

# Wave Propagation at an Interface of Elastic and Microstretch Thermoelastic Solids with Microtemperatures

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## ABSTRACT

In the present paper, the problem of reflection and transmission of waves at an interface of elastic and microstretch thermoelastic solids with microtemperatures has been studied. The amplitude ratios of various reflected and transmitted waves are functions of angle of incidence and frequency of incident wave. The expressions of amplitude ratios have been computed numerically for a particular model. The variations of amplitude ratios with angle of incidence are shown graphically to depict the effect of microrotation. Some particular cases of interest have been also deduced.

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## 1 INTRODUCTION

ERINGEN [1, 2] developed the theory of micromorphic bodies. The theory of microstretch elastic bodies developed by Eringen [3] is a generalization of the micropolar theory. Eringen [4] also developed the theory of thermomicrostretch elastic solids. The particles of microstretch materials have seven degree of freedom: three displacements, three microrotations and one microstretch. A microstretch continuum can model composite materials reinforced with chopped elastic fibres and various porous solids. The material points of these bodies can stretch and contract independently of their translations and rotations.

Grot [5] established a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. The Clausius–Duhem inequality is modified to include microtemperatures and the first-order moment of the energy equations are added to the usual balance laws of a continuum with microstructure.

Riha [6] presented a study of heat conduction in materials with microtemperatures. Experimental data for the silicone rubber containing spherical aluminium particles and for human blood were found to conform closely to predicted theoretical thermal conductivity. The linear theory of thermoelasticity with microtemperatures for materials with inner structure whose particles possess microtemperatures, in addition to the classical displacement and temperature fields, was constructed by Iesan and Quintanilla [7]. Various investigators have studied different types of problems in microstretch thermoelastic medium notable among them are Ciarletta and Scalia [8], Iesan and Quintanilla [9], Othman et al [10], Passarella and Tibullo [11], Marin [12,13], Kumar et al [14], Othman and Lofty [15,16], Kumar and Rupender [17,18], Shaw and Mukhopadhyay [19].

Iesan [20] discussed the theory of micromorphic elastic solids with microtemperatures, Iesan and Quintanilla [21] discussed various problems using thermoelasticity with microtemperatures. Exponential stability in thermoelasticity with microtemperatures was studied by Casas and Quintanilla [22]. Scalia and Svandze [23]

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presented the solutions of the theory of thermoelasticity with microtemperatures, Iesan [24] discussed thermoelasticity of bodies with microstructure and microtemperatures. Aouadi [25] studied some theorems in the isotropic theory of microstretch thermoelasticity with microtemperatures. Scalia et al [26] discussed basic theorems in the equilibrium theory of thermoelasticity with microtemperatures. The growth and continuous dependence in thermoelasticity with microtemperatures was discussed by Quintanilla [27]. Steeb et al [28] studied time harmonic waves in thermoelastic material with microtemperatures. Chirita et al [29] studied the theory of thermoelasticity with microtemperatures.

The main objective of the present investigation is to study the wave propagation at the boundary between a microstretch thermoelastic half-space with microtemperatures and elastic solid half-space. The amplitude ratios of longitudinal displacement wave (LD-wave), thermal wave (T-wave), microstretch wave (LM-wave), microtemperature wave (LT-wave), coupled transverse displacement, microrotational wave and microtemperature wave namely (CD-I wave, CD-II wave and CD-III wave) and transmitted longitudinal wave (P-wave) and transverse wave (SV-wave) have been obtained and plotted graphically with angle of incidence. Some special cases of interest have been deduced from the present investigation.

## 2 BASIC EQUATIONS

Following Eringen [5] and Iesan [30], the field equations and constitutive relations for a homogeneous, isotropic microstretch thermoelastic solid with microtemperatures without body forces, body couples, stretch force, heat sources and first heat source moment, can be written as:

$$(\lambda + 2\mu + K)\nabla(\nabla\mathbf{u}) - (\mu + K)\nabla \times (\nabla \times \mathbf{u}) + K(\nabla \times \boldsymbol{\phi}) + \lambda_0 \nabla \phi^* - \nu \nabla T = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \quad (1)$$

