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Review of Damage Tolerant Analysis of Laminated Composites

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ABSTRACT

With advanced composites increasing replacing traditional metallic materials, the material inhomogeneity and inherent anisotropy of such materials lead to not only new attributes for aerospace structures, but also introduce new technology to damage tolerant design and analysis. The deleterious effects of changes in material properties and initiation and growth of structural damage must be addressed. The anisotropic and brittle properties make this requirement a challenging to composite structural designers. Accurate, reliable and user-friendly computational methods, design and analysis methods are vital for more damage tolerant composite structures. Both durability and damage tolerant methodologies must address the possible changes in mechanical properties and the evolving damage accumulations that may occur during the vehicle's service lifetime. Delamination is a major failure mode in laminated composites and has received much research attention. It may arise out of manufacturing defects, free edge effects, structural discontinuities, low and high velocity impact damage, and even bird strikes. Early pioneering work established that the reduction in strength following delamination damages placed severe limits on the design allowable for highly loaded components such as aircraft wing and fuselage structure. In the present article, we provide a state-of-art survey on damage tolerant design correlated failure behavior and analysis methodologies of laminated composites. Particular emphasis is placed on some advanced formulations and numerical approaches for efficient computational modeling and damage tolerant analysis of laminated composites.
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Keywords: Damage tolerant analysis; Delamination; Virtual crack closure-integral technique; Cohesive zone model; Progressive failure analysis.

1 INTRODUCTION

ADVANCED composites are increasingly being used in aircraft, aerospace, marine, civil and automotive industries due to their high specific stiffness and strength, good fatigue resistance, and their tailorability. Although the need to consider damage tolerant during the design process is not unique to composite structures, material inhomogeneity and inherent anisotropy further complicated the design and analysis process. The relative successful experiences with design of metallic structures cannot be directly transferred to the design of composite structures because of the different failure mechanisms for composite and metallic materials. Several important factors lead to the difference [1]. Firstly, composite materials are not isotropic and homogenous. Secondly, composite materials are generally brittle and lack of ductility. Thirdly, the initiation and growth of damage and the failure modes of composites are not as well understood and still can not be predicted accurately. Due to these complications, composite structural design and manufacturing technology is not yet as mature as metallic structures, especially for heavily loaded aeronautic and astronautic structures.

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Durability and damage tolerant design have been a primary barrier to expand the application of composites to heavily loaded, primary structures. The current design models and numerical analysis methods are semi-empirical and rely heavily on the building-block approach to design and certification. Several bibliographical reviews dealing with damage tolerant design of composite structures are found in the specialized literature. As shown in the examples, Sierakowski [2] explored the damage tolerant issues based on design requirements, current state of the art design and analysis. Schmidt [3] summarized the major structural criteria and requirements to be considered in damage tolerant design and analysis of current and future aircraft structures. Ransom [4] presented an overview of the recent and planned future research in composites durability and damage tolerant analytical and experimental methods at NASA Langley Research Center. Tomblin [5-7], McGowan [8], and Moody [9] studied damage tolerant design methods for composite sandwich structures. Williams [10] discussed how the pioneering work in damage tolerant conducted at NASA has been applied in the oil industry. The major contributions in all of these researches are based on structural criteria, design requirements and design allowables for vehicle composite components. The present paper aims to review major numerical methods for damage tolerant analysis of laminated composites based on the understanding of failure mechanisms of laminated composites.

2 FAILURE BEHAVIORS UNDER STATIC LOADING

The prediction of the failure behavior of composite structures requires an accurate assessment of damage initiation, growth, and description of the evolving damage accumulation in service and adverse effect on structural performance, strength, and fatigue life. There have been a number of survey articles on the failure criterion of composite structures. These include reviews presented by Echaabi [11], Hinton and Soden [12-14], and Paris [15], where the failure criteria are given and compared. A more recent survey by Icardi [16] reviewed and discussed the most widely used and recently developed failure criteria, along with their applications. Based on the dominated failure modes, this paper will discuss the application of fracture mechanics for damage tolerant design and analysis of composites, and some issues associated with its application. The principal failure mode of laminated composites is the separation along the interfaces of the layers, via. delamination, which can be viewed as an interface crack between two anisotropic materials. Miller [17] indicated that approximately 60% of the damage phenomena observed in the composites parts of an aircraft are delaminations. Interlaminar fracture mechanics is widely used to predict the more dominant failure mode of delamination initiation and propagation [18-20]. Fig. 1 shows a flow diagram of the fracture mechanics method for damage tolerant design [21]. First, areas of possible damage should be decided according to stress-based criterion. Then, a fracture analysis is conducted to determine whether or not a delamination will initiate and grow. If a delamination does grow, the growth length and final failure should be predicted with suitable failure criterion.

