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Torsional Waves in Prestressed Fiber Reinforced Medium Subjected to Magnetic Field

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ABSTRACT

The propagation of torsional waves in a prestressed fiber-reinforced half-space under the effect of magnetic field and gravity has been discussed. The problem has been solved analytically using Whittaker function to obtain the exact solution frequency equations. Numerical results for stress, gravity and magnetic field are given and illustrated graphically. Comparisons are made with the results predicted by the boundary value condition for rigid boundary and for traction free boundary in the presence and in the absence of the effect of a magnetic field, gravity and stress. It is found that the reinforcement, gravity and magnetic field have great effects on the distribution.

Keywords: Magnetoelastic; Torsional waves; Initial stress; Fiber reinforced medium; Whittaker function

1 INTRODUCTION

INTERACTIONS between strain and electromagnetic fields are largely being undertaken due to its various applications in many branches of science and technology. Development of magneto elasticity also induces us to study various problems of geophysics, seismology and related topics. Bazer [12] Made a survey of linear and non-linear wave motion in a perfect magneto elastic medium.

The term "Initial stress" is meant by stresses developed in a medium before it is being used for study. The earth is an initially stressed medium. Due to presence of external loading, slow process of creep and gravitational field, considerable amount of stresses (called pre-stresses or initial stresses) remain naturally present in the layers. These stresses may have significant influence on elastic waves produced by earthquake or explosions and also in the stability of the medium. [16]

In most previous investigations, the effect of reinforcement has been neglected. The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield et al. [10]. The characteristic property of a reinforced concrete member is that its components, namely concrete and steel, act together as a single anisotropic unit as long as they remain in the elastic condition, i.e. the two components are bound together so that there can be no relative displacement between them.

The effect of gravity on wave propagation in an elastic solid medium was first considered by Bromwich [1], treating the force of gravity as a type of body force. Love [13-16] extended the work of Bromwich investigated the influence of gravity on superficial waves and showed that the Rayleigh wave velocity is affected by the gravity field. Sezawa [2] studied the dispersion of elastic waves propagated on curved surfaces.

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The problems of elastic wave propagation in the presence of a steady magnetic field have been investigated by Das and Bhattacharya [3], Andreou and Dassios [4] and Suhubi [6]. Some of the analogous results on magneto elastic waves propagation problems, but in an anisotropic medium, were obtained by Datta [8], Acharya et al. [9] and Belfield et al. [10] Investigated the effect of the transverse isotropy and magnetic field on the interface waves in a conducting medium subject to the initial state of stress of the form of hydrostatic tension or compression.

The effect of gravity or stress on the propagation of torsional waves in sandy medium and in other mediums has been discussed by many scientists Gupta et al. [14], but the effect of magnetic field on fiber reinforce medium is not yet considered.

In this paper, the authors study the existence of torsional waves in fiber reinforced medium, which depends upon or does not depend upon gravity, magnetic field and initial stress. The frequency equations has been derived for rigid as well as traction free boundaries in terms of Whittaker function and numerical results for stress, gravity and magnetic field are given and illustrated graphically.

2 BASIC EQUATIONS

The problem is dealing with magneto elasticity. Therefore, the basic equations will be electromagnetism and elasticity. The Maxwell equations of the electromagnetic field in a region with no charges (ρ = 0) and no currents (**J** = 0), such as in a vacuum, are [19]:

$$\vec{\nabla}\cdot\vec{\mathrm{E}} = 0, \tag{1a}$$

$$\vec{\nabla}\cdot\vec{\mathrm{B}} = 0, \tag{1b}$$

$$\vec{\nabla}\times\vec{\mathrm{E}} = -\frac{\overrightarrow{\partial \mathrm{B}}}{\partial t}, \tag{1c}$$

$$\vec{\nabla}\times\vec{\mathrm{B}} = \mu_0\varepsilon_0\frac{\overrightarrow{\partial \mathrm{E}}}{\partial t}. \tag{1d}$$

where, $\vec{\mathrm{E}}$, $\vec{\mathrm{B}}$, μ_0 and ε_0 are electric field, magnetic field induction, permeability and permittivity of the vacuum. For vacuum, $\mu_0 = 4\pi\times10^{-7}$ and $\varepsilon_0 = 8.85\times10^{-12}$ in SI units. These equations lead directly to $\vec{\mathrm{E}}$ and $\vec{\mathrm{B}}$ satisfying the wave equation for which the solutions are linear combinations of plane waves traveling at the speed of light, $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$. In addition, $\vec{\mathrm{E}}$ and $\vec{\mathrm{B}}$ are mutually perpendicular to each other to the direction of wave propagation.