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot \boldsymbol{\phi}) - \gamma \nabla \times (\nabla \times \boldsymbol{\phi}) + K(\nabla \times \mathbf{u}) - 2K\boldsymbol{\phi} - \mu_1(\nabla \times \mathbf{w}) = \rho j \frac{\partial^2 \boldsymbol{\phi}}{\partial t^2}, \quad (2)$$

$$\alpha_0 \nabla^2 \phi^* + \nu_1 T - \lambda_1 \phi^* - \lambda_0(\nabla \cdot \mathbf{u}) - \mu_2(\nabla \cdot \mathbf{w}) = \rho \frac{j_0}{2} \frac{\partial^2 \phi^*}{\partial t^2}, \quad (3)$$

$$K^* \nabla^2 T - \rho c^* \frac{\partial T}{\partial t} - \nu T_0 \frac{\partial \phi^*}{\partial t} - \nu T_0(\nabla \cdot \mathbf{u}) + k_1(\nabla \cdot \mathbf{w}) = 0, \quad (4)$$

$$k_6 \nabla^2 \mathbf{w} + (k_4 + k_5)\nabla(\nabla \cdot \mathbf{w}) + \mu_1 \frac{\partial}{\partial t}(\nabla \times \boldsymbol{\phi}) - \mu_2 \frac{\partial}{\partial t}(\nabla \phi^*) - b \frac{\partial \mathbf{w}}{\partial t} - k_2 \mathbf{w} - k_3 \nabla T = 0, \quad (5)$$

and the constitutive relations are

$$t_{ij} = \lambda u_{r,r} \delta_{ij} + \mu (u_{i,j} + u_{j,i}) + K(u_{j,i} - \epsilon_{ijr} \phi_r) - \nu T \delta_{ij} + \lambda_0 \phi^* \delta_{ij}, \quad (6)$$

$$m_{ij} = \alpha \phi_{r,r} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i} + b_0 \epsilon_{mji} \phi_{m}^*, \quad (7)$$

$$\lambda_i^* = \alpha_0 \phi_{,i}^* + b_0 \epsilon_{ijm} \phi_{j,m}^*, \quad (8)$$

$$q_{ij} = -k_4 w_{r,r} \delta_{ij} - k_5 w_{i,j} - k_6 w_{j,i}, \quad i, j, m = 1, 2, 3 \quad (9)$$

where  $K, \alpha, \beta, \gamma, \lambda, \mu, \alpha_0, \lambda_0, \lambda_1, \mu_1, \mu_2, j_0, k_i$  ( $i = 1, \dots, 6$ ) are constitutive coefficients.  $t_{ij}$  and  $m_{ij}$  are the components of stress tensor and couple stress tensor,  $\lambda_i^*$  is the microstress tensor,  $q_{ij}$  is the first heat flux moment tensor,  $\mathbf{u}$  and  $\boldsymbol{\phi}$  are the displacement and microrotation vectors,  $\mathbf{w}$  is the microtemperature vector and  $\phi^*$  is the microstretch scalar,  $\rho$  is the density,  $j$  is the microinertia,  $c^*$  is the specific heat at constant strain,  $K^*$  is the thermal conductivity,  $T$  is the thermodynamic temperature,  $T_0$  is the reference temperature,  $\nu = (3\lambda + 2\mu + K)\alpha_{T_1}$ ,  $\nu_1 = (3\lambda + 2\mu + K)\alpha_{T_2}$ , where  $\alpha_{T_1}$ ,  $\alpha_{T_2}$  are the coefficients of linear thermal expansion.

Following Bullen [31], the equation of motion and constitutive relation in an isotropic elastic solid are given by:

$$(\lambda^e + \mu^e)\nabla(\nabla\mathbf{u}^e) + \mu^e\nabla^2\mathbf{u}^e = \rho^e \frac{\partial^2\mathbf{u}^e}{\partial t^2}, \quad (10)$$

$$t_{ij}^e = \lambda^e u_{r,r}^e \delta_{ij} + \mu^e (u_{i,j}^e + u_{j,i}^e), \quad (11)$$

where  $\lambda^e$ ,  $\mu^e$  are Lamé's constants,  $\mathbf{u}^e$  is the displacement vector,  $\rho^e$  is density corresponding to isotropic elastic solid.

