

# Wave Propagation in Mixture of Generalized Thermoelastic Solids Half-Space

R. Kumar<sup>1,\*</sup>, S. Devi<sup>2</sup>

<sup>1</sup>Department of Mathematics, Kurukshetra University, Kurukshetra-136119, India

<sup>2</sup>Department of Mathematics, D.N. College, Hisar-125001, India

Received 4 March 2010; accepted 16 May 2010

## ABSTRACT

This paper concentrates on the reflection of plane waves in the mixture of generalized thermoelastic solid half-space. There exists quasi dilatational waves i.e.  $qP_1$ ,  $qP_2$ ,  $qT$  and two rotational waves  $S_1$ ,  $S_2$  in a two dimensional model of the solid. The boundary conditions are solved to obtain a system of five non-homogeneous equations for amplitude ratios. These amplitude ratios are found to depend on the angle of incidence of incident wave, mixture and thermal parameters and have been computed numerically and presented graphically. The appreciable effects of mixtures and thermal on the amplitude ratios are obtained.

© 2010 IAU, Arak Branch. All rights reserved.

**Keywords:** Dynamic theory of mixture; Amplitude ratios; Thermoelasticity; Wave propagation; Relaxation time; Reflection.

## 1 INTRODUCTION

**D**URING the past year there has been much effect to develop continuum theories of mixtures. The modern formulation of continuum theories of mixtures goes back to papers by Trusedell and Tupin [1], Eringen and Ingram [2], Green and Naghdi [3]. Bedford and Stern [4, 5] used for the first time the Lagrangian description in order to derive mixture theory of elastic solids. In this theory, the independent constitutive variables are the displacement gradients and the displacement fields. Much of the theoretical progress in the field is discussed in great detail in the review articles by Bowen [6], Atkin and Craine [7, 8], and in the book by Rajagopal and Tao [9].

The theory of binary mixtures of thermoelastic solids, in which the component interaction force depends on a difference of partial displacements, was constructed by Iesan [10]. He presented a theory for a binary mixture of thermoelastic solids in which the independent constitutive variables are the displacement gradients, relative displacement, temperature and temperature gradient whereas [11-13], the theories for mixture of thermoelastic solids studied the independent constitutive variables are the displacement gradients, relative velocity, temperature and temperature gradient. The existence theorems of weak solutions of dynamic problems for the linear theory of two thermoelastic solids are proved by the semigroup theory in [14] and obtain some qualitative results for the linear theory of binary mixtures of thermoelastic solids. Pompei and Scalia [15] addressed the linear dynamic theory of binary mixtures of thermoelastic solids and established the continuous dependence of solutions upon initial data and body loads and proved uniqueness theorems of the solutions for initial boundary value problems Burchuladze and Svanadze [16] investigating the boundary value problems of steady oscillations using the potential methods in the linear theory of binary mixtures of thermoelastic solids. Pompei and Scalia [17] study the continuous dependence of solutions on initial data and body sources in the linear theory of binary mixtures of thermoelastic solids and also establish the existence and uniqueness theorems. Passarella and Victoria [18] study the time-harmonic vibrations for some classes of homogeneous and isotropic thermoelastic mixtures for which the constitutive coefficients are

\* Corresponding author. Tel: 0941 612 0992, 0174 423 8644.  
E-mail address: rajneesh\_kuk@rediffmail.com (R. Kumar).

supposed to satisfy some mild positive definiteness conditions. Sharma and Gogna [19] studied the reflection and refraction of plane harmonic waves at an interface between elastic solid and porous solid saturated by viscous liquid and Vashisth, Sharma and Gogna [20] studied the reflection and transmission of elastic waves at a loosely bonded interface between an elastic solid and liquid-saturated porous solid. In the present we study the reflection of thermo elastic plane waves in the mixture theory of generalized thermoelastic solid half space.

## 2 BASIC EQUATIONS

The fundamental equations for the theory of mixtures are constituted from the following equation of motion Steel [12] in thermoelastic solid without body forces are given by

$$T_{ji,j} - p_i = \rho_1 \ddot{u}_i \tag{1}$$

$$S_{ji,j} + p_i = \rho_2 \ddot{w}_i \tag{2}$$

and the equation of energy

$$\rho T_0 \dot{\eta} = q_{i,i} \tag{3}$$

where  $\rho_1$  and  $\rho_2$  are the constants densities;  $T_{ij}$  and  $S_{ij}$  are the components of the partial stress tensors associated with the two constituents  $s_1$  and  $s_2$  respectively;  $p_i$  are the components of the diffusive force vector;  $\eta$  is the entropy per unit mass and  $q_i$  are the components of the heat flux vector. If  $u_i$  and  $w_i$  represents the components of the displacement vector fields associated with the two constituents  $s_1$  and  $s_2$ , respectively. Then, the infinitesimal strain measures are defined by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad h_{ij} = w_{i,j} + u_{j,i} \tag{4}$$

Following Apice et al. [21], Lord and Shulman [22] the constitutive equations for an isotropic and homogeneous mixture are Green and Steel [11-12].

$$T_{ij} = (\lambda + \nu)e_{ss}\delta_{ij} + 2(\mu + \zeta)e_{ij} + (\alpha + \nu)h_{ss}\delta_{ij} + (2\kappa + \zeta)h_{ij} + (2\gamma + \zeta)h_{ji} - (\beta + b)\theta\delta_{ij} \tag{5}$$

$$S_{ij} = \nu e_{ss}\delta_{ij} + 2\zeta e_{ij} + \alpha h_{ss}\delta_{ij} + 2\kappa h_{ji} + 2\gamma h_{ij} - b\theta\delta_{ij} \tag{6}$$

$$p_i = \xi v_i + \sigma g_i \tag{7}$$

$$\rho \dot{\eta} = \beta e_{ss} + b h_{ss} + a \theta \tag{8}$$

$$q_i = k g_i + m v_i \tag{9}$$

where

$$v_i = \dot{u}_i - \dot{w}_i, \quad g_i = \theta_{,i} \tag{10}$$

where  $\theta$  is the variation of the temperature measured from the reference temperature  $T_0$  and  $\lambda, \mu, \alpha, \beta, \kappa, \gamma, \nu, \zeta, \xi, \sigma, m, a,$  and  $b$  are constitutive constants. By substituting the relations (4) and (5)-(10) into the basic Eqs. (1)-(3), we obtain the following equations in terms of  $u_i, w_i$  and  $\theta$

$$(\mu + 2\zeta + 2\kappa)u_{i,jj} + (\lambda + \alpha + \mu + 2\nu + 2\zeta + 2\gamma)u_{j,ji} + (2\gamma + \zeta)w_{i,jj} + (\alpha + \nu + 2\kappa + 2\zeta)w_{j,ji} + \xi(\dot{u}_i - \dot{w}_i) - (\beta + b + \sigma)\theta_{,i} = \rho_1 \ddot{u}_i \tag{11}$$

$$(2\gamma + \zeta)u_{i,jj} + (\alpha + \nu + \zeta + 2\kappa)u_{j,ji} + 2\kappa w_{i,jj} + (2\gamma + \alpha)w_{j,ji} + \xi(\dot{u}_i - \dot{w}_i) - (b - \sigma)\theta_{,i} = \rho_2 \ddot{w}_i \tag{12}$$

$$K\theta_{,ii} + (m - (\beta + b)T_0(1 + \tau_0 \frac{\partial}{\partial t}))\dot{u}_{s,s} - (m + bT_0(1 + \tau_0 \frac{\partial}{\partial t}))\dot{w}_{s,s} - aT_0(1 + \tau_0 \frac{\partial}{\partial t})\dot{\theta} = 0 \tag{13}$$