Also, the term Ohm's law is used to refer to various generalizations. The simplest example of this is:

$$\vec{J} = \sigma\vec{E}, \tag{2a}$$

where, $\vec{J}$ is the current density at a given location in a resistive material $\vec{\mathrm{E}}$ is the electric field at that location, and σ is a material dependent parameter called the conductivity. If an external magnetic field induction $\vec{\mathrm{B}}$ is present and the conductor is not at rest but moving at velocity $\vec{V}$, then an extra term must be added to account for the current induced by the Lorentz force on the charge carriers [19].

$$\vec{J} = \sigma(\vec{E}+\vec{V}\times\vec{B}) = \sigma(\vec{E}+\frac{\partial v}{\partial t}\times\vec{B}). \tag{2b}$$

The electromagnetic wave equation is a second-order partial differential equation that describes the propagation of electromagnetic waves through a vacuum. The homogeneous form of the equation, written in terms of either the electric field $\vec{\mathrm{E}}$ or the magnetic field induction $\vec{\mathrm{B}}$, takes the form [19]:

$$\left(\nabla^2-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)\vec{\mathrm{E}}\;=\;0, \tag{3a}$$

$$\left(\nabla^2-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)\vec{\mathrm{B}}\;=\;0. \tag{3b}$$

where $\nabla^2=\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}$.

The dynamical equations of motion in cylindrical coordinate (r,θ,z) are Love [16]:

$$\frac{\partial s_{rr}}{\partial r}+\frac{1}{r}\frac{\partial s_{r\theta}}{\partial\theta}+\frac{\partial s_{rz}}{\partial z}+\frac{1}{r}(s_{rr}-s_{\theta\theta})+T_R=\rho\frac{\partial^2 u}{\partial t^2}, \tag{4a}$$

$$\frac{\partial s_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial s_{\theta\theta}}{\partial\theta}+\frac{\partial s_{\theta z}}{\partial z}+\frac{2s_{r\theta}}{r}+T_\theta=\rho\frac{\partial^2 v}{\partial t^2}, \tag{4b}$$

$$\frac{\partial s_{rz}}{\partial r}+\frac{1}{r}\frac{\partial s_{\theta z}}{\partial\theta}+\frac{\partial s_{zz}}{\partial z}+\frac{s_{rz}}{r}+T_Z=\rho\frac{\partial^2 w}{\partial t^2}. \tag{4c}$$

where $s_{rr},s_{r\theta},s_{rz},s_{rr}.s_{\theta\theta},s_{\theta z},s_{zz}$ are the respective stress components, T_R,T_θ,T_Z are the respective body forces and u,v,w are the respective displacement components.

The stress-strain relations are [16]:

$$s_{rr}=\delta_{11}^0 e_{rr}+\delta_{12}^0 e_{\theta\theta}+\delta_{13}^0 e_{zz}, \tag{5a}$$

$$s_{\theta\theta}=\delta_{21}^0 e_{rr}+\delta_{22}^0 e_{\theta\theta}+\delta_{23}^0 e_{zz}, \tag{5b}$$

$$s_{zz}=\delta_{31}^0 e_{rr}+\delta_{32}^0 e_{\theta\theta}+\delta_{33}^0 e_{zz}, \tag{5c}$$

$$s_{rz}=\delta_{44}^0 e_{rz}, \tag{5d}$$

$$s_{\theta z}=\delta_{55}^0 e_{\theta z}, \tag{5e}$$

$$s_{r\theta}=\delta_{66}^0 e_{r\theta}. \tag{5f}$$

where $\delta_{ij}=$ elastic constants ($ij=1,2......6$).

The constitutive equations for a fiber-reinforced linearly elastic anisotropic medium with respect to a preferred direction $\bar{a}$ are [10]:

$$s_{ij}=\lambda e_{kk}\delta_{ij}+2\mu_T e_{ij}+\alpha(a_k a_m a_{km}\delta_{ij}+e_{kk}a_i a_j)+2(\mu_L-\mu_T)(a_i a_k e_{kj}+a_j a_k e_{kk})+\beta(a_k a_m e_{km}a_i a_j) \tag{6}$$

where s_{ij} are components of stress; where, $e_{ij}=\dfrac{1}{2}(u_{i,j}+u_{ji})$ are components of strain; λ,μ_T,μ_L are elastic parameters; $\alpha,\beta,(\mu_L-\mu_T)$ reinforced anisotropic elastic parameters; u_i are the displacement vectors components and $\bar{a}=(a_1,a_2.a_3)$,where, $a_1^2+a_1^2+a_3^2=1$ If $\bar{a}$ has components that are (0, 0, 1) so that the preferred direction is the z axis, Eq. (6) simplifies, as given below

$$s_{rr}=(\lambda+2\alpha+4\mu_L-2\mu_T+\beta)e_{rr}+(\lambda+\alpha)e_{\theta\theta}+(\lambda+\alpha)e_{zz}, \tag{6a}$$

$$s_{\theta\theta}=(\lambda+\alpha)e_{rr}+(\lambda+2\mu_T)e_{\theta\theta}+\lambda e_{zz}, \tag{6b}$$

$$s_{zz}=(\lambda+\alpha)e_{rr}+\lambda e_{\theta\theta}+(\lambda+2\mu_T)e_{zz}, \tag{6c}$$

$$s_{\theta z} = 2\mu_T e_{\theta z}, \tag{6d}$$

$$s_{rz} = 2\mu_T e_{rz}, \tag{6e}$$

$$s_{r\theta} = 2\mu_T e_{r\theta}. \tag{6f}$$

The strain components are [18]:

$$e_{rr} = \frac{1}{2}\frac{\partial u}{\partial r}, \tag{7a}$$

$$e_{\theta\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\right), \tag{7b}$$