### 3 FORMULATION OF THE PROBLEM AND SOLUTION

We consider an isotropic elastic solid half-space lying over a homogeneous isotropic, microstretch thermoelastic half-space with microtemperatures. The origin of the Cartesian coordinate system  $Ox_1x_2x_3$  is taken at any point on the plane surface (interface) and  $x_3$ -axis points vertically downwards into the microstretch thermoelastic half-space with microtemperatures is introduced. The region  $x_3 \geq 0$  is occupied by microstretch thermoelastic half-space with microtemperatures (medium  $M_1$ ) and an elastic solid half-space occupies the region  $x_3 \leq 0$  (medium  $M_2$ ) as shown in Fig. 1. We consider plane waves in the  $x_1x_3$ -plane with wave front parallel to the  $x_2$ -axis.

For two dimensional problem, we take

$$\begin{aligned} \mathbf{u} &= (u_1(x_1, x_3), 0, u_3(x_1, x_3)), \quad \mathbf{w} = (w_1(x_1, x_3), 0, w_3(x_1, x_3)), \quad \boldsymbol{\phi} = (0, \phi_2(x_1, x_3), 0) \\ \mathbf{u}^e &= (u_1^e(x_1, x_3), 0, u_3^e(x_1, x_3)) \end{aligned} \quad (12)$$

We define the dimensionless quantities as:

$$\begin{aligned} x_1' &= \frac{x_1}{L}, \quad x_3' = \frac{x_3}{L}, \quad (u_1', u_3') = (u_1, u_3) \frac{1}{L}, \quad (u_1^e, u_3^e) = (u_1^e, u_3^e) \frac{1}{L}, \quad \phi_2' = \phi_2, \\ \phi^* &= \phi^*, \quad t_{ij}' = \frac{1}{\sqrt{T_0}} t_{ij}, \quad m_{ij}' = \frac{1}{L\sqrt{T_0}} m_{ij}, \quad \lambda_i^* = \frac{1}{L\sqrt{T_0}} \lambda_i, \quad q_{ij}' = \frac{1}{Lc_1\sqrt{T_0}} q_{ij}, \\ t_{ij}^e &= \frac{1}{\sqrt{T_0}} t_{ij}^e, \quad t' = \frac{c_1}{L} t, \quad T' = \frac{T}{T_0}, \quad w_i' = Lw_i, \quad L = \left(\frac{b}{\rho c^* T_0}\right)^{\frac{1}{2}}, \quad c_1^2 = \frac{\lambda + 2\mu + K}{\rho} \end{aligned} \quad (13)$$

The relations connecting displacement components and microtemperature components to the potential functions in dimensionless form are expressed as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}, \quad w_1 = \frac{\partial\phi_1}{\partial x_1} - \frac{\partial\psi_1}{\partial x_3}, \quad w_3 = \frac{\partial\phi_1}{\partial x_3} + \frac{\partial\psi_1}{\partial x_1}, \quad u_1^e = \frac{\partial\phi^e}{\partial x_1} - \frac{\partial\psi^e}{\partial x_3}, \quad u_3^e = \frac{\partial\phi^e}{\partial x_3} + \frac{\partial\psi^e}{\partial x_1} \quad (14)$$

where the primes have been suppressed. Using the dimensionless quantities given by Eq. (13) in Eqs. (1)-(5),(10),(11) and with the aid of Eq. (12) and (14), we obtain the following equations

$$[(a_1 + 1)\nabla^2 - a_5 \frac{\partial^2}{\partial t^2}]\phi + a_3\phi^* - a_4T = 0, \quad (15)$$

$$(\nabla^2 - a_5 \frac{\partial^2}{\partial t^2})\psi + a_2\phi_2 = 0, \quad (16)$$

$$(\nabla^2 - 2a_6 - a_8 \frac{\partial^2}{\partial t^2})\phi_2 - a_6 \nabla^2 \psi + a_7 \nabla^2 \psi_1 = 0, \quad (17)$$

$$(\nabla^2 - a_{10} - a_{13} \frac{\partial^2}{\partial t^2})\phi^* - a_{11} \nabla^2 \phi - a_{12} \nabla^2 \phi_1 + a_9 T = 0, \quad (18)$$

