). Figs. 4-6 show the wave fronts for the displacement and temperature distributions versus the nondimensional radius. The convergence of the solutions for these examples is achieved by consideration of 1200 eigenvalues used for the H-Fourier expansion. Choosing more than this number for eigenvalues, increases round-off and truncation errors affects the quality of the graphs.

Table 1
Material parameters [4]

Parameters	Value	Unit	Parameters	Value	Unit
N	0.4	-	α_s	1.5×10^{-5}	1/ °C
E	6×10^{5}	Pa	α_w	2×10^{-4}	1/ °C
ν	0.3	-	c_s	0.8	J/g °C
T_0	293	°K	c_w	4.2	J/g °C
K_s	2×10^{10}	Pa	ρ_s	2.6×10^{6}	g/m^3
K_w	5×10^{9}	Pa	ρ_w	1×10^{6}	g/m^3
K	0.5	W/m°C	α	1	-

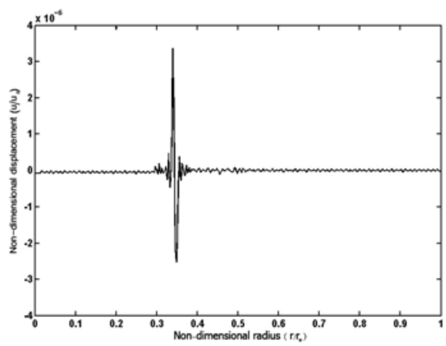

Fig. 1
Non-dimensional displacement distribution due to input $u(1,t) = 10^{-12} u_\circ \delta(t)$ at non-dimensional time $\hat{t} = 0.6$.

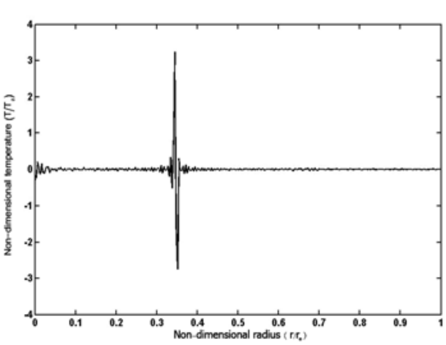

Fig. 2
Non-dimensional temperature distribution due to input $T(1,t) = 10^{-3} T_\circ \delta(t)$ at non-dimensional time $\hat{t} = 0.6$.

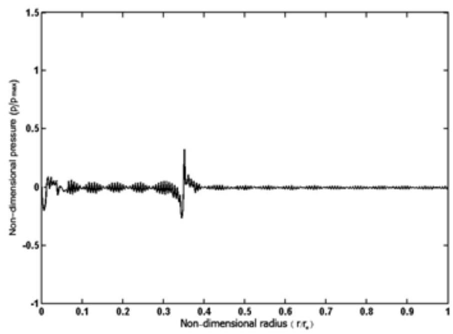

Fig. 3
Non-dimensional pressure distribution due to input $p(1,t)=10^{-3}p_{\circ}\delta(t)$ at non-dimensional time $\hat{t}=0.6$.

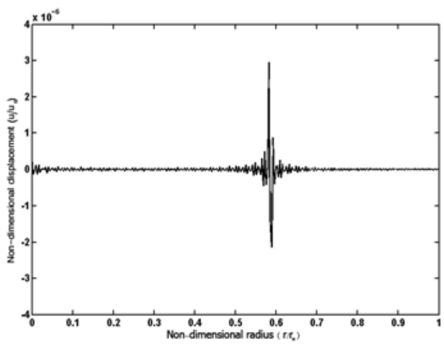

Fig. 4
Non-dimensional displacement distribution due to input $u(1,t)=10^{-12}u_{\circ}\delta(t)$ at non-dimensional time $\hat{t}=0.4$.

