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Third Order Formulation for Vibrating Non-Homogeneous Cylindrical Shells in Elastic Medium

M. Gheisari[1,*], H. Molatefi[2], S.S. Ahmadi[3]

[1]*Faculty of Engineering, Khomein Branch, Islamic Azad University, Khomein, Iran*
[2]*Railway Engineering School, Iran University of Science and Technology, Narmak, Tehran, Iran*
[3]*Department of Mechanical Engineering, Iran University of Science and Technology, Arak Branch, Arak, Iran*

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ABSTRACT

Third order shear deformation theory of cylindrical shells is employed to investigate the vibration characteristics of non-homogeneous cylindrical shells surrounded by an elastic medium. The kinematic relations are obtained using the strain-displacement relations of Donnell shell theory. The shell properties are considered to be dependent on both position and thermal environment. A suitable function through the thickness direction is assumed for the non-homogeneity property. The Winkler-Pasternak elastic foundation is used to model the elastic medium. Analytical solutions are presented for cylindrical shells with simply supported boundary conditions. From the numerical studies, it is revealed that, the natural frequencies are affected significantly by the elastic foundation coefficients and environmental temperature conditions.

Keywords: Vibration; Non-homogeneous cylindrical shell; Elastic medium; Third order shear deformation theory; Donnell shell theory

1 INTRODUCTION

CYLINDRICAL shells have many applications in different engineering disciplines such as mechanical, civil, aerospace and marine engineering. Unexpected frequencies due to various loads might cause a serious damage for these types of structures. In many cases, these structures are located on a foundation. Cylindrical shells surrounded by an elastic medium can be found in many engineering applicants especially in man-made structural components. The non-homogeneity of materials can be created due to various problems such as production techniques, radiation effect, and thermal polishing processes. In recent years, interest on non-homogeneous shell structures has raised. However, a few studies have focused on the vibration behavior of non-homogeneous cylindrical shells on elastic foundation.

Paliwal and Bhalla [1] presented the non-linear static analysis of a cylindrical shell on a Pasternak foundation using the variational principles and Galerkin techniques. Ng and Lam [2] examined the effects of elastic foundation on the instability regions of the cylindrical shell for transverse, longitudinal, and circumferential modes. The large deflection analysis of axisymmetric shells and plates on a non-linear tensionless elastic foundation and the behaviour of shallow spherical shells subjected to a central concentrated load on tensionless linear elastic foundations are studied by Hong et al. [3].

[*] Corresponding author.
E-mail address: gheisari@iaukhomein.ac.ir (M. Gheisari).

Sheng and Wang [4] examined the effect of thermal load on vibration, buckling and dynamic stability of functionally graded (FG) cylindrical shells embedded in an elastic medium, based on the first-order shear deformation theory considering rotary inertia and the transverse shear strains. They formulated the elastic foundation of the Winkler type- that reacts in compression as well as in tension. Shen and his co-workers employed a singular perturbation technique associated with a higher order shear deformation theory to study the post-buckling response of a FG cylindrical shell in thermal environments surrounded by an elastic medium subjected to axial compression [5] and internal pressure [6]. In their analysis, the surrounding elastic medium is modeled as a tensionless Pasternak foundation that reacts in compression only. Sofiyev et al. [7] investigated the free vibration of non-homogenous truncated conical shells on a Winkler foundation. They investigated the effects of the variation of truncated conical shell characteristics, non-homogeneity and a Winkler foundation on lowest values of the dimensionless frequency parameter.

To the author's knowledge there is no analytical solution for non-homogeneous cylindrical shells on elastic foundation based on higher order shear deformation theories. In this paper, natural frequencies of cylindrical shells with non-homogeneous properties across the thickness direction are studied using the third order shear deformation theory of cylindrical shells. The Donnell shell theory assumptions are used to determine of strain-displacement relations. The shell properties are dependent on both position and thermal environments. The Winkler-Pasternak mathematical model is employed to simulate the reaction of elastic medium on shell. Analytical solutions are presented for cylindrical shells with simply supported ends. The shear elastic foundation modulus and environmental temperature conditions have played significant roles on the lowest frequencies of shell in elastic medium.