$$(\nabla^2 - a_{14} \frac{\partial}{\partial t})T - a_{15} \frac{\partial \phi^*}{\partial t} - a_{16} \nabla^2 \phi + a_{17} \nabla^2 \phi_1 = 0, \quad (19)$$

$$[\nabla^2(1 + a_{18}) - a_{21} - a_{23} \frac{\partial}{\partial t}]\phi_1 - a_{20} \frac{\partial \phi^*}{\partial t} - a_{22} T = 0, \quad (20)$$

$$(\nabla^2 - a_{21} - a_{23} \frac{\partial}{\partial t})\psi_1 + a_{19} \frac{\partial \phi_2}{\partial t} = 0, \quad (21)$$

$$(\nabla^2 - \frac{\partial^2}{\partial t^2})\phi^e = 0, \quad (22)$$

$$(\nabla^2 - \bar{a}_1 \frac{\partial^2}{\partial t^2})\psi^e = 0, \quad (23)$$

where

$$\begin{aligned} a_1 &= \frac{\lambda + \mu}{\mu + K}, & a_2 &= \frac{K}{\mu + K}, & a_3 &= \frac{\lambda_0}{\mu + K}, & a_4 &= \frac{\nu T_0}{\mu + K}, & a_5 &= \frac{\rho c_1^2}{\mu + K}, & a_6 &= \frac{KL^2}{\gamma}, \\ a_7 &= \frac{\mu_1}{\gamma}, & a_8 &= \frac{\rho j c_1^2}{\gamma}, & a_9 &= \frac{\nu T_0 L^2}{\alpha_0}, & a_{10} &= \frac{\lambda_1 L^2}{\alpha_0}, & a_{11} &= \frac{\lambda_0 L^2}{\alpha_0}, & a_{12} &= \frac{\mu_2}{\alpha_0}, & a_{13} &= \frac{\rho j_0 c_1^2}{2\alpha_0}, \\ a_{14} &= \frac{\rho c^* c_1 L}{K^*}, & a_{15} &= \frac{\nu_1 c_1 L}{K^*}, & a_{16} &= \frac{\nu c_1 L}{K^*}, & a_{17} &= \frac{k_1}{K^* T_0}, & a_{18} &= \frac{k_4 + k_5}{k_6}, & a_{19} &= \frac{\mu_1 c_1 L}{k_6}, \\ a_{20} &= \frac{\mu_2 c_1 L}{k_6}, & a_{21} &= \frac{k_2 L^2}{k_6}, & a_{22} &= \frac{k_3 T_0 L^2}{k_6}, & a_{23} &= \frac{bc_1 L}{k_6}, & \bar{a}_1 &= \frac{\lambda^e + \mu^e}{\mu^e}, & \bar{a}_1 &= \frac{\rho^e c_1^2}{\mu^e} \end{aligned}$$

The boundary conditions at the interface  $x_3 = 0$  are given as:

$$\begin{aligned} t_{33} &= t_{33}^e, & t_{31} &= t_{31}^e, & m_{32} &= 0, & \lambda_3^* &= 0, & q_{33} &= 0, & q_{31} &= 0, & u_3 &= u_3^e, & u_1 &= u_1^e, \\ \frac{\partial T}{\partial x_3} &= 0 \end{aligned} \quad (24)$$

#### 4 REFLECTION AND TRANSMISSION

We consider longitudinal displacement wave (LD-wave), thermal wave (T-wave), microstretch wave (LM-wave), microtemperature wave (LT-wave), coupled transverse displacement, microrotational wave and microtemperature wave namely (CD-I wave, CD-II wave and CD-III wave) propagating through medium  $M_1$  and incident at the plane  $x_3 = 0$  with its direction of propagation with angle  $\theta_0$  normal to the surface. Corresponding to each incident wave, we get reflected LD-wave, T-wave, LM-wave, LT-wave, CD-I, CD-II and CD-III waves in medium  $M_1$  and transmitted P- wave and SV-wave in medium  $M_2$  as shown in Fig. 1.