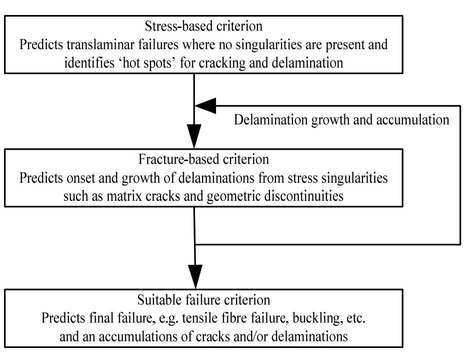

Fig. 1
A schematic of the fracture mechanics approach [19].

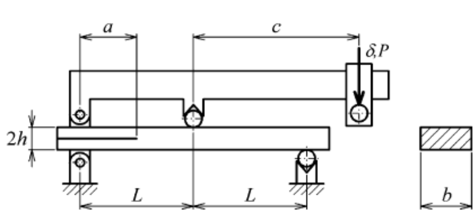

Fig. 2
Mixed-mode bending test.

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Fracture mechanics prediction procedures essentially have two steps: modeling the structure, and characterizing the delaminated materials properties [21]. For the first step, the most important part is to calculate strain energy release rate (SERR), which will be discussed in detail lately. For a given delamiantion length, a critical value of SERR is obtained. This critical SERR value is compared with the material's fracture data to give both a monotonic failure load and location to initiate the delamiantion. The delaminated material properties are usually obtained using standard composite fracture tests. Typically, delamination initiates and propagates under the combined influence of normal and shear stresses. Therefore, mixed-mode delamiantion tests are of great interest for the determination of interlaminar fracture toughness. There exist diverse experimental methods to obtain the mixed-mode I/II interlaminar fracture toughness of laminated composites, for example mixed-mode bending (MMB) [22, 23], cracked-lap shear (CLS) [24], edge delamination tension (EDT) [25], Arcan specimen [26], asymmetric double cantilever beam) (ADCB) [27], mixed-mode flexure (MMF) [28, 29]. The most commonly used test is the MMB method, which can simultaneously produce mode I and mode II bending loads by loading with a lever, as shown in Fig. 2 [23]. The main advantages of the MMB test method are the possibility of using virtually the same specimen configuration as for mode I tests, and the capability of obtaining different mixed-mode ratios, ranging from pure mode I to pure mode II, by changing the length of the loading lever.

It is important that accurate mixed mode delamination fracture criteria should be developed so that the extension of delaminations in structures can be predicted. Many delamination failure criteria based on fracture toughness have been suggested over the past few decades, such as the power law criterion [30, 31], Benzeggagh-Kenane criterion [32, 33], and Reeder criterion [34, 35]. But most of these criteria only covered mixed mode I/II or based on assumption that the relationship between mode I and mode III toughness is similar to the relation between mode I and mode II [35]. Extensive reviews of technical issues related to fracture toughness testing are provided by O'Brien [36], Davis [37], Brunner [38], and Reeder [35].

3 FAILURE BEHAVIORS UNDER IMPACT LOADING

Low transverse and interlaminar shear strength, no plastic deformation, and laminar construction make impact becomes the most dangerous loading conditions for laminated composites. Impact loads may result in a large internal damaged area of the laminated composites that is not detectable from visible observation, which can have a significant effect on the durability, damage tolerant, and stability of the laminated composites. Compression can continuously grow the damage area, possibly resulting in complete structural collapse of the damaged structure. In the aeronautic and astronautic industries, the residual compression strength of an impact damaged composite structure has become one of the most important design limiting factor, which is similar to current damage tolerant design philosophies in other industrial sectors.

The impact damage mechanism in laminated composites constitutes a very complex process. It is a combination of matrix cracking, surface buckling, delamination, fiber shear-out, and fiber fracture, etc., which usually all interact with each other. A typical impact damage mode for laminated composites is depicted in Fig. 3. An explanation of the basic mechanics and fundamental terms of impact can be found in several reviews [39-41]. In this section, low and high velocity impact on composites panels are discussed to investigate the impact behavior relevant to civil aircraft structures ranging from tool drop up to foreign object damage.

3.1 Low velocity impact

Low velocity impact is associated with delamination damage, especially that caused by blunt-headed projectiles. This interlaminar debonding primarily reduces the local bending stiffness and thus can affect the bending and buckling behavior of the structure. Such damage has been reported to cause as much as 40% reduction in static and fatigue strength [42, 43]. In order to measure the damage resistance of a fiber-reinforced polymer matrix composite to impact, a set of low velocity impact tests were carried out. Fig. 4 depicts the quasi-static indentation test (ASTM 7136-2007) [44] and the drop-weight impact test [45, 46].