2 METHOD

The cylindrical coordinate system (x,θ,z) is set on the middle surface of the shell (z=0). Fig. 1 shows a schematic view of the problem studied. The influence of Poisson's ratio on the deformation is much less than of elasticity modulus and mass density. Thus, the Poisson's ratio ν_0 is assumed to be constant. The elasticity modulus E and mass density ρ of non-homogeneous cylindrical shell are assumed to vary with respect to thickness coordinate according to the following functions [7];

$$\begin{aligned} E &= E_0(1+\mu\phi(z)) \\ \rho &= \rho_0(1+\mu\phi(z)) \end{aligned} \tag{1}$$

where E_0 and ρ_0 denote properties of homogeneous shell, μ is the non-homogeneity parameter satisfying $0 \le \mu < 1$, and $\phi(z)$ is a continuous function of the variation of material properties. The homogeneous elasticity modulus of shell can be expressed as a function of temperature form Reddy and Chin [8];

$$E_0 = 2.0104\times10^{11}(1+3.079\times10^{-4}T-6.534\times10^{-7}T^2) \tag{2}$$

where T is the environment temperature in Kelvin. The Poisson' ratio and mass density remain constant by increasing the temperature conditions. The Donnell shell theory is employed to obtain the basic relations. Based on this theory, the displacement field for the third order shear deformation is expressed as [9];

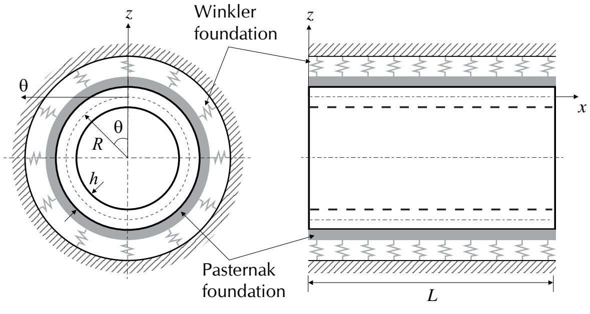

Fig. 1
Cylindrical shell surrounded by an elastic medium

$$u = u_0(x,\theta,t) + zu_1(x,\theta,t) - z^3\alpha(u_1(x,\theta,t) + w_{0,x})$$
$$v = v_0(x,\theta,t) + zv_1(x,\theta,t) - z^3\alpha\left(v_1(x,\theta,t) + \frac{1}{R}w_{0,\theta}\right) \tag{3}$$
$$w = w_0(x,\theta,t)$$

where $\alpha = 4/(3h^2)$ and u_0, v_0 and w_0 are the displacements of a point on the mid-surface of the cylindrical shell along the x-, θ- and z-axes, respectively and u_1 and v_1 describe the rotations about the θ- and x-axes, respectively. The Donnell non-linear strains are defined by the following relations [9]:

$$\varepsilon_x^0 = u_{0,x}, \qquad \varepsilon_x^1 = u_{1,x}, \qquad \varepsilon_x^2 = -\alpha(u_{1,x} + w_{0,xx})$$
$$\varepsilon_\theta^0 = \frac{v_{0,\theta} + w_0}{R}, \qquad \varepsilon_\theta^1 = \frac{v_{1,\theta}}{R}, \qquad \varepsilon_\theta^2 = -\frac{\alpha}{R}(v_{1,\theta} + \frac{w_{0,\theta\theta}}{R})$$
$$\gamma_{x\theta}^0 = \frac{u_{0,\theta}}{R} + v_{0,x}, \qquad \gamma_{x\theta}^1 = \frac{u_{1,\theta}}{R} + v_{1,x}, \qquad \gamma_{x\theta}^2 = -\alpha\left(\frac{u_{1,\theta}}{R} + v_{1,x} + 2\frac{w_{0,x\theta}}{R}\right) \tag{4}$$
$$\gamma_{xz}^0 = u_1 + w_{0,x}, \qquad \gamma_{xz}^1 = -3\alpha(u_1 + w_{0,x})$$
$$\gamma_{\theta z}^0 = v_1 + \frac{w_{0,\theta}}{R}, \qquad \gamma_{\theta z}^1 = -3\alpha\left(v_1 + \frac{w_{0,\theta}}{R}\right)$$

The force and moment resultants per unit length can be derived by integrating the stress components.