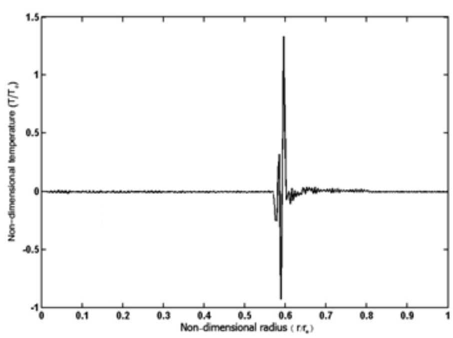

Fig. 5
Non-dimensional temperature distribution due to input $T(1,t)=10^{-3}T_{\circ}\delta(t)$ at non-dimensional time $\hat{t}=0.4$.

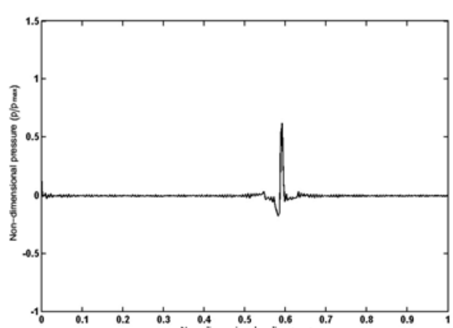

Fig. 6
Non-dimensional pressure distribution due to input $p(1,t)=10^{-3}p_{\circ}\delta(t)$ at non-dimensional time $\hat{t}=0.4$.

The convergence of the solution is better for the displacement results in comparison with the temperature. The small oscillations in Figs. (3) and (5) are due to the convergence errors of solutions.

5 CONCLUSION

In the present paper an analytical solution for the coupled thermoporoelasticity of thick cylinders under radial temperature is presented. The method is based on the eigenfunctions Fourier expansion, which is a classical and traditional method of solution of the typical initial and boundary value problems. The non-competetive strength of this method is its ability to reveal the fundamental mathematical and physical properties and interpretations of the problem under studying. In the coupled thermoporoelastic problem of radial-symmetric cylinder, the governing equations constitute a system of partial differential equations with two independent variables, radius (r) and time (t). The traditional procedure to solve this class of problems is to eliminate the time variable by using the Laplace transform. The resulting system is a set of ordinary differential equations in terms of the radius variable, whose solution falls in the Bessel functions family. This method of the analysis brings the Laplace parameter (s) in the argument of the Bessel functions, causing hardship or difficulties in carrying out the exact inverse of the Laplace transformation. As a result, the numerical inversion of the Laplase transformation is used in the papers dealing with this type of problems in literature. In the present paper, to prevent this problem, when the Laplace transform is applied to the particular solutions, it is postponed after eliminating the radius variable r by H-Fourier Expansion. Thus, the Laplace parameter (s) appears in polynomial function forms and hence the exact Laplace inversion transformation is possible.

