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Effect of Initial Stress on Propagation of Love Waves in an Anisotropic Porous Layer

S. Gupta*, A. Chattopadhyay, D.K. Majhi

Department of Applied Mathematics, Indian School of Mines, Dhanbad-826 004, India

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ABSTRACT

In the present paper, effect of initial stresses on the propagation of Love waves has been investigated in a fluid saturated, anisotropic, porous layer lying in welded contact over a prestressed, non-homogeneous elastic half space. The dispersion equation of phase velocity has been derived. It has been found that the phase velocity of Love waves is considerably influenced by porosity and anisotropy of the porous layer, inhomogeneity of the half-space and prestressing present in the media, the layer and the half-space. The effect of the medium characteristics on the propagation of Love waves has been discussed and results of numerical calculations have been presented graphically.

Keywords: Love wave; Anisotropic; Initial stress; Dispersion equation; Phase velocity

1 INTRODUCTION

SURFACE waves propagating over the surface of homogeneous and inhomogeneous elastic half-spaces are a well-known and prominent feature of wave theory. Surface waves carry the greatest amount of energy from shallow shocks and are of primary cause of destruction that can result from earthquakes. Biot [1-4] has first discussed the theory of propagation of elastic waves in a statically isotropic, fluid saturated, porous solid. Using the Biot's theory, several investigators (Deresiewicz H. [5-9]; Bose [10]; Deresiewicz and Rice [11]; Rao and Sarma [12]; Burridge and Vargas [13]) have studied extensively the propagation of surface waves such as Rayleigh and Love waves. Love [14] developed a mathematical model of surface waves known as Love waves. He predicted the existence of Love waves mathematically in 1911. The study of Love wave is important to seismologist for its possible application in prediction of earth structure and analysis of earth-quakes. Nowinski [15] has studied the effect of high initial stresses on Love waves in an isotropic, elastic, and incompressible medium. The naturally occurring medium is often porous and liquid filled. The size of pores is assumed to be small and, macroscopically speaking, their average distribution is uniform. The role of pore-water in seismology has been emphasized in many studies. The diffusion of water and readjustment of fluid pressure has been in vague as a triggering mechanism for earthquakes. Chattopadhyay and De [16] have studied the propagation of Love waves in a porous layer underlain by isotropic elastic medium with a rectangular irregularity at the interface.

The dispersion of love waves can be used to refine our knowledge of crustal and sub-crustal region near the earth surface. The earth's crust and subcrust can be anisotropic in nature, in addition to being porous. The most satisfactory theory has been given by Weiskopf [17], who considered the medium to have a particular kind of transverse-isotropy. Furthermore, the earth is an initially stressed medium. Due to the presence of external loading, variation in temperature, slow process of creep, and gravitational field, a considerable amount of stresses (called prestresses or initial stresses) remain naturally present in the layers. These stresses have much effect on the propagation of elastic waves (Biot [18]). It is, therefore, interesting to study the Love wave propagation in a medium which is formed by an initially stressed, anisotropic, porous layer lying over an elastic half-space under initial stress.

* Corresponding author.
E-mail address: shishir_ism@yahoo.com.

The half-space could be nonhomogeneous because non-homogeneity characteristics is one of the most generalized elastic condition inside the earth. Recently, Dey, Roy and Dutta [19] have investigated the propagation of Love waves in an initially stressed anisotropic porous layer lying over a pre-stressed non-homogeneous elastic half-space. In this paper [19], exponential variation of rigidity and density has been considered. Linear variation of rigidity and density has been taken and the effect of initial stress, anisotropy and porosity on the propagation of Love waves is studied. Ghasemi et al. [20, 21] described several ground-motion models for seismic hazard analysis. Keeping in mind the above ideas and using the theories of Biot ([3, 18]) and Weiskopf [17], the propagation of Love waves in a water-saturated, anisotropic, initially stressed, porous layer lying in welded contact over a prestressed, nonhomogeneous half-space has been studied in the present analysis. The fluid inside the pores could be viscous. But in order to keep the analysis simple here, the material is assumed to be non-dissipative liquid filled porous solid; for instance, the water saturated rocks or grains. The porous medium is assumed to have kinetic isotropy, but elastic anisotropy of Weiskopf type; the viscosity of water is neglected. The inhomogeneity in the half-space has been assumed to be of the form

$$\rho=\rho_0(1+bz) \text{ and } \mu=\mu_0(1+az)$$

where a and b are constants having dimension that are inverse of length, density ρ and rigidity μ vary linearly with space variable z, which is orthogonal to the x-axis i.e. direction of wave. It is also assumed that the shear velocity in the layer is constant and independent of the depth z. The dispersion equation of Love waves under such conditions has been derived. It has been observed that the phase velocity of Love waves is considerably influenced by porosity and anisotropy of the porous medium, inhomogeneity of the half-space and prestressing in both media, i.e., the layer and the half space. The effect of these characteristics of the media on the phase velocity of Love waves has been discussed and compared with the classical case when both media, i.e. the layer and the half-space, are homogeneous, isotropic and initially stress-free. The numerical values of the phase velocity have been calculated using the values of material constants given by Biot [4] from experiments.

