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# **Review of Damage Tolerant Analysis of Laminated Composites**

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## ABSTRACT

With advanced composites increasing replacing traditional metallic materials, the material inhomogeneity and inherent anisotropy of such materials lead to not only new attributes for aerospace structures, but also introduce new technology to damage tolerant design and analysis. The deleterious effects of changes in material properties and initiation and growth of structural damage must be addressed. The anisotropic and brittle properties make this requirement a challenging to composite structural designers. Accurate, reliable and user-friendly computational methods, design and analysis methods are vital for more damage tolerant composite structures. Both durability and damage tolerant methodologies must address the possible changes in mechanical properties and the evolving damage accumulations that may occur during the vehicle's service lifetime. Delamination is a major failure mode in laminated composites and has received much research attention. It may arise out of manufacturing defects, free edge effects, structural discontinuities, low and high velocity impact damage, and even bird strikes. Early pioneering work established that the reduction in strength following delamination damages placed severe limits on the design allowable for highly loaded components such as aircraft wing and fuselage structure. In the present article, we provide a state-of-art survey on damage tolerant design correlated failure behavior and analysis methodologies of laminated composites. Particular emphasis is placed on some advanced formulations and numerical approaches for efficient computational modeling and damage tolerant analysis of laminated composites. © 2010 IAU, Arak Branch. All rights reserved.

**Keywords:** Damage tolerant analysis; Delamination; Virtual crack closure-integral technique; Cohesive zone model; Progressive failure analysis.

#### **1 INTRODUCTION**

DVANCED composites are increasingly being used in aircraft, aerospace, marine, civil and automotive A industries due to their high specific stiffness and strength, good fatigue resistance, and their tailorability. Although the need to consider damage tolerant during the design process is not unique to composite structures, material inhomogeneity and inherent anisotropy further complicated the design and analysis process. The relative successful experiences with design of metallic structures cannot be directly transferred to the design of composite structures because of the different failure mechanisms for composite and metallic materials. Several important factors lead to the difference [1]. Firstly, composite materials are not isotropic and homogenous. Secondly, composite materials are generally brittle and lack of ductility. Thirdly, the initiation and growth of damage and the failure modes of composites are not as well understood and still can not be predicted accurately. Due to these complications, composite structural design and manufacturing technology is not yet as mature as metallic structures, especially for heavily loaded aeronautic and astronautic structures.

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Durability and damage tolerant design have been a primary barrier to expand the application of composites to heavily loaded, primary structures. The current design models and numerical analysis methods are semi-empirical and rely heavily on the building-block approach to design and certification. Several bibliographical reviews dealing with damage tolerant design of composite structures are found in the specialized literature. As shown in the examples, Sierakowski [2] explored the damage tolerant issues based on design requirements, current state of the art design and analysis. Schmidt [3] summarized the major structural criteria and requirements to be considered in damage tolerant design and analysis of current and future aircraft structures. Ransom [4] presented an overview of the recent and planned future research in composites durability and damage tolerant analytical and experimental methods at NASA Langley Research Center. Tomblin [5-7], McGowan [8], and Moody [9] studied damage tolerant design methods for composite sandwich structures. Williams [10] discussed how the pioneering work in damage tolerant conducted at NASA has been applied in the oil industry. The major contributions in all of these researches are based on structural criteria, design requirements and design allowables for vehicle composite components. The present paper aims to review major numerical methods for damage tolerant analysis of laminated composites based on the understanding of failure mechanisms of laminated composites.

## 2 FAILURE BEHAVIORS UNDER STATIC LOADING

The prediction of the failure behavior of composite structures requires an accurate assessment of damage initiation, growth, and description of the evolving damage accumulation in service and adverse effect on structural performance, strength, and fatigue life. There have been a number of survey articles on the failure criterion of composite structures. These include reviews presented by Echaabi [11], Hinton and Soden [12-14], and Paris [15], where the failure criteria are given and compared. A more recent survey by Icardi [16] reviewed and discussed the most widely used and recently developed failure criteria, along with their applications. Based on the dominated failure modes, this paper will discuss the application of fracture mechanics for damage tolerant design and analysis of composites, and some issues associated with its application. The principal failure mode of laminated composites is the separation along the interfaces of the layers, via. delamination, which can be viewed as an interface crack between two anisotropic materials. Miller [17] indicated that approximately 60% of the damage phenomena observed in the composites parts of an aircraft are delaminations. Interlaminar fracture mechanics is widely used to predict the more dominant failure mode of delamination initiation and propagation [18-20]. Fig. 1 shows a flow diagram of the fracture mechanics method for damage tolerant design [21]. First, areas of possible damage should be decided according to stress-based criterion. Then, a fracture analysis is conducted to determine whether or not a delamination will initiate and grow. If a delamination does grow, the growth length and final failure should be predicted with suitable failure criterion.





**Fig. 1** A schematic of the fracture mechanics approach [19].

Fig. 2 Mixed-mode bending test.

Fracture mechanics prediction procedures essentially have two steps: modeling the structure, and characterizing the delaminated materials properties [21]. For the first step, the most important part is to calculate strain energy release rate (SERR), which will be discussed in detail lately. For a given delamiantion length, a critical value of SERR is obtained. This critical SERR value is compared with the material's fracture data to give both a monotonic failure load and location to initiate the delamiantion. The delaminated material properties are usually obtained using standard composite fracture tests. Typically, delamination initiates and propagates under the combined influence of normal and shear stresses. Therefore, mixed-mode delamiantion tests are of great interest for the determination of interlaminar fracture toughness of laminated composites, for example mixed-mode bending (MMB) [22, 23], cracked-lap shear (CLS) [24], edge delamination tension (EDT) [25], Arcan specimen [26], asymmetric double cantilever beam) (ADCB) [27], mixed-mode flexure (MMF) [28, 29]. The most commonly used test is the MMB method, which can simultaneously produce mode I and mode II bending loads by loading with a lever, as shown in Fig. 2 [23]. The main advantages of the MMB test method are the possibility of using virtually the same specimen configuration as for mode I tests, and the capability of obtaining different mixed-mode ratios, ranging from pure mode I to pure mode II, by changing the length of the loading lever.

It is important that accurate mixed mode delamination fracture criteria should be developed so that the extension of delaminations in structures can be predicted. Many delamination failure criteria based on fracture toughness have been suggested over the past few decades, such as the power law criterion [30, 31], Benzeggagh-Kenane criterion [32, 33], and Reeder criterion [34, 35]. But most of these criteria only covered mixed mode I/II or based on assumption that the relationship between mode I and mode III toughness is similar to the relation between mode I and mode II [35]. Extensive reviews of technical issues related to fracture toughness testing are provided by O'Brien [36], Davis [37], Brunner [38], and Reeder [35].

