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Analysis of Laminated Soft Core Sandwich Plate Having Interfacial Imperfections by an Efficient C^0 FE Model

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ABSTRACT

An efficient C^0 continuous two dimensional (2D) finite element (FE) model is developed based on a refined higher order shear deformation theory (RHSDT) for the static analysis of soft core sandwich plate having imperfections at the layer interfaces. In this (RHSDT) theory, the in-plane displacement field for the face sheets and the core is obtained by superposing a globally varying cubic displacement field on a zig-zag linearly varying displacement field. The transverse displacement is assumed to have a quadratic variation within the core and it remains constant in the faces beyond the core. In this theory, the interfacial imperfection is represented by a liner spring-layer model. The proposed model satisfies the condition of transverse shear stress continuity at the layer interfaces and the zero transverse shear stress condition at the top and bottom of the sandwich plate. The nodal field variables are chosen in an efficient manner to circumvent the problem of C^1 continuity requirement of the transverse displacements associated with the RHSDT. The proposed model is implemented to analyze the laminated composites and sandwich plates having interfacial imperfection. Many new results are also presented which should be useful for the future research.

Keywords: Composites; Finite element methods; Interfacial imperfection; Sandwich plate; Soft core

1 INTRODUCTION

SANDWICH construction is a special type of laminated structure having low strength thicker core and high strength face sheets in the form of composite laminates. These structures are weak in shear due to their low shear modulus as compared to extensional rigidity as well as large variation of material properties between the core and face layers. Thus the effect of shear deformation is quite significant which may lead to failure.

The structural behavior of sandwich plates can not be predicted by a simple plate theories i.e. single layer plate theories [1-4], as the core and face sheets deform in different manners due to large variation of their material properties. In addition to that, the problem becomes much more complex if some inter-laminar imperfection is there in the form of weak bonding or something else. In this context , a number of plate theories have been developed for accurately modelling the shear deformation in laminated sandwich structures in a refined manner.

The typical feature of sandwich plate is that the variation of in-plane displacements across the thickness shows kinks at the interface between the core and stiff face layers, which gives discontinuity of transverse shear strains at these interfaces. This typical behavior is also detected in laminated composite plate due to its layered configuration but the order of discontinuity is not as prominent as in case of sandwich plate. Actually, it depends on the order of ifference in the values of transverse shear rigidity and thickness of adjacent layers, which is quite significant in the

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case of sandwich plate as compared to the composite laminates. However, the effect of this discontinuity cannot be ignored in a multi layer thick laminates.

Considering this aspect in view and to model some other features of thick laminate in a better manner, a number of layer wise plate theories [5-9] have been developed. In these theories, the unknown displacement components are taken at all the layer interfaces including top and bottom surfaces of the plate. Using these unknowns at the different layers, the displacement components at any intermediate level are interpolated with the help of piecewise linear or other functions. It gives a zigzag through the thickness variation of in-plane displacement, which represents the desired shear strain discontinuity at the layer interfaces. The performance of these plate theories is good but they require huge computational involvement, as the number of nodal unknowns is directly proportional to the number of layers.

The above problem of layer wise theories has been overcome by Di Sciuva [10], Murakami [11], Liu and Li [12] and some other researchers where the unknowns at the different interfaces are expressed in terms of those at a particular plane defined as reference plane. This can be achieved by satisfying transverse shear stress continuity at the layer interfaces. These plate theories may be identified as refined first order shear deformation theory (RFSDT) where the unknowns are similar to those of the first order shear deformation theory (FSDT) of Yang et.al. [3].

A further improvement on RFSDT is due to Bhaskar and Varadan [13], Di Sciuva [14], Lee and Liu [15] and Cho and Parmerter [16] who have combined the concepts of RFSDT and the higher order shear deformation theory (HSDT) of Reddy [4]. These plate theories may be defined as refined higher order shear deformation theory (RHSDT). It gives parabolic through the thickness variation of transverse shear strains with discontinuity at the layer interfaces as desired in a layered composite plate. Moreover, it satisfies the transverse shear stress free condition at the top and bottom surfaces of the plate. Thus RHSDT has all the merits required for an efficient modeling of sandwich plates. Though the basic features of all these plate theories are more or less the same but there are some refinements of one over the other [13-16]. In this group, the plate theory of Cho and Parmerter [16] seems to be more effective and possesses all the merits of the above mentioned theories. A lot of discussion on these plate theories addressing zig-zag and interlaminar shear stress continuity has been given by Carrera [17].

Therefore, these plate theories are more suitable for the analysis of sandwich plates as the discontinuity in transverse shear strain at the layer interfaces is more prominent in comparison to laminated plates. In addition to transverse shear strain jump at the core face sheet interfaces, transverse normal strain within the core becomes very much significant in some situations where the plate thickness/span ratio becomes quite high and core material is transversely flexible or soft (low transverse rigidity). The transverse flexibility of low-strength core seriously affects the overall response of the sandwich structure and also crucial for sandwich panels which are subjected to localized loads. Frostig [18] has presented the classical and the higher order computational models of unidirectional sandwich panels with incompressible and compressible cores to demonstrate the differences in overall response of the panels as well as in the vicinity of the localized loads and supports. Givil et al. [19] has presented the dynamic model based on higher order sandwich panel theory to study the behavior of soft core sandwich panel under dynamic loading. Therefore, more attention must be given for the accurate modeling of the variation of transverse deflection across the depth of a sandwich structure having soft core. Brischetto et al. [20] showed the effect of zig-zag function used to build higher order plate theories for the analysis of unsymmetrically laminated sandwich structures and highly recommended the use of zig-zag functions for analysis of softy-core sandwich structures.

The conventional modeling techniques adopted by different researchers [21-24] are not enough to accurately predict the behavior of soft core sandwich structures. A considerable amount of literature is available on the static analysis of the sandwich plate without taking into account the effect of transverse normal deformation for the core [25-29]. However, a limited work has been done on the analysis of the sandwich plate considering the effect of transverse normal deformation for the core [30-37]. Carrera and Ciuffreda [38] studied the response of composite and sandwich plates subjected to localized distribution of transverse pressure and to point loadings and concluded that an accurate description of transverse normal strain effect plays a fundamental role in order to capture the effect of localized bending. Carrera and Brischetto [39] has done the numerical assessment of various plate theories (e.g. classical, higher order, zig-zag, layerwise and mixed theories) for bending and vibration analysis of sandwich flat panels and concluded that classical and first order theory can not be used for the accurate analysis of sandwich structures.

Thus, the effect of transverse normal deformation of the core should be taken into consideration in order to accurately predict the behavior of soft core sandwich plate. Hence, it is required to introduce unknown transverse displacement fields across the depth in addition to that in the reference plane, to represent the variation of transverse deflection in a laminated sandwich structure. On the other hand, introduction of additional unknowns in the transverse displacement fields invites additional C^1 continuity requirements in its finite element implementation by using the above mentioned refined higher order shear deformation theory. Moreover, the application of a C^1

continuous finite element is not encouraged in a practical analysis. Recently Pandit et al. [40-41] proposed a higher order zigzag theory for the analysis of sandwich plates with soft compressible core. To circumvent the above problem of C^1 continuity, they have used separate shape functions to define the derivatives of transverse displacements in order to develop a C^0 finite element model for the implementation of the proposed higher order shear deformation theory. However, it has imposed some constrains, which are enforced variationally through penalty approach. The selection of suitable value for the penalty stiffness multiplier is quite arbitrary and is a well known problem in the finite element method. Recently Mantari et al. [42] has developed a new shear deformation theory for the analysis of sandwich and composite plates. This theory accounts for adequate distribution of the transverse shear strains through-the-thickness and transverse shear stress free boundary conditions, thus it does not require the use of shear correction factor. The transverse displacement has taken to be constant through-the-thickness in this theory.