For laminated composites under low velocity impact loading conditions, the relationships between the impact load and energy applied, the extent and modes of damage introduced and the residual properties are invaluable information for proper damage tolerant design of composite components and structures [47, 48]. Typical impact load-time and energy-time histories are shown in Fig. 5. Some investigators assume the first load drop during the force history corresponds to the occurrence of initial damage in the form of matrix cracking, fiber breakage, and local puncture or indentation [47-49]. Belingardi and Vadori [47] defined two thresholds from the load history.

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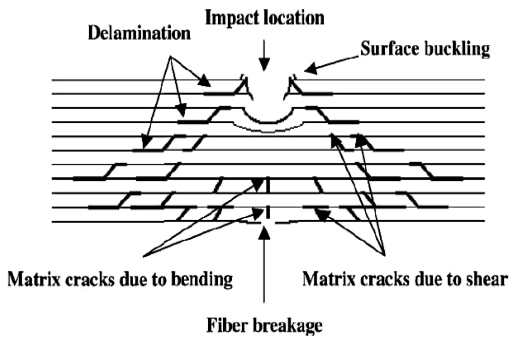

Fig. 3
Schematic representation shows typical impact damage modes for laminated composites.

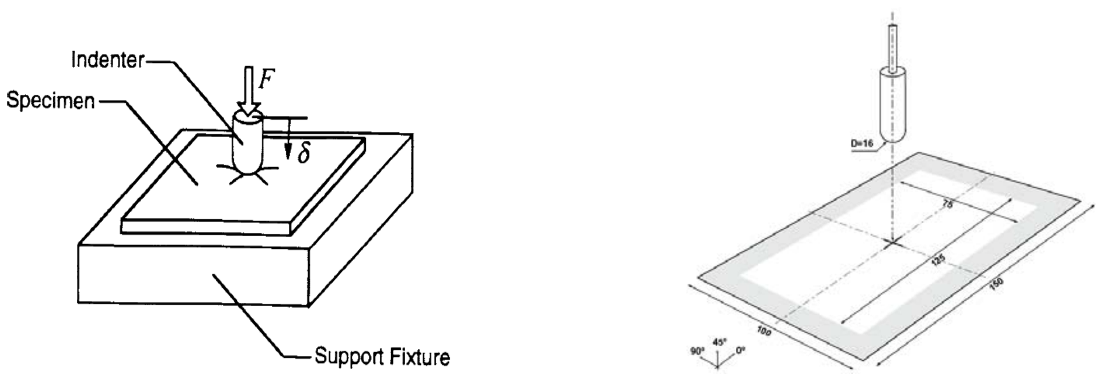

Fig. 4
Illustration of two low velocity impact tests setup, (a) quasi-static indentation test [44];(b) drop-weight impact test [45, 46].

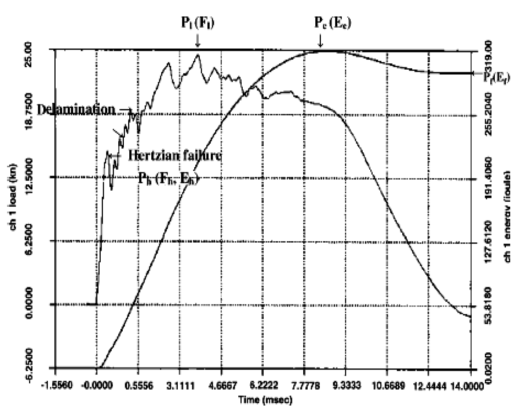

Fig. 5
Typical impact load-time and energy-time history for laminated composites.

The first one was at the first load drop for the first material damage, and the second one was the maximum force value for the first lamina failure. Gao and Zhang [49] pointed out that the first damage threshold is probably due to the initialization of delamination failure. They also proposed an equation for a critical force threshold [49]:

$$\sigma = (\mathbf{I} - \mathbf{E})\mathbf{D}\delta \;\; p_c^2 = \frac{8\pi^2 E h^3 G_{IIc}}{9(1-\nu^2)} \tag{1}$$

where P_c is the threshold load, E and ν are the equivalent in-plane modulus and Poisson's ratio, h is the laminated thickness, and G_{IIc} is the critical strain energy release rate. Based on the experimental study, Shen [50] proposed that