$$e_{zz} = \frac{1}{2}\frac{\partial w}{\partial z}, \tag{7c}$$

$$e_{\theta z} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial v}{\partial z}\right), \tag{7d}$$

$$e_{rz} = \frac{1}{2}\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right), \tag{7e}$$

$$e_{zz} = \frac{1}{2}\frac{\partial w}{\partial z}. \tag{7f}$$

The rotational components are [18]:

$$\Omega_r = \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial \theta} - \frac{\partial v}{\partial z}\right), \tag{8a}$$

$$\Omega_\theta = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}\right), \tag{8b}$$

$$\Omega_z = \frac{1}{r}\left(\frac{\partial (rv)}{\partial r} - \frac{\partial u}{\partial \theta}\right). \tag{8c}$$

Equations governing the propagation of small elastic disturbances in a perfectly conducting viscoelastic solid having electromagnetic force $\left(\vec{J} \times \vec{\mathrm{B}}\right)$ (the Lorentz force, $\vec{J}$ is the current density and $\vec{\mathrm{B}}$ being magnetic induction vector) as the only body force are (using Eq. (4))

$$\frac{\partial s_{rr}}{\partial r} + \frac{1}{r}\frac{\partial s_{r\theta}}{\partial \theta} + \frac{\partial s_{rz}}{\partial z} + \frac{1}{r}(s_{rr} - s_{\theta\theta}) + \left(\vec{J} \times \vec{\mathrm{B}}\right)_R = \rho\frac{\partial^2 u}{\partial t^2}, \tag{9a}$$

$$\frac{\partial s_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial s_{\theta\theta}}{\partial \theta} + \frac{\partial s_{\theta z}}{\partial z} + \frac{2s_{r\theta}}{r} + \left(\vec{J} \times \vec{\mathrm{B}}\right)_\theta = \rho\frac{\partial^2 v}{\partial t^2}, \tag{9b}$$

$$\frac{\partial s_{rz}}{\partial r} + \frac{1}{r}\frac{\partial s_{\theta z}}{\partial \theta} + \frac{\partial s_{zz}}{\partial z} + \frac{s_{rz}}{r} + \left(\vec{J} \times \vec{\mathrm{B}}\right)_Z = \rho\frac{\partial^2 w}{\partial t^2}. \tag{9c}$$

Let us assume the components of magnetic field intensity $\vec{\mathrm{H}}$ are $\mathrm{H}_r = \mathrm{H}_\theta = 0$ and $\mathrm{H}_z = \mathrm{H}$ constant. Therefore, the value of magnetic field intensity is [19]:

$$\vec{\mathrm{H}}\left(0,0,\mathrm{H}\right) = \vec{\mathrm{H}}_0 + \vec{\mathrm{H}}_i \tag{10}$$

where, $\vec{H}_0$ is the initial magnetic field intensity along z-axis and $\vec{H}_i$ is the perturbation in the magnetic field intensity.

The relation between magnetic field intensity $\vec{H}$ and magnetic field induction $\vec{B}$ is:

$$\vec{B} = \mu_0 \vec{H} \text{ (For vacuum, } \mu_0 = 4\pi\times10^{-7} \text{ SI units.)} \tag{11}$$

From Eq. (1), Eq. (2), Eq. (3) and Eq. (10), we get,

$$\nabla^2 \vec{H} = \mu_0 \sigma \left\{ \frac{\partial \vec{H}}{\partial t} + \vec{\nabla} \times \left(\frac{\partial v}{\partial t} \times \vec{H} \right) \right\} \tag{12}$$

The components of Eq. (12) can be written as [9]:

$$\frac{\partial H_r}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 H_{r,} \tag{13a}$$

$$\frac{\partial H_\theta}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 H_{\theta,} \tag{13b}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 H. \tag{13c}$$

3 FORMULATION OF THE PROBLEM

The cylindrical coordinate system has been considered with origin at the surface and z-axis towards the interior of the half space for studying the propagation of torsional surface wave in fiber reinforced elastic half space, the half space is assumed to be under compressive initial stress, magnetic field and gravitational field. If 'r' and 'θ' are radial and circumferential coordinates respectively and waves travel along the radial direction then from Eq. (1), Eq. (6) and Eq. (10) , the equation can be written (in terms of Gaussian System of units) as [5],

$$\nabla_{,r} s_{r\theta} + \nabla_{,z} s_{\theta z} + \frac{2}{r} s_{r\theta} - \nabla_{,z}\left(\left[\frac{P+\rho g z}{2} \right] \nabla_{,z} \right) - \frac{H^2}{4\pi} \nabla_{,,z} v - \frac{\rho g z}{2} \nabla_{,r} \left(\nabla_{,z} v + \frac{v}{r} \right) = \rho \nabla_{,,t} v \tag{14}$$

where 'P' is initial stress along radial direction, g, H and ρ are gravity, magnetic field and density of the medium respectively. $v = v(r,\theta,z)$ be the displacement along circumferential direction.