$$N_x = A_{11}\varepsilon_x^0 + A_{12}\varepsilon_\theta^0 + B_{11}\varepsilon_x^1 + B_{12}\varepsilon_\theta^1 + E_{11}\varepsilon_x^2 + E_{12}\varepsilon_\theta^2$$
$$N_\theta = A_{12}\varepsilon_x^0 + A_{11}\varepsilon_\theta^0 + B_{12}\varepsilon_x^1 + B_{11}\varepsilon_\theta^1 + E_{12}\varepsilon_x^2 + E_{11}\varepsilon_\theta^2$$
$$M_x = B_{11}\varepsilon_x^0 + B_{12}\varepsilon_\theta^0 + D_{11}\varepsilon_x^1 + D_{12}\varepsilon_\theta^1 + F_{11}\varepsilon_x^2 + F_{12}\varepsilon_\theta^2$$
$$M_\theta = B_{12}\varepsilon_x^0 + B_{11}\varepsilon_\theta^0 + D_{12}\varepsilon_x^1 + D_{11}\varepsilon_\theta^1 + F_{12}\varepsilon_x^2 + F_{11}\varepsilon_\theta^2$$
$$P_x = E_{11}\varepsilon_x^0 + E_{12}\varepsilon_\theta^0 + F_{11}\varepsilon_x^1 + F_{12}\varepsilon_\theta^1 + H_{11}\varepsilon_x^2 + H_{12}\varepsilon_\theta^2$$
$$P_\theta = E_{12}\varepsilon_x^0 + E_{11}\varepsilon_\theta^0 + F_{12}\varepsilon_x^1 + F_{11}\varepsilon_\theta^1 + H_{12}\varepsilon_x^2 + H_{11}\varepsilon_\theta^2$$
$$N_{x\theta} = A_{22}\gamma_{x\theta}^0 + B_{22}\gamma_{x\theta}^1 + E_{22}\gamma_{x\theta}^2 \tag{5}$$
$$M_{x\theta} = B_{22}\gamma_{x\theta}^0 + D_{22}\gamma_{x\theta}^1 + F_{22}\gamma_{x\theta}^2$$
$$P_{x\theta} = E_{22}\gamma_{x\theta}^0 + F_{22}\gamma_{x\theta}^1 + H_{22}\gamma_{x\theta}^2$$
$$Q_x = A_{22}\gamma_{xz}^0 + D_{22}\gamma_{xz}^1$$
$$Q_\theta = A_{22}\gamma_{\theta z}^0 + D_{22}\gamma_{\theta z}^1$$
$$R_x = D_{22}\gamma_{xz}^0 + F_{22}\gamma_{xz}^1$$
$$R_\theta = D_{22}\gamma_{\theta z}^0 + F_{22}\gamma_{\theta z}^1$$

where

$$(A_{11}, B_{11}, D_{11}, E_{11}, F_{11}, H_{11}) = \frac{1}{1-\nu^2}\int_{-h/2}^{h/2} E(1, z, z^2, z^3, z^4, z^6)\ \mathrm{d}z$$
$$(A_{12}, B_{12}, D_{12}, E_{12}, F_{12}, H_{12}) = \frac{1}{\nu(1-\nu^2)}\int_{-h/2}^{h/2} E(1, z, z^2, z^3, z^4, z^6)\ \mathrm{d}z \tag{6}$$
$$(A_{22}, B_{22}, D_{22}, E_{22}, F_{22}, H_{22}) = \frac{1}{2(1+\nu)}\int_{-h/2}^{h/2} E(1, z, z^2, z^3, z^4, z^6)\ \mathrm{d}z$$

The surrounding elastic medium is modeled as Winkler-Pasternak type foundation as follows [10]

$$q(x,\theta)=k_w w-k_p\left(w_{,xx}+\frac{w_{,\theta\theta}}{R^2}\right) \tag{7}$$

where q, k_w, and k_p are the reaction of foundation, vertical spring modulus and shear modulus of foundation, respectively. For the case k_p=0, the foundation reduces to the Winkler-type. The governing equations of motion appropriate for the displacement field, Eq. (3), can be derived using the dynamic version of the principle of virtual displacement [9] as