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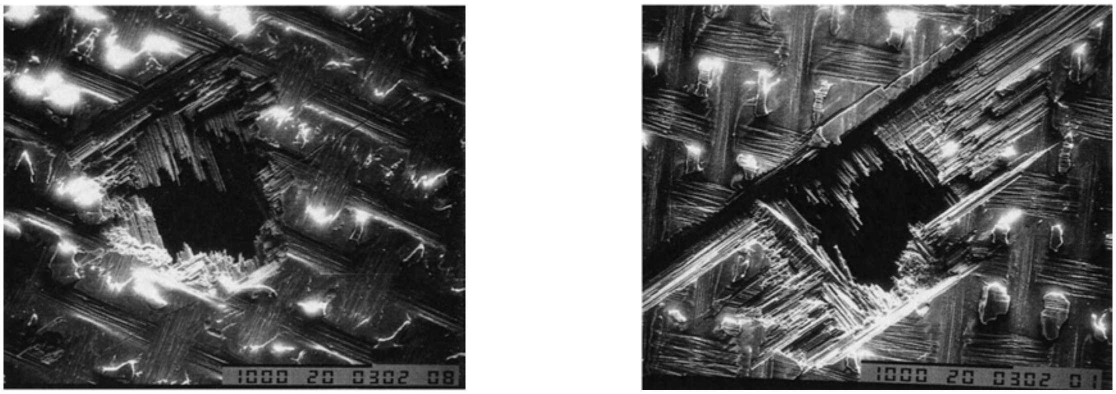

Fig. 6
SEM photographs of micrometeoroid/debris impact holes for SP-288/T300graphite/epoxy tube [41] (a) front (×35) (b) back (×35).

the damage tolerant behavior could be characterized by the threshold of compression failure strain after impact (CAIT) of the dent depth and compressive failure strain curve. The tests results indicate that there are knee points in *d-e* (dent depth-impact energy curve), *S-d* (damage area-dent depth curve) and *c-d* (compression failure strain-dent depth curve), and these points corresponding to the transformation of damage mechanisms. The dominant damage is delamination and matrix cracking before the point and it becomes fiber breakage after the point.

The current approaches to damage tolerant designs are based on a dominant failure mechanism associated with a critical crack size, such as that associated with delamination. For the case of damage induced by impact to aircraft composite structures, the presence of damage caused by a 2.54 cm (1.0 in) diameter hemispherical impactor delivered at 133 J (100 ft/lb) of kinetic energy, or a kinetic energy event causing a 0.54 cm (0.10 in) dent, have been used as damage thresholds [50].

3.2 High velocity impact

High velocity impact conditions are most likely to occur on spacecraft in low earth orbit, which can produce delamination coupled with spallation and eject plumes emanating from both sides of the laminated composites. Fig. 6 presents SEM photomicrographs of a crater hole viewed from the front and rear faces of a graphite/epoxy tube due to an actual micrometeoroid hypervelocity impact [41].

A larger number of laminated composites under high velocity impact have been investigated employing experimental and numerical methods. Yew et al. [51] studied a wide range of graphite/epoxy plates at the NASA Johnson Space Center (NASA JSC) using their light gas gun facilities. Lamontagne et al. [52] performed research on the effects of impact angle using their light gas gun. It was observed that for oblique angle impacts, the debris cloud does not follow the line of flight of the projectile. Moreover, the cone angle associated with the debris cloud is not symmetric about the projectile impact velocity vector. Christiansen [53] and Shortliffe [54] investigated the failure behavior of composite tubes and cylinders under high velocity impact. Taylor [55] and Vaidya [56] studied the response of composite sandwich panels under high velocity impact. Taylor also studied the effect of impact angle on the ballistic limit and the correlation of entry hole size and impactor energy [55]. Vaidya considered sandwich constructions with reinforced cores and made a conclusion that Z-pin reinforcement of the core suppressed core crushing effectively under the high strain rate impact loading [56]. A gas gun impact test program has been conducted within the EU HICAS project to study the impact resistance of stringer stiffened panels [57]. In their research, two cases were considered: impacts on a stringer and between two stringers, which show different failure behaviors. A recent survey by Jiang and Vecchio [58] contains the major progress in Hopkinson bar fracture test methods over the past 30 years, focused on dynamic fracture toughness measurement.

4 NUMERICAL ANALYSIS METHODS

There are various ways of analyzing the laminated failure process, in terms of the energy deposited and gross damage produced, micro energy dissipation or by considering the stresses acting on flaws in the material and the

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effects that are generated. Unlike homogenous metals, the discrete components of the composite structures may experience material damage and fracture concurrently. Elder [59] reviewed the current fracture mechanics and damage mechanics methods for predicting delamination. In this paper, several widely used or promising methods are reviewed, such as virtual crack closure-integral technique (VCCT) method [60, 61], cohesive zone model (CZM) method [62, 63], and the progressive failure analysis (PFA) method [64, 65].

