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Wave Propagation in Mixture of Generalized Thermoelastic Solids Half-Space

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ABSTRACT
This paper concentrates on the reflection of plane waves in the mixture of generalized thermo elastic solid half-space. There exists quasi dilatational waves i.e. qP_1, qP_2, qT and two rotational waves S_1, S_2 in a two dimensional model of the solid. The boundary conditions are solved to obtain a system of five non-homogeneous equations for amplitude ratios. These amplitude ratios are found to depend on the angle of incidence of incident wave, mixture and thermal parameters and have been computed numerically and presented graphically. The appreciable effects of mixtures and thermal on the amplitude ratios are obtained.

Keywords: Dynamic theory of mixture; Amplitude ratios; Thermoelasticity; Wave propagation; Relaxation time; Reflection.

1 INTRODUCTION

DURING the past year there has been much effect to develop continuum theories of mixtures. The modern formulation of continuum theories of mixtures goes back to papers by Trusedell and Tupin [1], Eringen and Ingram [2], Green and Naghdi [3]. Bedford and Stern [4, 5] used for the first time the Lagrangian description in order to derive mixture theory of elastic solids. In this theory, the independent constitutive variables are the displacement gradients and the displacement fields. Much of the theoretical progress in the field is discussed in great detail in the review articles by Bowen [6], Atkin and Craine [7, 8], and in the book by Rajagopal and Tao [9].

The theory of binary mixtures of thermoelastic solids, in which the component interaction force depends on a difference of partial displacements, was constructed by Iesan [10]. He presented a theory for a binary mixture of thermoelastic solids in which the independent constitutive variables are the displacement gradients, relative displacement, temperature and temperature gradient whereas [11-13], the theories for mixture of thermoelastic solids studied the independent constitutive variables are the displacement gradients, relative velocity, temperature and temperature gradient. The existence theorems of weak solutions of dynamic problems for the linear theory of two thermoelastic solids are proved by the semigroup theory in [14] and obtain some qualitative results for the linear theory of binary mixtures of thermoelastic solids. Pompei and Scalia [15] addressed the linear dynamic theory of binary mixtures of thermoelastic solids and established the continuous dependence of solutions upon initial data and body loads and proved uniqueness theorems of the solutions for initial boundary value problems Burchuladze and Svanadze [16] investigating the boundary value problems of steady oscillations using the potential methods in the linear theory of binary mixtures of thermoelastic solids. Pompei and Scalia [17] study the continuous dependence of solutions on initial data and body sources in the linear theory of binary mixtures of thermoelastic solids and also establish the existence and uniqueness theorems. Passarella and Victoria [18] study the time-harmonic vibrations for some classes of homogeneous and isotropic thermoelastic mixtures for which the constitutive coefficients are

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supposed to satisfy some mild positive definiteness conditions. Sharma and Gogna [19] studied the reflection and refraction of plane harmonic waves at an interface between elastic solid and orous solid saturated by viscous liquid and Vashisth, Sharma and Gogna [20] studied the reflection and transmission of elastic waves at a loosely bonded interface between an elastic solid and liquid-saturated porous solid. In the present we study the reflection of thermo elastic plane waves in the mixture theory of generalized thermoelastic solid half space.

2 BASIC EQUATIONS

The fundamental equations for the theory of mixtures are constituted from the following equation of motion Steel [12] in thermoelastic solid without body forces are given by

$$T_{ji,j} - p_i = \rho_1 \ddot{u}_i \tag{1}$$
$$S_{ji,j} + p_i = \rho_2 \ddot{w}_i \tag{2}$$

and the equation of energy

$$\rho T_0 \eta = q_{i,i} \tag{3}$$

where ρ_1 and ρ_2 are the constants densities; T_{ij} and S_{ij} are the components of the partial stress tensors associated with the two constituents s_1 and s_2 respectively; p_i are the components of the diffusive force vector; η is the entropy per unit mass and q_i are the components of the heat flux vector. If u_i and w_i represents the components of the displacement vector fields associated with the two constituents s_1 and s_2, respectively. Then, the infinitesimal strain measures are defined by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad h_{ij} = w_{i,j} + u_{j,i} \tag{4}$$

Following Apice et al. [21], Lord and Shulman [22] the constitutive equations for an isotropic and homogeneous mixture are Green and Steel [11-12].

$$T_{ij} = (\lambda + \upsilon)e_{ss}\delta_{ij} + 2(\mu + \zeta)e_{ij} + (\alpha + \upsilon)h_{ss}\delta_{ij} + (2\kappa + \zeta)h_{ij} + (2\gamma + \zeta)h_{ji} - (\beta + b)\theta\delta_{ij} \tag{5}$$
$$S_{ij} = \upsilon e_{ss}\delta_{ij} + 2\zeta e_{ij} + \alpha h_{ss}\delta_{ij} + 2\kappa h_{ji} + 2\gamma h_{ij} - b\theta\delta_{ij} \tag{6}$$
$$p_i = \xi v_i + \sigma g_i \tag{7}$$
$$\rho\eta = \beta e_{ss} + b h_{ss} + a\theta \tag{8}$$
$$q_i = k g_i + m v_i \tag{9}$$

where

$$v_i = \dot{u}_i - \dot{w}_i, \qquad g_i = \theta_{,i} \tag{10}$$

where θ is the variation of the temperature measured from the reference temperature T_0 and λ, μ, α, β, κ, γ, υ, ζ, ξ, σ, m, a, and b are constitutive constants. By substituting the relations (4) and (5)-(10) into the basic Eqs. (1)-(3), we obtain the following equations in terms of u_i, w_i and θ

$$\begin{aligned}(\mu + 2\varsigma + 2\kappa)u_{i,jj} + (\lambda + \alpha + \mu + 2\nu + 2\varsigma + 2\gamma)u_{j,ji} + (2\gamma + \varsigma)w_{i,jj} \\ + (\alpha + \nu + 2\kappa + 2\varsigma)w_{j,ji} + \xi(\dot{u}_i - \dot{w}_i) - (\beta + b + \sigma)\theta_{,i} = \rho_1 \ddot{u}_i\end{aligned} \tag{11}$$
$$(2\gamma + \varsigma)u_{i,jj} + (\alpha + \nu + \varsigma + 2\kappa)u_{j,ji} + 2\kappa w_{i,jj} + (2\gamma + \alpha)w_{j,ji} + \xi(\dot{u}_i - \dot{w}_i) - (b - \sigma)\theta_{,i} = \rho_2 \ddot{w}_i \tag{12}$$

$$K\theta_{,ii} + (m - (\beta + b)T_0(1 + \tau_0 \frac{\partial}{\partial t}))\dot{u}_{s,s} - (m + bT_0(1 + \tau_0 \frac{\partial}{\partial t}))\dot{w}_{s,s} - aT_0(1 + \tau_0 \frac{\partial}{\partial t})\dot{\theta} = 0 \tag{13}$$