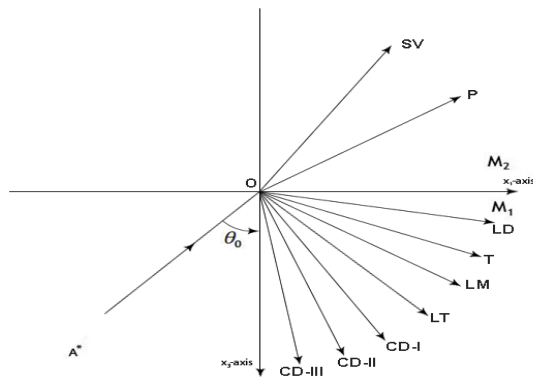


Fig. 1  
Geometry of the problem.

In order to solve the Eqs. (15)-(23), we assume the solutions of the form

$$\{\phi, T, \phi^*, \phi_1, \psi, \phi_2, \psi_1, \phi^e, \psi^e\} = \{\bar{\phi}, \bar{T}, \bar{\phi}^*, \bar{\phi}_1, \bar{\psi}, \bar{\phi}_2, \bar{\psi}_1, \bar{\phi}^e, \bar{\psi}^e\} e^{i\{k(x_1 \sin \theta - x_3 \cos \theta) - \omega t\}} \quad (25)$$

where  $k$  is the wave number and  $\omega$  is the angular frequency and  $\bar{\phi}, \bar{T}, \bar{\phi}^*, \bar{\phi}_1, \bar{\psi}, \bar{\phi}_2, \bar{\psi}_1, \bar{\phi}^e, \bar{\psi}^e$  are arbitrary constants. Making use of Eq. (25) in Eqs (15)-(23), yield

$$B_1 V^8 + B_2 V^6 + B_3 V^4 + B_4 V^2 + B_5 = 0, \quad (26)$$

$$D_1 V^6 + D_2 V^4 + D_3 V^2 + D_4 = 0, \quad (27)$$

where

$$B_1 = i a_{21}^* \frac{a_5}{\omega^5} (a_{13}^* a_{14}^* - a_9), \quad B_2 = -\frac{a_5}{\omega^2} a_{13}^* \delta_3 + i \frac{a_5}{\omega^3} \delta_4 + \frac{a_{12}}{\omega^2} \delta_5 - \frac{a_3}{\omega^5} a_{21}^* \delta_6 + \frac{a_4}{\omega^4} a_{21}^* (a_{13}^* a_{16} - a_{11} a_{15} \frac{1}{\omega}),$$

$$B_3 = -\frac{a_1^*}{\omega^2} a_{13}^* \delta_3 - i \frac{a_1^*}{\omega^3} \delta_4 - \frac{a_1^* a_{12}}{\omega^2} \delta_5 + a_5 (\delta_3 + \delta_7) + \frac{a_3 a_{16}}{\omega^4} (a_{12} a_{22} - a_9 a_{18}^*) + \frac{a_3 a_{11}}{\omega^2} \delta_3 - a_4 \delta_8,$$

$$B_4 = \frac{a_{18}^*}{\omega^2} (a_4 a_{16} - a_3 a_{11}) - a_5 a_{18}^* - \frac{i a_{11}^*}{\omega} (a_{12} a_{20} + a_{14} a_{18}^*) - \frac{a_1^* \delta_1}{\omega^2} - \frac{a_1^* a_{13}^* a_{18}^*}{\omega^2}, \quad B_5 = a_1 a_{18}^*,$$

$$a_1^* = a_1 + 1, \quad a_{13}^* = a_{13} \omega^2 - a_{10}, \quad a_{14}^* = a_{14} i \omega, \quad a_{15}^* = a_{15} i \omega, \quad a_{18}^* = a_{18} + 1, \quad a_{20}^* = a_{20} i \omega, \quad a_{21}^* = a_{23} i \omega^2 - a_{21},$$

$$\delta_1 = -a_{22} a_{17} + a_{21}^*, \quad \delta_2 = a_{15} a_{18}^* + a_{17} a_{20}, \quad \delta_3 = (a_{14} a_{18}^* \frac{1}{\omega} + \frac{\delta_1}{\omega^2}), \quad \delta_4 = (a_9 \delta_2 - a_{14} a_{21}^*), \quad \delta_5 = (a_{14} a_{20} - a_{15} a_{22} \frac{1}{\omega}),$$