For torsional surface waves propagate along radial direction and displacement of the particle along θ direction,

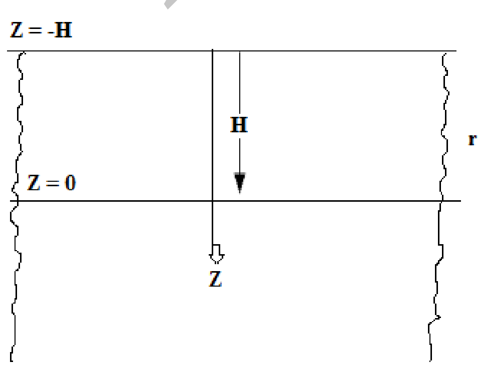

Fig. 1
Geometry of the problem.

We have

$$u_r = 0, \tag{15a}$$

$$u_z = 0, \tag{15b}$$

$$u_\theta = v(r,\ z,\ t). \tag{15c}$$

From Eq. (15), the Eq. (7) and Eq. (9) reduces to:

$$s_{rz} = 2\mu_T e_{\theta z}, \tag{16a}$$

$$s_{r\theta} = 2\mu_T e_{r\theta}. \tag{16b}$$

$$e_{\theta z} = \frac{1}{2}\left(\frac{\partial v}{\partial z}\right), \tag{16c}$$

$$e_{r\theta} = \frac{1}{2}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right). \tag{16d}$$

Using above relations, Eq. (14) becomes

$$\left(\mu_L - \frac{\rho g z}{2}\right)\left(\nabla_{,,r} - \frac{v}{r^2} + \frac{1}{r}\nabla_{,r}\right) - \frac{\rho g}{2}\nabla_{,z} + \left(\mu_T - \frac{P+\rho g z}{2} - \frac{H^2}{4\pi}\right)\nabla_{,,z} v = \rho \nabla_{,,t} v \tag{17}$$

4 SOLUTION OF THE PROBLEM

Assuming the time harmonic solution $e^{i\Omega t}$, the displacement in the circumferential direction in the medium can be written as [14]:

$$v = f(kr) q(z) e^{i\Omega t} \tag{18}$$

If $kr = \mathrm{T}$, where $k = \frac{2\pi}{\lambda'}$ is wave number

Put Eq. (18) in Eq. (17), we get

$$\left\{\begin{aligned} &\left(\mu_L - \frac{\rho g z}{2}\right)\left[\frac{d^2 f}{d\mathrm{T}^2} + \frac{1}{\mathrm{T}}\frac{df}{d\mathrm{T}} + \left(1 + \frac{1}{\mathrm{T}^2}\right) f\right] k^2 q \\ &+ \left[\left(\mu_T - \frac{P+\rho g z}{2} - \frac{H^2}{4\pi}\right)\frac{d^2 q}{dz^2} - \frac{\rho g}{2}\frac{dq}{dz} - \left(\mu_L - \rho\beta^2 - \frac{\rho g z}{2}\right) k^2 q\right] f \end{aligned}\right\} = 0 \tag{19}$$

Let

$$\Phi = \frac{d^2 f}{d\mathrm{T}^2} + \frac{1}{\mathrm{T}}\frac{df}{d\mathrm{T}} + \left(1 + \frac{1}{\mathrm{T}^2}\right) f, \tag{20a}$$

and

$$\Psi = \left(\mu_T - \frac{P+\rho g z}{2} - \frac{H^2}{4\pi}\right)\frac{d^2 q}{dz^2} - \frac{\rho g}{2}\frac{dq}{dz} - \left(\mu_L - \rho\beta^2 - \frac{\rho g z}{2}\right), \tag{20b}$$

Eq. (19) in terms of Eq. (20a) and Eq. (20b) $\Rightarrow\left(\mu_L-\dfrac{\rho gz}{2}\right)\Phi+\Psi f=0$

It is obvious $J_1(\mathrm{T})$, solution of $\Phi=\dfrac{d^2f}{d\mathrm{T}^2}+\dfrac{1}{\mathrm{T}}\dfrac{df}{d\mathrm{T}}+\left(1+\dfrac{1}{\mathrm{T}^2}\right)f=0$

Hence,

$$f=J_1(kr) \tag{21}$$

Now consider the equation

$$\Psi=\left(\mu_T-\frac{P+\rho gz}{2}-\frac{H^2}{4\pi}\right)\frac{d^2q}{dz^2}-\frac{\rho g}{2}\frac{dq}{dz}-\left(\mu_L-\rho\beta^2-\frac{\rho gz}{2}\right)k^2q=0 \tag{22}$$