3 FORMULATION OF THE PROBLEM

We consider an isotropic homogeneous binary mixture of generalized thermoelastic materials with one relaxation time. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface z=0 with z-axis pointing vertically in to the medium is introduced. For two dimensional problems, we assume

$$\vec{u} = (u_1, 0, u_3), \qquad \vec{w} = (w_1, 0, w_3) \tag{14}$$

Using the expression relating displacement components $u_i(x,z,t)$ and $w_i(x,z,t)$ to the scalar potential functions $\varphi(x,z,t)$, $\psi(x,z,t)$ and $\bar{\varphi}(x,z,t)$, $\bar{\psi}(x,z,t)$ in dimensionless form

$$u_1 = \frac{\partial\varphi}{\partial x} - \frac{\partial\psi}{\partial z}, \qquad u_3 = \frac{\partial\varphi}{\partial z} + \frac{\partial\psi}{\partial x}, \qquad w_1 = \frac{\partial\bar{\varphi}}{\partial x} - \frac{\partial\bar{\psi}}{\partial z}, \qquad w_3 = \frac{\partial\bar{\varphi}}{\partial z} + \frac{\partial\bar{\psi}}{\partial x} \tag{15}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$\begin{gathered} x' = \frac{\omega_1^* x}{c_1},\; z' = \frac{\omega_1^* z}{c_1},\; u_1' = \frac{\omega_1^* u}{c_1},\; u_3' = \frac{\omega_1^* u}{c_1},\; w_1' = \frac{\omega_1^* w}{c_1},\; w_3' = \frac{\omega_1^* w}{c_1}, \\ T_{33}' = \frac{T_{33}}{\mu},\; T_{31}' = \frac{T_{31}}{\mu},\; S_{33}' = \frac{S_{33}}{\mu},\; S_{31}' = \frac{T_{31}}{\mu},\; t' = \omega_1^* t,\; \tau_0' = \omega_1^* \tau_0,\; \theta' = \frac{\theta}{T_o} \end{gathered} \tag{16}$$

where

$$c_1 = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}}, \qquad \omega_1^* = \frac{\rho c_e c_1^2}{K}$$

Using Eqs. (14)-(16) in Eqs. (11)-(13) (after suppressing the primes), we obtain

$$(1 + a_1)\nabla^2\varphi + (a_2 + a_3)\nabla^2\bar{\varphi} - a_4(\dot{\varphi} - \dot{\bar{\varphi}}) - a_5\theta = a_6\ddot{\varphi} \tag{17}$$

$$\nabla^2\psi + a_2\nabla^2\bar{\psi} - a_4(\dot{\psi} - \dot{\bar{\psi}}) = a_6\ddot{\psi} \tag{18}$$

$$(1 + a_7)\nabla^2\varphi + (a_8 + a_9)\nabla^2\bar{\varphi} + a_{10}(\dot{\varphi} - \dot{\bar{\varphi}}) - a_{11}\theta = a_{12}\ddot{\bar{\varphi}} \tag{19}$$

$$\nabla^2\psi + a_8\nabla^2\bar{\psi} + a_{10}(\dot{\psi} - \dot{\bar{\psi}}) = a_{12}\ddot{\bar{\psi}} \tag{20}$$

$$\nabla^2\theta + \{a_{13} - a_{14}(1 + \tau_0 \frac{\partial}{\partial t})\}\nabla^2\dot{\varphi} - \{a_{13} + a_{15}(1 + \tau_0 \frac{\partial}{\partial t})\}\nabla^2\dot{\bar{\varphi}} - a_{16}(1 + \tau_0 \frac{\partial}{\partial t})\dot{\theta} = 0 \tag{21}$$

We assume the solution of Eqs. (17)-(21) as

$$\{\varphi, \bar{\varphi}, \theta, \psi, \bar{\psi}\} = \{\varphi^*, \bar{\varphi}^*, \theta^*, \psi^*, \bar{\psi}^*\}\exp\{\iota\{k(x\sin\theta - z\cos\theta) - \omega t\}\} \tag{22}$$

Using Eq. (22) in Eqs. (17), (19) and (21) (by omitting the star), we get

$$\{-(1 + a_1)k^2 + a_4\iota\omega + a_6\omega^2\}\varphi + \{-(a_2 + a_3)k^2 - a_4\iota\omega\}\bar{\varphi} - a_5\theta = 0 \tag{23}$$

$$\{-(1 + a_7)k^2 - a_{10}\iota\omega\}\varphi + \{-(a_8 + a_9)k^2 + a_{10}\iota\omega + a_{12}\omega^2\}\bar{\varphi} - a_{11}\theta = 0 \tag{24}$$

$$\iota\omega k^2(a_{13} + a_{14}\iota\omega\tau_0^*)\varphi + \iota\omega k^2(-a_{13} + a_{15}\iota\omega\tau_0^*)\bar{\varphi} + (k^2 - a_{16}\omega^2\tau_0^*)\theta = 0 \tag{25}$$

For non-trivial solution of the system of Eqs. (23)-(25), we have

$$\begin{vmatrix} -(1+a_1)k^2+a_4\iota\omega+a_6\omega^2 & -(a_2+a_3)k^2-a_4\iota\omega & -a_5 \\ -(1+a_7)k^2-a_{10}\iota\omega & -(a_8+a_9)k^2+a_{10}\iota\omega+a_{12}\omega^2 & -a_{11} \\ \iota\omega k^2(a_{13}+a_{14}\iota\omega\tau_0^*) & \iota\omega k^2(-a_{13}+a_{15}\iota\omega\tau_0^*) & -k^2+a_{16}\omega^2\tau_0^* \end{vmatrix} \begin{vmatrix} \varphi \\ \overline{\varphi} \\ \theta \end{vmatrix} = 0 \tag{26}$$

i.e. $(V^6+\omega^2AV^4+\omega^4BV^2+\omega^6C)(\phi,\overline{\phi},\theta)=0$ (27)

Eqs. (27) is cubic in V^2 therefore the roots of this equation give three values of V^2. Each value of V^2 corresponds to a velocity of propagation of three possible wave. The waves with velocities V_1, V_2, V_3 corresponds to qP_1, qP_2 and qT-waves, respectively.