**3 FORMULATION OF THE PROBLEM**

We consider an isotropic homogeneous binary mixture of generalized thermoelastic materials with one relaxation time. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface z=0 with z-axis pointing vertically in to the medium is introduced. For two dimensional problems, we assume

$$\bar{u} = (u_1, 0, u_3), \quad \bar{w} = (w_1, 0, w_3) \tag{14}$$

Using the expression relating displacement components  $u_i(x, z, t)$  and  $w_i(x, z, t)$  to the scalar potential functions  $\varphi(x, z, t)$ ,  $\psi(x, z, t)$  and  $\bar{\varphi}(x, z, t)$ ,  $\bar{\psi}(x, z, t)$  in dimensionless form

$$u_1 = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x}, \quad w_1 = \frac{\partial \bar{\varphi}}{\partial x} - \frac{\partial \bar{\psi}}{\partial z}, \quad w_3 = \frac{\partial \bar{\varphi}}{\partial z} + \frac{\partial \bar{\psi}}{\partial x} \tag{15}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$\begin{aligned} x' &= \frac{\omega_1^* x}{c_1}, \quad z' = \frac{\omega_1^* z}{c_1}, \quad u'_1 = \frac{\omega_1^* u}{c_1}, \quad u'_3 = \frac{\omega_1^* u}{c_1}, \quad w'_1 = \frac{\omega_1^* w}{c_1}, \quad w'_3 = \frac{\omega_1^* w}{c_1}, \\ T'_{33} &= \frac{T_{33}}{\mu}, \quad T'_{31} = \frac{T_{31}}{\mu}, \quad S'_{33} = \frac{S_{33}}{\mu}, \quad S'_{31} = \frac{T_{31}}{\mu}, \quad t' = \omega_1^* t, \quad \tau'_0 = \omega_1^* \tau_0, \quad \theta' = \frac{\theta}{T_0} \end{aligned} \tag{16}$$

where

$$c_1 = \left( \frac{\lambda + 2\mu}{\rho} \right)^{\frac{1}{2}}, \quad \omega_1^* = \frac{\rho c_e c_1^2}{K}$$

Using Eqs. (14)-(16) in Eqs. (11)-(13) (after suppressing the primes), we obtain

$$(1 + a_1)\nabla^2 \varphi + (a_2 + a_3)\nabla^2 \bar{\varphi} - a_4(\dot{\varphi} - \dot{\bar{\varphi}}) - a_5\theta = a_6\ddot{\varphi} \tag{17}$$

$$\nabla^2 \psi + a_2\nabla^2 \bar{\psi} - a_4(\dot{\psi} - \dot{\bar{\psi}}) = a_6\ddot{\psi} \tag{18}$$

$$(1 + a_7)\nabla^2 \varphi + (a_8 + a_9)\nabla^2 \bar{\varphi} + a_{10}(\dot{\varphi} - \dot{\bar{\varphi}}) - a_{11}\theta = a_{12}\ddot{\varphi} \tag{19}$$

$$\nabla^2 \psi + a_8\nabla^2 \bar{\psi} + a_{10}(\dot{\psi} - \dot{\bar{\psi}}) = a_{12}\ddot{\psi} \tag{20}$$

$$\nabla^2 \theta + \{a_{13} - a_{14}(1 + \tau_0 \frac{\partial}{\partial t})\}\nabla^2 \dot{\varphi} - \{a_{13} + a_{15}(1 + \tau_0 \frac{\partial}{\partial t})\}\nabla^2 \dot{\bar{\varphi}} - a_{16}(1 + \tau_0 \frac{\partial}{\partial t})\dot{\theta} = 0 \tag{21}$$

We assume the solution of Eqs. (17)-(21) as

$$\{\varphi, \bar{\varphi}, \theta, \psi, \bar{\psi}\} = \{\varphi^*, \bar{\varphi}^*, \theta^*, \psi^*, \bar{\psi}^*\} \exp\{t\{k(x \sin \theta - z \cos \theta) - \omega t\}\} \tag{22}$$

Using Eq. (22) in Eqs. (17), (19) and (21) (by omitting the star), we get

$$\{-(1 + a_1)k^2 + a_4 i\omega + a_6 \omega^2\}\varphi + \{-(a_2 + a_3)k^2 - a_4 i\omega\}\bar{\varphi} - a_5\theta = 0 \tag{23}$$

$$\{-(1 + a_7)k^2 - a_{10}i\omega\}\varphi + \{-(a_8 + a_9)k^2 + a_{10}i\omega + a_{12}\omega^2\}\bar{\varphi} - a_{11}\theta = 0 \tag{24}$$

$$i\omega k^2(a_{13} + a_{14}i\omega\tau_0^*)\varphi + i\omega k^2(-a_{13} + a_{15}i\omega\tau_0^*)\bar{\varphi} + (k^2 - a_{16}\omega^2\tau_0^*)\theta = 0 \tag{25}$$

For non-trivial solution of the system of Eqs. (23)-(25), we have

$$\begin{vmatrix} -(1+a_1)k^2 + a_4t\omega + a_6\omega^2 & -(a_2+a_3)k^2 - a_4t\omega & -a_5 \\ -(1+a_7)k^2 - a_{10}t\omega & -(a_8+a_9)k^2 + a_{10}t\omega + a_{12}\omega^2 & -a_{11} \\ t\omega k^2(a_{13} + a_{14}t\omega\tau_0^*) & t\omega k^2(-a_{13} + a_{15}t\omega\tau_0^*) & -k^2 + a_{16}\omega^2\tau_0^* \end{vmatrix} \begin{vmatrix} \phi \\ \bar{\phi} \\ \theta \end{vmatrix} = 0 \quad (26)$$

$$\text{i.e. } (V^6 + \omega^2 AV^4 + \omega^4 BV^2 + \omega^6 C)(\phi, \bar{\phi}, \theta) = 0 \quad (27)$$

Eqs. (27) is cubic in  $V^2$  therefore the roots of this equation give three values of  $V^2$ . Each value of  $V^2$  corresponds to a velocity of propagation of three possible wave. The waves with velocities  $V_1, V_2, V_3$  corresponds to  $qP_1, qP_2$  and  $qT$ -waves, respectively.