2 FORMULATION OF THE PROBLEM

Consider a water saturated porous layer of thickness H with anisotropy of the Weiskopf type under the compressive initial stress $P'=-S'_{11}$ along the direction of x over a non-homogeneous elastic half space under the compressive initial stress $P=-S_{11}$ as shown in Fig. 1. The surface of contact is the plane z=0, and the z-axis is directed vertically downwards. The wave is assumed to propagate along the x-direction. Neglecting the viscosity of water, the dynamic equations of motion in the porous layer under the compressive initial stress P', in the absence of body forces, can be written as (Biot[2, 3] and Biot[18])

$$\begin{aligned}
&\frac{\partial s'_{11}}{\partial x}+\frac{\partial s'_{12}}{\partial y}+\frac{\partial s'_{13}}{\partial z}-P'\frac{\partial \omega'_z}{\partial y}+P'\frac{\partial \omega'_y}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{11}u'_x+\rho_{12}U_x)\\
&\frac{\partial s'_{21}}{\partial x}+\frac{\partial s'_{22}}{\partial y}+\frac{\partial s'_{23}}{\partial z}-P'\frac{\partial \omega'_z}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{11}v'_y+\rho_{12}V_y)\\
&\frac{\partial s'_{31}}{\partial x}+\frac{\partial s'_{32}}{\partial y}+\frac{\partial s'_{33}}{\partial z}-P'\frac{\partial \omega'_y}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{11}w'_z+\rho_{12}W_z)\\
&\frac{\partial S}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{12}u'_x+\rho_{22}U_x),\frac{\partial S}{\partial y}=\frac{\partial^2}{\partial t^2}(\rho_{12}v'_y+\rho_{22}V_y),\frac{\partial S}{\partial z}=\frac{\partial^2}{\partial t^2}(\rho_{12}w'_z+\rho_{22}W_z)
\end{aligned} \tag{1}$$

where s'_{ij} $(i,j=1,2,3)$ are the incremental stress components, (u'_x,v'_y,w'_z) are the components of the displacement vector of the solid, (U_x,V_y,W_z) are the components of the displacement vector of the liquid, S is the stress vector due to the liquid

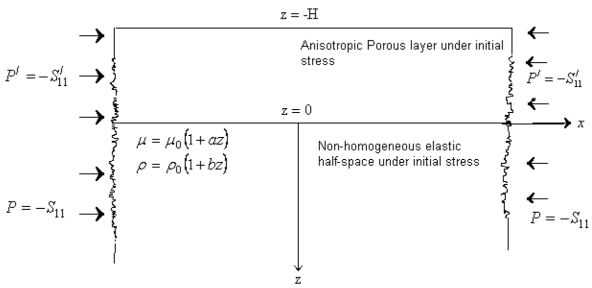

Fig.1
Geometry of the problem.

$$\omega_x^{/} = \frac{1}{2}\left(\frac{\partial w_z^{/}}{\partial y} - \frac{\partial v_y^{/}}{\partial z}\right),\ \omega_y^{/} = \frac{1}{2}\left(\frac{\partial u_x^{/}}{\partial z} - \frac{\partial w_z^{/}}{\partial x}\right),\ \omega_z^{/} = \frac{1}{2}\left(\frac{\partial v_y^{/}}{\partial x} - \frac{\partial u_x^{/}}{\partial y}\right) \tag{2}$$

where $\omega_x^{/}$, $\omega_y^{/}$, and $\omega_z^{/}$ are the components of the rotational vector $\omega^{/}$.

The stress-strain relations for the water saturated anisotropic porous layer under the normal initial stress $P^{/}$ are

$$\begin{aligned} s_{11}^{/} &= (A+P^{/})e_{xx} + (A-2N+P^{/})e_{yy} + (F+P^{/})e_{zz} + Q\varepsilon \\ s_{22}^{/} &= (A-2N)e_{xx} + Ae_{yy} + Fe_{zz} + Q\varepsilon \\ s_{33}^{/} &= Fe_{xx} + Fe_{yy} + Ce_{zz} + Q\varepsilon \\ s_{12}^{/} &= 2Ne_{xy}\ ,\ s_{23}^{/} = 2Le_{yz}\ ,\ s_{13}^{/} = 2Le_{zx} \end{aligned} \tag{3a}$$

where A, F, C, N, and L are elastic constants for the medium; N and L are, in particular, shear moduli of the anisotropic layer in the x and z direction respectively, and

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),\ \ \varepsilon = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z} \tag{3b}$$

Further, Q being the measure of coupling between the volume change of the solid and the liquid is a positive quantity, and S is the stress vector due to the liquid. This stress vector S is related to the fluid pressure p by the relation

$$-S=fp \tag{4}$$

where f is porosity of the layer. The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} are related to the densities $\rho^{/}$, ρ_s, ρ_w of the layer, solid and water, respectively, by (Biot[3])

$$\rho_{11} + \rho_{12} = (1-f)\rho_s,\ \rho_{12} + \rho_{22} = f\rho_w \tag{5}$$

So that the mass density of the aggregate is

$$\rho^{/} = \rho_{11} + 2\rho_{12} + \rho_{22} = \rho_s + f(\rho_w - \rho_s) \tag{6}$$

These mass coefficients also obey the following inequalities

$$\rho_{11} > 0,\ \ \rho_{22} > 0,\ \ \rho_{12} < 0,\ \ \rho_{11}\rho_{22} - \rho_{12}^2 > 0 \tag{7}$$

The dynamic Eqs. (1) have been constructed by coupling the Biot's dynamic equations in an initially stressed medium (Biot [18]) and his dynamic equations for a poro-elastic medium (Biot [2, 3]).