where

$$A=\frac{A_3}{A_4},\ B=\frac{A_2}{A_4},\ \ C=\frac{A_1}{A_4},\ V=\frac{\omega}{k}$$

with

$$\begin{aligned}
A_4 &= \{(\iota\omega a_4+\omega^2a_6)(\iota\omega^3\tau_0^*a_{10}a_{16}+\omega^4\tau_0^*a_{12}a_{16})+a_4a_{10}a_{16}\omega^4\tau_0^*\} \\
A_3 &= \{(a_4\iota\omega+a_6\omega^2)(-a_{10}\iota\omega-a_{12}\omega^2-(a_8+a_9)a_{16}\tau_0^*\omega^2-a_{11}a_{13}\iota\omega-a_{11}a_{15}\omega^2\tau_0^*) \\
&\quad -(1+a_1)\tau_0^*a_{16}(a_{10}\iota\omega^3+a_{12}\omega^4)+a_4\iota\omega(-a_{11}a_{13}\iota\omega-a_{11}a_{14}\omega^2\tau_0^*+a_{10}\iota\omega-(1+a_7)a_{16}\omega^2\tau_0^*) \\
&\quad -(a_2+a_3)a_{10}a_{16}\iota\omega^3\tau_0^*-a_5(a_{10}a_{15}\iota\omega^3\tau_0^*-a_{12}a_{13}\iota\omega^3+a_{10}a_{14}\iota\omega^3\tau_0^*+a_{12}a_{14}\omega^4\tau_0^*)\}, \\
A_2 &= \{(a_8+a_9)(a_4\iota\omega+a_6\omega^2)-(1+a_1)(-a_{10}\iota\omega-a_{12}\omega^2-(a_8+a_9)a_{16}\tau_0^*\omega^2-a_{11}a_{13}\iota\omega \\
&\quad +a_{11}a_{15}\omega^2\tau_0^*)+a_4(1+a_7)\tau_0^*\iota\omega-(a_2+a_3)(-a_{11}a_{13}\iota\omega+a_{11}a_{14}\omega^2\tau_0^*-a_{10}\iota\omega \\
&\quad +(1+a_7)a_{16}\omega^2\tau_0^*)-a_5((1+a_7)(a_{13}\iota\omega+a_{15}\tau_0^*\omega^2)+(a_8+a_9)a_{13}\iota\omega-(a_8+a_9)a_{14}\omega^2\tau_0^*)\} \\
A_1 &= \{-(1+a_1)(a_8+a_9)+(a_2+a_3)(1+a_7)\}
\end{aligned}$$

$$\tau_0^*=(\tau_0+\iota\omega^{-1}),\ a_1=\frac{\lambda+\alpha+\mu+2\nu+2\gamma+2\varsigma}{\mu+2\varsigma+2\kappa},\ a_2=\frac{2\gamma+\varsigma}{\mu+2\varsigma+2\kappa},\ \ a_3=\frac{\alpha+\nu+2\kappa+\varsigma}{\mu+2\varsigma+2\kappa},$$

$$a_4=\frac{\xi c_1^2}{\omega_1^*(\mu+2\varsigma+2\kappa)},\ \ a_5=\frac{(\beta+b+\sigma)T_0}{\mu+2\varsigma+2\kappa},\ a_6=\frac{\rho_1c_1^2}{\mu+2\varsigma+2\kappa},\ \ a_7=\frac{\alpha+\nu+2\kappa+\varsigma}{\varsigma+2\gamma},$$

$$a_8=\frac{2\kappa}{\varsigma+2\gamma},\ a_9=\frac{2\gamma+\alpha}{\varsigma+2\gamma},\ a_{10}=\frac{\xi c_1^2}{\omega_1^*(2\gamma+\varsigma)},\ a_{11}=\frac{(b-\sigma)T_0}{2\gamma+\varsigma},\ \ a_{12}=\frac{\rho_2c_1^2}{2\gamma+\varsigma},$$

$$a_{13}=\frac{mc_1^2}{K\omega_1^*T_0},\ \ a_{14}=\frac{(\beta+b)c_1^2}{K\omega_1^*},\ a_{15}=\frac{bc_1^2}{K\omega_1^*},\ \ a_{16}=\frac{aT_0c_1^2}{K\omega_1^*}$$

From Eq. (26), we obtain $\theta=m_{ii}\phi,\ \overline{\phi}=m_i\,\phi.$

with

$$m_{ii}=\frac{DetIII}{DetI},\qquad m_i=\frac{-DetII}{DetI}\qquad i=1,2,3$$

where

$$DetI=((a_8+a_9)k^2+\iota\omega a_{10}+a_{12}\omega^2)(-k^2+\omega^2\tau_0^*a_{16})+\iota\omega k^2a_{11}(-a_{13}+\iota\omega\tau_0^*a_{15})$$

$$DetII = (-(1+a_1)k^2 + a_4\iota\omega + a_6\omega^2)(-k^2 + \omega^2\tau_0^* a_{16}) + a_5\iota\omega k^2(a_{13} + \iota\omega\tau_0^* a_{14})$$
$$DetIII = (-(1+a_1)k^2 + \iota\omega a_4 + a_6\omega^2)(-(a_8+a_9)k^2 + \iota\omega a_{10} + a_{12}\omega^2) + (a_2+a_3)k^2 + \iota\omega a_4)(-(1+a_7)k^2 - \iota\omega a_{10})$$

Using Eq. (22) in Eqs. (18) and (20) (by omitting the star), we obtain the non-trivial solution of the system of Eqs. (18)-(20), as

$$\begin{vmatrix} -k^2 + a_4\iota\omega + a_6\omega^2 & -a_2k^2 - a_4\iota\omega \\ -k^2 - a_{10}\iota\omega & -a_8k^2 + a_{10}\iota\omega + a_{12}\omega^2 \end{vmatrix} \begin{vmatrix} \psi \\ \bar{\psi} \end{vmatrix} = 0 \tag{28}$$

i.e. $(V^4 + DV^2 + E)(\psi, \bar{\psi}) = 0$ (29)

Eq. (29) is quadratic in V^2 Therefore, two values of V will be velocity of propagation of S_1, S_2-waves, respectively, where

$$D = \frac{C_D}{C_0}, \qquad E = \frac{C_E}{C_0},$$

with

$$C_0 = \iota\omega^3(a_6a_{10} + a_4a_{12}) + \omega^4 a_6 a_{12}$$
$$C_D = -\iota\omega^3(a_4a_8 + a_{10} + a_4 + a_2a_{10}) - \omega^4(a_6a_8 + a_{12})$$
$$C_E = \omega^4(a_8 - a_2)$$

Also from Eq. (28), we have $\bar{\psi} = m_j\psi, \quad j = 4, 5$
where

$$m_j = \frac{DetII}{DetI}$$

with

$$DetI = -a_8k^2 + \iota\omega a_{10} + a_{12}\omega^2$$
$$DetII = k^2 - \iota\omega a_{10}$$

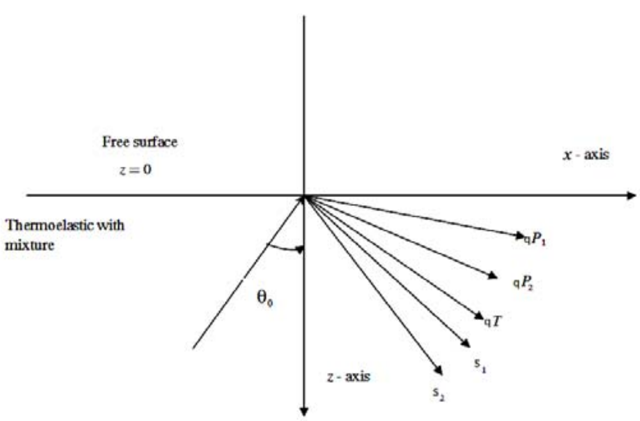

Fig. 1
Geometry of the Problem (Reflection).