where

$$A = \frac{A_3}{A_4}, \quad B = \frac{A_2}{A_4}, \quad C = \frac{A_1}{A_4}, \quad V = \frac{\omega}{k}$$

with

$$\begin{aligned} A_4 &= \{(\omega a_4 + \omega^2 a_6)(t\omega^3 \tau_0^* a_{10} a_{16} + \omega^4 \tau_0^* a_{12} a_{16}) + a_4 a_{10} a_{16} \omega^4 \tau_0^*\} \\ A_3 &= \{(a_4 t\omega + a_6 \omega^2)(-a_{10} t\omega - a_{12} \omega^2 - (a_8 + a_9) a_{16} \tau_0^* \omega^2 - a_{11} a_{13} t\omega - a_{11} a_{15} \omega^2 \tau_0^*) \\ &\quad - (1 + a_1) \tau_0^* a_{16} (a_{10} t\omega^3 + a_{12} \omega^4) + a_4 t\omega (-a_{11} a_{13} t\omega - a_{11} a_{14} \omega^2 \tau_0^* + a_{10} t\omega - (1 + a_7) a_{16} \omega^2 \tau_0^*) \\ &\quad - (a_2 + a_3) a_{10} a_{16} t\omega^3 \tau_0^* - a_5 (a_{10} a_{15} t\omega^3 \tau_0^* - a_{12} a_{13} t\omega^3 + a_{10} a_{14} t\omega^3 \tau_0^* + a_{12} a_{14} \omega^4 \tau_0^*)\}, \\ A_2 &= \{(a_8 + a_9)(a_4 t\omega + a_6 \omega^2) - (1 + a_1)(-a_{10} t\omega - a_{12} \omega^2 - (a_8 + a_9) a_{16} \tau_0^* \omega^2 - a_{11} a_{13} t\omega \\ &\quad + a_{11} a_{15} \omega^2 \tau_0^*) + a_4 (1 + a_7) \tau_0^* t\omega - (a_2 + a_3)(-a_{11} a_{13} t\omega + a_{11} a_{14} \omega^2 \tau_0^* - a_{10} t\omega \\ &\quad + (1 + a_7) a_{16} \omega^2 \tau_0^*) - a_5 ((1 + a_7)(a_{13} t\omega + a_{15} \tau_0^* \omega^2) + (a_8 + a_9) a_{13} t\omega - (a_8 + a_9) a_{14} \omega^2 \tau_0^*)\} \\ A_1 &= \{-(1 + a_1)(a_8 + a_9) + (a_2 + a_3)(1 + a_7)\} \\ \tau_0^* &= (\tau_0 + t\omega^{-1}), \quad a_1 = \frac{\lambda + \alpha + \mu + 2\nu + 2\gamma + 2\zeta}{\mu + 2\zeta + 2\kappa}, \quad a_2 = \frac{2\gamma + \zeta}{\mu + 2\zeta + 2\kappa}, \quad a_3 = \frac{\alpha + \nu + 2\kappa + \zeta}{\mu + 2\zeta + 2\kappa}, \\ a_4 &= \frac{\xi c_1^2}{\omega_1^* (\mu + 2\zeta + 2\kappa)}, \quad a_5 = \frac{(\beta + b + \sigma) T_0}{\mu + 2\zeta + 2\kappa}, \quad a_6 = \frac{\rho_1 c_1^2}{\mu + 2\zeta + 2\kappa}, \quad a_7 = \frac{\alpha + \nu + 2\kappa + \zeta}{\zeta + 2\gamma}, \\ a_8 &= \frac{2\kappa}{\zeta + 2\gamma}, \quad a_9 = \frac{2\gamma + \alpha}{\zeta + 2\gamma}, \quad a_{10} = \frac{\xi c_1^2}{\omega_1^* (2\gamma + \zeta)}, \quad a_{11} = \frac{(b - \sigma) T_0}{2\gamma + \zeta}, \quad a_{12} = \frac{\rho_2 c_1^2}{2\gamma + \zeta}, \\ a_{13} &= \frac{m c_1^2}{K \omega_1^* T_0}, \quad a_{14} = \frac{(\beta + b) c_1^2}{K \omega_1^*}, \quad a_{15} = \frac{b c_1^2}{K \omega_1^*}, \quad a_{16} = \frac{a T_0 c_1^2}{K \omega_1^*} \end{aligned}$$

From Eq. (26), we obtain  $\theta = m_{ii} \phi, \bar{\phi} = m_i \phi$ .

with

$$m_{ii} = \frac{DetIII}{DetI}, \quad m_i = \frac{-DetII}{DetI} \quad i = 1, 2, 3$$

where

$$DetI = ((a_8 + a_9)k^2 + t\omega a_{10} + a_{12}\omega^2)(-k^2 + \omega^2 \tau_0^* a_{16}) + t\omega k^2 a_{11} (-a_{13} + t\omega \tau_0^* a_{15})$$

$$DetII = -(1 + a_1)k^2 + a_4t\omega + a_6\omega^2)(-k^2 + \omega^2\tau_0^*a_{16}) + a_5t\omega k^2(a_{13} + t\omega\tau_0^*a_{14})$$

$$DetIII = -(1 + a_1)k^2 + t\omega a_4 + a_6\omega^2)(-a_8 + a_9)k^2 + t\omega a_{10} + a_{12}\omega^2) + (a_2 + a_3)k^2 + t\omega a_4)(-1 + a_7)k^2 - t\omega a_{10})$$

Using Eq. (22) in Eqs. (18) and (20) (by omitting the star), we obtain the non-trivial solution of the system of Eqs. (18)-(20), as

$$\begin{vmatrix} -k^2 + a_4t\omega + a_6\omega^2 & -a_2k^2 - a_4t\omega \\ -k^2 - a_{10}t\omega & -a_8k^2 + a_{10}t\omega + a_{12}\omega^2 \end{vmatrix} \begin{vmatrix} \psi \\ \bar{\psi} \end{vmatrix} = 0 \tag{28}$$

$$\text{i.e. } (V^4 + DV^2 + E)(\psi, \bar{\psi}) = 0 \tag{29}$$

Eq. (29) is quadratic in  $V^2$ . Therefore, two values of  $V$  will be velocity of propagation of  $S_1, S_2$ -waves, respectively, where

$$D = \frac{C_D}{C_0}, \quad E = \frac{C_E}{C_0},$$

with

$$\begin{aligned} C_0 &= t\omega^3(a_6a_{10} + a_4a_{12}) + \omega^4a_6a_{12} \\ C_D &= -t\omega^3(a_4a_8 + a_{10} + a_4 + a_2a_{10}) - \omega^4(a_6a_8 + a_{12}) \\ C_E &= \omega^4(a_8 - a_2) \end{aligned}$$

Also from Eq. (28), we have  $\bar{\psi} = m_j\psi, \quad j = 4, 5$

where

$$m_j = \frac{DetII}{DetI}$$

with

$$\begin{aligned} DetI &= -a_8k^2 + t\omega a_{10} + a_{12}\omega^2 \\ DetII &= k^2 - t\omega a_{10} \end{aligned}$$

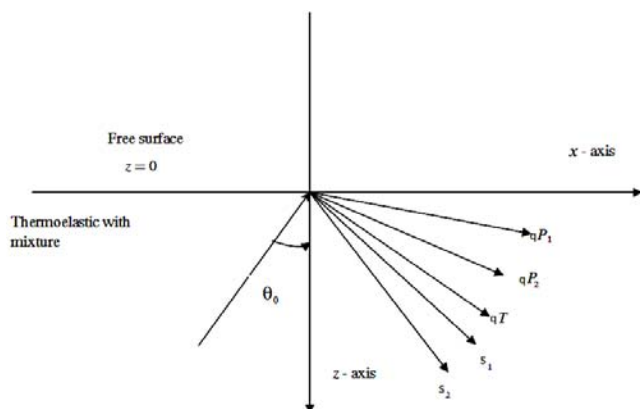


Fig. 1  
Geometry of the Problem (Reflection).

#### 4 REFLECTION

When we consider an incident quasi dilatational waves  $qP_1, qP_2, qT$  -waves or rotational waves  $S_1, S_2$  -waves incident on the free surface then five waves are reflected, namely  $qP_1, qP_2, qT, S_1$  and  $S_2$ , respectively at complex angles  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$  as shown in Fig. 1.

