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Free Vibration Analysis of Micropolar Thermoelastic Cylindrical Curved Plate in Circumferential Direction

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ABSTRACT

The free vibration analysis of homogeneous isotropic micropolar thermoelastic cylindrical curved plate in circumferential direction has been investigated in the context of generalized themoelasticity III, recently developed by Green and Naghdi. The model has been simplified using Helmholtz decomposition technique and the resulting equations have been solved using separation of variable method. Mathematical modeling of the problem to obtain dispersion curves for curved isotropic plate leads to coupled differential equations and solutions are obtained by using Bessel functions. The frequency equations connecting the frequency with circumferential wave number and other physical parameters are derived for stress free cylindrical plate. In order to illustrate theoretical development, numerical solutions are obtained and presented graphically for a magnesium crystal.

Keywords: Micropolar; Phase velocity; Circumferential wave number; Thermoelasticity type III; Thermoelasticity without energy dissipation.

1 INTRODUCTION

THE generalized thermoelasticity theories have been developed with the aim of removing the paradox of infinite speed of heat propagation inherent in the classical coupled dynamical thermoelasticity theory formulated by Biot [1]. In the generalized theories, the governing equations involve thermal relaxation times and they are of hyperbolic type. The extended thermoelasticity theory, which introduces one relaxation time in the thermoelastic process was proposed by Lord and Shulman [2] and the temperature - rate dependent theory of thermoelasticity was developed by Green and Lindsay [3], which takes into account two relaxation times. These theories are two well established generalized theory of thermoelasticity.

The most recent theoretical development in this subject made by Green and Naghdi [4-6] has been the centre of active research during the last few decades. In this development Green and Naghdi proposed three different models of thermoelasticity in an alternative way and provided sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat problems and labeled as thermoelasticity types I, II and III. The nature of these three types of constitutive equations is such that when the respective theories are linearized, type-I, corresponds to the classical heat equation (based on Fourier's law) whereas the linearized versions of type-II and type-III theories are of different nature. The entropy flux vector in type-II and type-III (i.e. thermoelasticity without energy dissipation (TEWOED) and thermoelasticity with energy dissipation (TEWED)) are determined in terms of potential that also determines stress. Micropolar theory introduced by Eringen and Suhubi [7] and Eringen [8]

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incorporates the local deformations and rotations of the material points of a body. The theory provides a model that can support body and surface couples and display a high frequency optical branch of the wave spectrum. For engineering applications, it can model composites with rigid chopped fibres, elastic solids with rigid granular inclusions and other industrial materials such as liquid crystals. The microplar theory was extended to include thermal effect by Nowacki [9] and Eringen [10]. Mathematical modeling of wave propagation in the axial direction of a cylinder has been studied extensively. However, for wave propagation in the circumferential direction, which is essential for nondestructive testing (NDT) of large diameter pipes, literature shows fewer investigations.

Viktorov [11] established the fundamental mathematical modeling of the problem for isotropic material properties. He introduced the concept of angular wave number and derived, decomposed and solved the governing equations. He obtained the solution for convex and concave cylindrical surfaces by considering only one curved surface. Qu et al. [12] solved the problems of guided wave propagation in isotropic curved plates. Different aspects of the circumferential direction wave propagation along one or multiple curved surfaces were analysed by Liu and Qu [13-14] and Valle et al. [15]. Towfighi et al. [16] discussed the elastic wave propagation in circumferential direction in anisotropic cylindrical curved plates. They solved coupled differential equations and presented the dispersion curves for anisotropic curved plates of different curvatures. Tajuddin and Shah [17] discussed circumferential waves of infinite hollow poroelastic cylinders. Sharma and Pathania [18] investigated generalized thermoelastic wave propagation in circumferential direction of transversely isotropic cylindrical curved plates. Tyutekin [19] studied circumferential normal modes in an empty elastic cylinder. Jiangong, Bin and Cunfu [20] discussed circumferential thermoelastic waves in orthotropic cylindrical curved plates without energy dissipation. Waves in hollow cylinders - such as piping and tubing - have long been a topic of considerable interest from the viewpoints of mechanics and ultrasonic inspection. Guided wave inspection using circumferential of longitudinal modes has received a great deal of attention. From a mechanics point of view, the problem can be tackled in a manner similar to that used for rods and plates.

The free vibration analysis of homogeneous isotropic micropolar thermoelastic cylindrical curved plate in circumferential direction has been carried out in the present work.