6 ACKNOWLEDGMENT

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7 APPENDIX A

$$
\begin{aligned}
& d_1=-\alpha\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}, \qquad d_2=-\beta\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}, \qquad d_3=-\rho\frac{(1+\nu)(1-2\nu)}{(1-\nu)E} \\
& d_4=-Z\frac{T_\circ}{K}, \qquad d_5=Y\frac{T_\circ}{K}, \qquad d_6=-\beta\frac{T_\circ}{K}, \qquad d_7=-\alpha_p\frac{\gamma_w}{k}\frac{1}{M}, \qquad d_8=Y\frac{\gamma_w}{k} \\
& d_9=-\alpha\frac{\eta}{k}, \qquad d_{10}=-\frac{(1+\nu)(1-2\nu)}{(1-\nu)E}, \qquad d_{11}=-\frac{1}{K}, \qquad d_{12}=\frac{\eta}{K} \\
& d_{13}=\int_0^1 3r\,\mathrm{d}r, \qquad d_{14}=\int_0^1 2d_1r^2\mathrm{d}r, \qquad d_{15}=\int_0^1 2d_2r^2\mathrm{d}r, \qquad d_{16}=\int_0^1 d_3r^3\mathrm{d}r, \\
& d_{17}=\int_0^1 4r\,\mathrm{d}r, \qquad d_{18}=\int_0^1 3d_5r^2\mathrm{d}r, \qquad d_{19}=\int_0^1 2d_4r^3\mathrm{d}r, \qquad d_{20}=\int_0^1 d_5r^3\mathrm{d}r, \\
& d_{21}=\int_0^1 4r\,\mathrm{d}r, \qquad d_{22}=\int_0^1 3d_9r^2\mathrm{d}r, \qquad d_{23}=\int_0^1 2d_7r^3\mathrm{d}r, \qquad d_{24}=\int_0^1 d_8r^3\mathrm{d}r, \\
& d_{25}=\int_0^1 F(r)r\,\mathrm{d}r, \qquad d_{26}=\int_0^1 G(r)r\,\mathrm{d}r, \qquad d_{27}=\int_0^1 W(r)r\,\mathrm{d}r \\
& m_1=m_{12}=m_{16}=m_{20}=m_{26}=m_{33}=m_{38}=m_{48}=-\beta^2 \\
& m_2=m_{11}=-d_1\beta, \qquad m_3=-d_2\beta, \qquad m_4=m_{10}=d_3\lambda^2 \\
& m_5=\beta^2, \qquad m_6=d_1\beta, \qquad m_7=d_2\beta, \qquad m_8=-d_3\lambda^2 \\
& m_9=-\frac{4}{\beta}d_3\lambda^2, \qquad m_{13}=-\frac{2}{\beta}d_3\lambda^2, \qquad m_{14}=\frac{8}{\beta^2}d_3\lambda^2 \\
& m_{15}=-d_2\beta, \qquad m_{17}=m_{21}=m_{27}=d_7\lambda, \\
& m_{18}=m_{22}=m_{29}=d_9\lambda\beta, \qquad m_{19}=m_{23}=d_8\lambda
\end{aligned}
\tag{A.1}
$$

$$m_{24} = m_{25} = m_{35} = m_{48} = 2\beta, \qquad m_{28} = \frac{2}{\beta} d_7 \lambda$$

$$m_{30} = 2 d_9 \lambda, \qquad m_{31} = d_8 \lambda, \qquad m_{32} = \frac{2}{\beta} d_8 \lambda$$

$$m_{34} = d_5 \lambda, \qquad m_{36} = m_{40} = m_{44} = d_6 \lambda \beta$$

$$m_{37} = m_{41} = m_{46} = d_4 \lambda, \qquad m_{39} = m_{42} = d_5 \lambda$$

$$m_{43} = \frac{2}{\beta} d_5 \lambda, \qquad m_{45} = 2 d_6 \lambda, \qquad m_{47} = \frac{2}{\beta} d_4 \lambda$$

$$\zeta_1 = \frac{\dfrac{\left((-\beta^2 + d_7\lambda) - \dfrac{d_2 d_8 \lambda}{d_1}\right)\left(2\beta\zeta_{12} + (-\beta^2 + d_5\lambda)\dfrac{2 d_3 \lambda^2}{d_1 \beta^2}\right)}{(\beta^2 - d_5\lambda)\dfrac{d_2}{d_1} + d_4\lambda} - \left(\dfrac{2 d_5 d_8 \lambda^3}{d_1 \beta^2} + 2\zeta_6 \beta\right)}{\left(d_9 \lambda \beta - (\beta^2 - d_3\lambda^2)\dfrac{d_8 \lambda}{d_1 \beta}\right) - \dfrac{\left((-\beta^2 + d_7\lambda) - \dfrac{d_2 d_8 \lambda}{d_1}\right)\left(d_6 \lambda \beta - \dfrac{(-\beta^2 + d_5\lambda)(\beta^2 - d_3 \lambda^2)}{d_1 \beta}\right)}{(-\beta^2 + d_5\lambda)\dfrac{d_2}{d_1} + d_4\lambda}}$$