#### 5 BOUNDARY CONDITIONS

The boundary conditions are given by vanishing of the stress component (normal and tangential), vanishing of relative velocities of the two constituents (normal and tangential) and also vanishing of the temperature gradient field. Mathematically these can be written as

$$T_{33} + S_{33} = 0, \quad T_{31} + S_{31} = 0, \quad \dot{u}_1 - \dot{w}_1 = 0, \quad \dot{u}_3 - \dot{w}_3 = 0, \quad \frac{\partial \theta}{\partial z} = 0 \quad \text{at } z = 0 \quad (30)$$

The appropriate potentials satisfying the boundary conditions can be written as

$$(\phi, \bar{\phi}, \theta) = \sum_{i=1}^3 (1, m_i, m_{ii}) (A_{0i} \text{Exp}(\iota P_i) + A_i \text{Exp}(\iota P'_i)) \quad (31)$$

$$(\psi, \bar{\psi}) = \sum_{j=4}^5 (1, m_j) (B_{0j} \text{Exp}(\iota P_j) + B_j \text{Exp}(\iota P'_j)) \quad (32)$$

where  $A_{0i}$  and  $B_{0j}$  are the amplitudes of the incident  $qP_1, qP_2, qT$  and  $S_1, S_2$  waves, respectively.

$$\begin{aligned} P_i &= k_i(x \sin \theta_0 - z \cos \theta_0) - \omega_i t \\ P'_i &= k_i(x \sin \theta_i + z \cos \theta_i) - \omega_i t, \quad i = 1, 2, 3 \\ P_j &= k_j(x \sin \theta_0 - z \cos \theta_0) - \omega_j t \\ P'_j &= k_j(x \sin \theta_j + z \cos \theta_j) - \omega_j t, \quad j = 4, 5 \\ A_{02} &= A_{03} = B_{04} = B_{05} = 0 \quad \text{for incident } qP_1 \text{ -wave} \\ A_{01} &= A_{03} = B_{04} = B_{05} = 0 \quad \text{for incident } qP_2 \text{ -wave,} \\ A_{01} &= A_{02} = A_{03} = B_{04} = 0 \quad \text{for incident } qT \text{ -wave} \\ A_{01} &= A_{02} = B_{04} = B_{05} = 0 \quad \text{for incident } S_1 \text{ -wave} \\ A_{01} &= A_{02} = A_{03} = B_{05} = 0 \quad \text{for incident } S_2 \text{ -wave, respectively} \end{aligned} \quad (32)$$

Making use of Eqs. (31)-(32), the boundary conditions (30) can be satisfied if the angles  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$  satisfy the relations i.e. Snell's law given as

$$\frac{\sin \theta_0}{v_0} = \frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} = \frac{\sin \theta_3}{v_3} = \frac{\sin \theta_4}{v_4} = \frac{\sin \theta_5}{v_5} \quad (33)$$

where

$$V_j = \frac{\omega}{k_j}, \quad (j = 1, 2, 3, 4, 5) \quad \text{at } z = 0 \quad (34)$$

and

$$v_0 = \begin{cases} v_1 & \text{for incident } qP - \text{ wave} \\ v_2 & \text{for incident } qP_2 - \text{ wave} \\ v_3 & \text{for incident } qT - \text{ wave} \\ v_4 & \text{for incident } S_1 - \text{ wave} \\ v_5 & \text{for incident } S_2 - \text{ wave} \end{cases} \quad (33c)$$

Making use of potentials given by Eqs. (31)-(32), and the Snell's law given by Eq. (33) in boundary condition and with the help of Eq. (16), we get a system of five non-homogeneous equations, which can be written as

$$\sum_{m=1}^5 c_{mn} Z_n = Y_m, \quad (m = 1, 2, 3, 4, 5) \quad (35)$$

where

$Z_i = A_i / A^*$ , ( $i = 1, 2, 3$ ) and  $Z_j = B_j / A^*$ , ( $j = 4, 5$ ) are the complex amplitude ratios of the reflected  $qP_1, qP_2, qT$  and  $S_1, S_2$  waves, respectively, and

$$\begin{aligned} c_{1i} &= -(q_1 + q_3 m_i) \sin^2 \theta_0 + (q_2 + q_4 m_i) [(V_0 / V_i)^2 - \sin^2 \theta_0] - q_5 m_{ii} (V_0 / \omega)^2 \\ c_{1j} &= (q_3 - q_4) m_j \sin \theta_0 [(V_0 / V_j)^2 - \sin^2 \theta_0]^{1/2} \\ c_{2i} &= \{-(R_1 + R_2) - (R_3 + R_4) m_i\} \sin \theta_0 [(V_0 / V_i)^2 - \sin^2 \theta_0]^{1/2} \\ c_{2j} &= -(R_1 + R_3 m_j) \sin^2 \theta_0 + (-R_2 + R_4 m_j) [(V_0 / V_j)^2 - \sin^2 \theta_0] \\ c_{3i} &= (1 - m_i) \sin \theta_0 \\ c_{3j} &= (-1 + m_j) [(V_0 / V_j)^2 - \sin^2 \theta_0]^{1/2} \\ c_{4i} &= (1 - m_i) [(V_0 / V_i)^2 - \sin^2 \theta_0]^{1/2} \\ c_{4j} &= (1 - m_j) [(V_0 / V_j)^2 - \sin^2 \theta_0]^{1/2} \\ c_{5i} &= m_{ii} [(V_0 / V_i)^2 - \sin^2 \theta_0]^{1/2} \\ c_{5j} &= 0 \end{aligned}$$

with

$$\begin{aligned} q_1 &= b_1 + b_3, \quad q_2 = b_2 + b_4, \quad q_3 = b_3 + b_{10}, \quad q_4 = b_4 + b_{11}, \quad q_5 = b_5 + b_{12}, \\ R_1 &= b_7 + b_9, \quad R_2 = b_6 + b_8, \quad R_3 = b_9 + b_{14}, \quad R_4 = b_8 + b_{13} \\ b_1 &= \frac{\lambda + \alpha + 2\nu}{\mu}, \quad b_2 = \frac{\lambda + \alpha + 2\nu + 2(\mu + \zeta) + (2\kappa + \zeta) + (2\gamma + \zeta)}{\mu}, \quad b_3 = \frac{\alpha + \nu}{\mu}, \\ b_4 &= \frac{\alpha + \nu + (2\kappa + \zeta) + (2\gamma + \zeta)}{\mu}, \quad b_5 = \frac{(\beta + b)T_0}{\mu}, \quad b_6 = \frac{(\mu + \zeta) + (2\kappa + \zeta)}{\mu}, \\ b_7 &= \frac{(\mu + \zeta) + (2\gamma + \zeta)}{\mu}, \quad b_8 = \frac{2\gamma + \zeta}{\mu}, \quad b_9 = \frac{2\kappa + \zeta}{\mu}, \quad b_{10} = \frac{\alpha}{\mu}, \quad b_{11} = \frac{\alpha + 2\kappa + 2\gamma}{\mu}, \\ b_{12} &= \frac{bT_0}{\mu}, \quad b_{13} = \frac{2\kappa}{\mu}, \quad b_{14} = \frac{2\gamma}{\mu} \end{aligned}$$

Considering the phase of the reflected waves can easily write using Eqs. (33)-(34)

$$\frac{\cos \theta_i}{V_i} = \frac{1}{V_0} [(V_0 / V_i)^2 - \sin^2 \theta_0]^{1/2}, \quad \frac{\cos \theta_j}{V_j} = \frac{1}{V_0} [(V_0 / V_j)^2 - \sin^2 \theta_0]^{1/2}.$$