4 REFLECTION

When we consider an incident quasi dilatational waves $qP_1,\ qP_2, qT$ -waves or rotational waves S_1, S_2 -waves incident on the free surface then five waves are reflected, namely qP_1, qP_2, qT, S_1 and S_2, respectively at complex angles $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ as shown in Fig. 1.

5 BOUNDARY CONDITIONS

The boundary conditions are given by vanishing of the stress component (normal and tangential), vanishing of relative velocities of the two constituents (normal and tangential) and also vanishing of the temperature gradient field. Mathematically these can be written as

$$T_{33}+S_{33}=0,\quad T_{31}+S_{31}=0,\quad \dot{u}_1-\dot{w}_1=0,\quad \dot{u}_3-\dot{w}_3=0,\quad \frac{\partial\theta}{\partial z}=0 \quad \text{at} \quad z=0 \tag{30}$$

The appropriate potentials satisfying the boundary conditions can be written as

$$(\phi,\ \overline{\phi},\ \theta)=\sum_{i=1}^{3}(1,m_i,m_{ii})\,(A_{0i}Exp(\iota P_i)+A_i Exp(\iota P_i')) \tag{31}$$

$$(\psi,\ \overline{\psi})=\sum_{j=4}^{5}(1,m_j)\ (B_{0j}Exp(\iota P_j)+B_j Exp(\iota P_j')) \tag{32}$$

where A_{0i} and B_{0j} are the amplitudes of the incident qP_1, qP_2, qT and S_1, S_2 waves, respectively.

$$\begin{aligned}
&P_i=k_i(x\sin\theta_0-z\cos\theta_0)-\omega_i t\\
&P_i'=k_i(x\sin\theta_i+z\cos\theta_i)-\omega_i t, \qquad i=1,2,3\\
&P_j=k_j(x\sin\theta_0-z\cos\theta_0)-\omega_j t\\
&P_j'=k_j(x\sin\theta_j+z\cos\theta_j)-\omega_j t, \qquad j=4,5\\
&A_{02}=A_{03}=B_{04}=B_{05}=0 \qquad \text{for incident } qP_1\text{-wave}\\
&A_{01}=A_{03}=B_{04}=B_{05}=0 \qquad \text{for incident } qP_2\text{-wave,}\\
&A_{01}=A_{02}=A_{03}=B_{04}=0 \qquad \text{for incident } qT\text{-wave}\\
&A_{01}=A_{02}=B_{04}=B_{05}=0 \qquad \text{for incident } S_1\text{-wave}\\
&A_{01}=A_{02}=A_{03}=B_{05}=0 \qquad \text{for incident } S_2\text{-wave, respectively}
\end{aligned} \tag{32}$$

Making use of Eqs. (31)-(32), the boundary conditions (30) can be satisfied if the angles $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ satisfy the relations i.e. Snell's law given as

$$\frac{\sin\theta_0}{v_0}=\frac{\sin\theta_1}{v_1}=\frac{\sin\theta_2}{v_2}=\frac{\sin\theta_3}{v_3}=\frac{\sin\theta_4}{v_4}=\frac{\sin\theta_5}{v_5} \tag{33}$$

where

$$V_j=\frac{\omega}{k_j}, \qquad (j=1,\ 2,\ 3,\ 4,\ 5) \quad \text{at} \quad z=0 \tag{34}$$

and

$$v_0 = \begin{cases} v_1 & \text{for incident } qP - \text{wave} \\ v_2 & \text{for incident } qP_2 - \text{wave} \\ v_3 & \text{for incident } qT - wave \\ v_4 & \text{for incident } S_1 - \text{wave} \\ v_5 & \text{for incident } S_2 - \text{wave} \end{cases} \tag{33c}$$

Making use of potentials given by Eqs. (31)-(32), and the Snell's law given by Eq. (33) in boundary condition and with the help of Eq. (16), we get a system of five non-homogeneous equations, which can be written as

$$\sum_{m=1}^{5} c_{mn} Z_n = Y_m, \qquad (m = 1,2,3,4,5) \tag{35}$$

where

$Z_i = A_i / A^*$, $(i = 1, 2, 3)$ and $Z_j = B_j / A^*$, $(j = 4, 5)$ are the complex amplitude ratios of the reflected qP_1, qP_2, qT and S_1, S_2 waves, respectively, and

$$\begin{aligned}
c_{1i} &= -(q_1 + q_3 m_i)\sin^2\theta_0 + (q_2 + q_4 m_i)[(V_0 / V_i)^2 - \sin^2\theta_0] - q_5 m_{ii} (V_0 / \omega)^2 \\
c_{1j} &= (q_3 - q_4) m_j \sin\theta_0 [(V_0 / V_j)^2 - \sin^2\theta_0]^{1/2} \\
c_{2i} &= \{-(R_1 + R_2) - (R_3 + R_4) m_i\} \sin\theta_0 [(V_0 / V_i)^2 - \sin^2\theta_0]^{1/2} \\
c_{2j} &= -(R_1 + R_3 m_j)\sin^2\theta_0 + (-R_2 + R_4 m_j)[(V_0 / V_j)^2 - \sin^2\theta_0] \\
c_{3i} &= (1 - m_i)\sin\theta_0 \\
c_{3j} &= (-1 + m_j)[(V_0 / V_j)^2 - \sin^2\theta_0]^{1/2} \\
c_{4i} &= (1 - m_i)[(V_0 / V_i)^2 - \sin^2\theta_0]^{1/2} \\
c_{4j} &= (1 - m_j)[(V_0 / V_j)^2 - \sin^2\theta_0]^{1/2} \\
c_{5i} &= m_{ii}[(V_0 / V_i)^2 - \sin^2\theta_0]^{1/2} \\
c_{5j} &= 0
\end{aligned}$$