# **3 FAILURE BEHAVIORS UNDER IMPACT LOADING**

Low transverse and interlaminar shear strength, no plastic deformation, and laminar construction make impact becomes the most dangerous loading conditions for laminated composites. Impact loads may result in a large internal damaged area of the laminated composites that is not detectable from visible observation, which can have a significant effect on the durability, damage tolerant, and stability of the laminated composites. Compression can continuously grow the damage area, possibly resulting in complete structural collapse of the damaged structure. In the aeronautic and astronautic industries, the residual compression strength of an impact damaged composite structure has become one of the most important design limiting factor, which is similar to current damage tolerant design philosophies in other industrial sectors.

The impact damage mechanism in laminated composites constitutes a very complex process. It is a combination of matrix cracking, surface buckling, delamination, fiber shear-out, and fiber fracture, etc., which usually all interact with each other. A typical impact damage mode for laminated composites is depicted in Fig. 3. An explanation of the basic mechanics and fundamental terms of impact can be found in several reviews [39-41]. In this section, low and high velocity impact on composites panels are discussed to investigate the impact behavior relevant to civil aircraft structures ranging from tool drop up to foreign object damage.

#### 3.1 Low velocity impact

Low velocity impact is associated with delamination damage, especially that caused by blunt-headed projectiles. This interlaminar debonding primarily reduces the local bending stiffness and thus can affect the bending and buckling behavior of the structure. Such damage has been reported to cause as much as 40% reduction in static and fatigue strength [42, 43]. In order to measure the damage resistance of a fiber-reinforced polymer matrix composite to impact, a set of low velocity impact tests were carried out. Fig. 4 depicts the quasi-static indentation test (ASTM 7136-2007) [44] and the drop-weight impact test [45, 46].

For laminated composites under low velocity impact loading conditions, the relationships between the impact load and energy applied, the extent and modes of damage introduced and the residual properties are invaluable information for proper damage tolerant design of composite components and structures [47, 48]. Typical impact load-time and energy-time histories are shown in Fig. 5. Some investigators assume the first load drop during the force history corresponds to the occurrence of initial damage in the form of matrix cracking, fiber breakage, and local puncture or indentation [47-49]. Belingardi and Vadori [47] defined two thresholds from the load history.



Fiber breakage







# Fig. 4 Illustration of two low velocity impact tests setup, (a) quasi-static indentation test [44];(b) drop-weight impact test [45, 46].





The first one was at the first load drop for the first material damage, and the second one was the maximum force value for the first lamina failure. Gao and Zhang [49] pointed out that the first damage threshold is probably due to the initialization of delamination failure. They also proposed an equation for a critical force threshold [49]:

$$\boldsymbol{\sigma} = (\mathbf{I} - \mathbf{E})\mathbf{D}\boldsymbol{\delta} \quad p_c^2 = \frac{8\pi^2 E h^3 G_{llc}}{9(1 - \nu^2)} \tag{1}$$

where  $P_c$  is the threshold load, E and v are the equivalent in-plane modulus and Poisson's ratio, h is the laminated thickness, and  $G_{IIc}$  is the critical strain energy release rate. Based on the experimental study, Shen [50] proposed that





Fig. 6

SEM photographs of micrometeoroid/debris impact holes for SP-288/T300graphite/epoxy tube [41] (a) front ( $\times$ 35) (b) back ( $\times$ 35).

the damage tolerant behavior could be characterized by the threshold of compression failure strain after impact (CAIT) of the dent depth and compressive failure strain curve. The tests results indicate that there are knee points in d-e (dent depth-impact energy curve), S-d (damage area-dent depth curve) and c-d (compression failure strain-dent depth curve), and these points corresponding to the transformation of damage mechanisms. The dominant damage is delamination and matrix cracking before the point and it becomes fiber breakage after the point.

The current approaches to damage tolerant designs are based on a dominant failure mechanism associated with a critical crack size, such as that associated with delamination. For the case of damage induced by impact to aircraft composite structures, the presence of damage caused by a 2.54 cm (1.0 in) diameter hemispherical impactor delivered at 133 J (100 ft/lb) of kinetic energy, or a kinetic energy event causing a 0.54 cm (0.10 in) dent, have been used as damage thresholds [50].

# 3.2 High velocity impact

High velocity impact conditions are most likely to occur on spacecraft in low earth orbit, which can produce delamination coupled with spallation and eject plumes emanating from both sides of the laminated composites. Fig. 6 presents SEM photomicrographs of a crater hole viewed from the front and rear faces of a graphite/epoxy tube due to an actual micrometeoroid hypervelocity impact [41].

A larger number of laminated composites under high velocity impact have been investigated employing experimental and numerical methods. Yew et al. [51] studied a wide range of graphite/epoxy plates at the NASA Johnson Space Center (NASA JSC) using their light gas gun facilities. Lamontagne et al. [52] performed research on the effects of impact angle using their light gas gun. It was observed that for oblique angle impacts, the debris cloud does not follow the line of flight of the projectile. Moreover, the cone angle associated with the debris cloud is not symmetric about the projectile impact velocity vector. Christiansen [53] and Shortliffe [54] investigated the failure behavior of composite tubes and cylinders under high velocity impact. Taylor [55] and Vaidya [56] studied the response of composite sandwich panels under high velocity impact. Taylor also studied the effect of impact angle on the ballistic limit and the correlation of entry hole size and impactor energy [55]. Vaidya considered sandwich constructions with reinforced cores and made a conclusion that Z-pin reinforcement of the core suppressed core crushing effectively under the high strain rate impact loading [56]. A gas gun impact test program has been conducted within the EU HICAS project to study the impact resistance of stringer stiffened panels [57]. In their research, two cases were considered: impacts on a stringer and between two stringers, which show different failure behaviors. A recent survey by Jiang and Vecchio [58] contains the major progress in Hopkinson bar fracture test methods over the past 30 years, focused on dynamic fracture toughness measurement.

# 4 NUMERICAL ANALYSIS METHODS

There are various ways of analyzing the laminated failure process, in terms of the energy deposited and gross damage produced, micro energy dissipation or by considering the stresses acting on flaws in the material and the

effects that are generated. Unlike homogenous metals, the discrete components of the composite structures may experience material damage and fracture concurrently. Elder [59] reviewed the current fracture mechanics and damage mechanics methods for predicting delamination. In this paper, several widely used or promising methods are reviewed, such as virtual crack closure-integral technique (VCCT) method [60, 61], cohesive zone model (CZM) method [62, 63], and the progressive failure analysis (PFA) method [64, 65].