2 BASIC EQUATIONS

The equations of motion and the constitutive relations in a homogeneous isotropic micropolar thermoelastic solid in the absence of body forces, body couples and heat sources are given by Eringen [21].

$$(\lambda+2\mu+K)\nabla(\nabla\cdot\vec{u})-(\mu+K)\nabla\times\nabla\times\vec{u}+K\ \nabla\times\vec{\phi}-\nu\ \nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla\cdot\vec{\phi})-\gamma\ \nabla\times(\nabla\times\vec{\phi})\ +K\ \nabla\times\vec{u}-2\ K\ \vec{\phi}=\rho\, j\frac{\partial^2\vec{\phi}}{\partial t^2} \tag{2}$$

$$t_{ij}=\lambda\ u_{r,r}\ \ \delta_{ij}+\mu\ (u_{i,j}+u_{j,i})+K\ (u_{j,i}-\varepsilon_{ijr}\phi_r)-\nu\ T\delta_{ij} \tag{3}$$

$$m_{ij}=\alpha\ \phi_{r,r}\ \ \delta_{ij}+\beta\ \phi_{i,j}+\gamma\ \phi_{j,i} \tag{4}$$

The heat conduction equation under G-N (type III) theory is

$$K^*\nabla^2 T+K_1^*\nabla^2\dot{T}=\rho\ C^*\frac{\partial^2 T}{\partial t^2}+\nu\ T_0\frac{\partial^2}{\partial t^2}(\nabla\cdot\vec{u}) \tag{5}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$ is the Laplacian operator, $\lambda,\mu,\alpha,\beta,\gamma$ and K are material constants, ρ is the density, j is the microinertia, t_{ij} and m_{ij} are the components of stress and couple stress tensors, $\vec{u}=(u_r,u_\theta,u_z)$ is the displacement vector, $\vec{\phi}=(\phi_r,\phi_\theta,\phi_z)$ is the microrotation vector, T_0 is the uniform temperature, T is the temperature change, $\nu=(3\lambda+2\mu+K)\alpha_t$, α_t is the coefficient of linear thermal expansion and K^* is an additional material

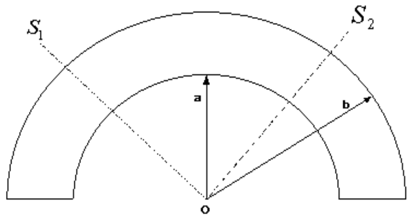

Fig. 1
Geometry of the problem.

constant characteristic of the theory of thermoelasticity without energy dissipation given by Green and Naghdi [6], K_1^* is the thermal conductivity, C^* is specific heat at constant strain, δ_{ij} is Kronecker delta. The comma notation denotes spatial derivatives.

3 FORMULATION OF THE PROBLEM

A homogeneous, isotropic, micropolar thermoelastic cylindrical plate with inner and outer radii a and b, respectively is considered. Let (r,θ,z) be the cylindrical polar coordinates such that z-axis coincides with the axis of the plate and consider the problem of wave propagation in the direction of curvature. The aim of this study is to analyse the dispersive waves in the curved plate for waves propagating from the section S_1 to S_2 as shown in Fig. 1. This analysis does not include the reflected guided waves from the plate boundary. The considered geometry of the problem can be a segment of a cylinder or a complete cylinder. For two dimensional problems, we take the displacement vector and microrotation vector as

$$\vec{u}=(u_r,u_\theta,0) \qquad \text{and} \qquad \vec{\phi}=(0,0,\phi_z) \tag{6}$$

We define the non-dimensional quantities

$$\begin{gathered} r'=\frac{\omega^* r}{c_1},\ a'=\frac{\omega^*}{c_1}a,\ b'=\frac{\omega^*}{c_1}b,\ u_r'=\frac{\rho\omega^* c_1}{\nu T_0}u_r,\ u_\theta'=\frac{\rho\omega^* c_1}{\nu T_0}u_\theta,\ \phi_z'=\frac{\rho\, c_1^2}{\nu T_0}\phi_z,\ T'=\frac{T}{T_0},\ t'=\omega^* t \\ t_{ij}'=\frac{1}{\nu T_0}t_{ij},\ m_{ij}'=\frac{\omega^* m_{ij}}{c_1\nu T_0},\ h'=\frac{c_1 h}{\omega^*},\ p=\frac{K}{\rho\, c_1^2},\ \delta^2=\frac{c_2^2}{c_1^2},\ \delta_1^2=\frac{c_3^2}{c_1^2},\ \delta^*=\frac{Kc_1^2}{\gamma\omega^{*2}} \end{gathered} \tag{7}$$