After partial simplification Eq. (22) can be restated as:

$$\frac{d^2q}{dz^2}-\frac{\xi k}{2\eta\left(1-\dfrac{P}{2\mu_L\eta}-\dfrac{H^2}{4\pi\eta}-\dfrac{\xi kz}{2\eta}\right)}\frac{dq}{dz}-\frac{\dfrac{1}{\eta}\left(1-\dfrac{\beta^2}{\beta_1^2}-\dfrac{\xi kz}{2}\right)k^2}{\left(1-\dfrac{P}{2\mu_L\eta}-\dfrac{H^2}{4\pi\eta}-\dfrac{\xi kz}{2\eta}\right)}q=0 \tag{23}$$

where

$$\eta=\frac{\mu_T}{\mu_L}\ \text{(Elastic parameter)} \tag{24a}$$

$$\xi=\frac{\rho g}{\mu_L k}\ \text{(Gravity parameter)} \tag{24b}$$

$$\beta=\frac{\Omega}{k}\ \text{(Phase velocity)} \tag{24c}$$

$$\beta_1=\sqrt{\frac{\mu_L}{\rho}}\ \text{(Radial shear wave velocity)} \tag{24d}$$

$$\frac{H^2}{4\pi\eta}\ \text{(Magneto parameter)} \tag{24e}$$

$$\frac{P}{2\mu_L\eta}\ \text{(Shear parameter)} \tag{24f}$$

Put $q=\dfrac{\Re(z)}{\sqrt{\left(1-\dfrac{P}{2\mu_L\eta}-\dfrac{H^2}{4\pi\eta}-\dfrac{\xi kz}{2\eta}\right)}}$ in Eq. (23), we get

$$\Re^{//}(z)+\left[\frac{\xi^2}{16\eta^2}\left(1-\frac{P}{2\mu_L\eta}-\frac{H^2}{4\pi\eta}-\frac{\xi kz}{2\eta}\right)^{-2}-\frac{1}{\eta}\left(1-\frac{\bar{\beta}^2}{\beta_1^2}-\frac{\eta kz}{2}\right)\left(1-\frac{P}{2\mu_L\eta}-\frac{\xi kz}{2\eta}\right)^{-1}\right]k^2\,\Re(z)=0 \tag{25}$$

Substitute $\Im=\dfrac{4\eta}{\xi}\left(1-\dfrac{P}{2\mu_L\eta}-\dfrac{H^2}{4\pi\eta}-\dfrac{\xi kz}{2\eta}\right)$ in Eq. (25) by setting $\Re(z)=D(\Im)$, we get Whittaker function as:

$$D(\Im)^{//}+\left[\frac{1}{2\Im^2}+\frac{t}{\Im}-\frac{1}{4}\right]D(\Im)=0 \tag{26}$$

The solution of Eq. (26) is:

$$D(\Im) = LW_{-t,0}D(-\Im) + MW_{t,0}D(\Im) \tag{27}$$

where

$$t = \frac{1}{\xi}\left(\eta - 1 + \frac{\beta^2}{\beta_1^2} - \frac{P}{2\mu_L} - \frac{H^2}{4\pi\eta}\right) \tag{28}$$

Since surface waves decay very rapidly with increase in depth which means $z \to \infty \Rightarrow (\Im \to -\infty)$, we get from Eq. (27)

$$D(\Im) = LW_{-t,0}D(-\Im) \tag{29}$$

The solution of Eq. (23) is given by

$$q = \frac{LW_{-t,0}\left[-\frac{4\eta}{\xi}\left(1 - \frac{P}{2\mu_L\eta} - \frac{H^2}{4\pi\eta} - \frac{\xi zk}{2\eta}\right)\right]}{\left(1 - \frac{P}{2\mu_L\eta} - \frac{H^2}{4\pi\eta} - \frac{\xi zk}{2\eta}\right)} \tag{30}$$

Using Eq. (21) and Eq. (30), the solution of Eq. (14) is:

$$v = \frac{LW_{-t,0}\left[-\frac{4\eta}{\xi}\left(1 - \frac{P}{2\mu_L\eta} - \frac{H^2}{4\pi\eta} - \frac{\xi zk}{2\eta}\right)\right]}{\sqrt{\left(1 - \frac{P}{2\mu_L\eta} - \frac{H^2}{4\pi\eta} - \frac{\xi zk}{2\eta}\right)}} J_1(kr)e^{i\Omega t} \tag{31}$$

5 BOUNDARY CONDITIONS AND FREQUENCY EQUATION

There are two boundary conditions

(1) For rigid boundary,

$$v = 0 \quad at \quad z = 0\,, \tag{32}$$

(2) For traction free boundary,

$$\mu_T \frac{\partial v}{\partial z} = 0 \quad at \quad z = 0\,, \tag{33}$$

Case.1

Using the boundary condition (1) in Eq. (31) and neglecting higher terms in Whittaker function, we get the velocity equation as:

$$\frac{\overline{\beta}^2}{\beta_1^2}=\frac{\left(\xi^2+\left(2-\xi\right)2\eta-4\eta^2\right)-2\left(\left(2-\xi\right)-4\eta\right)\left(\frac{P}{2\,\mu_L}+\frac{H^2}{4\pi}\right)-4\left(\frac{P}{2\,\mu_L}+\frac{H^2}{4\pi}\right)^2}{4\left(\eta-\frac{P}{2\,\mu_L}-\frac{H^2}{4\pi}\right)} \tag{34}$$