3 SOLUTIONS

3.1 Solution for the upper layer

For the Love waves propagating along the *x*-direction, having the displacement of particles along the *y*-direction, we have

$$u_x^{/} = 0,\ \ w_z^{/} = 0 \ \text{ and } \ \mathrm{v}_\mathrm{y}^{/} = v^{/}(x,z,t)$$
$$U_x = 0,\ \ W_z = 0 \ \text{ and } \ \mathrm{V}_\mathrm{y} = V(x,z,t) \tag{8}$$

These displacements will produce only the e_{yz} and e_{xy} strain components and the other strain components will be zero. Hence, the stress-strain relations useful in the problem are

$$s_{23}^{/} = 2Le_{yz} \ \text{ and } \ s_{12}^{/} = 2Ne_{xy} \tag{9a}$$

Introducing relations (9a) into system (1), the equations of motion which are not automatically satisfied are

$$\frac{\partial s_{21}^{/}}{\partial x} + \frac{\partial s_{22}^{/}}{\partial y} + \frac{\partial s_{23}^{/}}{\partial z} - P^{/}\frac{\partial \omega_z^{/}}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{11}v_y^{/} + \rho_{12}V_y)$$
$$\frac{\partial \mathrm{S}}{\partial \mathrm{y}} = \frac{\partial^2}{\partial t^2}(\rho_{12}v_y^{/} + \rho_{22}V_y) \tag{9b}$$

Since $\mathrm{v}_\mathrm{y}^{/} = v^{/}(x,z,t)$ and $V_y = V(x,z,t)$, with the help of Eqs. (8) and (9a), the above two equations reduce to

$$(N - \frac{P^{/}}{2})\frac{\partial^2 v^{/}}{\partial x^2} + L\frac{\partial^2 v^{/}}{\partial z^2} = \frac{\partial^2}{\partial t^2}(\rho_{11}v^{/} + \rho_{12}V) \tag{10}$$

$$\frac{\partial^2}{\partial t^2}(\rho_{12}v^{/} + \rho_{22}V) = 0 \tag{11}$$

which gives $V = (d^{//} - \rho_{12}v^{/}) / \rho_{22}$, where $d^{//} = \rho_{12}v^{/} + \rho_{22}V$. Now, $\partial^2 / \partial t^2(\rho_{11}v^{/} + \rho_{12}V) = d^{/}(\partial^2 v^{/} / \partial t^2)$, where $d^{/} = \rho_{11} - (\rho_{12}^2 / \rho_{22})$. Hence, Eq. (10) reduces to

$$\left(N - \frac{P^{/}}{2}\right)\frac{\partial^2 v^{/}}{\partial x^2} + L\frac{\partial^2 v^{/}}{\partial z^2} = d^{/}\frac{\partial^2 v^{/}}{\partial t^2} \tag{12}$$

From Eq. (12), it is clear that the velocity of shear wave along the *x*-direction is $[(N - P^{/}/2)/d^{/}]^{0.5}$ and that along the *z*-direction it is $(L/d^{/})^{0.5}$. The shear wave velocity in the porous medium along the *x*-direction can be expressed as

$$\beta' = \sqrt{(N - P'/2)/d'} = \beta_a\sqrt{1 - \xi'/d} \tag{13}$$

where $d = \gamma_{11} - (\gamma_{12}^2 / \gamma_{22})$, $\beta_a = \sqrt{N/\rho'}$ is the velocity of shear wave in the corresponding initial stress-free, non-porous, anisotropic, elastic medium along the direction of *x*, $\xi^{/} = P^{/}/2N$ is the non-dimensional parameter due to the initial stress $P^{/}$ and

$$\gamma_{11} = \frac{\rho_{11}}{\rho^{/}},\ \gamma_{12} = \frac{\rho_{12}}{\rho^{/}},\ \gamma_{22} = \frac{\rho_{22}}{\rho^{/}} \tag{14}$$

where γ_{11}, γ_{12}, and γ_{22} are the non-dimensional parameters for the material of the porous layer as obtained by Biot [2]. For the Love wave propagating along the *x*-direction, the solution of Eq. (12) may be taken as