## 4.1 Virtual Crack Closure-integral Technique (VCCT)

Linear elastic fracture mechanics is commonly applied for the prediction of laminated composites failure behavior. In fracture methods category, strain energy release rate (SERR) is compared to the material toughness to determine whether the material will fracture. The VCCT method has been widely used for computing SERR based on results from finite element analysis. A comprehensive review paper was published by Krueger [66].

VCCT is based on Irwin's crack closure integral concept [67], which assume that the energy  $\Delta E$  released when the crack is extended by  $\Delta a$  from a to  $a + \Delta a$  is identical to the energy required to close the crack between location Liand Ll, as shown in Fig. 7. At the same time, it is assumed that a crack extension of  $\Delta a$  from  $a + \Delta a$  to  $a + 2\Delta a$  does not significantly alter the state at the crack tip. For a crack modeled with three-dimensional, eight-node elements, the mode I, mode II and mode III SERR components,  $G_l$ ,  $G_{II}$  and  $G_{III}$ , are calculated as [68].

$$G_{I} = \frac{1}{2\Delta A} Z_{Li} (w_{Ll} - w_{Li}^{*})$$
<sup>(2)</sup>

$$G_{II} = \frac{1}{2\Delta A} X_{Li} (u_{Li} - u_{Li}^*)$$
(3)

$$G_{III} = \frac{1}{2\Delta A} Y_{Li} (v_{LI} - v_{LI}^*)$$
(4)

where *X*, *Y* and *Z* denote the forces at the delamination front, *u*, *v* and *w* are the corresponding displacements behind the delamination. And  $\triangle A = \triangle a \times b$  is the area virtually closed, as shown in Fig. 7.

The VCCT method has the advantages of providing accurate SERR assessment with a relatively coarse mesh, can be easily adapted to existing codes, has explicitly determined separated fracture modes and requires only one complete analysis of the structure to obtain the deformations. Therefore, VCCT based fracture mechanics has been widely used to assess the damage tolerant of composite structures in the design phase and during certification [19, 20, 66]. Recently, the ABAQUS/Standard commercial finite element code released their implementation of VCCT [69] which is based on a new interface element developed by Boeing [70] that performs the VCCT calculation internally and therefore allows the automation of delamination propagation analyses.

## 4.2 Cohesive Zone Model (CZM)

Another approach for the numerical simulation of delamination can be developed within the framework of damage mechanics, which are based on the concept of the cohesive zone model. The origin of the cohesive zone model goes back to Dugdale [62] who introduced the concept that stresses in the material are limited by the yield stress and that a thin plastic zone is generated in front of the notch. Barenblatt [63] introduced cohesive forces on a molecular scale in order to solve the problem of equilibrium in elastic bodies with cracks. Further developments are due to Williams [71], and Schapery [72] who modeled crack growth in visco-elastic media. Later, Hillerborg [73] extended the method to quasi-brittle materials. Unguwarungasri and Knauss [74] proposed a cohesive layer composed of a series of nonlinear springs. Needleman [75, 76] analyzed void nucleation and void coalescence in metals using cohesive layer models. Further significant contributions came, among others, from Shahwan and Wass [77], Ortiz and Pandolfi [78], Yu [79] and Corigliano, et al [80]. A majority of investigators have used cohesive layers of zero thickness [75, 76], finite-thickness [81], and line elements [82, 83].

In CZM method, a cohesive constitutive law relates the tractions to the displacement jumps at an interface where a crack may occur. The bilinear softening model which is chosen here for its simplicity is shown in Fig. 8. Damage initiation is related to the interfacial strength, i.e., the maximum traction in the traction-displacement jump relation. When the area under the relation curve is equal to the fracture toughness  $G_{cr}$ , the traction is reduced to zero and new crack surfaces are formed, that is



(b). Top view of upper surface (lower surface terms are omitted for clarity)



Fig. 7 VCCT for eight-node solid elements [68].

**Fig. 8** Bilinear cohesive law model.



The bilinear cohesive law model, which is shown in Fig. 8, can be implemented as follows: (1) for  $\delta < \delta_0$ , the constitutive equation is given by:

$$\boldsymbol{\sigma} = \begin{bmatrix} K_0 & 0 & 0\\ 0 & K_0 & 0\\ 0 & 0 & K_0 \end{bmatrix} \boldsymbol{\delta} = \mathbf{D}\boldsymbol{\delta}$$
(6)

(2) for  $\delta_0 < \delta < \delta_{cr}$ , the constitutive equation is given by:

$$\boldsymbol{\sigma} = (\mathbf{I} - \mathbf{E})\mathbf{D}\boldsymbol{\delta} \tag{7}$$

where I is the identity matrix and E is a diagonal matrix defining the position of the integration point in the softening curve.

(3) for  $\delta > \delta_{cr}$ , all the penalty stiffness values revert to zero. The contact problem should be addressed by adopting appreciate techniques when interpenetration is detected.

A bilinear cohesive law for mixed-mode delamination can be constructed by determining the initial damage threshold from the criterion for damage initiation and the final displacement from the formulation of the propagation criterion. For the B-K criterion [32, 33], the mixed-mode displacement jump for damage initiation is [84].

$$\delta_0 = \sqrt{(\delta_0^3)^2 + \left[ (\delta_0^{1,2})^2 - (\delta_0^3)^2 \right] \left( 1 - \frac{G_I}{G_T} \right)^{\eta}}$$
(8)

where  $\delta_0^3$  is the initiation displacement jump in mode I,  $\delta_0^{1,2} = \sqrt{(\delta_0^1)^2 + (\delta_0^2)^2}$  is the initiation displacement jump in mode II and mode III, and the parameter  $\eta$  is obtained by curve-fitting the fracture toughness of mixed-mode tests. CZM use failure criteria that combine aspects of strength-based analysis to predict the onset of the softening process at the interface between laminae, and fracture mechanics to predict delamination propagation. For the the B-K criterion, the displacement jump for final fracture is obtained as:

$$\delta_{cr} = \frac{1}{\delta_0} \left\{ \delta_0^3 \delta_{cr}^3 + (\delta_0^{1,2} \delta_{cr}^{1,2} - \delta_0^3 \delta_{cr}^3) \left( 1 - \frac{G_I}{G_T} \right)^\eta \right\}$$
(9)

It has been shown that CZM can be related to Griffith's theory of fracture if the area under the traction-relative displacement relation is equal to the corresponding fracture toughness, regardless of the constitutive equation [84,85]. Therefore, the mixed-mode delamination propagation criterion can also be established in terms of the energy release rate and fracture toughness, as presented in part II.

The need for an appropriate cohesive law in the formulation of the CZM is fundamental for an accurate simulation of the interlaminar cracking process. In addition to bilinear cohesive law, other cohesive laws proposed are [86]: linear elastic-perfectly plastic, linear elastic-progressive softening, linear elastic-regressive softening, etc. The main advantage of the use of CZM is the capability to predict both onset and propagation of delamination without previous knowledge of the crack location and propagation direction. Non-self-similar delamination growth, therefore, where the delamination front changes its shape throughout the loading history, can be predicted [87].