In all these refine higher order shear deformation theories, a perfect interface between the layers is taken, which is characterized by continuous displacements and tractions across the interfaces. But in case of imperfect interfaces, there should be jumps in the displacement components at the interfaces whereas traction would remain continuous from equilibrium point of view [44-45]. Di Sciuva and Gherlone [46, 47] have developed an analytical as well as a FE model based on third-order Hermitian zig-zag plate theory to study the damaged bonded interfaces. The simplest way to model this phenomenon is to use a linear spring layer model where the displacement jumps in a particular interface are proportional to the tractions at that interface. Such an attempt has been made by Cheng et al. [48], Di Sciuva [49], Chakrabarti and Sheikh [50] and few others where the linear spring layer model has been implemented in a plate model based on RHSDT. In all these studies it has been applied to ordinary composite laminates where the problem has been solved analytically.

Keeping all these aspects in view, an attempt has been made to study the behavior of soft core sandwich plates with inter-laminar imperfection of arbitrary variation at the different levels by modifying the FE model proposed by Khandelwal et al. [43] based on the RHSDT in combination with the linear spring model of Cheng et al. [48]. An efficient C^0 finite element model based on RHSDT has been proposed in this paper for the analysis of soft compressible core sandwich plate with inter-laminar imperfection. In this model, the in-plane displacement fields are assumed as a combination of a linear zigzag function with different slopes at each layer and a global cubically varying function over the entire thickness. The transverse displacement is considered to be quadratic within the core and constant in the face sheets. The proposed model satisfies the transverse shear stress continuity conditions at the layer interfaces and the zero transverse shear stress condition at the top and bottom of the plate. The isoparametric quadratic plate element has nine nodes with eleven field variables (i.e., in-plane displacements and transverse displacement at the reference mid surface, at the top and at the bottom of the plate along with rotational degrees of freedom at the reference mid surface and top and bottom of the plate) at each node. The displacement fields are chosen in an efficient manner that there is no need to impose any penalty stiffness in the formulation.

The proposed model is validated solving problems of sandwich plates with perfect interfaces and composite laminates with imperfect interfaces, as there are very few results available of laminated sandwich plates with inter-laminar imperfections. Finally the proposed model is applied to the actual problem where numerical results are generated by solving a number of problems to study the behavior of the present structure under different situations.

2 MATHEMATICAL FORMULATIONS

The in-plane displacement field (Fig. 1) is chosen as follows:

$$\bar{u} = u_0 + z\theta_x + \sum_{i=1}^{n_u-1}\left[\left(z-z_i^u\right)H\left(z-z_i^u\right)\alpha_{xu}^i + \Delta u_i\right] + \sum_{j=1}^{n_l-1}\left[\left(z-z_j^l\right)H\left(-z+z_j^l\right)\alpha_{xl}^j - \Delta u_j\right] + \beta_x z^2 + \eta_x z^3 \quad (1)$$

$$\bar{v} = v_0 + z\theta_y + \sum_{i=1}^{n_u-1}\left[\left(z-z_i^u\right)H\left(z-z_i^u\right)\alpha_{yu}^i + \Delta v_i\right] + \sum_{j=1}^{n_l-1}\left[\left(z-z_j^l\right)H\left(-z+z_j^l\right)\alpha_{yl}^j - \Delta v_j\right] + \beta_y z^2 + \eta_y z^3 \quad (2)$$

where u_0 and v_0 denotes the in-plane displacements of any point on the mid surface (i.e., u_0 along x-axis and v_0 along y-axis) of any point on the mid surface, θ_x and θ_y are the rotations of the normal to the middle plane about the x-axis and y-axis respectively, n_u and n_l are number of upper and lower layers respectively, β_x, β_y, η_x and η_y are the higher order unknown, $\alpha_{xu}^{i}, \alpha_{yu}^{i}, \alpha_{xl}^{j}$ and α_{yl}^{j} are the slopes of i-th / j-th layer corresponding to upper and lower layers respectively and $H(z - z_i^u)$ and $H(-z + z_j^l)$ are the unit step functions.

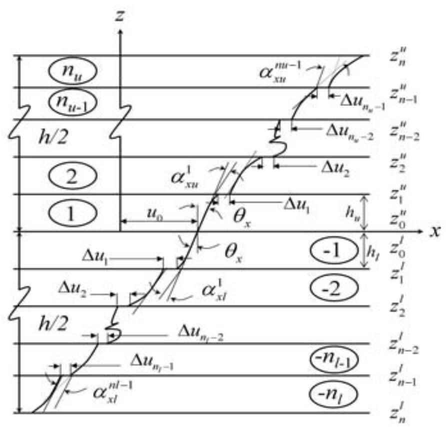

Fig. 1
General lamination lay-up and displacement configuration.

The transverse displacement is assumed to vary quadratically through the core thickness and constant over the face sheets (as shown in Fig. 2) and it may be expressed as:

$$\bar{w} = l_1 w_u + l_2 w_0 + l_3 w_l \quad \text{for core region} \tag{3a}$$

$$= w_u \quad \text{for upper face layers} \tag{3b}$$

$$= w_l \quad \text{for lower face layers} \tag{3c}$$

where w_u , w_0 and w_l are the values of the transverse displacement at the top layer, middle layer and bottom layer of the core, respectively, and l_1, l_2 and l_3 are Lagrangian interpolation functions in the thickness co-ordinate as defined below.

$$l_1 = \frac{z(z + h_l)}{h_u(h_u + h_u)} \;,\; l_2 = \frac{(h_l + z)(h_u - z)}{h_u h_l} \;,\; l_3 = \frac{z(h_u - z)}{-h_l(h_u + h_l)} \;. \tag{3d}$$

The stress-strain relationship of an orthotropic layer/ lamina (say *k*-th layer) having any fiber orientation with respect to structural axes system (*x-y-z*) may be expressed as:

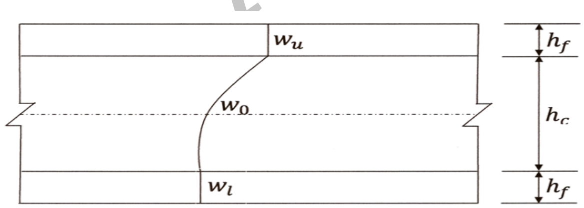

Fig. 2
Transverse displacement (*w*) variation through the thickness of sandwich plate.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & \overline{Q}_{14} & 0 & 0 \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{23} & \overline{Q}_{24} & 0 & 0 \\ \overline{Q}_{31} & \overline{Q}_{32} & \overline{Q}_{33} & \overline{Q}_{34} & 0 & 0 \\ \overline{Q}_{41} & \overline{Q}_{42} & \overline{Q}_{43} & \overline{Q}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{Q}_{55} & \overline{Q}_{56} \\ 0 & 0 & 0 & 0 & \overline{Q}_{65} & \overline{Q}_{66} \end{bmatrix}_K \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad or \quad \left\{\overline{\sigma}\right\} = \left[\overline{Q}_K\right]\left\{\overline{\varepsilon}\right\} \tag{4}$$

where $\left\{\overline{\sigma}\right\}$, $\left\{\overline{\varepsilon}\right\}$ and $\left[\overline{Q}_K\right]$ are the stress vector, the strain vector and the transformed rigidity matrix of *k*-th lamina, respectively.