$$\zeta_2 = -\frac{(m_{20} + m_{21})}{m_{22}}\zeta_7 - \frac{m_{24}}{m_{22}}\zeta_8 - \frac{m_{23}}{m_{22}}\zeta_{13}$$

$$\zeta_3 = -\frac{m_9}{(m_5 + m_8)} - \frac{m_7}{(m_5 + m_8)}\zeta_8 - \frac{m_6}{(m_5 + m_8)}\zeta_{14}$$

$$\zeta_4 = \zeta_6 = \zeta_9 = \frac{\left(-\beta^2 + d_3 \lambda^2\right)}{d_2 \beta} - \frac{d_1}{d_2}\zeta_{10}$$

$$\zeta_5 = \frac{-\left(d_6 \lambda \beta - \dfrac{\left(-\beta^2 + d_5 \lambda\right)\left(\beta^2 - d_3 \lambda^2\right)}{d_1 \beta}\right)\zeta_1 - \left(2\beta\zeta_{12} + \left(-\beta^2 + d_5\lambda\right)\dfrac{2 d_3 \lambda^2}{d_1 \beta^2}\right)}{\left(\left(\beta^2 - d_5 \lambda\right)\dfrac{d_2}{d_1} + d_4 \lambda\right)}$$

$$\zeta_7 = \frac{\left(\dfrac{(m_{33} + m_{34})}{m_{36}} - \dfrac{m_{23}}{m_{22}}\right)\zeta_{13} + \left(\dfrac{m_{35}\zeta_{15}}{m_{36}} - \dfrac{m_{24}\zeta_8}{m_{22}}\right)}{\left(-\dfrac{m_{37}}{m_{36}} + \dfrac{(m_{20} + m_{21})}{m_{22}}\right)}$$

$$\zeta_8 = \frac{\left(\dfrac{(m_{30} + m_{32}\zeta_{15}) + (m_{25} + m_{28})\zeta_9}{m_{29}} - \dfrac{m_9}{m_5 + m_8}\right) + \left(\dfrac{m_{31}}{m_{29}} - \dfrac{m_6}{m_5 + m_8}\right)\zeta_{15}}{\left(-\dfrac{(m_{26} + m_{27})}{m_{29}} + \dfrac{m_7}{m_5 + m_8}\right)}$$

$$\zeta_{10} = \zeta_{12} = \zeta_{15} = \frac{d_6 \lambda \beta + \dfrac{d_4 \lambda}{d_2 \beta}(-\beta^2 + d_3 \lambda^2)}{\dfrac{d_1 d_4 \lambda}{d_2} + (\beta^2 - d_5 \lambda)}$$

$$\zeta_{11} = -\frac{(\beta^2 - d_3 \lambda^2)}{d_1 \beta}\zeta_1 + \frac{2 d_3 \lambda^2}{d_1 \beta^2} - \frac{d_2}{d_1}\zeta_5 \tag{A.2}$$

$$\zeta_{13}=\frac{-\frac{-\left(\frac{(m_{10}+m_{12})(m_{20}+m_{21})}{m_{22}}+m_{15}\right)\left(-\frac{m_{35}\zeta_{15}}{m_{36}}+\frac{m_{24}\zeta_8}{m_{22}}\right)}{\left(-\frac{m_{37}}{m_{36}}+\frac{m_{20}+m_{21}}{m_{22}}\right)}-\left(\frac{(m_{10}+m_{12})m_{24}\zeta_8}{m_{22}}+m_{31}\zeta_3+m_{14}\right)}{\frac{-\left(\frac{(m_{10}+m_{12})(m_{20}+m_{21})}{m_{22}}-m_{15}\right)\left(-\frac{(m_{35}+m_{34})}{m_{36}}+\frac{m_{23}}{m_{22}}\right)}{\left(\frac{m_{37}}{m_{36}}+\frac{(m_{20}+m_{21})}{m_{22}}\right)}+\left(-\frac{(m_{10}+m_{12})m_{23}}{m_{22}}+m_{11}\right)}$$