#### 4.3 Progressive Failure Analysis (PFA)

Progressive failure analysis (PFA) has been developed over the past three decades and has been successfully implemented in predicting the initiation and propagation of damage while taking into account the damage evolution caused internal load redistributions[64,65]. Pandey and Reddy [88] developed a PFA procedure based on first-order, shear-deformation theory for first-ply failure analysis of laminated composite plates subjected to in-plane and/or transverse loads. Ochoa and Engblom [89] presented a PFA for composite laminated composites in uniaxial tension using a higher-order plate theory with shear deformable elements. Chang and Chang [90] developed a progressive failure damage model for laminated composites containing stress concentrations. Later, they applied the model to bolted composite joints [91] and a laminated composite plate containing an open hole [92]. Reddy and Reddy [93] calculated and compared the first-ply failure loads obtained by using both linear and nonlinear finite element analyses on composite plates. Then, they developed a three-dimensional progressive failure algorithm for composite laminated composites under axial tension [94]. Engelstad, Reddy, and Knight [95] investigated the postbuckling response and failure prediction of composite panels loaded in axial compression considering transverse shear deformation. Coats [96] developed a nonlinear progressive failure analysis for laminated composites that used a constitutive model describing the kinematics of matrix cracks via volume averaged internal state variables.

The simulation of progressive fracture has been verified to be in reasonable agreement with experimental data from tensile coupon test on graphite/epoxy laminated composites [97] and damage progression in carbon fiber reinforced plastic I-beams [98]. A variety of laminated fiber-reinforced composite structures are used to simulate the damage progression and fracture, such as: damage progression in laminated composite structures [99, 100], in stiffened adhesively bonded composite structures [101,102], in bolted composite structures [103], in notched composite panels [104], and damage tolerant of composite pressurized thin shell structures [105]. Ochoa and Reddy [106], Sleight [99], Garnich and Akula [107] presented excellent literature reviews on the basic steps for performing a progressive failure analysis.

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The typical PFA methods involves several key features, see Fig. 9 [99]: a suitable numerical scheme for performing the nonlinear structural analysis to establish equilibrium, an accurate stress recovery procedure to establish the local lamina stress state, a failure criterion to detect local lamina failure and determine the mode of failure, a strategy for modeling the effective properties of the damaged material, and a procedure to re-establish equilibrium after modifying local lamina properties.

The success of any progressive failure analysis is influenced by the failure criterion and the associated material degradation models. Various presented material failure criterion can be classified into two groups [99,107]: noninteractive failure criteria and interactive failure criteria. Several papers can be found which list the most commonly used composite failure theories [108-110]. A non-interactive failure criterion, sometimes called mode-independent failure criteria, is defined as one that directly compares the individual stress or strain components with the corresponding material allowable strength values. The maximum stress and maximum strain criteria belong to this category. An interactive failure criterion, sometimes called mode-dependent failure criteria, involves interactions between stress and strain components. The Tsai-Hill criterion [111, 112], the Tsai and Wu criterion [113], the Hashin criterion [114], the Hoffman criterion [115], and Chamis [116] are a few examples of mode-dependent failure criteria. Other popular mode-dependent failure criteria included which developed by Azzi and Tsai [117], Hashin–Rotem criterion [118], Christensen [119], and Mayes and Hansen [120]. A number of material properties degradation models have been proposed for PFA [108]. These models may be generally categorized into three main groups [121]: sudden unloading models where all properties are reduced, instantaneously, to some fraction of the undamaged properties [89, 94, 122-126], gradual unloading models where one or more of the properties are reduced based on some functional relation to other evolving variables [90, 91, 127-134], and constant stress at ply failure [135], as shown in Fig. 10.









## 5 RESULTS AND DISCUSSION

One of the most powerful computational methods for structural analysis of composites is the finite element method. A commonly employed approach is VCCT. VCCT is computationally effective since SERR can be obtained in one finite element analysis and SERR components corresponding to fracture models I, II and III can be obtained directly

instead of just the total values. However, stress singularity at delamination front between dissimilar materials becomes oscillatory and although convergence is achieved in total SERR values, SERR components values do not converge demonstrating that it requires usually definition of SERR components for a given mesh size or application of special techniques. Furthermore, its use in the simulation of delamination growth may require complex moving mesh techniques to advance the crack front when the local energy release rate reaches a critical value. Finally, an initial delamination must be defined and, for certain geometries and load cases, the location of the delamination front might be difficult to determine.

The use of cohesive elements placed at the interfaces between laminated composites can overcome some of the above difficulties. The CZM method incorporates damage initiation by a strength based criterion and propagation is controlled by fracture mechanics. Since the failure of the cohesive elements is explicitly modeled, this is particularly advantageous if tracking of delamination growth is desirable, and re-meshing of the finite element model with changing delamination front is to be avoided. CZM methods are powerful because they have the capability to predict both onset and propagation of delamination without previous knowledge of the crack location and propagation direction. Therefore, non-self-similar delamination growth can be predicted, and damage tolerant and strength analyses can be done with the same design tool. Nevertheless, a serious drawback of CZM methods is the considerable difficulty in identifying and determining model parameters. Since experiments cannot be performed directly on the interface to determine the properties, the parameters are often inferred indirectly from tests. Furthermore, cohesive elements are limited to very fine mesh size and can produce unacceptably inaccurate predictions when large elements are employed. Therefore, this method is numerically expensive and requires fine meshes in order to represent the damage zone adequately. Another problem is that the local softening employed in the model may also cause numerical instability and non-uniqueness of solutions.

Alternatives to fracture mechanics based models are PFA methods. The methodology relies on the failure criteria for the prediction of the occurrence of material damage and damage-dependent constitutive models that represent the effects of ply-level damage on the laminated stress-strain behavior. Sudden unloading and gradual unloading are two major strategies of material degradation models. The most attractive feature of the sudden degradation models is their simplicity. Therefore, only limited fidelity of this category can be achieved. Gradual degradation models generally describe the internal damage in the material by defining one or more internal state variables. As the number of damage variables increases, the complexity in formulation evolution laws and the numerical efforts also increase. Additionally, the complexity of modeling arises from the multiple failure modes, directionality of failure, and the interaction of failed and unfailed composite laminated composites. Therefore, these gradual degradation models generally have the disadvantage of higher computational cost due to the persistent need for iteration of the equilibrium equations.