with

$$\begin{aligned}
&q_1 = b_1 + b_3,\ q_2 = b_2 + b_4,\ \ q_3 = b_3 + b_{10},\ q_4 = b_4 + b_{11},\ q_5 = b_5 + b_{12}, \\
&R_1 = b_7 + b_9,\ \ R_2 = b_6 + b_8,\ \ R_3 = b_9 + b_{14},\ \ R_4 = b_8 + b_{13} \\
&b_1 = \frac{\lambda + \alpha + 2\nu}{\mu},\ b_2 = \frac{\lambda + \alpha + 2\nu + 2(\mu + \varsigma) + (2\kappa + \varsigma) + (2\gamma + \varsigma)}{\mu},\ \ b_3 = \frac{\alpha + \nu}{\mu}, \\
&b_4 = \frac{\alpha + \nu + (2\kappa + \varsigma) + (2\gamma + \varsigma)}{\mu},\ \ b_5 = \frac{(\beta + b)T_0}{\mu},\ \ b_6 = \frac{(\mu + \varsigma) + (2\kappa + \varsigma)}{\mu}, \\
&b_7 = \frac{(\mu + \varsigma) + (2\gamma + \varsigma)}{\mu},\ \ b_8 = \frac{2\gamma + \varsigma}{\mu},\ \ b_9 = \frac{2\kappa + \varsigma}{\mu},\ \ b_{10} = \frac{\alpha}{\mu},\ b_{11} = \frac{\alpha + 2\kappa + 2\gamma}{\mu}, \\
&b_{12} = \frac{bT_0}{\mu},\ b_{13} = \frac{2\kappa}{\mu},\ \ b_{14} = \frac{2\gamma}{\mu}
\end{aligned}$$

Considering the phase of the reflected waves can easily write using Eqs. (33)-(34)

$$\frac{\cos\theta_i}{V_i}=\frac{1}{V_0}[(V_0/V_i)^2-\sin^2\theta_0]^{1/2},\ \frac{\cos\theta_j}{V_j}=\frac{1}{V_0}[(V_0/V_j)^2-\sin^2\theta_0]^{1/2}.$$

Following Schoenberg [23], if we write

$$\frac{\cos\theta_i}{V_i}=\frac{\cos\theta_i^{'}}{V_i^{'}}+\iota\frac{c_i}{V_0 2\pi},\quad (i=1,\ 2,\ 3,\ 4,\ 5)$$

Then,

$$\frac{\cos\theta_i^{'}}{V_i^{'}}=\frac{1}{V_0}R\left\{[(V_0/V_i)^2-\sin^2\theta_0]^{1/2}\right\},\quad c_i=2\pi I\left\{[(V_0/V_i)^2-\sin^2\theta_0]^{1/2}\right\}$$

where $V_i^{'}$, , the real phase speed and $\theta_i^{'}$, , the angle of reflection are given by

$$\frac{V_i^{'}}{V_0}=\frac{\sin\theta_i^{'}}{\sin\theta_0}\left[\sin^2\theta_0+\left[R\left\{[(V_0/V_j)^2-\sin^2\theta_0]^{1/2}\right\}\right]^2\right]^{-1/2}$$

and c_i, the attenuation in a depth is equal to the wavelength of incident wave i.e. $2\pi V_0/\omega$.

(i) when qP_1-wave is incident $A^*=A_{01}$ $(\theta_0=\theta_1)$, $Y_i=(-1)^i c_{i1}$, $Y_j=c_{j4}$
(ii) when qP_2-wave is incident $A^*=A_{02}$ $(\theta_0=\theta_2)$, $Y_i=(-1)^i c_{i2}$, $Y_j=c_{j2}$
(iii) when qT (thermal)-wave is incident $A^*=A_{03}$ $(\theta_0=\theta_3)$, $Y_i=(-1)^i c_{i3}$, $Y_j=c_{j3}$, $\quad i=1,2,3;\ j=4,5$
(iv) when S_1-wave is incident $A^*=B_{04}$ $(\theta_0=\theta_4)$, $Y_i=(-1)^{i+1}c_{i4}$
(v) when S_2-wave is incident $A^*=B_{05}$ $(\theta_0=\theta_5)$, $Y_i=(-1)^{i+1}c_{i4}, Y_5=0$, $\quad i=1,2,3,4;$

The complex amplitude ratios of various reflected waves are $Z_i=A_i/A^*$, $Z_j=B_j/A^*$, $(i=1,2,3;\ j=4,5)$.

6 PARTICULAR CASES

Case I: In the absence of second constituent of mixture we have i.e. $\nu=\varsigma=\alpha=\kappa=\gamma=b=\xi=0$, our results reduce in generalized thermoelastic solid with one relaxation time as

$$\sum_{m=1}^{3}c_{nm}^{'}Z_n=Y_m,\qquad m=1,2,3 \tag{36}$$

where

$$c_{1i}^{''}=q_1^{**}\sin^2\theta_0+q_2^{**}[(V_0/V_i^{**})^2-\sin^2\theta_0]+q_5^{**}m_{ii}^{**}(V_0/\omega)^2$$
$$c_{13}^{''}=-(q_1^{**}-q_2^{**})\sin\theta_0[(V_0/V_3^{**})^2-\sin^2\theta_0]$$
$$c_{2i}^{''}=\sin\theta_0[(V_0/V_i^{**})^2-\sin^2\theta_0]^{1/2}$$
$$c_{23}^{''}=\sin^2\theta_0-[(V_0/V_i^{**})^2-\sin^2\theta_0]$$
$$c_{3i}^{''}=m_{ii}^{**}[(V_0/V_i^{**})^2-\sin^2\theta_0]$$

$c_{33}^{''}=0 \qquad i=1,\ 2$

and the complex amplitude ratios of three reflected waves are $Z_i = A_i / A^*,\ Z_3 = B_4 / A^*;\ i=1,\ 2.$