The imperfection at the *k*-th interface is characterized by the displacement jumps Δu_k and Δv_k as in Eqs. (1)-(2) and Fig. 1, which may be expressed in terms of tractions at the *k*-th interface following the concept of linear spring-layer model of Cheng et al. [48] as follows:

$$\Delta u_k = R_{11}^k \tau_{xz}^k + R_{12}^k \tau_{yz}^k \tag{5}$$

and

$$\Delta v_k = R_{21}^k \tau_{xz}^k + R_{22}^k \tau_{yz}^k \tag{6}$$

where $R_{11}^k, R_{12}^k, R_{21}^k$ and R_{21}^k are the compliance coefficients Cheng et al. [48] of the idealized linear spring layer at the *k*-th interface while τ_{xz}^k and τ_{yz}^k are the transverse shear stresses on that interface. Taking an adjacent layer of *k*-th interface τ_{xz}^k and τ_{yz}^k can be expressed in terms of γ_{xz} and γ_{yz} at the corresponding layer with the help of Eq. (4). Again Eqs. (1)-(3) may be used to express $\gamma_{xz}(=\partial\overline{w}/\partial x+\partial\overline{u}/\partial z)$ and $\gamma_{yz}(=\partial\overline{w}/\partial y+\partial\overline{v}/\partial z)$ where Δu_k and Δv_k will not appear. This will help to express Δu_k and Δv_k in terms of other terms.

Utilizing the conditions of zero transverse shear stress at the top and bottom surfaces of the plate and imposing the conditions of the transverse shear stress continuity at the interfaces between the layers along with the conditions, $\theta_x = \theta_x^u$ and $\theta_y = \theta_y^u$ at the top and $\theta_x = \theta_x^l$ and $\theta_y = \theta_y^l$ at the bottom of the plate, $\beta_x, \eta_x, \beta_y, \eta_y, \alpha_{xu}^i$, $\alpha_{xl}^i, \alpha_{yu}^i, \alpha_{yl}^i$, $(\partial w_u/\partial x)$, $(\partial w_l/\partial x)$ $(\partial w_u/\partial y)$ and $(\partial w_l/\partial y)$ may be expressed in terms of the displacements $u_0, v_0, \theta_x, \theta_y$, θ_x^u, θ_y^u, θ_x^l and θ_y^l as:

$$\{B\} = [A]\{\alpha\} \tag{7}$$

where $\{B\} = \left\{\beta_x, \eta_x, \beta_y, \eta_y, \alpha_{xu}^1, \alpha_{xu}^2, \ldots \alpha_{xu}^{nu-1}, \alpha_{xl}^1, \alpha_{xl}^2, \ldots \alpha_{xl}^{nl-1}, \alpha_{yu}^1, \alpha_{yu}^2, \ldots \alpha_{yu}^{nu-1}, \alpha_{yl}^1, \alpha_{yl}^2, \ldots \alpha_{yl}^{nl-1} \left(\partial w_u/\partial x\right),\right.$ $\left.\left(\partial w_u/\partial y\right), \left(\partial w_l/\partial x\right), \left(\partial w_l/\partial y\right)\right\}^{\mathrm{T}}$, $\{\alpha\} = \left\{u_0, v_0, \theta_x, \theta_y, \theta_x^u, \theta_y^u, \theta_x^l, \theta_y^l\right\}^{\mathrm{T}}$,and the elements of [*A*] are dependent on material properties. It is to be noted that last four entries of the vector {*B*} helps to define the derivatives of transverse displacement at the top and bottom faces of the plate in terms of the displacements $u_0, v_0, \theta_x, \theta_y$, θ_x^u, θ_y^u, θ_x^l and θ_y^l to overcome the problem of C^1 continuity as mentioned before [43].

Using the above equations, the in-plane displacement fields as given in Eqs. (1-2) may be expressed as:

$$\overline{u} = b_1 u_0 + b_2 v_0 + b_3 \theta_x + b_4 \theta_y + b_5 \theta_x^u + b_6 \theta_y^u + b_7 \theta_x^l + b_8 \theta_y^l \tag{8}$$

$$\bar{v} = c_1 u_0 + c_2 v_0 + c_3 \theta_x + c_4 \theta_y + c_5 \theta_x^u + c_6 \theta_y^u + c_7 \theta_x^l + c_8 \theta_y^l \tag{9}$$

where the coefficients b_i's and c_i's are function of thickness coordinates, unit step functions and material properties. The generalized displacement vector { δ } for the present plate model can now be written with the help of Eqs. (3), (8) and (9) as:

$$\{\delta\} = \left\{u_0, v_0, w_0, \theta_x, \theta_y, \theta_x^u, \theta_y^u, w_u, \theta_x^l, \theta_y^l, w_l\right\}^{\mathrm{T}} \tag{10}$$

Using linear strain-displacement relation and Eqs.(1)-(5), the strain field may be expressed in terms of unknowns (for the structural deformation) as:

$$\{\bar{\varepsilon}\} = \left[\frac{\partial \bar{u}}{\partial x} \;\; \frac{\partial \bar{v}}{\partial y} \;\; \frac{\partial \bar{w}}{\partial z} \;\; \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \;\; \frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x} \;\; \frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial x}\right] \quad or \quad \{\bar{\varepsilon}\} = [H_0]\{\varepsilon\} \tag{11a}$$

where

$$\begin{aligned}\{\varepsilon\} = [&u_0, v_0, w_0, \theta_x, \theta_y, u_u, v_u, w_u, u_l, v_l, w_l, (\partial u_0/\partial x), (\partial u_0/\partial y), (\partial v_0/\partial x), (\partial v_0/\partial y), (\partial w_0/\partial x),\\ &(\partial w_0/\partial y), (\partial \theta_x/\partial x), (\partial \theta_x/\partial y), (\partial \theta_y/\partial x), (\partial \theta_y/\partial y), (\partial u_u/\partial x), (\partial u_u/\partial y), (\partial v_u/\partial x), (\partial v_u/\partial y),\\ &(\partial w_u/\partial x), (\partial w_u/\partial y), (\partial u_l/\partial x), (\partial u_l/\partial y), (\partial v_l/\partial x), (\partial v_l/\partial y), (\partial w_l/\partial x), (\partial w_l/\partial y)]\end{aligned} \tag{11b}$$

and the elements of $[H_0]$ having an order of (6×33) contains z and some constant quantities found above.