## 6 CONCLSIONS

Durability and damage tolerant design is very challenging and requires expertise in damage mechanics, fracture mechanics, structural mechanics, material science, and physics to guide the experimental and analytical work. Although some advances have been made in damage tolerant design and analysis, the current methodology for composite laminated composites remains semi-empirical with bias toward reliance on experimental data and far from nature for application to all aircraft and aerospace structural components. Laminated composite structures can develop local failures or exhibit local damage such as matrix cracks, fiber breakage, fiber-matrix debonds, and delaminations under normal operating conditions which may contribute to their failure. There is therefore a need to better understand and predict the multiple complex failure mechanisms in composite structures, and to devise more reliable failure theories and damage progression models.

Although finite element method can be used to perform numerical damage tolerance analysis either within the framework of fracture mechanics or the continuum computational damage mechanics. The design must start with the creation of complex discretized model representing the structure and stress singularities exist. Moreover, it is generally necessary to re-mesh large portions of the problem domain in order to accommodate the changes of discontinuities. Despite all achievements such as singular elements, adaptive finite element procedure and combined finite/discrete element methodologies, the continuum basis of finite element method remained a source of relative disadvantage for discontinuous fracture mechanics. A number of research issues require further studies including:

- Understanding how the material parameters GI, GII and GIII relate to damage tolerance[3]
- Continuum methods such as CZM method for damage tolerance analysis in laminated composites
- Interactive models for predicting fatigue crack growth and residual strength analysis

- New techniques to model cracks and crack growth without re-meshing like extended finite element method (XFEM) [136-138]
- Advanced methods for describing discontinuity and tracking moving boundaries

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## REFERENCES

- [1] Harris C.E., Starnes J.H., Shuart M.J., 2003, Advanced Durability and Damage Tolerance Design and Analysis Methods for Composite Structures, NASA/TM-2003-212420.
- [2] Schmidt H.J., Brandecker B.S., 2003, Damage tolerance design and analysis of current and future aircraft structure, in: *AIAA/ICAS International Air and Space Symposium and Exposition: the next 100 years*, Dayton, Ohio, Paper No. 2003-2784.
- [3] Sierakowski R.L., 2005, Damage tolerance: a status report, in: 46th AIAA/ ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, 2005, Austin, Texas.
- [4] Ransom J.B., Glaessgen E.H., Raju I.S., Harris, C.E., 2007, Recent advances in durability and damage tolerance methodology at NASA Langley Research Center, in: *Proceedings of the 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference,* Paper No. AIAA 2007-2377-CP, April 23-26, Honolulu, HI, USA.
- [5] Tomblin J., Lacy T., Smith B., Hooper S., Vizzini A., Lee S., 1999, Review of damage tolerance for composite sandwich airframe structures, *FAA*, Report No. DOT/FAA/AR-99/49.
- [6] Tomblin J.S., Raju K.S., Walker T., Acosta J.F., 2005, Damage tolerance of composite sandwich airframe structuresadditional results, *FAA*, Report No. DOT/FAA/AR-05/33.
- [7] Tomblin J.S., Raju K.S., Liew J., Smith, B.L., 2001, Impact damage characterization and damage tolerance of composite sandwich airframe structures-final report, *FAA*, Report No. DOT/FAA/AR-00/44.
- [8] McGowan D.M., Ambur D.R., 1997, Damage-Tolerance Characteristics of Composite Fuselage Sandwich Structures with Thick Facesheets, NASA TM 110303.
- [9] Moody R.C., Vizzini A.J., 1999, Damage tolerance of composite sandwich structures, *FAA*, Report No. DOT/FAA/AR-99/91.
- [10] Williams J.G., 2005, NASA research in composite structure damage tolerance and composite applications in the oil industry, in: *46th AIAA Structures, Structural Dynamics & Materials Conference*, Austin, Texas.
- [11] Echaabi J.F., Trochu F., 1996, Review of failure criteria of fibrous composite materials, *Polymer Composites* **17**: 786-798.
- [12] Hinton M.J., Soden P.D., 1998, Predicting failure in composite laminated composites: the background to the exercise, *Composite Science and Technology* **58**: 1001-1010.
- [13] Soden P.D., Hinton M.J., Kaddour A.S., 1998, A comparison of the redictive capabilities of current failure theories for composite laminated composites, *Composite Science and Technology* **58**: 1225-1254.
- [14] Hinton M.J., Kaddour A.S., Soden P.D., 2002, A comparison of the predictive capabilities of current failure theories for composite laminated composites: Judged against experimental evidence, *Composite Science and Technology* 62: 1725-1797.
- [15] Paris F., 2001, A Study of Failure Criteria of Fibrous Composite Materials, NASA/CR-2001-210661.
- [16] Icardi U., Locatto S., Longo A., 2007, Assessment of recent theories for predicting failure of composite laminated composites, *Applied Mechanics Review* **60**(3): 76-86.
- [17] Miller A.G., Lowell D.T., Seferis J.C., 1994, The evolution of an aerospace material: influence of design, manufacturing and in-service performance, *Composite Structures* **27**: 193-206.
- [18] Bolotin V.V., 1996, Delaminations in composite structures: its origin, buckling, growth and stability, *Composites: Part B* **27A**: 129-145.
- [19] Miravete A., Jimenez M. A., 2002, Application of the finite element method to prediction of onset of delamination growth, *Applied Mechanics Review* **55**(2): 89-106.
- [20] Tay T.E., 2003, Characterization and analysis of delamination fracture in composites an overview of developments from 1990 to 2001, *Applied Mechanics Review* **56**(1): 1-32.
- [21] Harris B., 2003, Fatigue in Composites, edited by Bryan Harris, CRC Press, New York.
- [22] Reeder J.R., Crews J.H.Jr., 1990, Mixed-mode bending method for delamination testing, *AIAA Journal* 28: 1270-1276.
- [23] Test Method D6671-01, Standard test method for mixed mode I-mode II interlaminar fracture toughness of unidirectional fiber reinforced polymer matrix composites, *American Society for Testing and Materials (ASTM)*, West Conshohocken, PA, USA.