(i) when qP_1-wave is incident $A^* = A_{01}\ (\theta_0=\theta_1),\ Y_i=(-1)^i c_{i1}^{'}$
(ii) when qT (thermal)-wave is incident $A^* = A_{03}\ (\theta_0=\theta_3),\ Y_i=(-1)^i c_{i2}^{'}$
(iii) when S_1-wave is incident $A^* = B_{04}\ (\theta_0=\theta_4), Y_i=(-1)^{i+1} c_{i3}^{'}, \qquad Y_3=0, \qquad i=1,2$

where

$$m_{ii}^* = \frac{-a_{14}^* \iota k V_i^{**}}{(1+a_{16}^* V_i^* \tau_0^*)},\ a_1^{**}=\frac{\lambda+\mu}{\mu},\ a_5^{**}=\frac{\beta T_0}{\mu},\ a_6^{**}=\frac{\rho_1 c_1^2}{\mu},\ a_{14}^{**}=\frac{\beta c_1^2}{K\omega_1^*},$$

$$b_1^{**}=\frac{\lambda}{\mu},\ b_2^{**}=\frac{\lambda+2\mu}{\mu}, b_5^{**}=\frac{\beta T_0}{\mu},\ b_6^{**}=b_7^{**}=1$$

and

$V_i^{**};\ (i=1,\ 2)$ can be obtain from the equation $(V^{*4}+A^{**}V^{*2}+C^{**})(\phi,\theta)=0.$

where

$$A^{**}=\frac{\frac{\iota}{\omega} a_5^{**} a_{14}^{**} - a_6^{**} + a_{16}(1+a_1^{**})\tau_0^*}{-a_6^{**} a_{16}\tau_0^*},\ C^{**}=\frac{(1+a_1^{**})}{-a_6^{**} a_{16}\tau_0^*}$$

and

$$V_4^{**}=\sqrt{\frac{1}{a_6^{**}}}$$

Case II: neglecting thermal effect i.e. $b=\sigma=m=K=a=\beta=aT_0=0,$, our results reduce in elastic solid with mixture as

$$\sum_{n=1}^{4} c_{nm}^{''} Z_n = Y_m, \qquad m=1,2,3,4 \tag{37}$$

where

$$\begin{aligned}
c_{1i}^{'} &= -(q_1+q_3 m_i^*)\sin^2\theta_0 + (q_2+q_4 m_i^*)[(V_0/V_i^*)^2-\sin^2\theta_0] \\
c_{1j}^{'} &= (q_3-q_4)m_j \sin\theta_0[(V_0/V_j)^2-\sin^2\theta_0]^{1/2} \\
c_{2i}^{'} &= \{-(R_1+R_2)-(R_3+R_4)m_i^*\}\sin\theta_0[(V_0/V_i^*)^2-\sin^2\theta_0]^{1/2} \\
c_{2j}^{'} &= -(R_1+R_3 m_j)\sin^2\theta_0 + (-R_2+R_4 m_j)[(V_0/V_j)^2-\sin^2\theta_0] \\
c_{3i}^{'} &= (1-m_i^*)\sin\theta_0 \\
c_{3j}^{'} &= (-1+m_j)[(V_0/V_j)^2-\sin^2\theta_0]^{1/2} \\
c_{4i}^{'} &= (1-m_i^*)[(V_0/V_i^*)^2-\sin^2\theta_0]^{1/2} \\
c_{4j}^{'} &= (1-m_j)[(V_0/V_j)^2-\sin^2\theta_0]^{1/2}, \qquad (i=1,\ 2;\quad \mathrm{j}=3,\ 4\ )
\end{aligned}$$

where

$$m_i^* = \frac{(1+a_7)k^2 + a_{10}\iota k\vartheta_i^*}{(a_8+a_9)k^2 - \iota k a_{10}\vartheta_i^* + a_{12}k^2\vartheta_i^{*2}}$$

and $V_i^*;\ i=1,\ 2$ can be obtain from $(V^{*4} + A^* V^{*2} + C^*)(\phi,\ \overline{\phi}) = 0,$ with

$$A^* = \frac{-\{(a_8+a_9)(a_4\iota\omega^3 + a_6\omega^4) + \iota\omega^3 a_{10}a_{12}(1+a_1) + \iota\omega^3 a_4(1+a_7) + \iota\omega^3 a_{10}(a_2+a_3)\}}{\{(a_4\iota\omega + a_6\iota\omega^2)\iota\omega a_{10}a_{12} + a_4 a_{10}\omega^2\}}$$

$$C^* = \frac{\{(a_8+a_9)(1+a_1)\omega^4 - (1+a_7)(a_2+a_3)\omega^4\}}{\{(a_4\iota\omega + a_6\iota\omega^2)\iota\omega a_{10}a_{12} + a_4 a_{10}\omega^2\}}$$

and

(i) when qP_1-wave is incident $A^* = A_{01}\ (\theta_0 = \theta_1),\ Y_i = (-1)^i c''_{i1}$
(ii) when qP_2-wave is incident $A^* = A_{02}\ (\theta_0 = \theta_2),\ Y_i = (-1)^i c''_{i2}$
(iii) when S_1-wave is incident $A^* = B_{04}\ (\theta_0 = \theta_4),\ Y_i = (-1)^{i+1} c''_{i3}$
(iv) when S_2-wave is incident $A^* = B_{05}(\theta_0 = \theta_5), Y_i = (-1)^{i+1} c'_{i4}$, $\quad (i = 1, 2, 3, 4).$

The complex amplitude ratios of four reflected waves are

$$Z_1 = \frac{A_1}{A^*},\ Z_2 = \frac{A_2}{A^*},\ Z_3 = \frac{B_4}{A^*},\ Z_4 = \frac{B_5}{A^*} \tag{36c}$$

7 NUMERICAL RESULT AND DISCUSSION

In order to illustrate theoretical results obtained in the proceeding sections, we now present some numerical results. Following Dhaliwal and Singh [24] We take the following values of relevant parameters as

$\lambda = 2.17\times10^{10}\,\mathrm{Nm}^{-2},\ \mu = 3.278\times10^{10}\,\mathrm{Nm}^{-2},\ \nu = 0.75\times10^{10}\,\mathrm{Nm}^{-2},\ \rho_1 = 1.74\times10^3\,\mathrm{Kg\,m}^{-3},$
$\rho_2 = 1.35\times10^3\,\mathrm{Kg\,m}^{-3},\ T_o = 298\mathrm{K},\ C_e = 1.04\times10^3\,\mathrm{JKg}^{-1}\mathrm{deg}^{-1},\ K = 1.7\times10^2\,\mathrm{w\,m}^{-1}\mathrm{deg}^{-1},$
$\varsigma = 1.85\times10^{10}\,\mathrm{Nm}^{-2},\ \alpha = 0.5\times10^{10}\,\mathrm{Nm}^{-2},\ \kappa = 0.85\times10^{10}\,\mathrm{Nm}^{-2},\ \gamma = 1.95\times10^{10}\,\mathrm{Nm}^{-2},$
$m = 0.75\times10^{10}\,\mathrm{Nm}^{-2}, \beta = 2.68\times10^6\,\mathrm{Nm}^{-2}\mathrm{K}^{-1},\ b = 1.85\times10^6\,\mathrm{Nm}^{-2}\mathrm{K}^{-1},\ \sigma = 2.05\times10^6\,\mathrm{Nm}^{-2}\mathrm{K}^{-1},$
$a = 1.85\times10^6\,\mathrm{Nm}^{-2}\mathrm{K}^{-2},\ \xi = .075\times10^{12}\,\mathrm{Nsm}^{-4}$

The comparison were carried out for nondimensional frequency and relaxation times i.e. $(\omega/\omega_1^*) = 4$, and $\tau_o = 0.02$. A computer programme has been developed and an amplitude ratio of various reflected waves has been computed. The variations of amplitude ratios for thermoelastic with mixture (TWM), thermoelastic in the absence of second constituent of mixture(TWIM) and Elastic with mixture (EWM) have been shown by solid line, small dashed line and long dashed line, respectively. The variations of the amplitude ratios $|Z_i|(i = 1,2,3,4,5)$ for TWM, EWM, TWIM with the angle of incidence of the incident qP_1-wave, incident qT-wave and incident S_1-wave are shown graphically in the Figs. 2-14.