where $c_1^2=(\lambda+2\mu+K)/\rho$, $c_2^2=(\mu+K)/\rho$, $c_3^2=\gamma/\rho j$, $\omega^*=\rho\, C^* c_1^2/K^*$, ω^* is the characteristic frequency of the medium. We introduce the potential functions ϕ and ψ through the relations

$$u_r=\frac{\partial\phi}{\partial r}+\frac{1}{r}\frac{\partial\psi}{\partial\theta},\ u_\theta=\frac{1}{r}\frac{\partial\phi}{\partial\theta}-\frac{\partial\psi}{\partial r} \tag{8}$$

where ϕ and ψ are velocity potential functions of longitudinal and shear waves. Using equations (6)-(8) in equations (1)-(2), (5) and after suppressing the primes for convenience, we obtain

$$\nabla^2\phi-T-\frac{\partial^2\phi}{\partial t^2}=0 \tag{9}$$

$$\nabla^2\psi+\frac{p\phi_z}{\delta^2}-\frac{1}{\delta^2}\frac{\partial^2\psi}{\partial t^2}=0 \tag{10}$$

$$\nabla^2\phi_z-\delta^*\nabla^2\psi-2\delta^*\phi_z-\frac{1}{\delta_1^2}\frac{\partial^2\phi_z}{\partial t^2}=0 \tag{11}$$

$$\in_2\nabla^2 T+\in_3\nabla^2\dot{T}-\ddot{T}=\in_1\nabla^2\ddot{\phi} \tag{12}$$

where $\in_1 = \nu^2 T_0 / \rho^2 C^* c_1^2$, $\in_2 = K^* / \rho\, C^* c_1^2$, $\in_3 = K_1^* \omega^* / \rho\, C^* c_1^2$, $\in_1$ is the thermoelastic coupling factor, $\in_2$ is the characteristic parameter of the G-N theory (of type II) and $\in_3$ is the characteristic parameter of the G-N theory (of type III). We assume the solutions of Eqs. (9)-(12) of the form

$$\phi = \bar{\phi}(r) \,{}^{Cos}_{Sin}(n\theta)\, e^{-i\omega t}, \; \psi = \bar{\psi}(r) \,{}^{Cos}_{Sin}(n\theta)\, e^{-i\omega t}, \; \phi_z = \bar{\phi}_z(r) \,{}^{Cos}_{Sin}(n\theta)\, e^{-i\omega t}, \; T = \bar{T}(r) \,{}^{Cos}_{Sin}(n\theta)\, e^{-i\omega t} \tag{13}$$

where n is the integer number of waves around the circumference or angular wave number, ω is the angular frequency of the wave. Towfighi et al. [16] pointed out that in cylindrical geometry, the generation of surface waves in the circumferential direction with a plane wave front requires the circumferential wave speed to be a function of the radial distance. We also adopt the same formulation here and hence assume that the phase velocity is not constant but changes with radius. The phase velocity at a point having radius r is given by

$$v_{ph}(r) = c_b r / b \tag{14}$$

where c_b is the phase velocity at the outer surface having a radius b. The angular wave number n, which is independent of r, is defined as

$$n = \omega / (v_{ph}(r)/r) = \omega\, b / c_b \tag{15}$$

Substitution of Eq. (13) in Eqs. (9)- (12) gives us

$$(\nabla_1^2 + \omega^2)\bar{\phi}(r) = \bar{T}(r) \tag{16}$$

$$(\nabla_1^2 + \frac{\omega^2}{\delta^2})\bar{\psi}(r) + \frac{p}{\delta^2}\bar{\phi}_z(r) = 0 \tag{17}$$

$$(\nabla_1^2 + \frac{\omega^2}{\delta_1^2} - 2\delta^*)\bar{\phi}_z(r) - \delta^* \nabla_1^2 \bar{\psi}(r) = 0 \tag{18}$$

$$(\in_2 \nabla_1^2 - i\omega \in_3 \nabla_1^2 + \omega^2)\bar{T}(r) = -\in_1 \omega^2 \nabla_1^2\, \bar{\phi}(r) \tag{19}$$

where $\nabla_1^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} - \dfrac{n^2}{r^2}$.