Case-2

Using the boundary condition (2) in Eq. (31), we get

$$\left(\frac{d}{dz}\left\{w_{-t,0}\left[-\frac{4\eta}{\xi}\left(1-\frac{P}{2\mu_L\,\eta}-\frac{H^2}{4\pi\eta}-\frac{\xi zk}{2\eta}\right)\right]\right\}\right)_{z=0}+\frac{\xi k}{4\eta}\frac{1}{\left(1-\frac{P}{2\mu_L\,\eta}-\frac{H^2}{4\pi\eta}\right)}\left(w_{-t,0}\left[-\frac{4\eta}{G}\left(1-\frac{P}{2\mu_L\,\eta}-\frac{H^2}{4\pi\eta}-\frac{\xi zk}{2\eta}\right)\right]\right)_{z=0}=0 \tag{35}$$

Neglecting higher terms of Eq. (35), we get the velocity equation as:

$$\frac{\overline{\beta}^2}{\beta_1^2}=\frac{\left\{\xi+\left(1-\xi\right)2\eta-2\eta^2\right\}-2\left\{\left(1-\xi\right)-2\eta\right\}\left(\frac{P}{2\mu_L}+\frac{H^2}{4\pi}\right)-2\left(\frac{P}{2\mu_L}+\frac{H^2}{4\pi}\right)^2}{\xi+2\left(\eta-\frac{P}{2\mu_L}-\frac{H^2}{4\pi}\right)} \tag{36}$$

6 DISCUSSION

6.1 In the absence of magnetic field, i.e.; H = 0

The velocity equation of fiber reinforced half space is for rigid boundary given by from Eq. (34)

$$\frac{\overline{\beta}^2}{\beta_1^2}=\frac{\left(\xi^2+\left(2-\xi\right)2\eta-4\eta^2\right)-2\left(\left(2-\xi\right)-4\eta\right)\left(\frac{P}{2\,\mu_L}\right)-4\left(\frac{P}{2\,\mu_L}\right)^2}{4\left(\eta-\frac{P}{2\,\mu_L}\right)} \tag{37}$$

The torsional surface wave exists. The velocity equation of fiber reinforced half space is for traction free boundary given by from Eq. (36)

$$\frac{\overline{\beta}^2}{\beta_1^2}=\frac{\left\{\xi+\left(1-\xi\right)2\eta-2\eta^2\right\}-2\left\{\left(1-\xi\right)-2\eta\right\}\left(\frac{P}{2\mu_L}\right)-2\left(\frac{P}{2\mu_L}\right)^2}{\xi+2\left(\eta-\frac{P}{2\mu_L}\right)} \tag{38}$$

The torsional surface wave exists.

6.2 In the absence of initial stress, i.e.; P = 0

The velocity equation of fiber reinforced half space is for rigid boundary given by from Eq. (34)

$$\frac{\overline{\beta}^2}{\beta_1^2}=\frac{\left(\xi^2+\left(2-\xi\right)2\eta-4\eta^2\right)-2\left(\left(2-\xi\right)-4\eta\right)\left(\frac{H^2}{4\pi}\right)-4\left(\frac{H^2}{4\pi}\right)^2}{4\left(\eta-\frac{H^2}{4\pi}\right)} \tag{39}$$

The torsional surface wave exists. The velocity equation of fiber reinforced half space is for traction free boundary given by from Eq. (36)

$$\frac{\overline{\beta}^2}{\beta_1{}^2}=\frac{\left\{\xi+\left(1-\xi\right)2\eta-2\eta^2\right\}-2\left\{\left(1-\xi\right)-2\eta\right\}\left(\frac{H^2}{4\pi}\right)-2\left(\frac{H^2}{4\pi}\right)^2}{\xi+2\left(\eta-\frac{H^2}{4\pi}\right)} \tag{40}$$

The torsional surface wave exists.

6.3 In the absence of magnetic field and initial stress, i.e.; H=0, P = 0

The velocity equation of fiber reinforced half space is for rigid boundary given by from Eq. (34)

$$\frac{\overline{\beta}^2}{\beta_1^2}=\frac{\left(\xi^2+\left(2-\xi\right)2\eta-4\eta^2\right)}{4\eta}=\frac{\xi}{2}\left(\frac{\xi}{\eta}-1\right)+1-\xi \tag{41}$$

The torsional surface wave exists. The velocity equation of fiber reinforced half space is for traction free boundary given by from Eq. (36)

$$\frac{\overline{\beta}^2}{\beta_1{}^2}=\frac{\left\{\xi+\left(1-\xi\right)2\eta-2\eta^2\right\}-2\left\{\left(1-\xi\right)-2\eta\right\}}{\xi+2\left(\eta\right)} \tag{42}$$

The torsional surface wave exists.

6.4 In the absence of gravity, i.e.; ξ =0

The velocity equation of fiber reinforced half space is for rigid boundary as well as for free boundary is same (from Eq. (34) and Eq. (36)) i.e.

$$\frac{\overline{\beta}^2}{\beta_1{}^2}=\frac{\left(\eta-\eta^2\right)-\left(\frac{P}{2\mu_L}+\frac{H^2}{4\pi}\right)-\left(\frac{P}{2\mu_L}+\frac{H^2}{4\pi}\right)^2}{\left(\eta-\frac{P}{2\mu_L}-\frac{H^2}{4\pi}\right)} \tag{43}$$

The torsional surface wave exists.