4.1 Virtual Crack Closure-integral Technique (VCCT)

Linear elastic fracture mechanics is commonly applied for the prediction of laminated composites failure behavior. In fracture methods category, strain energy release rate (SERR) is compared to the material toughness to determine whether the material will fracture. The VCCT method has been widely used for computing SERR based on results from finite element analysis. A comprehensive review paper was published by Krueger [66].

VCCT is based on Irwin's crack closure integral concept [67], which assume that the energy ΔE released when the crack is extended by Δa from a to $a+\Delta a$ is identical to the energy required to close the crack between location Li and Ll, as shown in Fig. 7. At the same time, it is assumed that a crack extension of Δa from $a+\Delta a$ to $a+2\Delta a$ does not significantly alter the state at the crack tip. For a crack modeled with three-dimensional, eight-node elements, the mode I, mode II and mode III SERR components, G_I, G_{II} and G_{III}, are calculated as [68].

$$G_I = \frac{1}{2\Delta A} Z_{Li}(w_{Ll} - w_{Ll}^*) \tag{2}$$

$$G_{II} = \frac{1}{2\Delta A} X_{Li}(u_{Ll} - u_{Ll}^*) \tag{3}$$

$$G_{III} = \frac{1}{2\Delta A} Y_{Li}(v_{Ll} - v_{Ll}^*) \tag{4}$$

where X, Y and Z denote the forces at the delamination front, u, v and w are the corresponding displacements behind the delamination. And $\Delta A=\Delta a\times b$ is the area virtually closed, as shown in Fig. 7.

The VCCT method has the advantages of providing accurate SERR assessment with a relatively coarse mesh, can be easily adapted to existing codes, has explicitly determined separated fracture modes and requires only one complete analysis of the structure to obtain the deformations. Therefore, VCCT based fracture mechanics has been widely used to assess the damage tolerant of composite structures in the design phase and during certification [19, 20, 66]. Recently, the ABAQUS/Standard commercial finite element code released their implementation of VCCT [69] which is based on a new interface element developed by Boeing [70] that performs the VCCT calculation internally and therefore allows the automation of delamination propagation analyses.

4.2 Cohesive Zone Model (CZM)

Another approach for the numerical simulation of delamination can be developed within the framework of damage mechanics, which are based on the concept of the cohesive zone model. The origin of the cohesive zone model goes back to Dugdale [62] who introduced the concept that stresses in the material are limited by the yield stress and that a thin plastic zone is generated in front of the notch. Barenblatt [63] introduced cohesive forces on a molecular scale in order to solve the problem of equilibrium in elastic bodies with cracks. Further developments are due to Williams [71], and Schapery [72] who modeled crack growth in visco-elastic media. Later, Hillerborg [73] extended the method to quasi-brittle materials. Unguwarungasri and Knauss [74] proposed a cohesive layer composed of a series of nonlinear springs. Needleman [75, 76] analyzed void nucleation and void coalescence in metals using cohesive layer models. Further significant contributions came, among others, from Shahwan and Wass [77], Ortiz and Pandolfi [78], Yu [79] and Corigliano, et al [80]. A majority of investigators have used cohesive layers of zero thickness [75, 76], finite-thickness [81], and line elements [82, 83].

In CZM method, a cohesive constitutive law relates the tractions to the displacement jumps at an interface where a crack may occur. The bilinear softening model which is chosen here for its simplicity is shown in Fig. 8. Damage initiation is related to the interfacial strength, i.e., the maximum traction in the traction-displacement jump relation. When the area under the relation curve is equal to the fracture toughness G_{cr}, the traction is reduced to zero and new crack surfaces are formed, that is

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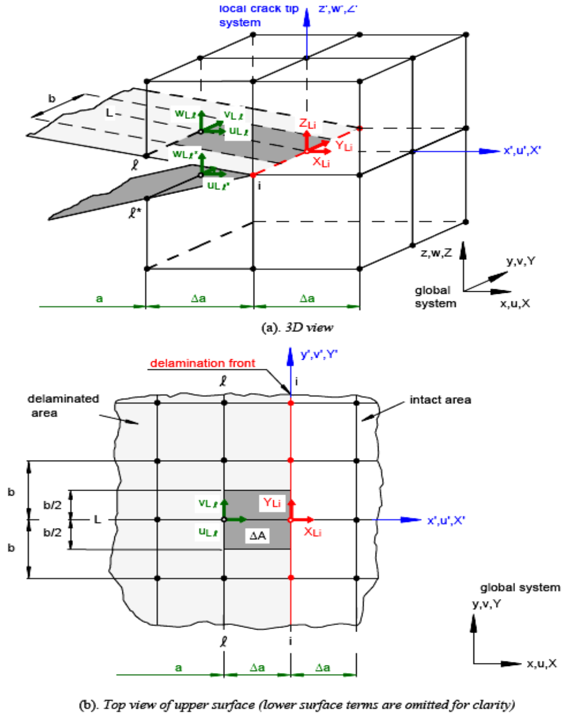