7.1 Incident qP_1-wave

Fig. 2 shows the variations of $|Z_1|$ with angle of incidence θ_0. The amplitude ratio $|Z_1|$ for TWM, EWM, is generally small as compared to $|Z_1|$ for TWM but the trend of variations of $|Z_1|$ for TWM, TWIM, EWM is similar whereas the corresponding values are different in magnitude, respectively. The presence of constituent of mixture decreases the amplitude ratio at $\theta_0 \approx 45^\circ$ and increases as θ_0 increases further. Fig.3. shows the variations of amplitude ratio $|Z_2|$ with angle of incidence θ_0. It is observed that the absence of the thermal parameters the amplitude ratio $|Z_2|$ for TWIM is slightly large in the range $0^\circ \le \theta_0 \le 10^\circ$ as compared to TWM and decrease in the increasing direction of θ_0. The behavior of variations of amplitude ratios $|Z_3|$, $|Z_4|$ and $|Z_5|$ are oscillatory in the whole range of θ_0 but the magnitude of oscillation is different for different amplitude ratios, these variations are shown in Figs.4-6, respectively.

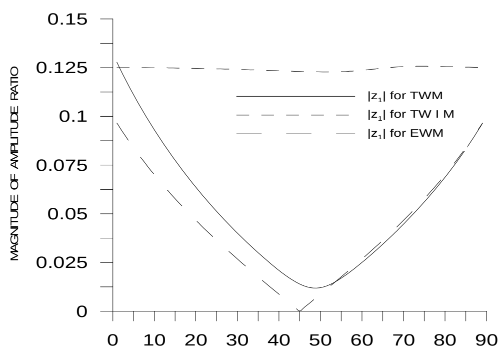

Fig. 2
The variations of amplitude ratios of reflected waves when qP_1-wave is incident.

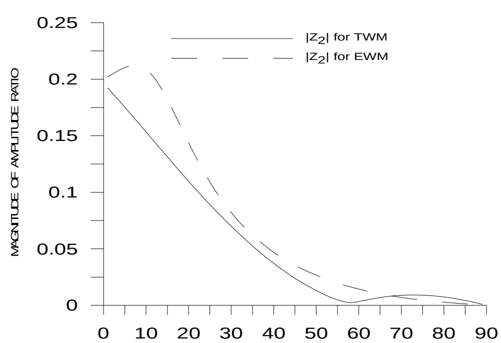

Fig. 3
The variations of amplitude ratios of reflected waves when qP_1-wave is incident.

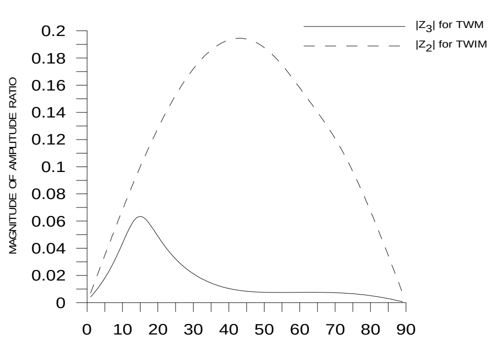

Fig. 4
The variations of amplitude ratios of reflected waves when qP_1-wave is incident.

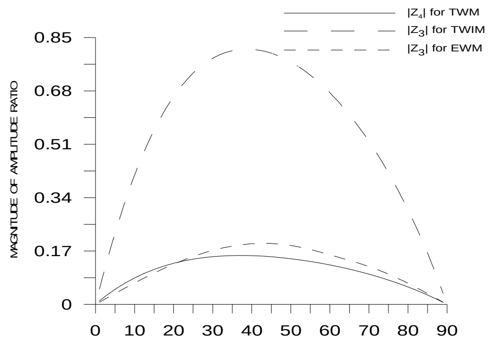

Fig. 5
The variations of amplitude ratios of reflected waves when qP_1-wave is incident.

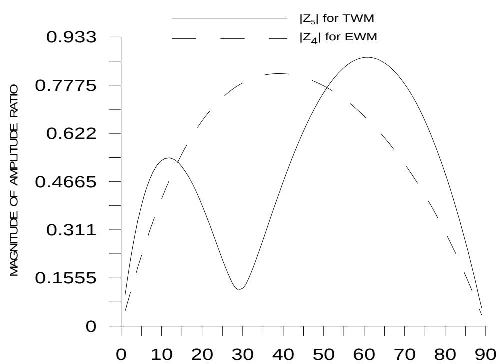

Fig. 6
The variations of amplitude ratios of reflected waves when qP_1-wave is incident.

7.2 Incident qT -wave

Figs. 7-9 represent the variations of amplitude ratios $|Z_1|, |Z_2|, |Z_3|$ with angle of incidence θ_0. The behavior of variations of amplitude ratios $|Z_1|, |Z_3|$ is similar i.e. the values of amplitude ratios for TWM decrease in the range $0^\circ \le \theta_0 \le 9^\circ$ and oscillate around zero as θ_0 increases further and for TWIM increase in the range $0^\circ \le \theta_0 \le 49^\circ$ and decrease in the rest range of θ_0, shown in Figs. 7-9, respectively. But the trend of variations of amplitude ratio $|Z_2|$ for TWM, TWIM is similar for all values of θ_0, as depicts graphically in Fig. 8. In the case of incident qT -wave, resulting reflecting waves are same as that of incident qP_1 **-**wave.

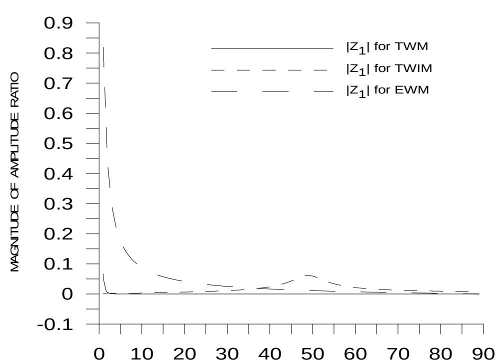

Fig. 7
The variations of amplitude ratios of reflected waves when qT-wave is incident.