6.5 In the absence of gravity, stress and magnetic field i.e. ; ξ =0, P=0 and H=0

$$\frac{\overline{\beta}^2}{\beta_1^2}=\frac{\eta-\eta^2}{\eta}=1-\eta \tag{44}$$

The torsional surface wave does not exist if $\mu_L=\mu_T$.

6.6 If the medium is homogeneous and isotropic, then

$$\alpha = \beta = 0 \qquad and \qquad \eta = \frac{\mu_T}{\mu_L} = 1, \tag{45}$$

All the results reduce to the classical isotropic results when the anisotropic parameters for the fiber reinforced medium tend to zero. The torsional surface wave does not exist. If η is less than 1 then torsional waves < shear waves in reinforced medium.

7 NUMERICAL VALUES AND RESULTS

Numerical values have been calculated for Eq. (34) and Eq. (36) for rigid boundary and for traction free boundary using MATLAB. It is quite clear from the diagram that torsional waves are more stable for traction free boundary as compared to rigid boundary. All the curves are obtained by taking the expansion of Whittaker function up to linear term. [14] Taken the case of propagation of torsional surface waves in gravitating anisotropic porous half-space with rigid boundary and study the variation of gravity parameter with no magnetic field but in our study magneto parameter is fixed at constant value. When, the boundary is rigid, Figs. (2-4) show the change in phase velocity with depth in the reinforce medium for different values of stress and magnetic field. It is also seen the complex values are obtained when we put gravity and initial stress equal to zero, we do not obtain curves, it implies that torsional waves does not exist for that case because complex values are obtained. For $\mu_L = \mu_T$, the increase in value of the gravity parameter means decrease in value of phase velocity as obtained in all Figs. (2-4). Also, stress parameter plays very important role in the fiber reinforce medium, in Figs. (2-4), decrease in phase velocity with the increase in stress parameter. When, the boundary is traction free, Figs. (5-7) show the change in phase velocity with depth in the reinforce medium for different values of stress and magnetic field. If we compare these curves with the curves obtained for rigid boundary, the curves are not stable. It means that the torsional waves are less stable in traction free surface. It has been observed that when the value of gravity factor is 0.8, the stability in the curves increases.

Table 1
Material properties

	Z=0	b= $H^2/4\pi$ (Gauss)	a= $P/2\mu_L$
For Rigid Boundary	v= 0	0.3	0.2,0.4,0.6
For Traction free Boundary	$\mu_T \frac{\partial v}{\partial z} = 0$	0.3	0.2,0.4,0.6

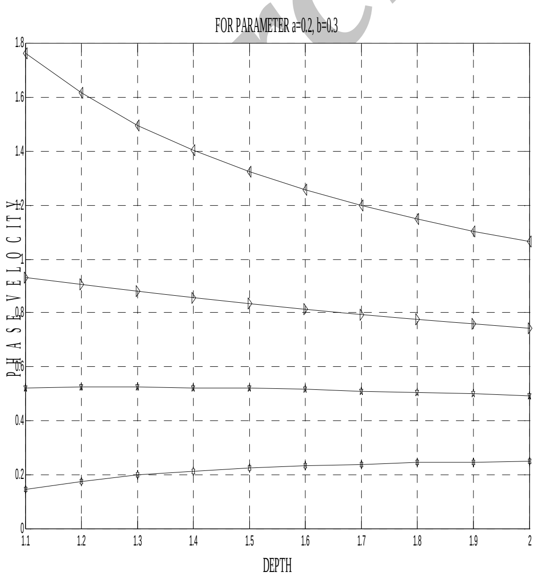

Fig. 2
Velocities of torsional surface waves showing the effect of initial stress, gravity and magnetic field in its propagation with parameters a= $P / 2\mu_L$ =0.2, b= $H^2 / 4\pi$ =0.3.

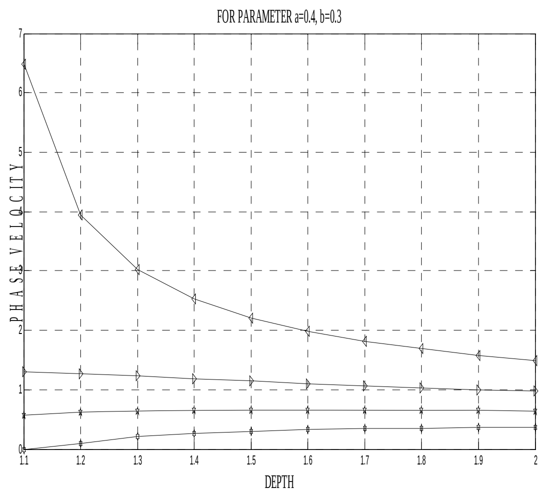

Fig.3
Velocities of torsional surface waves showing the effect of initial stress, gravity and magnetic field in its propagation with parameters a= $P/2\mu_L$ =0.4, b= $H^2/4\pi\eta$ =0.3.