$$\delta_6 = (a_{11} a_{14} i - a_9 a_{16}), \quad \delta_7 = (a_{13} a_{18}^* \frac{1}{\omega^2} + a_{12} a_{20} \frac{1}{\omega}), \quad \delta_8 = (a_{13}^* a_{18}^* + a_{21}^*) \frac{a_{16}}{\omega^4} + (a_{12} a_{16} a_{20} - a_{11} \delta_2) \frac{1}{\omega^3},$$

$$D_1 = a_5 \left( \frac{2a_6}{\omega^2} - a_8 \right) \left( \frac{a_{21}}{\omega^2} + \frac{i a_{23}}{\omega} \right), \quad D_2 = -\left( \frac{2a_6}{\omega^2} - a_8 \right) \left( \frac{a_{21}}{\omega^2} - \frac{i a_{23}}{\omega} \right) + i a_5 \left( \frac{-a_{23} - a_7 a_{19}}{\omega} \right) - a_5 a_8 + \frac{a_2 a_6}{\omega^3} (-i a_{23} + \frac{a_{21}}{\omega}) + a_5 \left( \frac{a_{21} + 2a_6}{\omega^2} \right),$$

$$D_3 = \frac{a_2 a_6}{\omega^2} - \left( \frac{a_{21}}{\omega^2} - \frac{i a_{23}}{\omega} \right) - \left( \frac{2a_6 - i a_7 a_{19} \omega}{\omega^2} \right) + a_5 + a_8, \quad D_4 = -1$$

and

$$V^2 = \frac{\omega^2}{k^2}$$

Eq. (26) is biquadratic in  $V^2$ , therefore the roots of this equation gives four values of  $V^2$ . Corresponding to each value of  $V^2$ , there exist four types of waves in medium  $M_1$  in decreasing order of their velocities, namely LD-wave, T-wave, LM-wave, LT-wave. Eq. (27) is cubic in  $V^2$ , corresponding to each value of  $V^2$ , there exist three types of waves in medium  $M_1$ , namely CD-I wave, CD-II wave and CD-III wave. Let  $V_1, V_2, V_3, V_4$  are the velocities of reflected LD-wave, T-wave, LM-wave, LT-wave and  $V_5, V_6, V_7$  are the velocities of reflected CD-I wave, CD-II wave, CD-III wave in medium  $M_1$ . Similarly,  $\bar{V}_1 = 1$  is the velocity of transmitted P-wave and  $\bar{V}_2 = \frac{1}{a_2}$  is the velocity of SV-wave in medium  $M_2$ .

In view of Eq. (25), the appropriate solutions of Eqs. (15)-(23) for medium  $M_1$  and medium  $M_2$  take the form  
Medium  $M_1$ :

$$\{\phi, T, \phi^*, \phi_1\} = \sum_{i=1}^4 \{1, l_i, g_i, f_i\} [S_{0i} e^{i k_i (x_1 \sin \theta_{0i} - x_3 \cos \theta_{0i}) - \omega_i t} + P_i], \quad (28)$$

$$\{\psi, \phi_2, \psi_1\} = \sum_{j=5}^7 \{1, n_j, m_j\} [S_{0j} e^{i k_j (x_1 \sin \theta_{0j} - x_3 \cos \theta_{0j}) - \omega_j t} + P_j], \quad (29)$$

where

$$g_i = \frac{[a_1 - a_5 V_i^2 - a_4 (a_{11} a_{17} + a_{12} a_{16})] [-a_{12} + i a_{14} a_{12} \frac{V_i^2}{\omega_i} + a_9 a_{17} \frac{V_i^2}{\omega_i^2}]}{a_3 a_{12} \frac{V_i^2}{\omega_i^2} (1 + \frac{V_i^2}{\omega_i^2}) + a_{17} \frac{V_i^2}{\omega_i^2} [(a_3 a_9 - a_4 a_{10}) \frac{V_i^2}{\omega_i^2} - a_4] + i a_4 a_{12} a_{15} \frac{V_i^4}{\omega_i^3} + a_4 a_{13} a_{17} \frac{V_i^4}{\omega_i^2}},$$