The solution of Eqs. (16)-(19) is written as

$$\bar{\phi}(r) = [A_1 J_n(m_1 r) + B_1 Y_n(m_1 r) + A_2 J_n(m_2 r) + B_2 Y_n(m_2 r)]e^{i(n\theta - \omega t)} \tag{20}$$

$$\bar{T}(r) = [g_1 A_1 J_n(m_1 r) + g_1 B_1 Y_n(m_1 r) + g_2 A_2 J_n(m_2 r) + g_2 B_2 Y_n(m_2 r)]e^{i(n\theta - \omega t)} \tag{21}$$

$$\bar{\psi}(r) = [A_3 J_n(m_3 r) + B_3 Y_n(m_3 r) + A_4 J_n(m_4 r) + B_4 Y_n(m_4 r)]e^{i(n\theta - \omega t)} \tag{22}$$

$$\bar{\phi}_z(r) = [f_3 A_3 J_n(m_3 r) + f_3 B_3 Y_n(m_3 r) + f_4 A_4 J_n(m_4 r) + f_4 B_4 Y_n(m_4 r)]e^{i(n\theta - \omega t)} \tag{23}$$

where $m_i^2 = -\omega^2 a_i^2$, $i = 1,2,3,4$;

$$(a_1^2, a_2^2) = \frac{1}{2(\in_2 - i\,\omega \in_3)}\left\{ \left[\in_2 + 1 - i\omega \in + \in_1\right] \pm \left[(\in_2 + 1 - i\omega \in + \in_1)^2 - 4(\in_2 - i\,\omega \in_3)\right]^{1/2} \right\}$$

$$(a_3^2, a_4^2) = \frac{1}{2}\left\{\left[\frac{1}{\delta_1^2}+\frac{1}{\delta^2}+\frac{\delta^*}{\omega^2\delta^2}(p-2\delta^2)\right]\pm\left[\begin{array}{l}\left\{\frac{1}{\delta^2}-\frac{1}{\delta_1^2}+\frac{\delta^*}{\omega^2\delta^2}(p-2\delta^2)\right\}^2 \\ +\frac{4\delta^*}{\omega^2\delta_1^2\delta^2}\left\{p-2(\delta^2-\delta_1^2)\right\}\end{array}\right]^{\frac{1}{2}}\right\}$$

$$g_i = m_i^2+\omega^2, \mathrm{i}=1,2 \quad , f_i = -\frac{\delta^2}{p}(m_i^2+\frac{\omega^2}{\delta^2}), \mathrm{i}=3,4$$

Here J_n and Y_n are the Bessel functions of first and second kind of order *n*. The displacements, microrotation, temperature and stresses are obtained as

$$u_r = (\bar{\phi}' + \frac{i\,n}{r}\bar{\psi})e^{i(n\theta-\omega t)} \tag{24}$$

$$u_\theta = (\frac{i\,n}{r}\bar{\phi} - \bar{\psi}')e^{i(n\theta-\omega t)} \tag{25}$$

$$\phi_z = \bar{\phi}_z e^{i(n\theta-\omega t)} \tag{26}$$

$$T = \bar{T}e^{i(n\theta-\omega t)} \tag{27}$$

$$t_{rr} = [-\omega^2\bar{\phi} - (2\delta^2-p)(\frac{\bar{\phi}'}{r} - \frac{n^2}{r^2}\bar{\phi}) + (2\delta^2-p)\frac{i\,n}{r}(\bar{\psi} - \frac{\bar{\psi}}{r})]\ e^{i(n\theta-\omega t)} \tag{28}$$

$$t_{r\theta} = [(\delta^2-p)(\bar{\psi}'' + \frac{\bar{\psi}'}{r} - \frac{n^2}{r^2}\bar{\psi}) - (2\delta^2-p)(\bar{\psi}'' - \frac{i\,n}{r}\bar{\phi}' + \frac{i\,n}{r^2}\bar{\phi}) - p\bar{\phi}_z]\ e^{i(n\theta-\omega t)} \tag{29}$$

$$m_{rz} = \gamma\ \bar{\phi}_z'\ e^{i(n\theta-\omega t)} \tag{30}$$

where prime denotes differentiation with respect to radial coordinate *r*. The Eqs. (24)-(30) contain real as well as imaginary parts. However, only real part has been considered in depicting numerical results.

3.1 Boundary Conditions

Let us consider the following types of boundary conditions. The lower and upper surfaces $r = a$ and $r = b$ of the plate are assumed to be

(i) stress free, which leads to $t_{rr} = t_{r\theta} = m_{rz} = 0$ (31)

(ii) thermal condition $T_{,r} + hT = 0$ (32)

where *h* is the Biot's heat transfer coefficient. Here $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ refers to isothermal one.