With the quantities found in the above equations, the total potential energy of the system under the action of transverse load may be expressed as:

$$\Pi_e = U_s - W_{ext} \tag{12}$$

where U_s is the strain energy and W_{ext} is the energy due to the external transverse static load.

Using Eqs. (11), the strain energy (U_s) is given by:

$$U_s = \frac{1}{2}\sum_{k=1}^{n}\iiint \{\bar{\varepsilon}\}^T [\bar{Q}_k]\{\bar{\varepsilon}\} dxdydz = \frac{1}{2}\iint \{\varepsilon\}^T [D]\{\varepsilon\} dxdy \tag{13}$$

where

$$[D] = \sum_{k=1}^{n}\int [H_0]^T [\bar{Q}_k][H_0] dz \tag{14}$$

and the energy due to externally applied distributed transverse static load of intensity $q(x,y)$ can be calculated as:

$$W_{ext.} = \iint wq dxdy \tag{15}$$

In the present problem, a nine-node quadratic element with eleven field variables ($u_0, v_0, w_0, \theta_x, \theta_y$, u_u, v_u, w_u, u_l, v_l and w_l) per node is employed. Using finite element method ,the generalized displacement vector { δ } at any point may be expressed as:

$$\{\delta\} = \sum_{i=1}^{n} N_i \delta_i \tag{16}$$

where, $\{\delta\} = \left\{u_0, v_0, w_0, \theta_x, \theta_y, \theta_x^u, \theta_y^u, w_u, \theta_x^l, \theta_y^l, w_l\right\}^{\mathrm{T}}$ as defined earlier, δ_i is the displacement vector corresponding to node *i*, N_i is the shape function associated with the node *i* and *N* is the number of nodes per element, which is nine in the present study.

With the help of Eq. (16), the strain vector { ε } that appeared in Eq. (11) may be expressed in terms of unknowns (for the structural deformation) as:

$$\{\varepsilon\} = [B]\{\delta\} \tag{17}$$

where [*B*] is the strain-displacement matrix in the Cartesian coordinate system.

The elemental potential energy as given in Eq. (12) may be rewritten with the help of Eqs. (13)-(17) as:

$$\Pi_{\mathrm{e}} = \frac{1}{2}\iint \{\delta\}^T [B]^T [D][B]\{\delta\}\, dxdy - \frac{1}{2}\iint \{\delta\}^T [B]^T \left[N^w\right]^T q dxdy \tag{18a}$$

$$= \frac{1}{2}\{\delta\}^T [K_e]\{\delta\} - \frac{1}{2}\{\delta\}^T \{P_e\} \tag{18b}$$

where

$$[K_e] = \int [B]^T [D][B]\, dxdy \tag{19}$$

$$\{P_e\} = \iint [N^w]^T q dxdy \tag{20}$$

where, $[N^w]$ is the shape function like matrix with non-zero terms associated only with the corresponding transverse nodal displacements.

The equilibrium equation can be obtained by minimizing Π_{e} as given in Eq. (18) with respect to { δ } as:

$$[K_e]\{\delta\} = \{P_e\} \tag{21}$$

where $[K_e]$ is the element stiffness matrix and $\{P_e\}$ is the nodal load vector.

The global stiffness matrix and global load vector for the whole plate is then formed by taking the contribution of all the plate elements. Finally, the global linear simultaneous equations are formed and solved for the problem of the sandwich plate after incorporation of appropriate boundary conditions. The in-plane stresses are calculated with the help of constitutive relationship by using the condition of stress continuity as in Eq. (4).

A numerical code is developed to implement the above mentioned operations involved in the proposed FE model to calculate deflections and stresses in the sandwich plate. The skyline technique has been used to store the global stiffness matrix in a single array and Gaussian decomposition scheme is adopted for the solution.

The following different boundary conditions are used:

1. Boundary line parallel to *x* axis
   - Simply supported condition: The degrees of freedoms u_0, v_0, w_0, θ_x, θ_x^u, θ_x^l, w_u, w_l are restrained while θ_y, θ_y^u and θ_y^l are unrestrained.
   - Clamped condition: All the nodal degrees of freedoms at the boundary are fully restrained.
   - Free boundary condition: All the nodal degrees of freedoms at the boundary are unrestrained.

2. Boundary line parallel to y axis
   - Simply supported condition: The degrees of freedoms u_0, v_0, w_0, θ_y, θ_y^u, θ_y^l, w_u, w_l are restrained while θ_x, θ_x^u and θ_x^l are unrestrained.
   - Clamped condition: All the nodal degrees of freedoms at the boundary are fully restrained.

Free boundary condition: All the nodal degrees of freedoms at the boundary are unrestrained.

3 NUMERICAL RESULTS AND DISCUSSIONS

The behavior of laminated sandwich plates having imperfect interfaces is studied by solving a number of numerical examples having different features using the proposed C^0 plate FE model based on refined higher order shear deformation theory (RHSDT). The validation of the proposed model is carried out with the available results of ordinary composite plates Cheng et al. [51]-[52], Cheng and Kitiporncahi [53] and laminated sandwich plates Chakrabarti and Sheikh [50] with or without imperfection. Finally, the proposed model is applied to solve many problems to generate results in the form of deflections and stresses. The geometric details of the plate problem considered in different cases are shown in Fig. 3. The results obtained are presented in the form of different tables and figures.

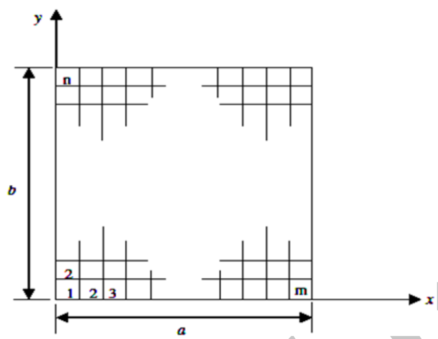

Fig.3
Rectangular plate having a mesh of m×n.

3.1 Cylindrical bending of a cross ply (0/90/0) laminate subjected to a distributed load of sinusoidal variation

The problem have been solved by Cheng et al. [51] and Chakrabarti and Sheikh [50] using a plate theory, which is more or less similar to that used in the present study except that in the present model the variation of the transverse displacement is also considered. They Cheng et al. [51] and Chakrabarti and Sheikh [50] have taken imperfection at all the interfaces and solved the problem. To assess the performance of the proposed element, it is used to solve this plate problem. The plate Fig. 3, b/a=3 is subjected to a distributed load of intensity $q=q_0 \sin(\pi x/a)$. In this example, all the layers are of equal thickness and possess the same material properties in their material axis system ($E_1=25E$, $E_2=E_3=E$, $G_{12}=G_{13}=0.5E$, $G_{23}=0.2E$, $\nu_{12}=\nu_{13}=\nu_{23}=0.25$). The imperfections at the layer interfaces excepting the reference middle plane (i.e., a continuous part of the core) are defined by the parameters: $R_{11}^k = R_{22}^k = Rh/E$ and $R_{12}^k = R_{21}^k = 0.0$ where the non-dimensional parameter R is varied from 0.0 to 0.6 (R=0.0 represents perfect interface). Taking thickness ratio (h/a) = 0.25, the plate is analyzed by the present finite element model using mesh sizes Fig. 3 of 2×2, 4×4, 6×6, 8×8, 12×12 and 16×16. The values of the non-dimensional central deflection, $\overline{w}_c = 100wEh^3/(q_0a^4)$ obtained by the proposed element are presented in Table 2. with the analytical solution of Cheng et al. [51] and finite element solution of Chakrabarti and Sheikh [50] for the cases of perfect as well as imperfect interfaces. For the case of perfect interfaces, the results based on three-dimensional elasticity solution of Pagano [30] are also included in Table 2. , which shows that the present results are in excellent agreement with those of Pagano [30] and Cheng et al. [51]. It also shows that the convergence of present results with mesh refinement is very good.