$$f_i = \frac{a_{11} a_{22} [\eta_1 \eta_2 - i a_9 a_{15} \frac{V_i^4}{\omega_i^3}] + [-a_9 a_{16} \frac{V_i^2}{\omega_i^2} - a_{11} \eta_2] [a_{22} \eta_1 + i a_9 a_{20} \frac{V_i^2}{\omega_i}]}{[a_9 \eta_3 + a_{21} a_{22}] [\eta_1 \eta_2 - i a_9 a_{15} \frac{V_i^4}{\omega_i^3}] - [a_{12} \eta_2 + a_9 a_{17} \frac{V_i^2}{\omega_i^2}] [a_{22} \eta_1 + i a_9 a_{20} \frac{V_i^2}{\omega_i}]},$$

$$\eta_1 = (-1 - a_{10} \frac{V_i^2}{\omega_i^2} + a_{13} V_i^2), \quad \eta_2 = (-1 + i a_{14} \frac{V_i^2}{\omega_i}), \quad \eta_3 = (-a_{18}^* - a_{21} \frac{V_i^2}{\omega_i^2} + i a_{23} \frac{V_i^2}{\omega_i}),$$

$$l_i = \frac{-i \omega_i (a_5 V_i^2 - a_1^*) [(-a_{17} a_{20} + a_{15} a_{21}) \frac{V_i^2}{\omega_i^2} - a_{15} \eta_3] - a_3 a_6 \eta_3}{-(-a_{17} a_{20} + a_{15} a_{21}) a_4 \frac{V_i^2}{\omega_i^2} - i a_4 a_{15} \eta_3 \frac{V_i^2}{\omega_i} + a_3 \eta_3 (-1 + i a_{14} \frac{V_i^2}{\omega_i}) - a_3 a_{17} a_{22} \frac{V_i^2}{\omega_i^2}},$$

$$m_j = \frac{-a_2 a_6 \frac{V_j^2}{\omega_j^2} + (-1 + a_3 V_j^2) (-1 - 2a_6 \frac{V_j^2}{\omega_j^2} + a_8 V_j^2) + a_7 a_{19} i \frac{V_j^2}{\omega_j}}{a_2 a_7 \frac{V_j^2}{\omega_j^2} (a_{21} \frac{V_j^2}{\omega_j^2} - i a_{23} \frac{V_j^2}{\omega_j})}, \quad n_j = \frac{a_6 (1 + a_{21} \frac{V_j^2}{\omega_j^2} - i a_{23} \frac{V_j^2}{\omega_j}) + i a_7 a_{19} \frac{\omega_j}{a_2} (-1 + a_3 V_j^2)}{(1 + 2a_6 \frac{V_j^2}{\omega_j^2} - a_8 V_j^2) (1 + a_{21} \frac{V_j^2}{\omega_j^2} - i a_{23} \frac{V_j^2}{\omega_j})},$$

and

$$P_i = S_i e^{i\{k_i(x_1 \sin \theta_i + x_3 \cos \theta_i) - \omega_i t\}}, \quad P_j = S_j e^{i\{k_j(x_1 \sin \theta_j + x_3 \cos \theta_j) - \omega_j t\}}$$

Medium  $M_2$  :

$$\bar{\phi} = \bar{S}_1 e^{i\{k_1(x_1 \sin \bar{\theta}_1 - x_3 \cos \bar{\theta}_1) - \bar{\omega}_1 t\}}, \quad (30)$$

$$\bar{\psi} = \bar{R}_1 e^{i\{k_2(x_1 \sin \bar{\theta}_2 - x_3 \cos \bar{\theta}_2) - \bar{\omega}_2 t\}}, \quad (31)$$

where  $S_{0i}$ ,  $S_{0j}$  are the amplitudes of incident (LD-wave, T-wave, LM-wave, LT-wave) and (CD-I, CD-II, CD-III) waves respectively.  $S_i$  and  $S_j$  are the amplitudes of reflected (LD-wave, T-wave, LM-wave, LT-wave) and (CD-I, CD-II, CD-III) waves and  $\bar{S}_1$ ,  $\bar{R}_1$  are the amplitudes of transmitted P-wave and SV-wave respectively.