Following Schoenberg [23], if we write

$$\frac{\cos \theta_i}{V_i} = \frac{\cos \theta'_i}{V'_i} + \iota \frac{c_i}{V_0 2\pi}, \quad (i = 1, 2, 3, 4, 5)$$

Then,

$$\frac{\cos \theta'_i}{V'_i} = \frac{1}{V_0} R \{ [(V_0 / V_i)^2 - \sin^2 \theta_0]^{1/2} \}, \quad c_i = 2\pi I \{ [(V_0 / V_i)^2 - \sin^2 \theta_0]^{1/2} \}$$

where  $V'_i$ , the real phase speed and  $\theta'_i$ , the angle of reflection are given by

$$\frac{V'_i}{V_0} = \frac{\sin \theta'_i}{\sin \theta_0} \left[ \sin^2 \theta_0 + \left[ R \{ [(V_0 / V_i)^2 - \sin^2 \theta_0]^{1/2} \} \right]^2 \right]^{-1/2}$$

and  $c_i$ , the attenuation in a depth is equal to the wavelength of incident wave i.e.  $2\pi V_0 / \omega$ .

- (i) when  $qP_1$ -wave is incident  $A^* = A_{01} (\theta_0 = \theta_1)$ ,  $Y_i = (-1)^i c_{i1}$ ,  $Y_j = c_{j4}$
- (ii) when  $qP_2$ -wave is incident  $A^* = A_{02} (\theta_0 = \theta_2)$ ,  $Y_i = (-1)^i c_{i2}$ ,  $Y_j = c_{j2}$
- (iii) when  $qT$  (thermal)-wave is incident  $A^* = A_{03} (\theta_0 = \theta_3)$ ,  $Y_i = (-1)^i c_{i3}$ ,  $Y_j = c_{j3}$ ,  $i = 1, 2, 3; j = 4, 5$
- (iv) when  $S_1$ -wave is incident  $A^* = B_{04} (\theta_0 = \theta_4)$ ,  $Y_i = (-1)^{i+1} c_{i4}$
- (v) when  $S_2$ -wave is incident  $A^* = B_{05} (\theta_0 = \theta_5)$ ,  $Y_i = (-1)^{i+1} c_{i4}$ ,  $Y_5 = 0$ ,  $i = 1, 2, 3, 4$ ;

The complex amplitude ratios of various reflected waves are  $Z_i = A_i / A^*$ ,  $Z_j = B_j / A^*$ , ( $i = 1, 2, 3; j = 4, 5$ ).

## 6 PARTICULAR CASES

*Case I:* In the absence of second constituent of mixture we have i.e.  $\nu = \zeta = \alpha = \kappa = \gamma = b = \xi = 0$ , our results reduce in generalized thermoelastic solid with one relaxation time as

$$\sum_{m=1}^3 c'_{nm} Z_n = Y_m, \quad m = 1, 2, 3 \tag{36}$$

where

$$\begin{aligned} c''_{1i} &= q_1^{**} \sin^2 \theta_0 + q_2^{**} [(V_0 / V_i^{**})^2 - \sin^2 \theta_0] + q_5^{**} m_{ii}^{**} (V_0 / \omega)^2 \\ c''_{13} &= -(q_1^{**} - q_2^{**}) \sin \theta_0 [(V_0 / V_3^{**})^2 - \sin^2 \theta_0] \\ c''_{2i} &= \sin \theta_0 [(V_0 / V_i^{**})^2 - \sin^2 \theta_0]^{1/2} \\ c''_{23} &= \sin^2 \theta_0 - [(V_0 / V_i^{**})^2 - \sin^2 \theta_0] \\ c''_{3i} &= m_{ii}^{**} [(V_0 / V_i^{**})^2 - \sin^2 \theta_0] \end{aligned}$$



$$c_{33}^* = 0 \quad i = 1, 2$$

and the complex amplitude ratios of three reflected waves are  $Z_i = A_i / A^*$ ,  $Z_3 = B_4 / A^*$ ;  $i = 1, 2$ .

- (i) when  $qP_1$ -wave is incident  $A^* = A_{01}$  ( $\theta_0 = \theta_1$ ),  $Y_i = (-1)^i c_{i1}^*$
- (ii) when  $qT$  (thermal)-wave is incident  $A^* = A_{03}$  ( $\theta_0 = \theta_3$ ),  $Y_i = (-1)^i c_{i2}^*$
- (iii) when  $S_1$ -wave is incident  $A^* = B_{04}$  ( $\theta_0 = \theta_4$ ),  $Y_i = (-1)^{i+1} c_{i3}^*$ ,  $Y_3 = 0$ ,  $i = 1, 2$

where

$$m_{ii}^* = \frac{-a_{14}^* k V_i^{**}}{(1 + a_{16}^* V_i^* \tau_0)}, \quad a_1^{**} = \frac{\lambda + \mu}{\mu}, \quad a_5^{**} = \frac{\beta T_0}{\mu}, \quad a_6^{**} = \frac{\rho_1 c_1^2}{\mu}, \quad a_{14}^{**} = \frac{\beta c_1^2}{K \omega_1},$$

$$b_1^{**} = \frac{\lambda}{\mu}, \quad b_2^{**} = \frac{\lambda + 2\mu}{\mu}, \quad b_5^{**} = \frac{\beta T_0}{\mu}, \quad b_6^{**} = b_7^{**} = 1$$

and

$V_i^{**}$ ; ( $i = 1, 2$ ) can be obtain from the equation  $(V^{*4} + A^{**} V^{*2} + C^{**})(\phi, \theta) = 0$ .

where

$$A^{**} = \frac{\frac{l}{\omega} a_5^{**} a_{14}^{**} - a_6^{**} + a_{16}^* (1 + a_1^{**}) \tau_0}{-a_6^{**} a_{16}^* \tau_0}, \quad C^{**} = \frac{(1 + a_1^{**})}{-a_6^{**} a_{16}^* \tau_0}$$

and

$$V_4^{**} = \sqrt{\frac{1}{a_6^{**}}}$$

Case II: neglecting thermal effect i.e.  $b = \sigma = m = K = a = \beta = aT_0 = 0$ , , our results reduce in elastic solid with mixture as

$$\sum_{n=1}^4 c_{nm}^* Z_n = Y_m, \quad m = 1, 2, 3, 4 \tag{37}$$

where

$$\begin{aligned} \dot{c}_{1i} &= -(q_1 + q_3 m_i^*) \sin^2 \theta_0 + (q_2 + q_4 m_i^*) [(V_0 / V_i^*)^2 - \sin^2 \theta_0] \\ \dot{c}_{1j} &= (q_3 - q_4) m_j \sin \theta_0 [(V_0 / V_j)^2 - \sin^2 \theta_0]^{1/2} \\ \dot{c}_{2i} &= \{-(R_1 + R_2) - (R_3 + R_4) m_i^*\} \sin \theta_0 [(V_0 / V_i^*)^2 - \sin^2 \theta_0]^{1/2} \\ \dot{c}_{2j} &= -(R_1 + R_3 m_j) \sin^2 \theta_0 + (-R_2 + R_4 m_j) [(V_0 / V_j)^2 - \sin^2 \theta_0] \\ \dot{c}_{3i} &= (1 - m_i^*) \sin \theta_0 \\ \dot{c}_{3j} &= (-1 + m_j) [(V_0 / V_j)^2 - \sin^2 \theta_0]^{1/2} \\ \dot{c}_{4i} &= (1 - m_i^*) [(V_0 / V_i^*)^2 - \sin^2 \theta_0]^{1/2} \\ \dot{c}_{4j} &= (1 - m_j) [(V_0 / V_j)^2 - \sin^2 \theta_0]^{1/2}, \quad (i = 1, 2; \quad j = 3, 4) \end{aligned}$$