$$v'(x,z,t) = V(z)\, e^{ik(x-ct)} \tag{15}$$

Using (15) in Eq. (12), we get

$$\frac{d^2V}{dz^2} + q'^2 V = 0 \tag{16}$$

where $q'^2 = \frac{k^2}{L}\left[c^2 d' - \left(N - \frac{P'}{2}\right)\right]$. Solution of Eq. (16) is $V = A\, e^{iq'z} + B\, e^{-iq'z}$. Hence, Eq. (15) gives

$$v' = (Ae^{iq'z} + Be^{-iq'z})e^{ik(x-ct)}, \quad -H \le z \le 0 \tag{17a}$$

where

$$q' = k\sqrt{\frac{c^2 d' - (N - P'/2)}{L}} = k\sqrt{\gamma d\left\{\frac{c^2}{(N/d')d} - \frac{1-\xi'}{d}\right\}} = k\sqrt{\gamma d\left\{\frac{c^2}{\beta_a^2} - \frac{1-\xi'}{d}\right\}} \tag{17b}$$

in which $\gamma = N/L$, $\xi' = P'/2N$, where *k* is wave number.

3.2 Solution for the half-space

The lower medium is considered as non-homogeneous elastic half space. The equation of motion corresponding to the displacement due to Love waves can be written as (Biot [18])

$$\frac{\partial s_{21}}{\partial x} + \frac{\partial s_{23}}{\partial z} - \frac{P}{2}\left(\frac{\partial^2 v}{\partial x^2}\right) = \frac{\partial^2}{\partial t^2}(\rho\, v) \tag{18}$$

where s_{ij} are the incremental stress components in the half-space, *P* is the initial compressive stress along the *x*-direction, and ρ is the density of the material of the half-space. The non-homogeneity in medium are taken as

$$\mu = \mu_0(1+az), \quad \rho = \rho_0(1+bz) \tag{19}$$

where μ_0 and ρ_0 are the values of μ and ρ at *z* =0, and *a*, *b* are constants having dimensions that are inverse of length. Using the stress-strain relations

$$s_{21} = 2\mu e_{xy}, \qquad s_{23} = 2\mu e_{yz} \tag{20}$$

and the relation (19), the equations of motion (18) can be written as

$$\left[1 - \frac{P}{2\mu_0(1+az)}\right]\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + \frac{a}{1+az}\frac{\partial v}{\partial z} = \frac{\rho_0(1+bz)}{\mu_0(1+az)}\frac{\partial^2 v}{\partial t^2} \tag{21}$$

Let $v = V(z)\, e^{ik(x-ct)}$ be the solution of (21), then Eq. (21) gives

$$\frac{d^2V}{dz^2}+\frac{a}{1+az}\frac{dV}{dz}+\left[\frac{\rho_0(1+bz)}{\mu_0(1+az)}c^2-\left\{1-\frac{P}{2\mu_0(1+az)}\right\}\right]k^2V=0 \tag{22}$$

Now then, we put $V=\varphi(z)/(1+az)^{1/2}$ in Eq. (22) to eliminate the term d*V*/d*z*, we get

$$\phi''(z)+\left[\frac{a^2}{4(1+az)^2}-k^2\left\{\left(1-\frac{P}{2\mu_0(1+az)}\right)-\frac{c^2}{c_{os}^2}\frac{(1+bz)}{(1+az)}\right\}\right]\phi(z)=0 \tag{23}$$

where $c_{os}=\sqrt{\mu_0/\rho_0}$ and c is phase-velocity.

Now putting $\gamma_1=\left[1-\frac{P}{2\mu_0(1+az)}-\frac{c^2}{c_{os}^2}\frac{b}{a}\right]^{0.5}$, $\eta=\frac{2\gamma_1k\left(1+az\right)}{a}$, $\omega=kc$ in Eq. (23) we get

$$\frac{d^2\phi}{d\eta^2}+\left[\frac{R}{2\eta}+\frac{1}{4\eta^2}-\frac{1}{4}\right]\phi(\eta)=0 \tag{24}$$

where $R=\frac{\omega^2(a-b)}{c_{os}^2a^2\gamma_1k}$

The solution of Whittaker's Eq. (24) is given by $\phi(\eta)=D_1W_{R/2,0}(\eta)+D_2W_{-R/2,0}(-\eta)$, where D_1 and D_2 are arbitrary constants and $W_{R/2,0}(\eta)$ is the Whittaker's function. The solution of Eq. (24) satisfying the condition $\lim z\to\infty$ when $V(z)\to 0$ i.e. lim $\eta\to\infty$ when $\varphi(\eta)\to 0$ may be taken as

$$\phi(\eta)=D_1W_{R/2,o}(\eta) \tag{25}$$

Hence, the displacement component in the heterogeneous medium is given by

$$v=V(z)\,e^{ik(x-ct)}=\frac{D_1W_{R/2,0}(\eta)}{(1+az)^{0.5}}e^{ik(x-ct)} \tag{26}$$

Expanding Whittaker's function up to linear terms eq. (26) reduce to

$$v=D_1e^{\frac{-\gamma_1k(1+az)}{a}}\left\{\frac{2\gamma_1(1+az)}{a}\right\}^{R/2}\left[\frac{1}{(1+az)^{1/2}}-\frac{(R/2-1/2)^2a}{2\gamma_1k(1+az)^{3/2}}\right]e^{ik(x-ct)} \tag{27}$$