Table 1
Material properties (dimensionless) used for the Core and Face Sheets

Location	Elastic properties								
	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	υ_{12}	υ_{13}	υ_{32}
Face	25.0	1.0	1.0	0.5	0.5	0.2	0.25	0.25	0.25
Core	0.04	0.04	0.5	0.016	0.06	0.06	0.25	0.02	0.25

Table 2
Normalized central deflection *($\bar{w}_c$)* of a simply supported cross ply (0/90/0) laminate subjected to a distributed load of sinusoidal variation (cylindrical bending) (*b*/*a*=3, *h*/*a*=0.25)

Reference	Theory	Central deflection ($\bar{w}_c$)			
		R=0.0	*R*=0.2	*R*=0.4	*R*=0.6
Present (2x2)[b]	RHSDT	2.7678	2.8690	3.3971	3.9777
Present (4x4)	RHSDT	2.8320	3.3702	3.8940	4.4490
Present (6x6)	RHSDT	2.8360	3.3938	3.9207	4.4785
Present (8x8)	RHSDT	2.8367	3.3981	3.9257	4.4838
Present (12x12)	RHSDT	2.8370	3.3997	3.9275	4.4858
Present (16x16)	RHSDT	2.8371	3.4000	3.9277	4.4861
Pagano [30]	3D-Elasticity	2.8200	-	-	-
Cheng et al. [45]	Zigzag theory	2.7567	3.3419	3.8825	4.3622
Chakrabarti and Sheikh [44]	RHSDT	2.7575	3.3429	3.8833	4.3630

[b]Quantities inside the parenthesis indicate mesh division

3.2 A cross ply (0/90/90/0) square laminate simply supported at the four edges and subjected to a distributed load of sinusoidal variation in both the direction

The problem of a simply supported cross ply (0/90/90/0) square laminate Fig. 3, *a*=*b* having imperfections at all the layer interfaces studied by Cheng et al. [51] and Chakrabarti and Sheikh [50] is taken in this example. The plate is subjected to a distributed load of sinusoidal variation $q=q_0\sin(\pi x/a)\times\sin(\pi y/b)$. The analysis is carried out by the proposed element using mesh sizes Fig. 3 of 2×2, 4×4, 6×6, 8×8, 12×12 and 16×16 taking *h*/*a* = 0.25 and 0.10 and *R* = 0.0 and 0.2. The value of non-dimensional central deflection $\bar{w}_c$; transverse shear stress $\bar{\tau}_{xz}=\tau_{xz}h/(q_0a)$ at *x* = 0, *y* = *b*/2 and *z* = 0; transverse shear stress $\bar{\tau}_{yz}=\tau_{yz}h/(q_0a)$ at *x* = *a*/2, *y* = 0 and *z* = 0; in-plane normal stress $\bar{\sigma}_{xx}=\sigma_{xx}h^2/(q_0a^2)$ at *x* = *a*/2, *y* = *b*/2 and *z* = *h*/2; in-plane normal stress $\bar{\sigma}_{yy}=\sigma_{yy}h^2/(q_0a^2)$ at *x* = *a*/2, *y* = *b*/2 and *z* = *h*/4 obtained are presented in Table 3. with those of Cheng et al. [51] , Chakrabarti and Sheikh [50] and Pagano [30]. Table 3. shows that the agreement between the results is quite good.

Now the through the thickness variation of non-dimensional in-plane displacement $E_2\bar{u}/(q_0h)$ at *x*=0 and *y*=*b*/2; in-plane normal stress σ_x/q_0 at *x*=*a*/2 and *y*=*b*/2 obtained in the present analysis (mesh size: 12×12) are plotted in the Figs. 4-7 where *R*= 0.0, 0.2, 0.4 and 0.6; and *h*/*a*= 0.25 and 0.10. In all these plots, results based on elasticity solution of Pagano [30] are included to compare the present results for the perfect case (*R* = 0.0). The plots are found to follow a proper trend as expected.

Table 3

Normalized central deflection $\bar{w}_c$, in-plane normal stresses $\bar{\sigma}_{xx}$, $\bar{\sigma}_{yy}$ and transverse shear stresses $\bar{\tau}_{xz}$, $\bar{\tau}_{yz}$ of a simply supported square cross ply (0/90/90/0) laminate subjected to a distributed load of sinusoidal variation

h/*a*	Reference	Theory	$\bar{w}_c$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
			R=0.0				
0.25	Present (2x2)[b]	RHSDT	1.9446	0.8101	0.8178	0.2947	0.3144
	Present (4x4)	RHSDT	1.9291	0.7219	0.7313	0.2442	0.2582
	Present (6x6)	RHSDT	1.9282	0.6985	0.7116	0.2323	0.2456
	Present (8x8)	RHSDT	1.9281	0.6904	0.7046	0.2280	0.2410
	Present (12x12)	RHSDT	1.9281	0.6850	0.6996	0.2249	0.2377
	Present (16x16)	RHSDT	1.9281	0.6832	0.6979	0.2238	0.2365
	Pagano [30]	3D Elasticity	1.9540	0.7200	0.6630	0.2190	0.2920
	Cheng et al. [45]	Zigzag	1.9060	0.7368	0.7000	0.2109	0.3148
	Chakrabarti and Sheikh [44]	RHSDT	1.9065	0.7461	0.7044	0.2079	0.3158
0.10	Present (2x2)[b]	RHSDT	0.7378	0.6599	0.4685	0.3587	0.1973
	Present (4x4)	RHSDT	0.7319	0.5890	0.4238	0.3392	0.1636
	Present (6x6)	RHSDT	0.7314	0.5707	0.4126	0.3231	0.1557
	Present (8x8)	RHSDT	0.7313	0.5642	0.4086	0.3171	0.1528
	Present (12x12)	RHSDT	0.7313	0.5597	0.4057	0.3128	0.1507
	Present (16x16)	RHSDT	0.7313	0.5582	0.4047	0.3113	0.1499
	Pagano [30]	3D Elasticity	0.7430	0.5590	0.4010	0.3010	0.1960
	Cheng et al. [45]	Zigzag	0.7359	0.5611	0.4081	0.3002	0.1995
	Chakrabarti and Sheikh [44]	RHSDT	0.7364	0.5681	0.4106	0.3006	0.2006
			R=0.2				
0.25	Present (2x2)[b]	RHSDT	2.1575	0.9141	0.9061	0.3032	0.3288
	Present (4x4)	RHSDT	2.1387	0.8128	0.8017	0.2514	0.2709
	Present (6x6)	RHSDT	2.1377	0.7883	0.7775	0.2392	0.2575
	Present (8x8)	RHSDT	2.1375	0.7799	0.7696	0.2347	0.2526
	Present (12x12)	RHSDT	2.1375	0.7922	0.7640	0.2315	0.2490
	Present (16x16)	RHSDT	2.1375	0.7721	0.7620	0.2304	0.2477
	Cheng et al. [45]	Zigzag	2.4811	0.885	0.7761	0.1931	0.2818
	Chakrabarti and Sheikh [44]	RHSDT	2.4816	0.8960	0.7815	0.1950	0.2834
0.10	Present (2x2)[b]	RHSDT	0.8669	0.7404	0.5494	0.4367	0.2144
	Present (4x4)	RHSDT	0.8598	0.6610	0.4972	0.3654	0.1783
	Present (6x6)	RHSDT	0.8593	0.6412	0.4833	0.3480	0.1698
	Present (8x8)	RHSDT	0.8592	0.6343	0.4783	0.3416	0.1667
	Present (12x12)	RHSDT	0.8591	0.6316	0.4748	0.3369	0.1643
	Present (16x16)	RHSDT	0.8591	0.6279	0.4736	0.3352	0.1635
	Cheng et al. [45]	Zigzag	0.8615	0.5820	0.4498	0.2887	0.2108
	Chakrabarti and Sheikh [44]	RHSDT	0.8621	0.5893	0.4525	0.2913	0.2121