where

$$m_i^* = \frac{(1+a_7)k^2 + a_{10}tk\vartheta_i^*}{(a_8+a_9)k^2 - \iota ka_{10}\vartheta_i^* + a_{12}k^2\vartheta_i^{*2}}$$

and  $V_i^*$ ;  $i=1, 2$  can be obtain from  $(V^{*4} + A^*V^{*2} + C^*)(\phi, \bar{\phi}) = 0$ , with

$$A^* = \frac{-\{(a_8+a_9)(a_4\iota\omega^3 + a_6\omega^4) + \iota\omega^3 a_{10}a_{12}(1+a_1) + \iota\omega^3 a_4(1+a_7) + \iota\omega^3 a_{10}(a_2+a_3)\}}{\{(a_4\iota\omega + a_6\iota\omega^2)\iota\omega a_{10}a_{12} + a_4 a_{10}\omega^2\}}$$

$$C^* = \frac{\{(a_8+a_9)(1+a_1)\omega^4 - (1+a_7)(a_2+a_3)\omega^4\}}{\{(a_4\iota\omega + a_6\iota\omega^2)\iota\omega a_{10}a_{12} + a_4 a_{10}\omega^2\}}$$

and

- (i) when  $qP_1$ -wave is incident  $A^* = A_{01}$  ( $\theta_0 = \theta_1$ ),  $Y_i = (-1)^i c_{i1}''$
- (ii) when  $qP_2$ -wave is incident  $A^* = A_{02}$  ( $\theta_0 = \theta_2$ ),  $Y_i = (-1)^i c_{i2}''$
- (iii) when  $S_1$ -wave is incident  $A^* = B_{04}$  ( $\theta_0 = \theta_4$ ),  $Y_i = (-1)^{i+1} c_{i3}''$
- (iv) when  $S_2$ -wave is incident  $A^* = B_{05}$  ( $\theta_0 = \theta_5$ ),  $Y_i = (-1)^{i+1} c_{i4}'$ , ( $i = 1, 2, 3, 4$ ).

The complex amplitude ratios of four reflected waves are

$$Z_1 = \frac{A_1}{A^*}, Z_2 = \frac{A_2}{A^*}, Z_3 = \frac{B_4}{A^*}, Z_4 = \frac{B_5}{A^*} \tag{36c}$$

## 7 NUMERICAL RESULT AND DISCUSSION

In order to illustrate theoretical results obtained in the proceeding sections, we now present some numerical results. Following Dhaliwal and Singh [24] We take the following values of relevant parameters as

$$\lambda = 2.17 \times 10^{10} \text{ Nm}^{-2}, \mu = 3.278 \times 10^{10} \text{ Nm}^{-2}, \nu = 0.75 \times 10^{10} \text{ Nm}^{-2}, \rho_1 = 1.74 \times 10^3 \text{ Kg m}^{-3},$$

$$\rho_2 = 1.35 \times 10^3 \text{ Kg m}^{-3}, T_o = 298\text{K}, C_e = 1.04 \times 10^3 \text{ JKg}^{-1}\text{deg}^{-1}, K = 1.7 \times 10^2 \text{ w m}^{-1}\text{deg}^{-1},$$

$$\zeta = 1.85 \times 10^{10} \text{ Nm}^{-2}, \alpha = 0.5 \times 10^{10} \text{ Nm}^{-2}, \kappa = 0.85 \times 10^{10} \text{ Nm}^{-2}, \gamma = 1.95 \times 10^{10} \text{ Nm}^{-2},$$

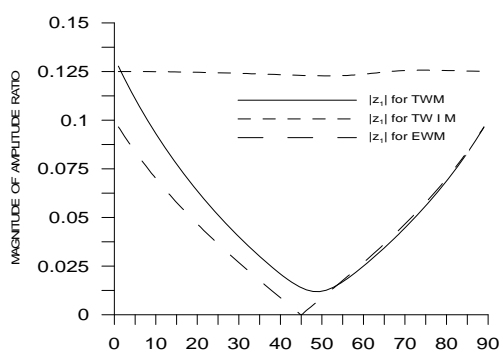
$$m = 0.75 \times 10^{10} \text{ Nm}^{-2}, \beta = 2.68 \times 10^6 \text{ Nm}^{-2}\text{K}^{-1}, b = 1.85 \times 10^6 \text{ Nm}^{-2}\text{K}^{-1}, \sigma = 2.05 \times 10^6 \text{ Nm}^{-2}\text{K}^{-1},$$

$$a = 1.85 \times 10^6 \text{ Nm}^{-2}\text{K}^{-2}, \xi = .075 \times 10^{12} \text{ Nsm}^{-4}$$

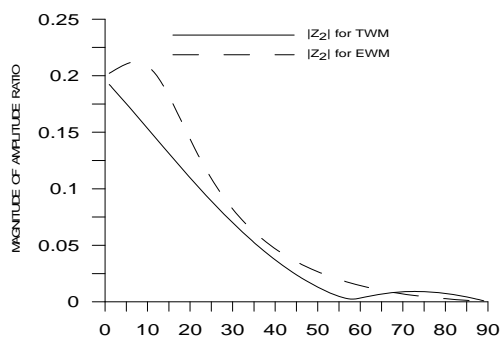
The comparison were carried out for nondimensional frequency and relaxation times i.e.  $(\omega/\omega_1^*) = 4$ , and  $\tau_o = 0.02$ . A computer programme has been developed and an amplitude ratio of various reflected waves has been computed. The variations of amplitude ratios for thermoelastic with mixture (TWM), thermoelastic in the absence of second constituent of mixture(TWIM) and Elastic with mixture (EWM) have been shown by solid line, small dashed line and long dashed line, respectively. The variations of the amplitude ratios  $|Z_i|$  ( $i = 1, 2, 3, 4, 5$ ) for TWM, EWM, TWIM with the angle of incidence of the incident  $qP_1$ -wave, incident  $qT$ -wave and incident  $S_1$ -wave are shown graphically in the Figs. 2-14.

7.1 Incident  $qP_1$ -wave

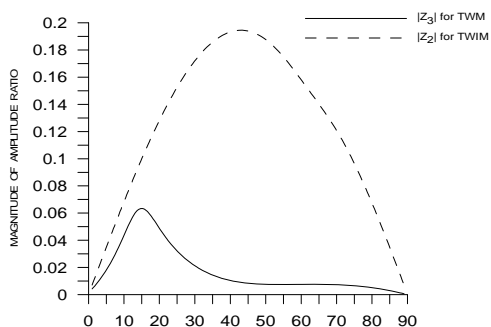
Fig. 2 shows the variations of  $|Z_1|$  with angle of incidence  $\theta_0$ . The amplitude ratio  $|Z_1|$  for TWM, EWM, is generally small as compared to  $|Z_1|$  for TWIM but the trend of variations of  $|Z_1|$  for TWM, TWIM, EWM is similar whereas the corresponding values are different in magnitude, respectively. The presence of constituent of mixture decreases the amplitude ratio at  $\theta_0 \approx 45^\circ$  and increases as  $\theta_0$  increases further. Fig.3. shows the variations of amplitude ratio  $|Z_2|$  with angle of incidence  $\theta_0$ . It is observed that the absence of the thermal parameters the amplitude ratio  $|Z_2|$  for TWIM is slightly large in the range  $0^\circ \leq \theta_0 \leq 10^\circ$  as compared to TWM and decrease in the increasing direction of  $\theta_0$ . The behavior of variations of amplitude ratios  $|Z_3|$ ,  $|Z_4|$  and  $|Z_5|$  are oscillatory in the whole range of  $\theta_0$  but the magnitude of oscillation is different for different amplitude ratios, these variations are shown in Figs.4-6, respectively.