- [24] Ramkumar R.L., Whitcomb J.D., 1985, Characterization of mode I and mixed mode delamination growth in T300/5028 graphite epoxy, *ASTM STP* **876**: 315-335.
- [25] O'Brien T.K., 1984, Mixed-mode strain energy release rate effects on the edge delamination of composites, *ASTM STP* **836**: 125-142.
- [26] Arcan M., Hashin Z., Voloshin A., 1978, A method to produce uniform plane-stress states with application to fiberreinforced materials, *Experimental Mechanics* **18**(4): 141-146.
- [27] Bradley W.L., Cohen R.N., 1985, Matrix deformation and fracture in graphite reinforced epoxies, Delamination and Debonding of Materials, *ASTM STP* **876**: 389-410.
- [28] Russell A.I., Street K.N., 1987, The effect of matrix toughness on delamination: static and fatigue fracture under mode II shear loading of graphite fiber composites, *ASTM STP* **937**: 275-294.
- [29] Hashemi S., Kinloch A.I., Williams J.G., 1987, Interlaminar fracture of composite materials, in: *6th ICCM & 2nd ECCM Conference Proceedings*, London, **3**: 254-264.
- [30] Whitcomb J.D., 1984, Analysis of instability-related growth of a through-width delamination, NASA TM-86301.
- [31] Goyal V.K., Johnson E.R., Davila C.G., 2004, Irreversible constitutive law for modeling the delamination process using interfacial surface discontinuities, *Computers and Structures* **65**(3-4): 289-305.
- [32] Benzeggagh M.L., Kenane M., 1996, Measurement of mixed-mode delamination fracture toughness of unidirectional glass/epoxy composites with mixed-mode bending apparatus, *Composite Science and Technology* **56**(4): 439-449.
- [33] Gong X.J., Benszeggagh M., 1995, Mixed mode interlaminar fracture toughness of unidirectional glass/epoxy composite, Composite Materials: Fatigue and Fracture, *ASTM STP* 1230, **3**: 100-123.
- [34] Reeder J., Kyongchan S., Chunchu P.B., Ambur D.R., 2002, Postbuckling and growth of delaminations in composite plates subjected to axial compression, in: *Proceeding of the 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Denver, Colorado, Paper No. AIAA-2002-1746.
- [35] Reeder J.R., 2006, 3D mixed-mode delamination fracture criteria—an experimentalist's perspective, in: *Proceedings of the 21st Annual Technical Conference of the American Society for Composites*, Lancaster, PA, 17-20.
- [36] Davies P., Blackman B.R.K., Brunner A.J., 1998, Standard test methods for delamination resistance of composite materials: current status, *Applied Composite Materials* **5**: 345-364.
- [37] O'Brien T.K., 1998, Interlaminar fracture toughness: the long and winding road to standardization, *Composites: Part B* **29B**: 57-62.
- [38] Brunner A.J., 2000, Experimental aspects of mode I and mode II fracture toughness testing of fiber-reinforced polymermatrix composites, *Computer methods in applied mechanics and engineering* **185**(2-4): 161-172.
- [39] Abrate S., 1998, *Impact on Composite Structures*, Cambridge University Press, Cambridge, UK.
- [40] Schoeppner G.A., Abrate S., 2000, Delamination threshold loads for low velocity impact on composite laminated composites, *Composites: Part A: Applied Science and Manufacturing* **31**: 903-915.
- [41] Reid S.R., Zhou G., 2000, *Impact Behavior of Fiber-Reinforced Composite Materials and Structures*, edited by S.R. Reid, and G. Zhou, Loughborough University, UK.
- [42] Poe C.C., Illg W., 1987, Strength of a Thick Graphite/Epoxy Rocket Motor Case after Impact by a Blunt Object, NASA TM-89099.
- [43] Christoforou A.P., Swanson S. R., 1988, Strength loss in composite cylinders under impact, ASME Journal of Engineering Materials and Technology **110**(2): 180-184.
- [44] ASTM D6264/D6264M, 2007, Standard test method for measuring the damage resistance of a fibre reinforced polymer matrix composite to a concentrated quasi-static indentation force, *American Society for Testing and Materials*.
- [45] ASTM D 7136/D 7136M, 2007, Standard test method for measuring the damage resistance of a fibre reinforced polymer matrix composite to a drop-weight impact event, *American Society for Testing and Materials*.
- [46] Lopes C.S., Seresta O., Coquet Y., Gurdal Z., Camanho P.P., Thuis B., 2009, Low-velocity impact damage on dispersed stacking sequence laminated composites: Part I experiments, *Composites Science and Technology* **69**(7-8): 926-936.
- [47] Belingardi G., Vadori R., 2002, Low velocity impact tests of laminated glass-fiber-epoxy matrix composite material plates, *International Journal of Impact Engineering* **27**: 213-229.
- [48] Shyr T.W., Pan Y.H., 2003, Impact resistance and damage characteristics of composite laminated composites, *Composite Structures* **62**:193-203.
- [49] Gao D., Zhang X., 1994, Impact damage prediction in carbon composite structure, *International Journal of Impact Engineering* **16**(1): 149-170.
- [50] Shen Z., Zhang Z.L., Wang J., Yang S.C., Ye L., 2004, Characterization of damage resistance and damage tolerance behaviour of composite laminates, *Acta Materiae Composite Sinica* **21**(5): 140-145.
- [51] Yew C.H., Kendrick R.B.,1987, A study of damage in composite panels produced by hypervelocity impact, *International Journal of Impact Engineering* **5**: 729-738.
- [52] Lamontagne C.G., Manuelpillai G.N., Taylor E.A., Tennyson R.C., 1999, Normal and oblique hypervelocity impacts on carbon fibre/PEEK composites, *International Journal of Impact Engineering* **23**(1): 519-532.
- [53] Christiansen E.L., 1990, Investigation of hypervelocity impact damage to space station truss tubes, *International Journal of Impact Engineering* **10**: 125-133.
- [54] Shortliffe G.D., Tennyson R.C., 1997, Hypervelocity impact tests on composite boom structures for space robot applications, *Canadian Aeronautics and Space Journal* **43**(3): 195-202.