[b]Quantities inside the parenthesis indicate mesh division

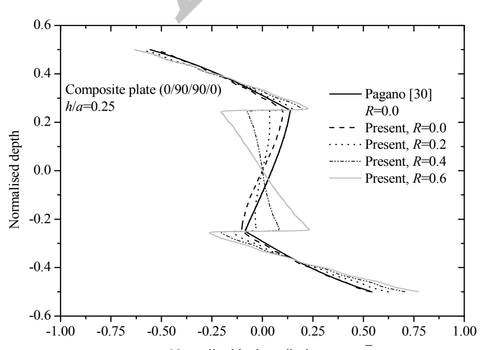

Fig. 4
Through the thickness variation of normalized in-plane displacement $\bar{u}$.

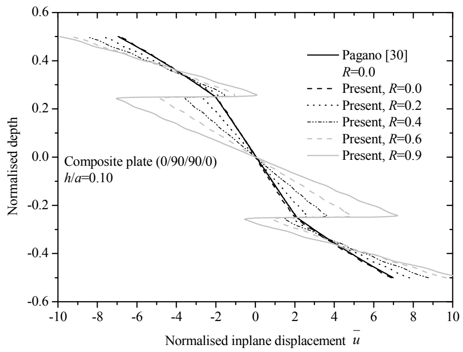

Fig. 5
Through the thickness variation of normalized in-plane displacement $\bar{u}$.

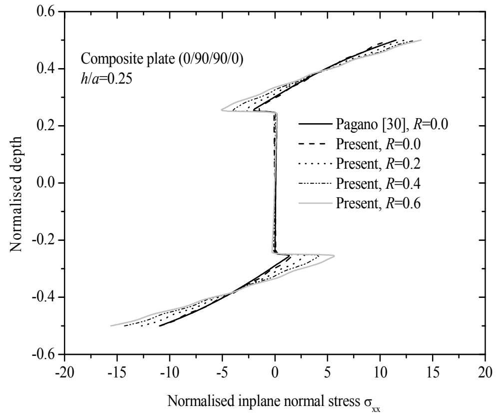

Fig. 6
Through the thickness variation of normalized in-plane stress σ_{xx} .

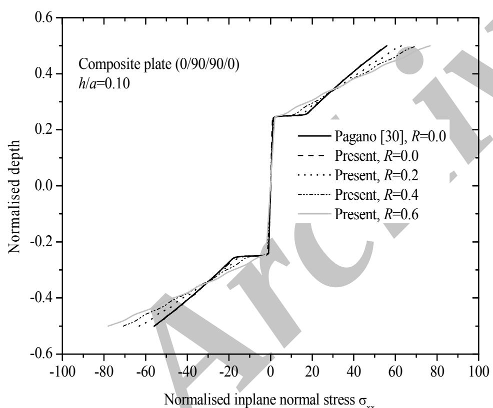

Fig. 7
Through the thickness variation of normalized in-plane stress σ_{xx} .

3.3 A simply supported square sandwich plate (f/C/f) having orthotropic face sheets under distributed load of sinusoidal variation in both the direction

A square sandwich plate (Fig. 3, a=b) simply supported at the four sides and subjected to distributed load of intensity $q=q_0 \sin(\pi x/a) \times \sin(\pi y/b)$ is analyzed by the proposed element using mesh sizes Fig. 3 of 4×4, 6×6, 8×8, 12×12 and 16×16 taking h/a = 0.25, 0.10 and 0.01. The plate has a central core of thickness 0.8 h and an orthotropic stiff layer of thickness 0.1 h at each face. The material properties of face sheet and core are given in Table 1. The imperfections at the interfaces between core and stiff face layers are defined by $R_{11}^k = R_{22}^k = Rh/E$ and $R_{12}^k = R_{21}^k = 0.0$ where the non-dimensional parameter R is taken as 0.0, 0.3 and 0.6 in the present study. The values of non-dimensional central deflection $\bar{w}_c$; in-plane normal stress $\bar{\sigma}_{xx}$ ($x = a/2$, $y = b/2$ and $z = h/2$), in-plane shear stress $\bar{\tau}_{xy} = \tau_{xy} h^2/(q_0 a^2)$ ($x = 0$, $y = 0$ and $z = h/2$) and transverse shear stress $\bar{\tau}_{xz}$ ($x = 0$, $y = b/2$ and $z = 0.4h$) obtained by

Table 4

Normalized central deflection $\bar{w}_c$, in-plane normal stress $\bar{\sigma}_{xx}$, in-plane shear stress $\bar{\tau}_{xy}$ and transverse shear stress $\bar{\tau}_{xz}$ of a simply supported square sandwich plate (*f/C/f*) having orthotropic face sheets under distributed load of sinusoidal variation