4 DERIVATION OF THE SECULAR EQUATIONS

Invoking the stress free and thermal boundary conditions (31)-(32) at the lower and upper surfaces *r=a, b* of the plate and using Eqs. (24)-(30), one can get the free vibration equation as

$$\left|E_{ij}\right| = 0, \qquad i,\ j = 1,2,3,4,5,6,7,8 \tag{33}$$

where

$$E_{11}=-\omega^2 J_n(m_1\eta_1)-\frac{(2\delta^2-p)}{\eta_1}[m_1J_n'(m_1\eta_1)+\frac{n^2}{\eta_1}J_n(m_1\eta_1)]$$
$$E_{13}=-\omega^2 J_n(m_2\eta_1)-\frac{(2\delta^2-p)}{\eta_1}[m_2J_n'(m_2\eta_1)+\frac{n^2}{\eta_1}J_n(m_2\eta_1)]$$
$$E_{15}=\frac{i\,n(2\delta^2-p)}{\eta_1}m_3J_n'(m_3\eta_1)-\frac{1}{\eta_1}J_n(m_3\eta_1)$$
$$E_{17}=\frac{i\,n(2\delta^2-p)}{\eta_1}m_4J_n'(m_4\eta_1)-\frac{1}{\eta_1}J_n(m_4\eta_1)$$
$$E_{21}=\frac{i\,n(2\delta^2-p)}{\eta_1}[m_1J_n'(m_1\eta_1)-\frac{1}{\eta_1}J_n(m_1\eta_1)]$$
$$E_{23}=\frac{i\,n(2\delta^2-p)}{\eta_1}[m_2J_n'(m_2\eta_1)-\frac{1}{\eta_1}J_n(m_2\eta_1)]$$
$$E_{25}=-\delta^2m_3^2J_n''(m_3\eta_1)+\frac{(\delta^2-p)}{\eta_1}m_3J_n'(m_3\eta_1)-[(\delta^2-p)\frac{n^2}{\eta_1^2}+pf_3]J_n(m_3\eta_1)$$
$$E_{27}=-\delta^2m_4^2J_n''(m_4\eta_1)+\frac{(\delta^2-p)}{\eta_1}m_4J_n'(m_4\eta_1)-[(\delta^2-p)\frac{n^2}{\eta_1^2}+pf_4]J_n(m_4\eta_1)$$
$$E_{31}=m_1J_n'(m_1\eta_1), E_{33}=m_2J_n'(m_2\eta_1), E_{35}=E_{37}=0$$
$$E_{41}=g_1m_1J_n'(m_1\eta_1)+hg_1J_n(m_1\eta_1), E_{43}=g_2m_2J_n'(m_2\eta_1)+hg_2J_n(m_2\eta_1), E_{45}=E_{47}=0$$

Here $E_{ij}\,(j=2,4,6,8)$ can be obtained by just replacing the Bessel functions of first kind in $E_{ij}\,(j=1,3,5,7)$ with those of second kind, while $E_{ij}\,(i=5,6,7,8)$ can be obtained by replacing η_1 in $E_{ij}\,(i=1,2,3,4)$ with η_2 respectively, where $\eta_1=a/R=1-\eta^*/2$ and $\eta_2=b/R=1+\eta^*/2$ and $\eta^*=(b-a)/R$ is the thickness to mean radius ratio of the plate.

4.1 Particular case

4.1.1 Micropolar thermoelasticity without energy dissipation (the linearized G-N theory of type II)

In this case, $K_1^*=0$ and $K^*=\frac{C^*(\lambda+2\mu)}{4}$.

5 NUMERICAL RESULTS AND DISCUSSION

With the view of illustrating theoretical results obtained in the preceding sections and comparing these in the context of various theories of thermoelasticity, we now present some numerical results. The material chosen for this purpose is Magnesium crystal (micropolar thermoelastic solid), the physical data for which is given below. Following Eringen [22], Micropolar parameters are

$$\rho=1.74\times10^3 \text{ Kg/m}^3,\ \lambda=9.4\times10^{10} \text{ N/m}^2,\ \mu=4.0\times10^{10} \text{ N/m}^2,$$
$$K=1.0\times10^{10} \text{ N/m}^2,\ \gamma=0.779\times10^{-9} \text{ N},\ j=0.2\times10^{-19} \text{ m}^2$$

Following Dhaliwal and Singh [23], thermal parameters are

$$\in_1=0.028,\ T_0=298^0\text{K},\ C^*=1.04\times10^3 \text{ J/Kgdeg},\ K_1^*=1.7\times10^2 \text{ J/msecdeg},\ \nu=2.68\times10^6 \text{ N/m}^2\text{deg}$$

The phase velocity of various modes of wave propagation has been computed for various values of circumferential wave number from secular equation (33) for stress free thermally insulated and stress free isothermal boundaries. The corresponding numerically computed values of phase velocity are shown graphically in Figs. 2-3 for different modes (m=0 to m=2). The solid curves correspond to G-N theory of thermoelasticity of type III (GN-III) and dotted curves refer to G-N theory of thermoelasticity of type II (GN-II).