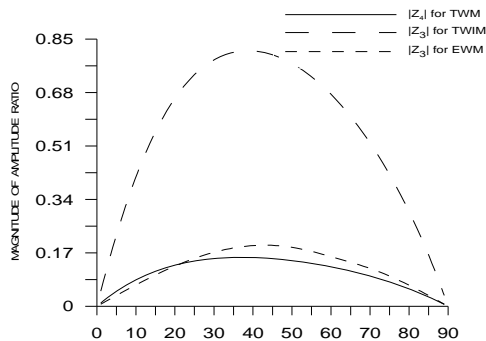
**Fig. 2**  
The variations of amplitude ratios of reflected waves when  $qP_1$ -wave is incident.



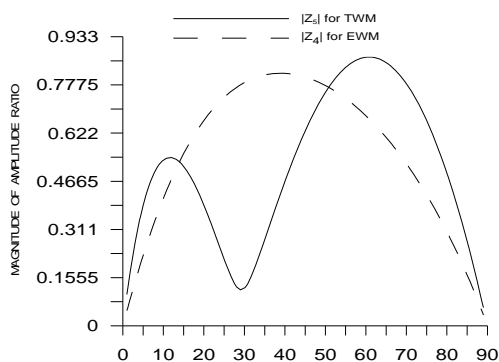
**Fig. 3**  
The variations of amplitude ratios of reflected waves when  $qP_1$ -wave is incident.



**Fig. 4**  
The variations of amplitude ratios of reflected waves when  $qP_1$ -wave is incident.



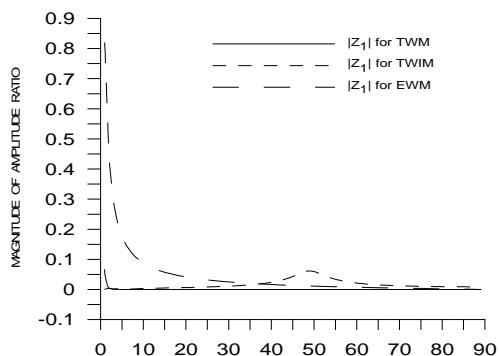
**Fig. 5**  
 The variations of amplitude ratios of reflected waves when  $qP_1$ -wave is incident.



**Fig. 6**  
 The variations of amplitude ratios of reflected waves when  $qP_1$ -wave is incident.

### 7.2 Incident $qT$ -wave

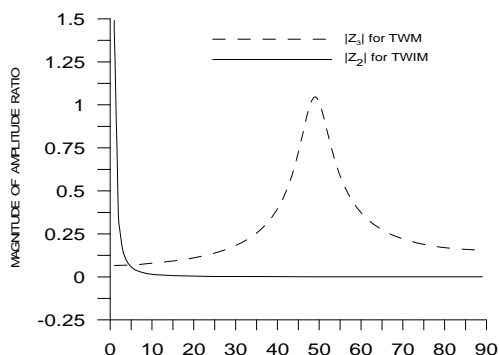
Figs. 7-9 represent the variations of amplitude ratios  $|Z_1|$ ,  $|Z_2|$ ,  $|Z_3|$  with angle of incidence  $\theta_0$ . The behavior of variations of amplitude ratios  $|Z_1|$ ,  $|Z_3|$  is similar i.e. the values of amplitude ratios for TWM decrease in the range  $0^\circ \leq \theta_0 \leq 9^\circ$  and oscillate around zero as  $\theta_0$  increases further and for TWIM increase in the range  $0^\circ \leq \theta_0 \leq 49^\circ$  and decrease in the rest range of  $\theta_0$ , shown in Figs. 7-9, respectively. But the trend of variations of amplitude ratio  $|Z_2|$  for TWM, TWIM is similar for all values of  $\theta_0$ , as depicts graphically in Fig. 8. In the case of incident  $qT$ -wave, resulting reflecting waves are same as that of incident  $qP_1$ -wave.



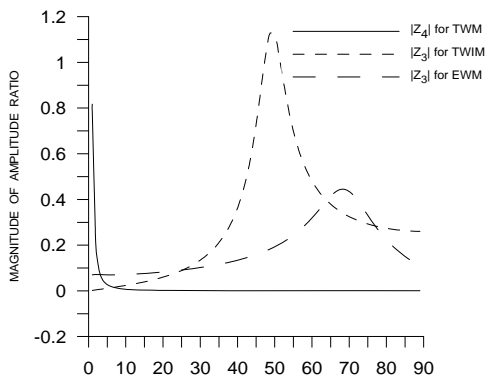
**Fig. 7**  
 The variations of amplitude ratios of reflected waves when  $qT$ -wave is incident.

7.3 Incident  $S_1$ -wave

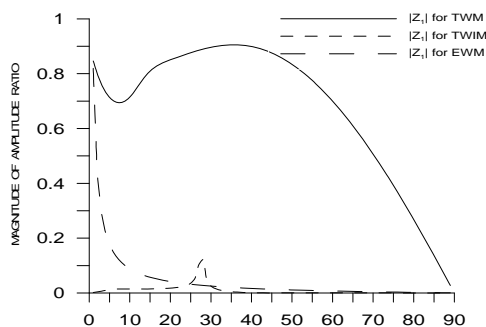
At  $\theta_0 \approx 40^\circ$  the values amplitude ratios  $|Z_1|$ ,  $|Z_2|$ ,  $|Z_3|$ , show a sudden fall and vanish as the angle of incidence exceeds this angle but the amplitude ratio  $|Z_4|$ , shows a sudden rise for TWM. The values of amplitude ratios  $|Z_1|$ ,  $|Z_3|$ ,  $|Z_4|$  for TWIM initially start from their minima reach upto their maxima at  $\theta_0 \approx 25^\circ$  and decrease, again reach up to their minima. These variations are shown graphically in Figs. 10-13, respectively for the absence of the constituent of mixture. The trend of variations of  $|Z_2|$ ,  $|Z_5|$  for EWM is similar i.e. oscillatory in the whole range of  $\theta_0$ , but the magnitude of oscillation of  $|Z_2|$  is large as compared to  $|Z_5|$ , shown in Figs. 11-14, respectively.



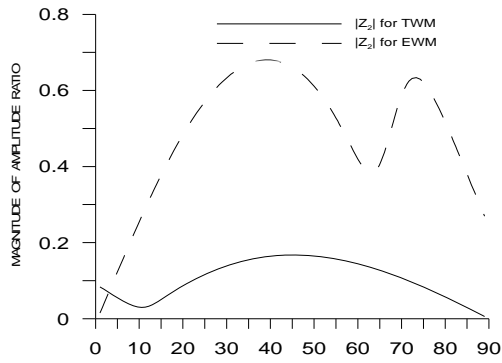
**Fig. 8**  
The variations of amplitude ratios of reflected waves when  $qT$ -wave is incident.