h/*a*	Reference	Theory	$\bar{w}_c$	$\bar{\sigma}_{xx}$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
		R=0.0				
0.25	Present (4x4) [b]	RHSDT	7.5926	1.6080	0.1503	0.2893
	Present (6x6)	RHSDT	7.5882	1.5608	0.1468	0.2752
	Present (8x8)	RHSDT	7.5875	1.5436	0.1456	0.2701
	Present (12x12)	RHSDT	7.5873	1.5316	0.1446	0.2665
	Present (16x16)	RHSDT	7.5873	1.5276	0.1443	0.2651
	Pagano [30]	3D Elasticity	7.5960	1.5120	0.1481	0.2354
	Chakrabarti and Sheikh [44]	RHSDT	7.6262	1.5775	0.1484	0.2322
0.10	Present (4x4)	RHSDT	2.1804	1.2147	0.0731	0.3174
	Present (6x6)	RHSDT	2.1789	1.1761	0.0716	0.3018
	Present (8x8)	RHSDT	2.1787	1.1630	0.0710	0.2962
	Present (12x12)	RHSDT	2.1786	1.1539	0.0705	0.2922
	Present (16x16)	RHSDT	2.1786	1.1509	0.0704	0.2908
	Pagano [30]	3D Elasticity	2.2004	1.1520	0.0717	0.2974
	Chakrabarti and Sheikh [44]	RHSDT	2.2011	1.1686	0.0716	0.2968
0.01	Present (4x4)	RHSDT	0.8823	1.1476	0.0451	0.3291
	Present (6x6)	RHSDT	0.8820	1.1186	0.0440	0.3161
	Present (8x8)	RHSDT	0.8819	1.1074	0.0437	0.3112
	Present (12x12)	RHSDT	0.8819	1.0991	0.0434	0.3073
	Present (16x16)	RHSDT	0.8819	1.0963	0.0433	0.3059
	Pagano [30]	3D Elasticity	0.8924	1.0980	0.0437	0.3222
	Chakrabarti and Sheikh [44]	RHSDT	0.8935	1.1095	0.0438	0.3259
		R=0.3				
0.25	Present (4x4)	RHSDT	8.0047	1.6950	0.1459	0.2617
	Present (6x6)	RHSDT	8.0000	1.6455	0.1387	0.2491
	Present (8x8)	RHSDT	7.9993	1.6277	0.1348	0.2446
	Present (12x12)	RHSDT	7.9991	1.6151	0.1301	0.2413
	Present (16x16)	RHSDT	7.9991	1.6108	0.1272	0.2401
	Chakrabarti and Sheikh [44]	RHSDT	8.0586	1.6159	0.1531	0.1336
0.10	Present (4x4)	RHSDT	2.3293	1.2720	0.0739	0.2808
	Present (6x6)	RHSDT	2.3277	1.2314	0.0713	0.2673
	Present (8x8)	RHSDT	2.3275	1.2177	0.0698	0.2625
	Present (12x12)	RHSDT	2.3273	1.2083	0.0681	0.2591
	Present (16x16)	RHSDT	2.3273	1.2051	0.0670	0.2579
	Chakrabarti and Sheikh [44]	RHSDT	2.2982	1.1735	0.0735	0.276
0.01	Present (4x4)	RHSDT	0.9540	1.1968	0.0470	0.2895
	Present (6x6)	RHSDT	0.9537	1.1666	0.0458	0.2786
	Present (8x8)	RHSDT	0.9536	1.1550	0.0453	0.2747
	Present (12x12)	RHSDT	0.9536	1.1463	0.0449	0.2720
	Present (16x16)	RHSDT	0.9536	1.1433	0.0445	0.2714
	Chakrabarti and Sheikh [44]	RHSDT	0.8946	1.1096	0.0438	0.3259
		R=0.6				
0.25	Present (4x4)	RHSDT	8.4643	1.7888	0.1408	0.2352
	Present (6x6)	RHSDT	8.4593	1.7369	0.1301	0.2243
	Present (8x8)	RHSDT	8.4585	1.7184	0.1238	0.2204
	Present (12x12)	RHSDT	8.4583	1.7052	0.1161	0.2175
	Present (16x16)	RHSDT	8.4583	1.7007	0.1112	0.2164
	Chakrabarti and Sheikh [44]	RHSDT	8.4983	1.6557	0.1578	0.0309
0.01	Present (4x4)	RHSDT	2.4972	1.3343	0.0748	0.2428
	Present (6x6)	RHSDT	2.4955	1.2914	0.0709	0.2315
	Present (8x8)	RHSDT	2.4952	1.2770	0.0686	0.2278
	Present (12x12)	RHSDT	2.4951	1.2672	0.0657	0.2253
	Present (16x16)	RHSDT	2.4951	1.2639	0.0638	0.2244
	Chakrabarti and Sheikh [44]	RHSDT	2.3980	1.1785	0.0755	0.2540
0.01	Present (4x4)	RHSDT	1.0348	1.2501	0.0489	0.2473
	Present (6x6)	RHSDT	1.0343	1.2187	0.0476	0.2387
	Present (8x8)	RHSDT	1.0343	1.2065	0.0471	0.2361
	Present (12x12)	RHSDT	1.0342	1.1975	0.0464	0.2350
	Present (16x16)	RHSDT	1.0342	1.1943	0.0458	0.2357
	Chakrabarti and Sheikh [44]	RHSDT	0.8957	1.1097	0.0439	0.3258

[b]Quantities inside the parenthesis indicate mesh division

proposed element are presented in Table 4. For the perfect cases (R = 0.0), the present results are compared with those obtained from the elasticity solution of Pagano [30] in Table 4., which shows the good agreement between the results. Now the through the thickness variation of non-dimensional in-plane normal stress σ_y / q_0 at $x=a/2$ and $y=b/2$ obtained in the present analysis (mesh size: 12×12) are plotted in the Figs. 8-9 where R= 0.0, 0.3 and 0.6; and h/a= 0.25 and 0.10. In all these plots, results based on elasticity solution of Pagano [30] are included to compare the present results for the perfect case (R = 0.0). The plots are found to follow the proper trend as expected.

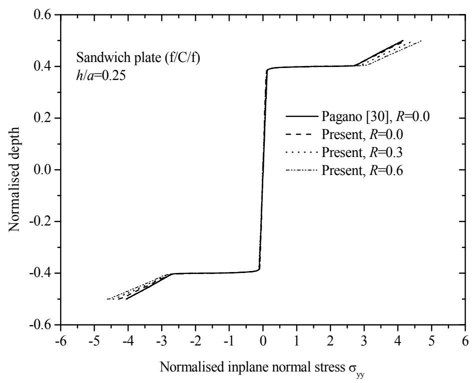

Fig. 8
Through the thickness variation of normalized in-plane stress σ_{yy} .

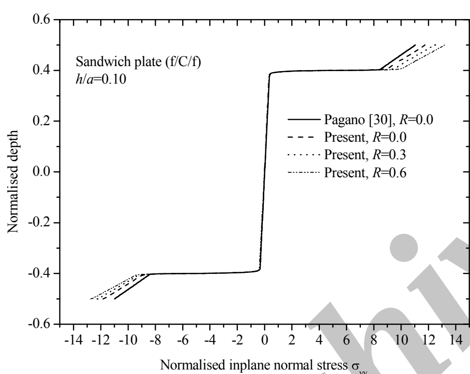

Fig.9
Through the thickness variation of normalized in-plane stress σ_{yy} .