$$\begin{aligned}
&N_{x,x}+\frac{N_{x\theta,\theta}}{R}=I_0\ddot{u}_0+\bar{I}_1\ddot{u}_1-\alpha I_3\ddot{w}_{0,x}\\
&\bar{M}_{x,x}+\frac{\bar{M}_{x\theta,\theta}}{R}-\bar{Q}_x=\bar{I}_1\ddot{u}_0+\bar{I}_2\ddot{u}_1-\alpha\bar{I}_4\ddot{w}_{0,x}\\
&N_{x\theta,x}+\frac{N_{\theta,\theta}}{R}=I_0\ddot{v}_0+\bar{I}_1\ddot{v}_1-\frac{\alpha I_3}{R}\ddot{w}_{0,\theta}\\
&\bar{M}_{x\theta,x}+\frac{\bar{M}_{\theta,\theta}}{R}-\bar{Q}_\theta=\bar{I}_1\ddot{v}_0+\bar{I}_2\ddot{v}_1-\frac{\alpha\bar{I}_4}{R}\ddot{w}_{0,\theta}\\
&\bar{Q}_{x,x}+\frac{\bar{Q}_{\theta,\theta}}{R}+\alpha\left(P_{x,xx}+2\frac{P_{x\theta,x\theta}}{R}+\frac{P_{\theta,\theta\theta}}{R^2}\right)-\frac{N_\theta}{R}+q(x,\theta)=\\
&\qquad I_0\ddot{w}_0+\alpha I_3\left(\ddot{u}_{0,x}+\frac{\ddot{v}_{0,\theta}}{R}\right)+\alpha\bar{I}_4\left(\ddot{u}_{1,x}+\frac{\ddot{v}_{1,\theta}}{R}\right)-\alpha^2 I_6\left(\ddot{w}_{0,xx}+\frac{\ddot{w}_{0,\theta\theta}}{R}\right)
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
&(I_0,I_1,I_2,I_3,I_4,I_6)=\int_{-h/2}^{h/2}\rho(1,z,z^2,z^3,z^4,z^6)\,dz\\
&(\bar{M}_x,\bar{M}_\theta,\bar{M}_{x\theta})=(M_x,M_\theta,M_{x\theta})-\alpha(P_x,P_\theta,P_{x\theta})\\
&(\bar{Q}_x,\bar{Q}_\theta)=(Q_x,Q_\theta)-3\alpha(R_x,R_\theta)\\
&(\bar{I}_1,\bar{I}_4)=(I_1,I_4)-\alpha(I_3,I_6)\\
&\bar{I}_2=I_2-2\alpha I_4+\alpha^2 I_6
\end{aligned} \tag{9}$$

Solution of equations of motion is expressed as products of undetermined functions and known trigonometric functions so as to satisfy identically the simply supported boundary conditions at ends [9]

$$\begin{aligned}
&u_0=\sum_m\sum_n U_{mn}\cos\left(\frac{m\pi}{L}x\right)\cos(n\theta)\cos\omega t\\
&v_0=\sum_m\sum_n V_{mn}\sin\left(\frac{m\pi}{L}x\right)\sin(n\theta)\cos\omega t\\
&w_0=\sum_m\sum_n W_{mn}\sin\left(\frac{m\pi}{L}x\right)\cos(n\theta)\cos\omega t\\
&u_1=\sum_m\sum_n X_{mn}\cos\left(\frac{m\pi}{L}x\right)\cos(n\theta)\cos\omega t\\
&v_1=\sum_m\sum_n Y_{mn}\sin\left(\frac{m\pi}{L}x\right)\sin(n\theta)\cos\omega t
\end{aligned} \tag{10}$$

where ω denotes the natural frequency, m and n are the axial and circumferential wave numbers and U_{mn}, V_{mn}, W_{mn}, X_{mn}, and Y_{mn} are unknown coefficients. Substitution of relations (10) into Eqs. (8) obtained in terms of displacement components, leads to an equation which is called frequency equation. The lowest root of that equation is the natural frequency.

3 NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results are discussed in details. As mentioned before, natural frequencies are obtained for cylindrical shells with non-homogeneous properties and simply supported boundary conditions on Winkler-Pasternak elastic foundation. The stainless steel (SUS304) is used as homogeneous material with ρ_0=8166 kg m^{-3} and ν=0.317756 [8]. The elasticity modulus is dependent on thermal environment as indicated in Eq. (2).

To investigate the accuracy of the present analysis, comparison studies are presented. Table 1 shows the dimensionless frequencies for a homogeneous isotropic cylindrical shell on Winkler-type foundation. The dimensionless frequency is considered as $\Omega = \omega R\sqrt{(1-\nu_0^2)\rho_0 / E_0}$. The frequencies are obtained for values R/h=100, L/R=2, and k_w=10^{-4} N m^{-3}. The results of references [10] and [7] are based on classical shell theory and reference [11] is based on first order shear deformation theory. This is the main reason to satisfy the difference between the results of references [10] and [7]. A quadratic function is considered for variations of non-homogeneous shells across the thickness which is given by $\phi(z)=(z/h)^2$ [7]. This function is plotted in Fig 2.

Three thermal environmental conditions are considered. The natural frequencies (Hz) of non-homogeneous cylindrical shells are given in Tables 2-4. for three different temperature changes. Results are calculated for R/h=100, L/R=5. It is found that for room temperature (T=300K) the values of natural frequencies for non-homogeneous cylindrical shell in elastic medium are slightly lower than isotropic cylindrical shells. However, as listed in Table 2. for high temperatures the difference becomes more.