4 BOUNDARY CONDITIONS AND DISPERSIVE EQUATION

The boundary conditions suitable for the problem are

$\Delta f_y'=0$ at $z=-H$

$\Delta f_y'=\Delta f_y$ at $z=0$

$v'=v$ at $z=0$

where $\Delta f_y'$ and Δf_y are the incremental boundary forces per unit initial area (Biot [18]), which in the present problem take the form $\Delta f_y'=s_{23}'$ and $\Delta f_y=s_{23}$. Now, using $\Delta f_y'=0$ at $z=-H$, i.e., $s_{23}'=0$ at $z=-H$, we get

$$Ae^{-\mathrm{i}q^{/}H} - Be^{\mathrm{i}q^{/}H} = 0 \tag{28}$$

Using $\Delta f_y^{/} = \Delta f_y$ at $z = 0$, i.e., $s_{23}^{/} = s_{23}$ at $z = 0$, we get

$$\mathrm{i}q^{/} L(A - B) = D_1 K_1 \tag{29}$$

where

$$K_1 = \mu_0 \mathrm{e}^{\frac{-\gamma_1 k}{a}} \left\{\frac{2\gamma_1 k}{a}\right\}^{R/2} \left[\left\{\left(\frac{R-1}{2}\right) - \frac{(R/2-1/2)^2 a}{2\gamma_1 k}\left(\frac{R-3}{2}\right)\right\} a - \gamma_1 k \left\{1 - \frac{(R/2-1/2)^2 a}{2\gamma_1 k}\right\}\right]$$

Using $v^{/} = v$ at $z = 0$, we get

$$A + B = D_1 K_2 \tag{30}$$

where

$$K_2 = \mathrm{e}^{\frac{-\gamma_1 k}{a}} \left\{\frac{2\gamma_1 k}{a}\right\}^{R/2} \left[1 - \frac{(R/2-1/2)^2 a}{2\gamma_1 k}\right]$$

Eliminating A, B, and D_1 from Eqs. (28), (29) and (30) we get

$$\begin{vmatrix} \mathrm{e}^{-\mathrm{i}q^{/}H} & -\mathrm{e}^{\mathrm{i}q^{/}H} & 0 \\ \mathrm{i}q^{/}L & -\mathrm{i}q^{/}L & -K_1 \\ 1 & 1 & -K_2 \end{vmatrix} = 0$$

or

$$\tan\left[\left\{\gamma \mathrm{d}\left(\frac{\mathrm{c}^2}{\beta_a^2} - \frac{1-\xi^{/}}{d}\right)\right\}^{1/2} kH\right] = \frac{\mu_0}{Lk\left[\gamma d\left(\frac{c^2}{\beta_a^2} - \frac{1-\xi^{/}}{d}\right)\right]^{1/2}} \left[\frac{\gamma_1 k\left\{1 - \frac{\left(\frac{R}{2}-0.5\right)^2 a}{2\gamma_1 k}\right\} - a\left\{\left(\frac{R-1}{2}\right) - \frac{\left(\frac{R}{2}-0.5\right)^2 a}{2\gamma_1 k}\left(\frac{R-3}{2}\right)\right\}}{1 - \frac{\left(\frac{R}{2}-0.5\right)^2 a}{2\gamma_1 k}}\right] \tag{31}$$

where $\gamma_1 = \left[1 - \frac{\xi}{1+az} - \frac{c^2}{c_{os}^2}\frac{b}{a}\right]^{1/2}$, $\xi = \frac{P}{2\mu_0}$, c is the phase velocity of Love waves and $R = \frac{\omega^2(a-b)}{c_{os}^2 a^2 \gamma_1 k}$.

Eq. (31) is the dispersive equation for Love waves in the initially stressed, water saturated, anisotropic porous layer over the initially stressed, nonhomogeneous, elastic half space. The presence of a, b, $\xi^{/}$, ξ, γ and d in Eq. (31) shows that the Love wave is affected by the parameters under study. It should be noted that (1-d) gives the fraction of porosity in the layer. If the layer is non-porous then $f \to 0$ and hence $\rho_s \to \rho^{/}$. From relation (5) one gets $\gamma_{11} + \gamma_{12} \to 1$ and $\gamma_{12} + \gamma_{22} \to 0$, which leads to $(\gamma_{11} - \gamma_{12}^2/\gamma_{22}) \to 1$, i.e. $d \to 1$. Again, if $f \to 1$, then $\rho_w \to \rho^{/}$ and the layer becomes a fluid, and in that case the shear velocity in the layer cannot exist which happens when $\gamma_{11}-\gamma_{12}^2/\gamma_{22}$, i.e., $d\to 0.$ Thus we have:

(I) $d\to 1,$ when the layer is non-porous

(II) $d\to 0,$ when the layer tends to be fluid

(III) $0<d<1,$, when the layer is porous

5 PARTICULAR CASES

(I) In the case when both the layer and half-space are free from initial stresses, i.e. $\xi^{/}=\xi=0$, the dispersive eq. (31) takes the form

$$\tan\left[\left\{\gamma d\left(\frac{c^2}{\beta_a^2}-\frac{1}{d}\right)\right\}^{1/2}kH\right]=\frac{\mu_0}{L}\frac{\left[\gamma_1\left\{1-\frac{\left(\frac{R}{2}-0.5\right)^2 a}{2\gamma_1 k}\right\}-\frac{a}{k}\left\{\left(\frac{R-1}{2}\right)-\frac{\left(\frac{R}{2}-0.5\right)^2 a}{2\gamma_1 k}\left(\frac{R-3}{2}\right)\right\}\right]}{\left[\gamma d\left(\frac{c^2}{\beta_a^2}-\frac{1}{d}\right)\right]^{1/2}\left[1-\frac{a\left(\frac{R}{2}-0.5\right)^2}{2\gamma_1 k}\right]}$$

where $\gamma_1=\left[1-\frac{c^2}{c_{os}^2}\frac{b}{a}\right]^{1/2}$

(II) In the case when the layer is non-porous, i.e. $f=0$, then d=1, then the dispersion Eq. (31) takes the form

$$\tan\left[\left\{\gamma\left(\frac{c^2}{\beta_a^2}-(1-\xi^{/})\right)\right\}^{1/2}kH\right]=\frac{\mu_0}{L}\frac{\left[\gamma_1\left\{1-\frac{\left(\frac{R}{2}-0.5\right)^2 a}{2\gamma_1 k}\right\}-\frac{a}{k}\left\{\left(\frac{R-1}{2}\right)-\frac{\left(\frac{R}{2}-0.5\right)^2 a}{2\gamma_1 k}\left(\frac{R-3}{2}\right)\right\}\right]}{\left\{\gamma\left(\frac{c^2}{\beta_a^2}-(1-\xi^{/})\right)\right\}^{1/2}\left\{1-\frac{a\left(\frac{R}{2}-0.5\right)^2}{2\gamma_1 k}\right\}}$$

This is dispersive equation of Love waves in an initially stressed, non-porous anisotropic medium.

(III) If the lower half-space is homogeneous, i.e. $a\to 0$ and $b\to 0$ then the Eq. (31) is reduced to

$$\tan\left\{\left[\gamma\left\{\frac{c^2}{\beta_a^2}-(1-\xi^{/})\right\}\right]^{1/2}kH\right\}=\frac{\mu_0}{L}\frac{\left[1-\xi-\frac{c^2}{c_{os}^2}\right]^{1/2}}{\left[\gamma\left\{\frac{c^2}{\beta_a^2}-(1-\xi^{/})\right\}\right]^{1/2}}$$

which for an initial stress-free, isotropic, homogeneous, elastic layer (i.e. $\xi^{/}=0,\ \gamma=1,\ N=L=\mu^{/}$) lying over an initial stress-free, isotropic, homogeneous half-space (i.e. ξ=0, b=0, a=0) is reduced to

$$\tan\left[\left\{\frac{c^2}{\beta_a^2}-1\right\}^{1/2} kH\right]=\frac{\mu_0}{\mu^{/}}\frac{\left[1-\frac{c^2}{c_{os}^2}\right]^{1/2}}{\left[\frac{c^2}{\beta_a^2}-1\right]^{1/2}}$$

which is well-known classical result.

6 EXISTENCE OF LOVE WAVES

From Eq. (31), it follows that Love waves can propagate in the porous layer if

$$\sqrt{\frac{1-\xi^{/}}{d}}\beta_a < c < \sqrt{1-\xi+\frac{a}{k}.kH}\sqrt{\frac{a}{b}\frac{1}{1+\frac{a}{k}.kH}}c_{os} \tag{32}$$

Relation (32) indicates the role of initial stresses, nonhomogeneity and porosity of the media for the existence and non-existence of Love waves. From this condition, particular cases can be obtained as follows

6.1 Particular cases

(i) When both the layer and the half-space are free from initial stresses, i.e., $\xi^{/}=\xi=0,$ then condition (32) takes the form

$$\frac{1}{d}<\frac{c^2}{\beta_a^2}<\frac{1}{\frac{\beta_a^2}{c_{os}^2}}\left(\frac{a}{b}\right) \tag{33}$$

This shows that the propagation of Love waves depends on both the porosity and the ratio of shear velocities in the layer and in the half-space. The propagation also depends on the nonhomogeneity of the half-space. It is clear from (33) that as d decreases, i.e. as the porosity of the layer increases, the phase velocity of Love waves increases.
(ii) When the porosity and β_a / c_{os} are constant and $a/k \to 0,\ b/k \to 0$ then (32) takes the form

$$\frac{1-\xi^{/}}{d}<\frac{c^2}{\beta_a^2}<\frac{1}{\frac{\beta_a^2}{c_{os}^2}}(1-\xi) \tag{34}$$

Taking $\beta_a / c_{os} = 0.7$ and d=0.6154 as a particular case, relation (34) is changed to

$$(1-\xi^{/})1.625<\frac{c^2}{\beta_a^2}<2.041(1-\xi) \tag{35}$$

Hence, the propagation is favoured by positive value of $\xi^{/}$ and negative value of ξ, i.e. compressive initial stress in the porous layer and tensile initial stress in the half-space.
(iii) If $\xi^{/}=0,\ \xi=0,\ a\to 0,\ b\to 0$ and d=1 then inequality (32) takes the form $\beta_a<c<c_{os}$ which agrees with the well-known classical result.