Fig. 7
VCCT for eight-node solid elements [68].

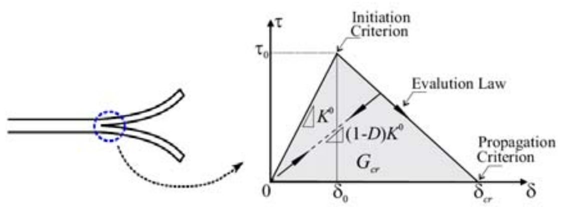

Fig. 8
Bilinear cohesive law model.

$$\int_0^{\delta_{cr}} \tau(\delta)d\delta = G_{cr} \tag{5}$$

The bilinear cohesive law model, which is shown in Fig. 8, can be implemented as follows:

(1) for $\delta < \delta_0$, the constitutive equation is given by:

$$\sigma = \begin{bmatrix} K_0 & 0 & 0 \\ 0 & K_0 & 0 \\ 0 & 0 & K_0 \end{bmatrix} \delta = \mathbf{D}\delta \tag{6}$$

(2) for $\delta_0 < \delta < \delta_{cr}$, the constitutive equation is given by:

$$\sigma = (\mathbf{I} - \mathbf{E})\mathbf{D}\delta \tag{7}$$

where $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{E}$ is a diagonal matrix defining the position of the integration point in the softening curve.

(3) for $\delta > \delta_{cr}$, all the penalty stiffness values revert to zero. The contact problem should be addressed by adopting appreciate techniques when interpenetration is detected.

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A bilinear cohesive law for mixed-mode delamination can be constructed by determining the initial damage threshold from the criterion for damage initiation and the final displacement from the formulation of the propagation criterion. For the B-K criterion [32, 33], the mixed-mode displacement jump for damage initiation is [84].

$$\delta_0 = \sqrt{(\delta_0^3)^2 + \left[(\delta_0^{1,2})^2 - (\delta_0^3)^2\right]\left(1 - \frac{G_I}{G_T}\right)^{\eta}} \tag{8}$$

where δ_0^3 is the initiation displacement jump in mode I, $\delta_0^{1,2} = \sqrt{(\delta_0^1)^2 + (\delta_0^2)^2}$ is the initiation displacement jump in mode II and mode III, and the parameter η is obtained by curve-fitting the fracture toughness of mixed-mode tests. CZM use failure criteria that combine aspects of strength-based analysis to predict the onset of the softening process at the interface between laminae, and fracture mechanics to predict delamination propagation. For the the B-K criterion, the displacement jump for final fracture is obtained as:

$$\delta_{cr} = \frac{1}{\delta_0}\left\{\delta_0^3\delta_{cr}^3 + (\delta_0^{1,2}\delta_{cr}^{1,2} - \delta_0^3\delta_{cr}^3)\left(1 - \frac{G_I}{G_T}\right)^{\eta}\right\} \tag{9}$$

It has been shown that CZM can be related to Griffith's theory of fracture if the area under the traction-relative displacement relation is equal to the corresponding fracture toughness, regardless of the constitutive equation [84,85]. Therefore, the mixed-mode delamination propagation criterion can also be established in terms of the energy release rate and fracture toughness, as presented in part II.

The need for an appropriate cohesive law in the formulation of the CZM is fundamental for an accurate simulation of the interlaminar cracking process. In addition to bilinear cohesive law, other cohesive laws proposed are [86]: linear elastic-perfectly plastic, linear elastic-progressive softening, linear elastic-regressive softening, etc. The main advantage of the use of CZM is the capability to predict both onset and propagation of delamination without previous knowledge of the crack location and propagation direction. Non-self-similar delamination growth, therefore, where the delamination front changes its shape throughout the loading history, can be predicted [87].