- [55] Taylor E.A., Herbert M.K., Gardner D.J., Thomson R., Burchell M.J., 1998, Hypervelocity impact on spacecraft carbon fibre-reinforced plastic/aluminum honeycomb, *Journal of Aerospace Engineering* **221**: 355-366.
- [56] Vaidya U.K., Nelson S., Sinn B., Mathew B., 2001, Processing and high strain rate impact response of multi-functional sandwich composites, *Composite Structures* **52**: 429-440.
- [57] HICAS: High Velocity Impact of Composite Aircraft Structures, 1998-2000, CEC DG XII BRITE-EURAM Project BE 96-4238.
- [58] Jiang F.C., Vecchio K.S., 2009, Hopkinson bar loaded fracture experimental technique: a critical review of dynamic fracture toughness tests, *Applied Mechanics Review* **62**(6): 060802-060839.
- [59] Elder D.J., Thomson R.S., Nguyen M.Q., Scott M.L., 2004, Review of delamination predictive methods for low speed impact of composite laminated composites, *Composite Structures* **66**: 677-683.
- [60] Rybichi E.F., Kanninen M.F., 1977, A finite element calculation of stress intensity factors by a modified crack closure integral, *Engineering Fracture Mechanics* **9**: 931-938.
- [61] Raju I.S., Shivakumar K.N., 1988, Three-dimensional elastic analysis of a composite double cantilever beam specimen, *AIAA Journal* **26**(12): 1493-1498.
- [62] Dugdale D.S., 1960, Yielding of steel sheets containing slits, *Journal of the Mechanics and Physics of Solids* 8: 100-104
- [63] Barenblatt G.I., 1962, The mathematical theory of equilibrium cracks in brittle fracture, *Advances in Applied Mechanics* **7**: 55-129.
- [64] Murthy P.L.N., Chamis C.C., 1986, *Integrated Composite Analyzer (ICAN): Users and Programmers Manual*, NASA Technical Paper 2515.
- [65] Allen D.H., Groves S.E., Harris C.E., 1988, A cumulative damage model for continuous fiber composite laminated composites with matrix cracking and interply delamination, *ASTM STP* **972**: 57-80.
- [66] Krueger R., 2004, Virtual crack closure technique: history, approach, and application, *Applied Mechanics Review* **57**(2): 109-143.
- [67] Irwin G.R.,1958, *Fracture I, Handbuch der Physik VI*, edited by S. Flügge, Springer Verlag, Berlin, Germany, 558-590.
- [68] Krueger R., Minguet P.J., 2005, Skin-Stiffener Debond Prediction Based on Computational Fracture Analysis, NASA/CR-2005-213915.
- [69] ABAQUS Analysis User's Manual Version 6.5, Volume I, 2005, ABAQUS Inc.
- [70] Mabson G., Doper B., Deobald L., 2004, *User Manual for Fracture and Traction Interface Elements* Version 1.3, The Boeing Company.
- [71] Williams M.L., 1963, *The Fracture of Visco-Elastic Material*, Wiley Interscience Publishers, Drucker.
- [72] Schapery R.A., 1975, A theory of crack initiation and growth in visco-elastic media, *International Journal of Fracture* **11**(1): 141-159.
- [73] Hillerborg A., Modeer M., Petersson P.E., 1976, Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements, *Cement Concrete Research* **6**: 773-782.
- [74] Unguwarungasri T., Knauss W.G., 1987, The role of damage-softened material behavior in the fracture of composites and adhesive, *International Journal of Fracture* **35**:221-241.
- [75] Needleman A., 1987, A continuum model for void nucleation by inclusion debonding, *Journal of Applied Mechanics* **54**: 525-531.
- [76] Needleman A., 1990, An analysis of decohesion along an imperfect interface, *International Journal of Fracture* **42**: 21-40.
- [77] Shahwan K.L., Wass A. M., 1997, Non-self-similar decohesion along a finite interface of unilaterally constrained delaminations, in: *Proceeding of the Royal Society of London* **453**: 515-550.
- [78] Ortiz M., Pandolfi A., 1999, Finite-deformation irreversible cohesive elements for three-dimensional crack-propagation analysis, *International Journal for Numerical Methods in Engineering* **44**(9): 1267-1282.
- [79] Yu C., 2001, Three Dimensional Cohesive Modeling of Impact Damage of Composites, PhD Thesis, Pasadena, CA.
- [80] Corigliano A., Mariani S., Pandolfi A., 2006, Numerical analysis of rate-dependent dynamic composite delamination, *Composites Science and Technology* **66**: 766-775.
- [81] Reddy Jr.E.D., Mello F.J., Guess T.R., 1997, Modeling the initiation and growth of delaminations in composite structures, *Journal of Composite Materials* **31**: 812-831.
- [82] Petrossian Z., Wisnom M.R., 1998, Prediction of delamination initiation and growth from discontinuous plies using interface elements, *Composites: Part A* **29**: 503-515.
- [83] Mi Y., Crisfield M.A., Davies G.A.O., Hellweg H.B., 1998, Progressive delamination using interface elements, *Journal* of Composite Materials **32**: 1246-1273.
- [84] Turon A., Camanho P.P., Costa J., Davila C.G., 2006, A damage model for the simulation of delamination in advanced composites under variable-mode loading, *Mechanics of Materials* **38**(11): 1072-1089.
- [85] Rice J.R., 1968, A path independent integral and the approximate analysis of strain concentration by notches and cracks, *Journal of Applied Mechanics* **31**: 379-386.
- [86] Crisfield M.A., Hellweg H.B., Davies G.A.O., 1997, Failure analysis of composite structures using interface elements, in: *Proceedings of the NAFEMS Conference on Application of Finite Elements to Composite Materials*, London, U.K.
- [87] Camanho P.P., Dávila C.G., Ambur D.R., 2001, Numerical Simulation of Delamination Growth in Composite Materials, NASA TP-2001-211041.