7.3 Incident S_1 -wave

At $\theta_0 \approx 40^\circ$ the values amplitude ratios $|Z_1|, |Z_2|, |Z_3|$, show a sudden fall and vanish as the angle of incidence exceeds this angle but the amplitude ratio $|Z_4|$, shows a sudden rise for TWM. The values of amplitude ratios $|Z_1|, |Z_3|, |Z_4|$ for TWIM initially start from their minima reach upto their maxima at $\theta_0 \approx 25^\circ$ and decrease, again reach up to their minima. These variations are shown graphically in Figs. 10-13, respectively for the absence of the constituent of mixture. The trend of variations of $|Z_2|, |Z_5|$ for EWM is similar i.e. oscillatory in the whole range of θ_0, but the magnitude of oscillation of $|Z_2|$ is large as compared to $|Z_5|$, shown in Figs. 11-14, respectively.

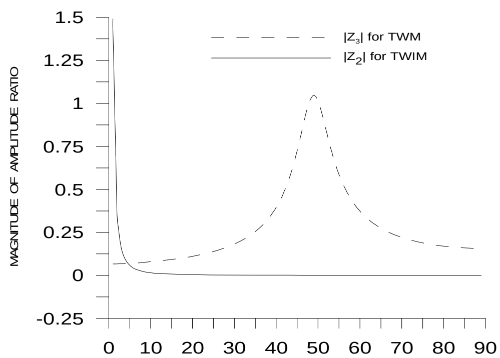

Fig. 8
The variations of amplitude ratios of reflected waves when *qT*-wave is incident.

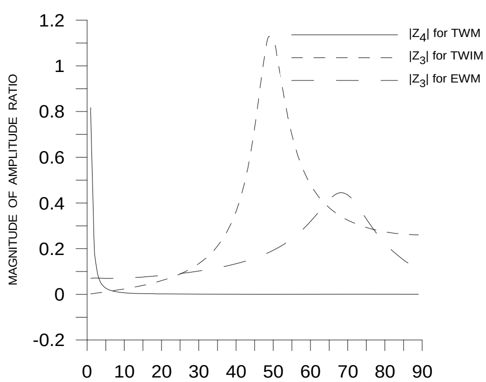

Fig. 9
The variations of amplitude ratios of reflected waves when *qT*-wave is incident.

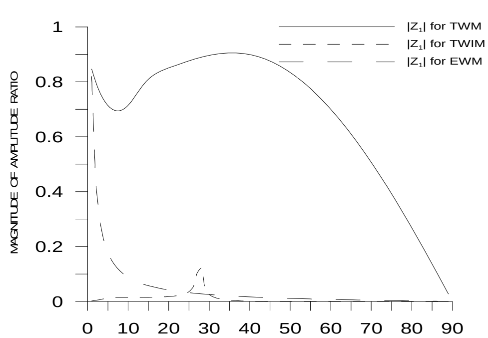

Fig. 10
The variations of amplitude ratios of reflected waves when S_1-wave is incident.

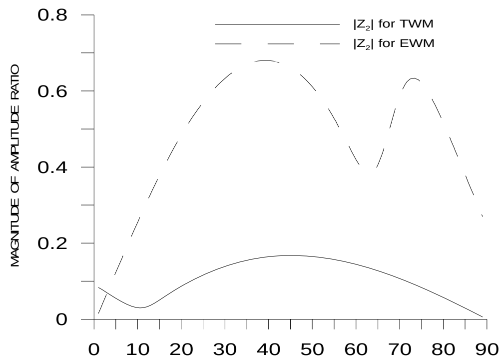

Fig. 11
The variations of amplitude ratios of reflected waves when S_1-wave is incident.

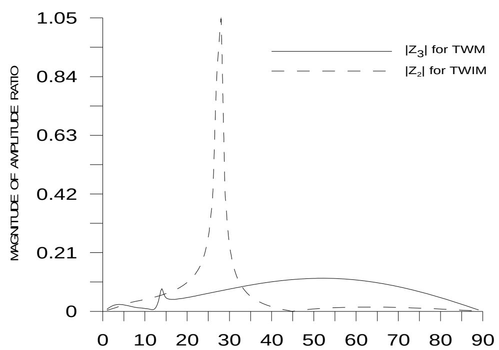

Fig. 12
The variations of amplitude ratios of reflected waves when S_1-wave is incident.

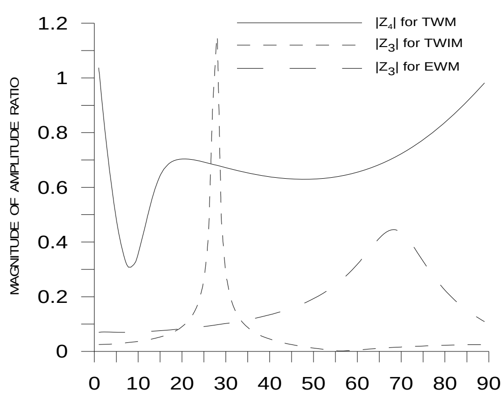

Fig. 13
The variations of amplitude ratios of reflected waves when S_1-wave is incident.

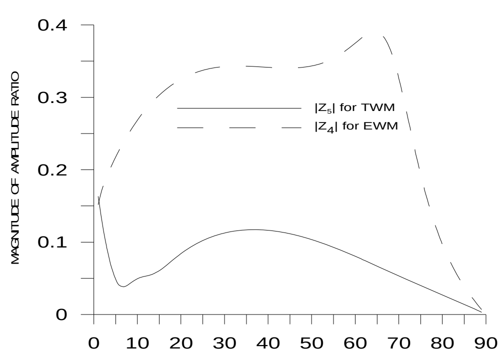

Fig. 14
The variations of amplitude ratios of reflected waves when S_1-wave is incident.

8 CONCLUSIONS

The numerical calculation for amplitude ratios of various reflected waves have been derived in thermoelastic solid half-space with mixture, without thermal and in the absence of second constituent of mixture for the incident qP_1, qT, S_1-waves. It is observed that due to incident qP_1, qT-waves the values of amplitude ratio $|Z_1|$ for TWIM are maximum at the incident angle $\theta_0 \approx 50^\circ$ and $\theta_0 \approx 25^\circ$ for the incident S_1-wave and the amplitude ratio decrease at these incident angles for TWM. It is also conclude that due to different incident waves we obtain oscillatory behavior of amplitude ratios. We observe that the effect of mixture and thermal are prodigious.

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