The phase velocity of higher modes of wave propagation attains quite large values at vanishing wave number, which sharply slashes down to become steady with increasing wave number. The phase velocity of lowest mode (m =0) in stress free thermally insulated plate varies at lower wave number and becomes constant at higher wave number. It is observed that for modes m=1, 2 in stress free thermally insulated plate, the values of phase velocity are smaller in GN-III than in GN-II. In case of lowest mode (m=0), phase velocity in GN-II is more than in case of GN-III for wave number $\xi \leq 0.8$; the values of phase velocity are smaller in GN-II than in GN-III for wave number lying between 0.8 and 3.8; and phase velocity in GN-III and GN-II is nearly same for wave number $\xi \geq 3.8$.

For stress free isothermal plate, we notice the following from Fig. 3 for m=0, phase velocity in GN-III is less than in case of GN-II for wave number $\xi \leq 1.1$; phase velocity in GN-III is more than in case of GN-II for wave number lying between 1.1 and 2.4; the phase velocity profiles in respect of GN-III and GN-II coincide for wave number lying between 2.4 and 2.9; phase velocity in GN-III is slightly more than in case of GN-II for wave number $\xi \geq 2.9$ (b) for m=1, phase velocity in GN-III is more than in case of GN-II for wave number $\xi \leq 2.2$ and $\xi \geq 3.9$; phase velocity in GN-III is less than in case of GN-II for wave number lying between 2.2 and 3.9 (iii) for m=2, phase velocity in GN-III is slightly more than in case of GN-II for wave number $\xi \leq 0.8$; phase velocity in GN-III is less than in case of GN-II for wave number lying between 0.8 and 4.2; the phase velocity profiles in respect of GN-III and GN-II coincide for wave number $\xi \geq 4.2$.

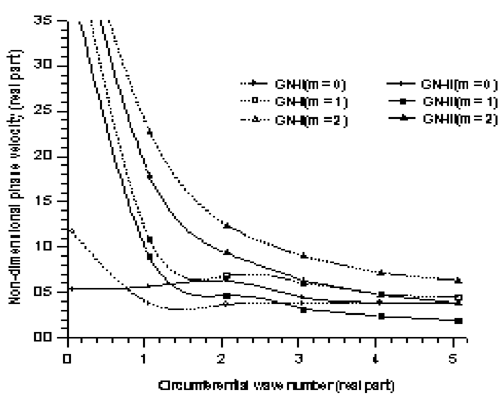

Fig. 2
Phase velocity profiles of wave modes in a stress free thermally insulated plate with circumferential wave number.

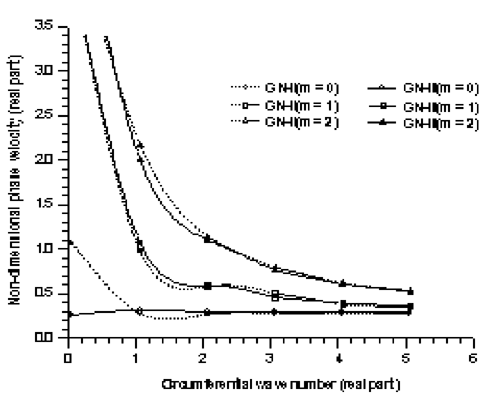

Fig. 3
Phase velocity profiles of wave modes in a stress free isothermal plate with circumferential wave number.

6 CONCLUSIONS

The Bessel functions have been directly used to study the free vibration analysis along circumferential direction in homogeneous isotropic micropolar thermoelastic cylindrical curved plate in the context of Green and Naghdi (G-N) theories of themoelasticity. The phase velocity of various modes of wave propagation has been computed for various values of circumferential wave number from dispersion equation for stress free thermally insulated boundaries and stress free isothermal boundaries and has been represented graphically for different modes (m=0 to m=2). The phase velocities of higher modes of propagation attain quite large values at vanishing wave number which sharply slashes down to become steady and asymptotic with increasing wave number

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