4.3 Progressive Failure Analysis (PFA)

Progressive failure analysis (PFA) has been developed over the past three decades and has been successfully implemented in predicting the initiation and propagation of damage while taking into account the damage evolution caused internal load redistributions[64,65]. Pandey and Reddy [88] developed a PFA procedure based on first-order, shear-deformation theory for first-ply failure analysis of laminated composite plates subjected to in-plane and/or transverse loads. Ochoa and Engblom [89] presented a PFA for composite laminated composites in uniaxial tension using a higher-order plate theory with shear deformable elements. Chang and Chang [90] developed a progressive failure damage model for laminated composites containing stress concentrations. Later, they applied the model to bolted composite joints [91] and a laminated composite plate containing an open hole [92]. Reddy and Reddy [93] calculated and compared the first-ply failure loads obtained by using both linear and nonlinear finite element analyses on composite plates. Then, they developed a three-dimensional progressive failure algorithm for composite laminated composites under axial tension [94]. Engelstad, Reddy, and Knight [95] investigated the postbuckling response and failure prediction of composite panels loaded in axial compression considering transverse shear deformation. Coats [96] developed a nonlinear progressive failure analysis for laminated composites that used a constitutive model describing the kinematics of matrix cracks via volume averaged internal state variables.

The simulation of progressive fracture has been verified to be in reasonable agreement with experimental data from tensile coupon test on graphite/epoxy laminated composites [97] and damage progression in carbon fiber reinforced plastic I-beams [98]. A variety of laminated fiber-reinforced composite structures are used to simulate the damage progression and fracture, such as: damage progression in laminated composite structures[99, 100], in stiffened adhesively bonded composite structures [101,102], in bolted composite structures [103], in notched composite panels [104], and damage tolerant of composite pressurized thin shell structures [105]. Ochoa and Reddy [106], Sleight [99], Garnich and Akula [107] presented excellent literature reviews on the basic steps for performing a progressive failure analysis.

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The typical PFA methods involves several key features, see Fig. 9 [99]: a suitable numerical scheme for performing the nonlinear structural analysis to establish equilibrium, an accurate stress recovery procedure to establish the local lamina stress state, a failure criterion to detect local lamina failure and determine the mode of failure, a strategy for modeling the effective properties of the damaged material, and a procedure to re-establish equilibrium after modifying local lamina properties.

The success of any progressive failure analysis is influenced by the failure criterion and the associated material degradation models. Various presented material failure criterion can be classified into two groups [99,107]: non-interactive failure criteria and interactive failure criteria. Several papers can be found which list the most commonly used composite failure theories [108-110]. A non-interactive failure criterion, sometimes called mode-independent failure criteria, is defined as one that directly compares the individual stress or strain components with the corresponding material allowable strength values. The maximum stress and maximum strain criteria belong to this category. An interactive failure criterion, sometimes called mode-dependent failure criteria, involves interactions between stress and strain components. The Tsai-Hill criterion [111, 112], the Tsai and Wu criterion [113], the Hashin criterion [114], the Hoffman criterion [115], and Chamis [116] are a few examples of mode-dependent failure criteria. Other popular mode-dependent failure criteria included which developed by Azzi and Tsai [117], Hashin–Rotem criterion [118], Christensen [119], and Mayes and Hansen [120]. A number of material properties degradation models have been proposed for PFA [108]. These models may be generally categorized into three main groups [121]: sudden unloading models where all properties are reduced, instantaneously, to some fraction of the undamaged properties [89, 94, 122-126], gradual unloading models where one or more of the properties are reduced based on some functional relation to other evolving variables [90, 91, 127-134], and constant stress at ply failure [135], as shown in Fig. 10.

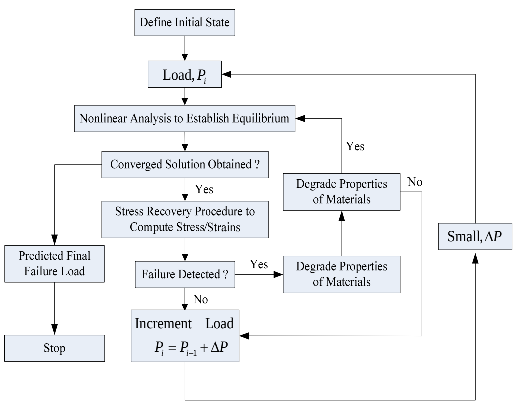

Fig. 9
Typical progressive failure analysis methodology [99].

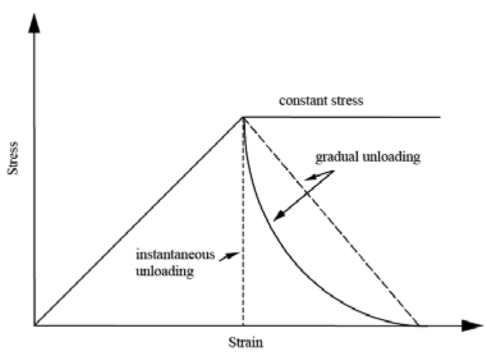

Fig. 10
Material degradation models in composite laminated composites [99].