7 RESULTS AND DISCUSSION

To show the effect of porosity, anisotropy, nonhomogeneity and initial stresses on the propagation of Love waves, numerical computations of eq. (31) were performed with different values of the parameters representing the above characteristics. The values of μ_0 / L and β_a / c_{os} were taken -in all these calculations- as 2.5 and 0.7, respectively. The results are presented in Figs. 2-7. Fig. 2 shows the effect of porosity on the propagation of Love waves in a nonhomogeneous, anisotropic, initially stress free medium. It has been observed that as the porosity increases, i.e. as μ_0 the value of d $(=\gamma_{11} - \gamma_{12}^2 / \gamma_{22})$ decreases, the velocity of Love waves increases. This effect becomes more prominent for higher frequencies. Fig. 3 shows the effect of anisotropy on the propagation of Love waves in a porous medium. The square of the velocity of Love waves in a medium corresponding to $\gamma_{11} = 0.65$, $\gamma_{12} = -0.15$, and $\gamma_{22} = 0.65$, i.e. d=0.6154 in homogeneous or nonhomogeneous case, is always greater than 1.625 for all frequencies. It has been observed that as the anisotropy increases, the velocity of Love waves in the porous medium decreases. We now turn our attention to show the effect of initial stresses present in the porous layer. The conditions for propagation of Love waves under initial stresses have already been discussed in connection with Eq. (35). Fig. 4 shows the effect of initial stresses (ξ') present in the porous layer while the half-space is initially stress-free, i.e. $\xi = 0$. In comparison with curve2 ($\xi' = 0$ and ξ=0) it has been observed from curves 3-6 that an increase in compressive initial stresses $(\xi' > 0)$ in the porous layer decreases the velocity of Love waves for the same frequency. The tensile initial stresses $(\xi' < 0)$ of small magnitude in the porous medium, however, increase the velocity (curve-1), but the large magnitude of tensile stress ξ' does not allow Love waves to propagate.

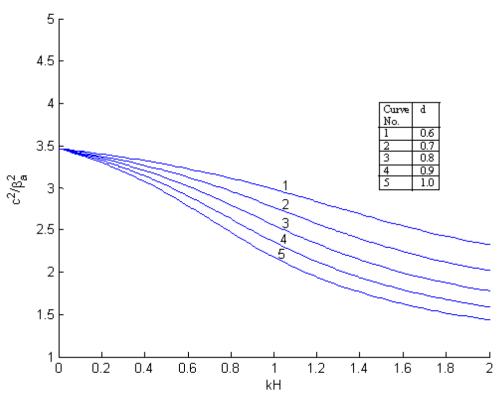

Fig. 2
Velocity dispersion curves of Love waves in a porous layer for various porosity in terms of parameter d and for $\mu_0 / L = 2.50$, $\gamma = 1$, b/k=0.4, a/k=1, $\xi = \xi' = 0$, and $\beta_a / c_{os} = 0.7$.

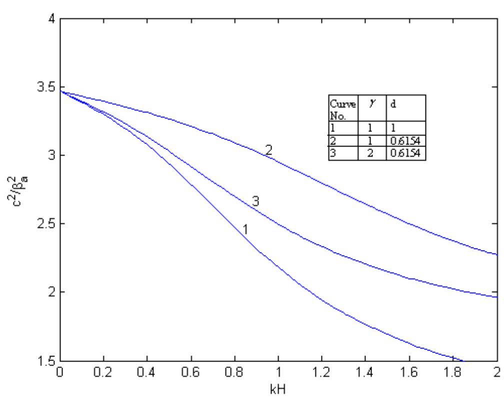

Fig. 3
Effect of anisotropy (γ) in the porous layer on the propagation of Love waves for $\mu_0 / L = 2.50$, b/k=0.4, a/k=1, $\xi = \xi' = 0$, and $\beta_a / c_{os} = 0.7$.

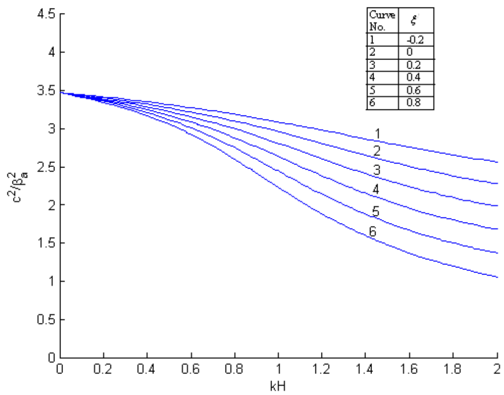

Fig. 4
Effect of initial stresses ($\xi^{/}$) in the porous layer on the propagation of Love waves for $\mu_0 / L = 2.50$, b/k=0.4, a/k=1, $\gamma = 1$, $d = 0.6154$, $\xi = 0$, and $\beta_a / c_{os} = 0.7$.