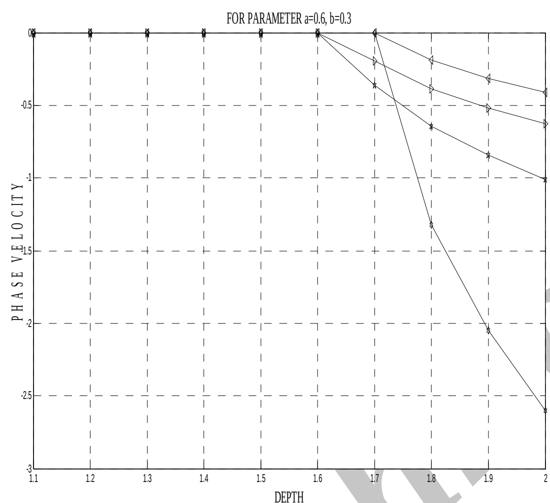

Fig.4
Velocities of torsional surface waves showing the effect of initial stress, gravity and magnetic field in its propagation (taking the expansion of Whittaker function up to linear term.) with parameters a= $P/2\mu_L$ =0.6, b= $H^2/4\pi$ =0.3.

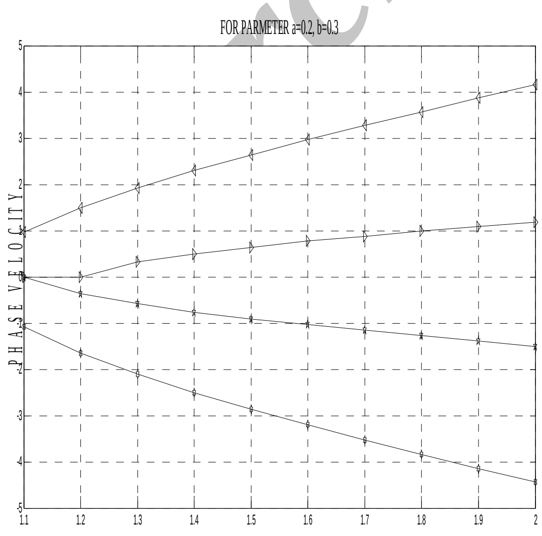

Fig.5
Velocities of torsional surface waves showing the effect of initial stress, gravity and magnetic field in its propagation (taking the expansion of Whittaker function up to linear term.) with parameters a= $P/2\mu_L$ =0.2, b= $H^2/4\pi$ =0.3.

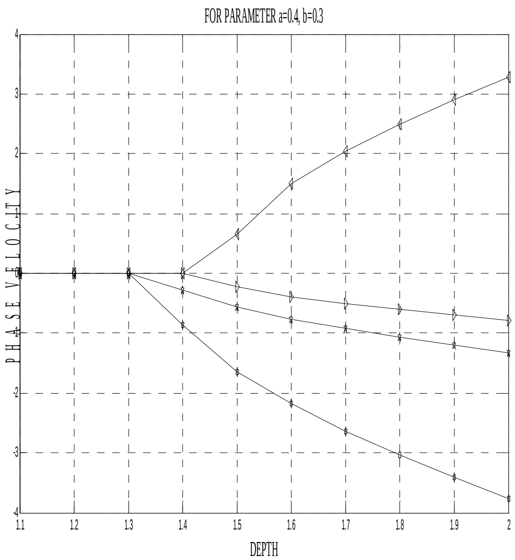

Fig.6
Velocities of torsional surface waves showing the effect of initial stress, gravity and magnetic field in its propagation (taking the expansion of Whittaker function up to linear term.) with parameters a= $P/2\mu_L$ =0.4, b= $H^2/4\pi$ =0.3.

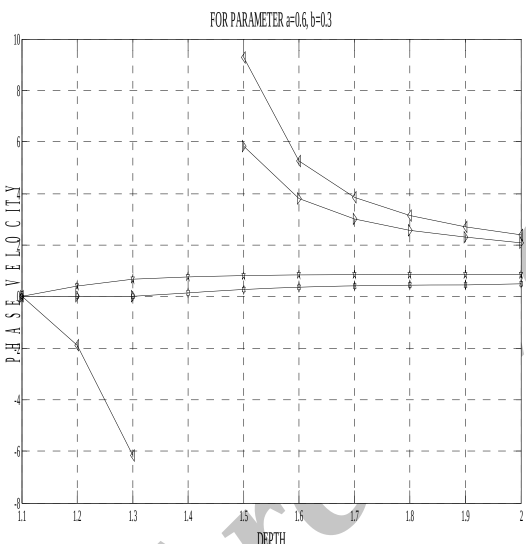

Fig.7
Velocities of torsional surface waves showing the effect of initial stress, gravity and magnetic field in its propagation (taking the expansion of Whittaker function up to linear term.) with parameters a= $P/2\mu_L$ =0.6, b= $H^2/4\pi$ =0.3.

8 CONCLUSIONS

A. The torsional surface wave exists in presence of gravity, stress and magnetic field.
B. The torsional surface wave exists for absence of gravity, stress and magnetic field.
C. The torsional surface wave does not exist for the condition $\mu_L = \mu_T$, gravity= 0, stress= 0 and magnetic field= 0.
D. Phase velocity increases with increase in initial stress and decreases with increase in gravity below certain value of α and β.
E. It has also been noticed that the medium is stable for higher value of gravity and lower value of initial stress.
F. Also the stability of the medium is more for traction free boundary in comparison to rigid boundary for higher values of gravity.

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