Table 1
Dimensionless frequency parameter for a cylindrical shell resting on a Winkler foundation

n	Ref. [10]	Ref. [7]	Ref. [11]	Present
1	0.67882	0.67921	0.58306	0.56828
2	0.36394	0.36463	0.34931	0.32517
3	0.20526	0.20804	0.23491	0.19419
4	0.12745	0.13824	0.19019	0.13059

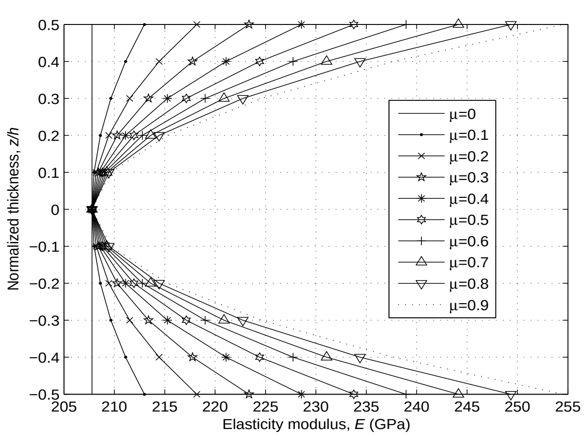

Fig. 2
Quadratic variations of elasticity modulus for non-homogeneous shell.

Table 2
Natural frequencies (Hz) of non-homogeneous cylindrical shells in an elastic medium for three environmental temperature conditions (k_w=k_p=1×10^6 N m^{-3})

μ	T=300 K	T=500 K	T=900 K
0	10235.027	10036.417	8830.661
0.1	10231.841	10033.169	8826.974
0.2	10228.707	10029.973	8823.345
0.3	10225.622	10026.827	8819.774
0.4	10222.586	10023.731	8816.259
0.5	10219.598	10020.684	8812.798
0.6	10216.656	10017.685	8809.391
0.7	10213.760	10014.731	8806.036
0.8	10210.907	10011.822	8802.732
0.9	10208.099	10008.958	8799.478

Table 3
Effects of shear modulus of foundation on natural frequencies (Hz) of non-homogeneous cylindrical shells in thermal environments (k_w= 1×10^6 N m^{-3})

μ	k_p	T=300 K	T=500 K	T=700 K
0	1×10^4	9847.294	9640.743	9166.315
	1×10^5	9883.249	9677.466	9204.930
	1×10^7	13183.628	13026.839	12671.238
0.3	1×10^4	9847.143	9640.588	9166.151
	1×10^5	9882.224	9676.418	9203.826
	1×10^7	13114.005	12956.531	12599.358
0.5	1×10^4	9847.046	9640.489	9166.045
	1×10^5	9881.567	9675.747	9203.120
	1×10^7	13069.205	12911.283	12553.083
0.7	1×10^4	9846.952	9640.392	9165.943
	1×10^5	9880.931	9675.097	9202.436
	1×10^7	13025.634	12867.272	12508.061

Table 4
Effects of vertical modulus of foundation on natural frequencies (Hz) of non-homogeneous cylindrical shells in thermal environments (k_p=1×10^6 N m^{-3}, T=500 K, μ=0.5)

m	n	k_w=1×10^4	k_w=1×10^5	k_w=1×10^7
1	1	10018.016	10018.256	10044.903
	2	7497.056	7497.607	7558.048
2	1	24076.484	24076.607	24090.039
	2	14349.184	14349.472	14381.173

It can be concluded that for quadratic distribution of material properties, the non-homogeneous cylindrical shells in elastic medium vibrate earlier than isotropic cylindrical shells. The frequencies are also decreased for higher thermal environmental conditions. It should be interesting to note that this increasing becomes more as the temperature increases. Tables 3 and 4 shows the effect of vertical and shear elastic foundation moduli on natural frequencies of non-homogeneous cylindrical shells. By increasing the values of foundation coefficients k_w and k_p, the natural frequencies are slightly increased. This increase is more visible for higher values of elastic foundation coefficients.

4 CONCLUSIONS

Third order shear deformation theory is developed to obtain the lowest natural frequencies of non-homogeneous cylindrical shells embedded in an elastic medium. The numerical results are presented for different thermal environmental an elastic foundation conditions. The results demonstrate that the temperature-dependency of shell property has a significant effect on the vibration behavior for non-homogeneous cylindrical shells in elastic medium, especially for shells in high thermal environment.

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