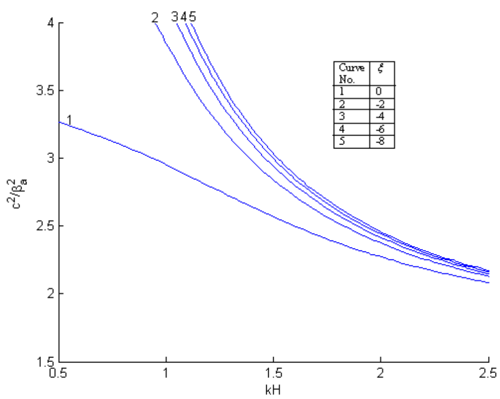

Fig. 5
Effect of tensile initial stresses (ξ <0) in the half-space, in the absence of initial stresses in the porous layer ($\xi^{/}$=0) on the propagation of Love waves for $\mu_0 / L = 2.50$, b/k=0.4, a/k=1, $\gamma = 1$, $d = 0.6154$, $\xi' = 0$, and $\beta_a / c_{os} = 0.7$.

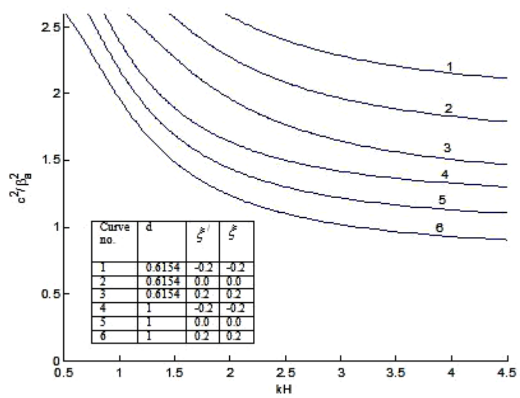

Fig. 6
Love wave dispersion curves in the presence of equal initial stress ($\xi^{/} = \xi$) in both the mediums when the layer is porous (d=0.6154) or non-porous (d=1) for $\mu_0 / L = 2.50$, b/k=0.4, a/k=1, $\gamma = 1$, and $\beta_a / c_{os} = 0.7$.

In Fig. 5 the effect of tensile initial stresses $(\xi < 0)$ in the half-space, in the absence of initial stresses in the porous layer, is shown. The curves show that the tensile initial stresses in the half-space increase the velocity of Love waves remarkably at low frequency; for compressive initial stresses in the half-space, the Love waves do not propagate. Fig. 6 shows the values of c^2 / β_a^2 in the porous medium (curves1-3) and non-porous medium (curves 4-6) in the presence of initial stresses of equal magnitude in the upper layer and half-space. Fig. 6 shows that porosity has significant effect on the propagation of Love waves. In Fig. 7 the comparison has been made with the velocities of Love waves in the classical non-porous case $(d = 1,\ \xi^{/} = 0,\ \xi = 0,\ \text{curve1})$ and initial stress-free porous case $(d = 0.6154,\ \xi^{/} = 0,\ \xi = 0,\ \text{curve2})$ in anisotropic medium.

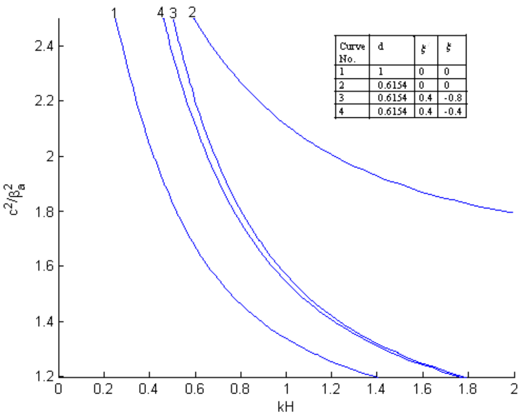

Fig. 7
Effect of different combinations of the initial stresses (ξ', ξ) on the propagation of Love waves for $\mu_0 / L = 2.50$, *b*/*k*=0.4, *a*/*k*=1, $\gamma = 4$, and $\beta_a / c_{os} = 0.7$.

8 CONCLUSIONS

As the porosity of the layer increases, the phase velocity of Love waves increases. As the anisotropy increases, the velocity of Love waves in the porous medium decreases. An increase in compressive initial stresses in the porous layer decreases the velocity of Love waves. The tensile initial stresses of small magnitude in the porous medium, however, increase the velocity, but the large magnitude of tensile stress does not allow Love waves to propagate. The tensile initial stresses in the half space increase the velocity of Love waves remarkably at low frequency; for compressive initial stresses in the half-space, the Love waves do not propagate.

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