5 RESULTS AND DISCUSSION

One of the most powerful computational methods for structural analysis of composites is the finite element method. A commonly employed approach is VCCT. VCCT is computationally effective since SERR can be obtained in one finite element analysis and SERR components corresponding to fracture models I, II and III can be obtained directly

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instead of just the total values. However, stress singularity at delamination front between dissimilar materials becomes oscillatory and although convergence is achieved in total SERR values, SERR components values do not converge demonstrating that it requires usually definition of SERR components for a given mesh size or application of special techniques. Furthermore, its use in the simulation of delamination growth may require complex moving mesh techniques to advance the crack front when the local energy release rate reaches a critical value. Finally, an initial delamination must be defined and, for certain geometries and load cases, the location of the delamination front might be difficult to determine.

The use of cohesive elements placed at the interfaces between laminated composites can overcome some of the above difficulties. The CZM method incorporates damage initiation by a strength based criterion and propagation is controlled by fracture mechanics. Since the failure of the cohesive elements is explicitly modeled, this is particularly advantageous if tracking of delamination growth is desirable, and re-meshing of the finite element model with changing delamination front is to be avoided. CZM methods are powerful because they have the capability to predict both onset and propagation of delamination without previous knowledge of the crack location and propagation direction. Therefore, non-self-similar delamination growth can be predicted, and damage tolerant and strength analyses can be done with the same design tool. Nevertheless, a serious drawback of CZM methods is the considerable difficulty in identifying and determining model parameters. Since experiments cannot be performed directly on the interface to determine the properties, the parameters are often inferred indirectly from tests. Furthermore, cohesive elements are limited to very fine mesh size and can produce unacceptably inaccurate predictions when large elements are employed. Therefore, this method is numerically expensive and requires fine meshes in order to represent the damage zone adequately. Another problem is that the local softening employed in the model may also cause numerical instability and non-uniqueness of solutions.

Alternatives to fracture mechanics based models are PFA methods. The methodology relies on the failure criteria for the prediction of the occurrence of material damage and damage-dependent constitutive models that represent the effects of ply-level damage on the laminated stress-strain behavior. Sudden unloading and gradual unloading are two major strategies of material degradation models. The most attractive feature of the sudden degradation models is their simplicity. Therefore, only limited fidelity of this category can be achieved. Gradual degradation models generally describe the internal damage in the material by defining one or more internal state variables. As the number of damage variables increases, the complexity in formulation evolution laws and the numerical efforts also increase. Additionally, the complexity of modeling arises from the multiple failure modes, directionality of failure, and the interaction of failed and unfailed composite laminated composites. Therefore, these gradual degradation models generally have the disadvantage of higher computational cost due to the persistent need for iteration of the equilibrium equations.

6 CONCLSIONS

Durability and damage tolerant design is very challenging and requires expertise in damage mechanics, fracture mechanics, structural mechanics, material science, and physics to guide the experimental and analytical work. Although some advances have been made in damage tolerant design and analysis, the current methodology for composite laminated composites remains semi-empirical with bias toward reliance on experimental data and far from nature for application to all aircraft and aerospace structural components. Laminated composite structures can develop local failures or exhibit local damage such as matrix cracks, fiber breakage, fiber-matrix debonds, and delaminations under normal operating conditions which may contribute to their failure. There is therefore a need to better understand and predict the multiple complex failure mechanisms in composite structures, and to devise more reliable failure theories and damage progression models.

Although finite element method can be used to perform numerical damage tolerance analysis either within the framework of fracture mechanics or the continuum computational damage mechanics. The design must start with the creation of complex discretized model representing the structure and stress singularities exist. Moreover, it is generally necessary to re-mesh large portions of the problem domain in order to accommodate the changes of discontinuities. Despite all achievements such as singular elements, adaptive finite element procedure and combined finite/discrete element methodologies, the continuum basis of finite element method remained a source of relative disadvantage for discontinuous fracture mechanics. A number of research issues require further studies including:

- Understanding how the material parameters GI, GII and GIII relate to damage tolerance[3]
- Continuum methods such as CZM method for damage tolerance analysis in laminated composites
- Interactive models for predicting fatigue crack growth and residual strength analysis

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- New techniques to model cracks and crack growth without re-meshing like extended finite element method (XFEM) [136-138]
- Advanced methods for describing discontinuity and tracking moving boundaries

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