$$\zeta_{14}=\frac{\left(\frac{-m_{44}m_9}{m_5+m_8}+m_{45}+(m_{49}+m_{43})\zeta_{15}+m_{47}\zeta_9\right)}{\frac{\left(\frac{m_{44}m_7}{m_5+m_8}+m_{46}\right)\left(-\frac{m_{31}}{m_{29}}+\frac{m_6}{m_5+m_8}\right)}{\left(-\frac{m_{26}+m_{27}}{m_{29}}+\frac{m_7}{m_5+m_8}\right)}+\left(-\frac{m_{44}m_6}{m_5+m_8}+m_{48}+m_{42}\right)}$$

$$+\frac{\frac{\left(\frac{m_{44}m_7}{m_5+m_8}-m_{46}\right)\left(-\frac{((m_{30}+m_{32}\zeta_{15})+(m_{25}+m_{28})\zeta_9)}{m_{29}}+\frac{m_9}{m_5+m_8}\right)}{\left(-\frac{m_{26}+m_{27}}{m_{29}}+\frac{m_7}{m_5+m_8}\right)}}{\frac{\left(\frac{m_{44}m_7}{m_5+m_8}+m_{46}\right)\left(-\frac{m_{31}}{m_{29}}+\frac{m_6}{m_5+m_8}\right)}{\left(-\frac{m_{26}+m_{27}}{m_{29}}+\frac{m_7}{m_5+m_8}\right)}+\left(-\frac{m_{44}m_6}{m_5+m_8}+m_{48}+m_{42}\right)}$$

$$\zeta_{16}=-[-J_1(\beta)(\zeta_7J_0(\beta)+\zeta_8J_1(\beta)+\zeta_9J_2(\beta))+J_0(\beta)(\zeta_2J_1(\beta)+\zeta_3J_2(\beta)+J_3(\beta))]/$$
$$[-(\zeta_1J_1(\beta)+J_2(\beta))(\zeta_7J_0(\beta)+\zeta_8J_1(\beta)+\zeta_9J_2(\beta))$$
$$+(\zeta_5J_0(\beta)+\zeta_6J_1(\beta))(\zeta_2J_1(\beta)+\zeta_3J_2(\beta)+J_3(\beta))]$$

$$\zeta_{17}=\frac{J_1(\beta)+(\zeta_1J_1(\beta)+\zeta_{16}J_2(\beta))}{\zeta_2J_1(\beta)+\zeta_3J_2(\beta)+J_3(\beta)}$$

$$C_0=1+\zeta_{16}+\zeta_{17},\qquad C_1=\zeta_1\zeta_{16}+\zeta_2\zeta_{17},\qquad C_2=\zeta_3\zeta_{17},$$
$$E_0=\zeta_4+\zeta_5\zeta_{16}+\zeta_7\zeta_{17},\qquad E_1=\zeta_6\zeta_{16}+\zeta_8\zeta_{17},\qquad E_2=\zeta_9\zeta_{17},$$
$$D_0=\zeta_{10}+\zeta_{11}\zeta_{16}+\zeta_{13}\zeta_{17},\qquad D_1=\zeta_{12}\zeta_{16}+\zeta_{14}\zeta_{17},\qquad D_2=\zeta_{15}\zeta_{17},$$
$$H_0=C_0J_1(\beta r)+C_1rJ_2(\beta r)+C_2r^2J_3(\beta r),\qquad H_1=E_0J_0(\beta r)+E_1rJ_1(\beta r)+E_2r^2J_2(\beta r)$$
$$H_2=D_0J_0(\beta r)+D_1rJ_1(\beta r)+D_2r^2J_2(\beta r)\tag{A.3}$$

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