3.4 A square laminated sandwich plate (0/90/0/C/0/90/0) having different boundary conditions under uniformly distributed load

The problem of a square (Fig. 3, $b = a$) sandwich plate (0/90/0/C/0/90/0) having stiff laminated face sheets (0/90/0) under uniformly distributed load of intensity q_0 is studied for different boundary conditions and imperfections at the interfaces between core and face sheets. The core has a thickness of 0.85 h while it is 0.025 h for each ply in the face sheets. The material properties of the core and those of ply in the face sheets are identical to those used in the previous example. The different boundary conditions taken are SSSS (all the sides simply supported), SCSC (simply supported at x = 0 and $x = a$; and clamped at the other sides) and CCCC (all the sides clamped). The imperfections are taken only at the interfaces between core and face sheets where these are defined by $R_{11}^k = R_{22}^k = R_1 h/E$ for the upper interface, $R_{11}^k = R_{22}^k = R_2 h/E$ for the lower interface and $R_{12}^k = R_{21}^k = 0.0$ for both these interfaces. The analysis is carried out with a mesh size of 12×12 taking h/a = 0.20 and different values of R_1 and R_2 to have a number of combinations for the imperfections (see Table 5.) The values of non-dimensional central deflection $\overline{w}_c$; in-plane normal stress $\overline{\sigma}_{xx}$ ($x = a/2$, $y = b/2$ and $z = h/2$) and transverse shear stress $\overline{\tau}_{yz}$ ($x = a/2$, y = 0 and z = 0.425 h) obtained in the present analysis are presented in Table 5. For SSSS boundary condition, the present results corresponding to $R_1 = R_2 = 0.0$ (perfect cases) are compared with those obtained from the elasticity solution of Pagano [30] (see Table 5.), which shows these results have very good agreement.

Table 5
Normalized central deflection $\bar{w}_c$, in-plane normal stress $\bar{\sigma}_{xx}$, and transverse shear stress $\bar{\tau}_{yz}$ of a laminated sandwich plate (0/90/0/*C/0/90/0*) subjected to uniformly distributed load having different boundary conditions

Interfacial parameters	Boundary conditions	References	Theory	$\bar{w}_c$	$\bar{\sigma}_{xx}$	$\bar{\tau}_{yz}$
R1=0.0 *R2*=0.0	SSSS	Present element	RHSDT	6.7337	2.0271	0.2914
		Pagano [30]	3D Elasticity	6.7576	1.9816	-
		Chakrabarti and Sheikh [44]	RHSDT	6.7444	2.0479	-
	SCSC	Present element	RHSDT	5.7640	1.7477	0.5451
		Chakrabarti and Sheikh [44]	RHSDT	5.6843	1.7504	-
	CCCC	Present element	RHSDT	5.0650	0.8663	0.4960
		Chakrabarti and Sheikh [44]	RHSDT	4.8696	0.9028	-
R1=0.0 *R2*=0.2	SSSS	Present element	RHSDT	6.8742	2.0775	0.3031
		Chakrabarti and Sheikh [44]	RHSDT	6.8494	2.0579	-
	SCSC	Present element	RHSDT	5.8725	1.7875	0.5519
		Chakrabarti and Sheikh [44]	RHSDT	5.7863	1.7588	-
	CCCC	Present element	RHSDT	5.1510	0.8825	0.5017
		Chakrabarti and Sheikh [44]	RHSDT	4.9626	0.9106	-
R1=0.2 *R2*=0.2	SSSS	Present element	RHSDT	6.9250	2.0701	0.3207
		Chakrabarti and Sheikh [44]	RHSDT	6.9556	2.0600	-
	SCSC	Present element	RHSDT	5.8959	1.7738	0.5680
		Chakrabarti and Sheikh [44]	RHSDT	5.8898	1.7652	-
	CCCC	Present element	RHSDT	5.1724	0.8885	0.5162
		Chakrabarti and Sheikh [44]	RHSDT	5.0569	0.9168	-
R1=0.4 *R2*=0.2	SSSS	Present element	RHSDT	6.9777	2.0596	0.3378
		Chakrabarti and Sheikh [44]	RHSDT	7.0627	2.0623	-
	SCSC	Present element	RHSDT	5.9205	1.7569	0.5828
		Chakrabarti and Sheikh [44]	RHSDT	5.9936	1.7714	-
	CCCC	Present element	RHSDT	5.1959	0.8945	0.5296
		Chakrabarti and Sheikh [44]	RHSDT	5.1513	0.9231	-
R1=0.6 *R2*=0.2	SSSS	Present element	RHSDT	7.0320	2.0462	0.3544
		Chakrabarti and Sheikh [44]	RHSDT	7.1708	2.0647	-
	SCSC	Present element	RHSDT	5.9461	1.7370	0.5962
		Chakrabarti and Sheikh [44]	RHSDT	6.0678	1.7773	-
	CCCC	Present element	RHSDT	5.2219	0.9008	0.5419
		Chakrabarti and Sheikh [44]	RHSDT	5.2457	0.9293	-
R1=0.8 *R2*=0.2	SSSS	Present element	RHSDT	7.0877	2.0298	0.3705
		Chakrabarti and Sheikh [44]	RHSDT	7.2799	2.0671	-
	SCSC	Present element	RHSDT	5.9728	1.7142	0.6081
		Chakrabarti and Sheikh [44]	RHSDT	6.2022	1.7829	-
	CCCC	Present element	RHSDT	5.2502	0.9072	0.5529
		Chakrabarti and Sheikh [44]	RHSDT	5.3401	0.9354	-
R1=1.0 *R2*=0.2	SSSS	Present element	RHSDT	7.1449	2.0107	0.3859
		Chakrabarti and Sheikh [44]	RHSDT	7.3899	2.0697	-
	SCSC	Present element	RHSDT	6.0004	1.6887	0.6185
		Chakrabarti and Sheikh [44]	RHSDT	6.3069	1.7883	-
	CCCC	Present element	RHSDT	5.2809	0.9139	0.5627
		Chakrabarti and Sheikh [44]	RHSDT	5.4346	0.9416	-

For SSSS, SCSC and CCCC boundary condition, the present results corresponding to different values of R_1 and R_2 are compared with those of Chakrabarti and Sheikh [51]. Table 5. shows that the agreement between the results is quite good.

4 CONCLUSIONS

In this paper an efficient C^0 plate finite element (FE) model based on refined higher order shear deformation theory (RHSDT) is used to represent the behavior of soft core sandwich plate having different degrees of inter-laminar imperfections. The imperfections are incorporated taking displacement jumps at the layer interfaces, which are characterized by a linear spring-layer model. The analysis is carried out by displacement based finite element technique using a nine node quadratic element with eleven field variables. As very less number of investigations is carried out on sandwich plates with inter-laminar imperfections, the performance of the proposed element is initially tested by solving some benchmark numerical examples of laminated composites and sandwich plates with inter-laminar imperfections and comparing the present results with some published results. It is found that the results obtained by this element are quite consistent with good accuracy and convergence.

The major contributions of the present paper are as given below.

- The present C^0 FE model based on RHSDT, has been developed in such a way that it does not require to include the C^1 continuity of the transverse displacement in the formulation.
- There is no need to impose any penalty parameter for the formation of the element stiffness matrix.
- The compressibility of the core of sandwich plates having a soft core is also taken care by taking variation of transverse displacement within the core.
- The proposed model satisfies the transverse shear stress continuity conditions at the layer interfaces and the conditions of zero transverse shear stress at the top and bottom of the plate.
- The imperfection at the layer interfaces are very efficiently combined using a linear spring layer model with the proposed C^0 FE model based on RHSDT.

Therefore, the proposed FE model should be recommended for the accurate analysis of laminated soft core sandwich plates having inter-laminar imperfections.

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