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- [88] Pandey A.K., Reddy J.N., 1987, A post first-ply failure analysis of composite laminated composites, in: *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 28th Structures, Structural Dynamics, and Materials Conference*, 788-797.
- [89] Ochoa O.O., Engblom J.J., 1987, Analysis of failure in composites, *Composites Science and Technology* 28: 87-102.
- [90] Chang F.K., Chang K.Y., 1987, A progressive damage model for laminated composites containing stress concentrations, *Journal of Composite Materials* **21**: 834-855.
- [91] Chang F.K., Chang K.Y., 1987, Post-failure analysis of bolted composite joints in tension or shear-out Mode failure, *Journal of Composite Materials* **21**: 809-833.
- [92] Chang F.K., Lessard L., 1989, Modeling compression failure in laminated composite plates containing an open hole, in: *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 30th Structures, Structural Dynamics and Materials Conference*, 979-988.
- [93] Reddy Y.S., Reddy J.N., 1992, Linear and non-linear failure analysis of composite laminated composites with transverse shear, *Composites Science and Technology* **44**: 227-255.
- [94] Reddy Y.S., Reddy J.N., 1993, Three-dimensional finite element progressive failure analysis of composite laminated composites under axial extension, *Journal of Composites Technology and Research* **15**(2): 73-87.
- [95] Engelstad S.P., Reddy J.N., Knight N.F., 1992, Postbuckling response and failure prediction of graphite-epoxy plates loaded in compression, *AIAA Journal* **30**(8):2106-2113.
- [96] Coats T.W., 1996, A Progressive Damage Methodology for Residual Strength Predictions of Center-Crack Tension Composite Panels, PhD Dissertation, Old Dominion University.
- [97] Irvin F.B., Ginty C.A., 1986, Progressive fracture of fiber composites, *Journal of Composite Materials* 20: 166-184.
- [98] Huang D., Minnetyan L., 1998, Damage progression in carbon-fiber reinforced I-beams, ASCE Journal of Composites for Construction 2: 38-45.
- [99] Sleight D.W., 1999, *Progressive Failure Analysis Methodology for Laminated Composite Structures*, NASA/TP-209107.
- [100] Knight Jr.N.F., 2006, User-Defined Material Model for Progressive Failure Analysis, NASA/CR-214526.
- [101] Gotsis P.K., Chamis C.C., Minnetyan L., 1995, Effect of combined loads in the durability of a stiffened adhesively bonded composite structure, in: *Proceedings of the 36th AIAA/ASME/ASCE/AHS/ ASC Structures, Structural Dynamics, and Material Conference*, AIAA-95-1283-CP **2**: 1083-1092.
- [102] Gotsis P.K., Chamis C.C., David K., Abdi F., 2007, *Progressive Fracture of Laminated Composite Stiffened Plate*, NASA/TM-2007-214927.
- [103] Chamis C.C., Gotsis P.K., Minnetyan L., 1996, Damage progression in bolted composite structures, in: Proceedings of the 1995 USAF Structural Integrity Program Conference ASIP II: 663-679.
- [104] Coats T.W., Harris C.E, 1998, A Progressive Damage Methodology for Residual Strength Predictions of Notched Composite Panels, NASA TM-1998-207646.
- [105] Gotsis P.K., Chamis C.C., Minnetyan L., 1996, *Progressive Fracture of Fiber Composite Shell Structures Under Internal Pressure and Axial Loads*, NASA TM-07234.
- [106] Ochoa O., Reddy J.N., 1992, *Finite Element Analysis of Composite Lamaintes*, Kluwer Academic Publishers, Dordrecht, Netherlands.
- [107] Garnich M.R., Akula M.K., 2009, Review of degradation models for progressive failure analysis of fiber reinforced polymer composites, *Applied Mechanics Review* 62: 010801.
- [108] Nahas M.N., 1986, Survey of failure and post-failure theories of laminated fiber-reinforced composites, *Journal of Composites Technology and Research* **8**(4): 138-153.
- [109] Tsai S.W., 1984, A survey of macroscopic failure criteria for composite materials, *Journal of Reinforced Plastics and Composites* **3**: 40-63.
- [110] Icardi U., Locatto S., Longo A., 2007, Assessment of recent theories for predicting failures of composite laminated composites, *Applied Mechanics Review* **60**(2): 76-86.
- [111] Tsai S. W., 1965, Strength Characteristics of Composite Materials, NASA CR-224.
- [112] Hill R., 1948, A theory of the yielding and plastic flow of anisotropic metals, in: *Proceedings of the Royal Society of London*, Series A, **193**: 281-297.
- [113] Tsai S.W., Wu E.M., 1971, A general theory of strength for anisotropic materials, *Journal of Composite Materials* **5**: 58-80.
- [114] Hashin Z., 1980, Failure criteria for unidirectional fiber composites, *ASME Journal of Applied Mechanics* **47**(2): 329-334.
- [115] Hoffman O., 1967, The brittle strength of orthotropic materials, *Journal of Composite Materials* 1: 200-206.
- [116] Chamis C.C., 1969, Failure criteria for filamentary composites, *Composite Materials: Testing and Design*, ASTM STP **460**: 336-351.
- [117] Azzi V.D., Tsai S.W., 1965, Anisotropic Strength of Composites, *Experimental Mechanics*, 283-288.
- [118] Hashin Z., Rotem A., 1973, A fatigue failure criterion for fiber reinforced materials, *Journal of Composite Materials* 7(4): 448-464.
- [119] Christensen R.M., 1997, Stress based yield/failure criteria for fiber composites, *International Journal of Solids and Structures* **34**(5): 529-543.
- [120] Mayes J.S., Hansen A.C., 2001, Multicontinuum failure analysis of composite structural laminated composites, *Mechanics of Composite Materials and Structures* **8**(4): 249-262.

- [121] Murray Y., Schwer L., 1990, Implementation and verification of fiber-composite damage models, Failure Criteria and Analysis in Dynamic Response, *ASME AMD*, **107**: 21-30.
- [122] Lee J.D., 1982, Three dimensional finite element analysis of damage accumulation in composite laminated, *Computers and Structures* **15**(3): 335-350.
- [123] Engblom J.J., Ochoa O.O., 1986, Finite element formulation including interlaminar stress calculations, *Computers and Structures* **23**(2): 241-249.
- [124] Hwang W.C., Sun C.T., 1989, Failure analysis of laminated composites by using iterative three-dimensional finite element method, *Computers and Structures* **33**(1): 41-47.
- [125] Kweon J.H., 2002, Crippling analysis of composite stringers based on complete unloading method, *Computers and Structures* **80**(27–30): 2167-2175.
- [126] Huang Z.M., 2004, A bridging model prediction of the ultimate strength of composite laminated composites subjected to biaxial loads, *Composite Science and Technology* **64**(3–4): 395-448.
- [127] Sandhu R.S., 1974, Nonlinear behavior of unidirectional and angle ply laminated composites, *AIAA Journal of Aircraft* **13**: 104-111.
- [128] Kwon Y.W., Berner J.M., 1994, Analysis of matrix damage evolution in laminated composite plates, *Engineering Fracture Mechanics* **48**(6): 811-817.
- [129] Reddy Y.S.N., Moorthy C.M.D., Reddy J.N., 1995, Non-linear progressive failure analysis of laminated composite plates, *International Journal of Non-Linear Mechanics* **30**(5): 629-649.
- [130] Puck A., Schürmann H., 1998, Failure analysis of FRP laminated composites by means of physically based phenomenological models, *Composite Science and Technology* **58**(7): 1045-1067.
- [131] Joo S.G., Hong C.S., 2000, Progressive failure analysis of composite laminated composites using 3-D finite element method, *Key Engineering Materials* **183-187**: 535-540.
- [132] Puck A., Schürmann H., 2002, Failure analysis of FRP laminated composites by means of physically based phenomenological models—Part B, *Composite Science and Technology* **62**(12–13): 1633-1662.
- [133] Voyiadjis G.Z., Kattan P.I., 2005, *Damage Mechanics*, CRC Press, New York.
- [134] Knops M., Bogle C., 2006, Gradual failure in fibre/polymer laminated composites, *Composite Science and Technology* **66**(5): 616-625.
- [135] Hahn H.T., Tsai S.W., 1983, On the behavior of composite laminated composites after initial failures, *Astronautics and Aeronautics* **21**: 58-62.
- [136] Belytschko T., Black T., 1999, Elastic crack growth in finite elements with minimal remeshing, *International Journal* of Fracture Mechanics **45**: 601-620.
- [137] Moës N., Dolbow J., Belytschko T.,1999, A finite element method for crack growth without remeshing, *International Journal for Numerical Methods in Engineering* **46**: 131-150.
- [138] Dolbow J.E., 1999, *An Extended Finite Element Method with Discontinuous Enrichment for Applied Mechanics*, PhD dissertation, Theoretical and Applied Mechanics, Northwestern University, USA.