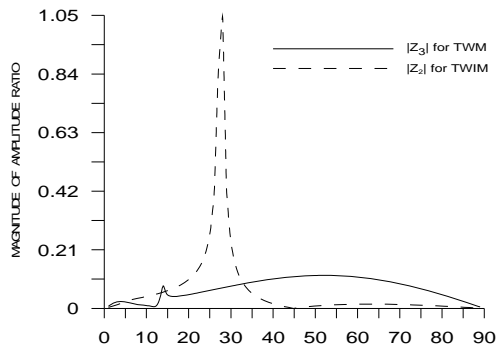
**Fig. 9**  
The variations of amplitude ratios of reflected waves when  $qT$ -wave is incident.



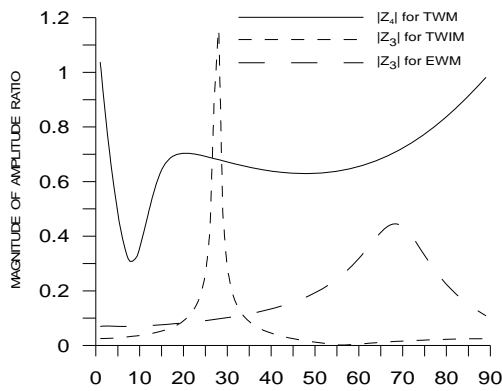
**Fig. 10**  
The variations of amplitude ratios of reflected waves when  $S_1$ -wave is incident.



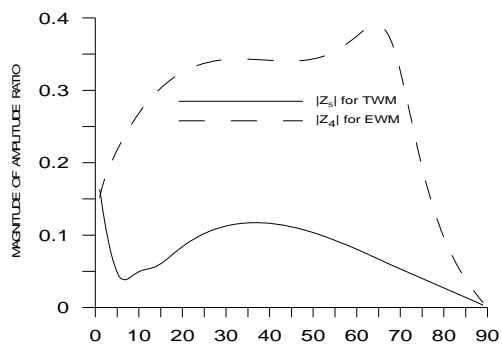
**Fig. 11**  
 The variations of amplitude ratios of reflected waves when  $S_1$ -wave is incident.



**Fig. 12**  
 The variations of amplitude ratios of reflected waves when  $S_1$ -wave is incident.



**Fig. 13**  
 The variations of amplitude ratios of reflected waves when  $S_1$ -wave is incident.



**Fig. 14**  
 The variations of amplitude ratios of reflected waves when  $S_1$ -wave is incident.

## 8 CONCLUSIONS

The numerical calculation for amplitude ratios of various reflected waves have been derived in thermoelastic solid half-space with mixture, without thermal and in the absence of second constituent of mixture for the incident  $qP_1$ ,  $qT$ ,  $S_1$ -waves. It is observed that due to incident  $qP_1$ ,  $qT$ -waves the values of amplitude ratio  $|Z_1|$  for TWIM are maximum at the incident angle  $\theta_0 \approx 50^\circ$  and  $\theta_0 \approx 25^\circ$  for the incident  $S_1$ -wave and the amplitude ratio decrease at these incident angles for TWM. It is also conclude that due to different incident waves we obtain oscillatory behavior of amplitude ratios. We observe that the effect of mixture and thermal are prodigious.

## REFERENCES

- [1] Trusdell C., Tupin R.A., 1960, *The Classical Field Theories*, Volume III/1, Springer-Verlag, Berlin.
- [2] Eringen A.C., Ingram J.D., 1965, A continuum theory of chemically reacting media-I, *International Journal of Engineering Science* **3**: 197-212.
- [3] Green A.E., Naghdi P.M., 1965, A dynamical theory of interacting continua, *International Journal of Engineering Science* **3**: 231-241.
- [4] Bedford A., Stern M., 1972, A multi-continuum theory of elastic materials, *Acta Mechanica* **14**: 85-102.
- [5] Bedford A., Stern M., 1972, Toward a diffusing continuum theory of composite elastic materials, *ASME Journal of Applied Mechanics* **38**: 8-14.
- [6] Bowen R.M., 1976, *Theory of mixtures*, in: Continuum Physics, Volume III, edited by A.C. Eringen, Academic Press, New York.
- [7] Atkin R.J., Craine R.E., 1976, Continuum theories of mixtures: Applications, *Journal of the Institute of Mathematical Applications* **17**: 153-207.
- [8] Atkin R.J., Craine R.E., 1976, Continuum theories of mixtures: Basic theory and Historical Development, *Quarterly Journal of Mechanics and Applied Mathematics* **29**: 209-245.
- [9] Rajgopal K.R., Tao L., 1995, *Mechanics of Mixtures*, World Scientific, Singapore.
- [10] Iesan D., 1991, On the theory of mixtures of thermoelastic solids, *Journal of Thermal Stresses* **14**: 389-408.
- [11] Green A.E., Steel T.R., 1966, Constitutive equations for interacting continua, *International Journal of Engineering Science* **4**: 483-500.
- [12] Steel T.R., 1967, Applications of theory of interacting continua, *Quarterly Journal of Mechanics and Applied Mathematics* **20**: 57-72.
- [13] Bowen R.M., Wiese J.C., 1969, Diffusion in mixtures of elastic materials, *International Journal of Engineering Science* **7**: 689-722.
- [14] Martinez F., Quintanilla R., 1995, Some qualitative results for the linear theory of binary mixtures of thermoelastic solids, *Collectanea Mathematica* **46**: 263-277.
- [15] Pompei A. Scalia A., 1999, On the theory of mixtures of thermoelastic solids, *Journal of Thermal Stresses* **22**: 23-34.
- [16] Burchuladze T., Svanadze M., 2000, Potential method in the linear theory of binary mixtures of thermoelastic solids, *Journal of Thermal Stresses* **23**: 601-626.
- [17] Pompei A., 2003, On the dynamical theory of mixtures of thermoelastic solids, *Journal of Thermal Stresses* **26**: 873-888.
- [18] Passarella F., Vittorio Z., 2007, Some exponential decay estimates for the thermoelastic mixtures, *Journal of Thermal Stresses* **30**: 25-41.
- [19] Sharma M. D., Gogna M. L., 1992, Reflection and refraction of plane harmonic waves at an interface between elastic solid and porous solid saturated by viscous liquid, *Pageoph* **138**: 249-266.
- [20] Vashisth A. K., Sharma M. D., Gogna M. L., 1991, Reflection and transmission of elastic waves at a loosely bonded interface between an elastic solid and liquid-saturated porous solids, *Geophysics International Journal* **105**: 601-617.
- [21] Apice C.D., Chirita S., Tibullo V., 2005, On the spatial behavior in dynamic theory of mixtures of thermoelastic solids, *Journal of Thermal Stresses* **28**: 63-82.
- [22] Lord H., Shulman Y.A., 1967, A generalized dynamical theory of thermoelasticity, *Journal of the Mechanics and Physics of Solids* **15**: 299-309.
- [23] Schoenberg M., 1971, Transmission and reflection of plane waves at an elastic-viscoelastic interface, *Geophysical Journal Royal Astronomical Society* **25**:35-47.
- [24] Dhaliwal R. S., Singh A., 1980, *Dynamic Coupled Thermoelasticity*, Hindustan Publishers, Delhi.