We use the following extension of the Snell's law

$$\frac{\sin \theta_0}{V_0} = \frac{\sin \theta_1}{V_1} = \frac{\sin \theta_2}{V_2} = \frac{\sin \theta_3}{V_3} = \frac{\sin \theta_4}{V_4} = \frac{\sin \theta_5}{V_5} = \frac{\sin \theta_6}{V_6} = \frac{\sin \theta_7}{V_7} = \frac{\sin \bar{\theta}_1}{\bar{V}_1} = \frac{\sin \bar{\theta}_2}{\bar{V}_2} \quad (32)$$

where

$$V_j = \frac{\omega}{k_j}, \bar{V}_1 = \frac{\omega}{k_1}, \bar{V}_2 = \frac{\omega}{k_2} \quad (j=1, 2, 3, 4, 5, 6, 7) \text{ at } x_3 = 0 \quad (33)$$

Making use of Eqs. (28)-(31) in boundary conditions (24) and with the help of Eqs. (32) and (33), we obtain a system of nine non-homogeneous equations which can be written as:

$$\sum_{j=1}^9 a_{ij} Z_j = Y_i; \quad (i=1, 2, 3, 4, 5, 6, 7, 8, 9) \quad (34)$$

$$a_{1i} = -\left( d_1 \left( 1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0 \right) + d_2 \frac{V_i^2}{V_0^2} \sin^2 \theta_0 \right) \frac{\omega^2}{V_i^2} - l_i + d_3 g_i, \quad a_{1j} = (d_2 - d_1) \frac{\omega^2}{V_j V_0} \sin \theta_0 \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0},$$

$$a_{18} = \left[ \left( \bar{d}_1 \left( 1 - \frac{\bar{V}_1^2}{V_0^2} \sin^2 \theta_0 \right) + \bar{d}_2 \frac{\bar{V}_1^2}{V_0^2} \sin^2 \theta_0 \right) \frac{\omega^2}{\bar{V}_1^2} \right], \quad a_{19} = (\bar{d}_2 - \bar{d}_1) \frac{\omega^2}{\bar{V}_2 V_0} \sin \theta_0 \sqrt{1 - \frac{\bar{V}_2^2}{V_0^2} \sin^2 \theta_0},$$

$$a_{2i} = -(2d_4 + d_5) \frac{\omega^2}{V_i V_0} \sin \theta_0 \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0},$$

$$a_{2j} = d_4 \frac{\omega^2}{V_j^2} \left( 1 - 2 \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) + d_5 \frac{\omega^2}{V_j^2} \left( 1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) - d_5 n_j,$$

$$a_{28} = -2\bar{d}_3 \frac{\omega^2}{\bar{V}_1 V_0} \sin \theta_0 \sqrt{1 - \frac{\bar{V}_1^2}{V_0^2} \sin^2 \theta_0}, \quad a_{29} = -\left[ \bar{d}_3 \frac{\omega^2}{\bar{V}_2^2} \left( 1 - 2 \frac{\bar{V}_2^2}{V_0^2} \sin^2 \theta_0 \right) \right],$$

$$a_{3i} = t d_7 g_i \frac{\omega}{V_0} \sin \theta_0, \quad a_{3j} = u n_j \frac{\omega}{V_j} \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0}, \quad a_{38} = a_{39} = 0,$$

$$\begin{aligned}
a_{4i} &= \iota d_8 \frac{\omega}{V_i} \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0} g_i, & a_{4j} &= -\iota d_9 \frac{\omega}{V_0} \sin \theta_0 n_j, & a_{48} &= a_{49} = 0, \\
a_{5i} &= -\left( d_{10} + d_{11} \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0} \right) \frac{\omega^2}{V_i^2} f_i, & a_{5j} &= -d_{11} \frac{\omega^2}{V_j V_0} m_j \sin \theta_0 \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0}, & a_{58} &= a_{59} = 0, \\
a_{6i} &= -(d_{12} + d_{13}) f_i \frac{\omega^2}{V_i V_0} \sin \theta_0 \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0}, & a_{6j} &= (d_{13} (1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0) - d_{12} \frac{V_j^2}{V_0^2} \sin^2 \theta_0) m_j \frac{\omega^2}{V_j^2}, \\
a_{68} &= a_{69} = 0, \\
a_{7i} &= \iota \frac{\omega}{V_0} \sin \theta_0, & a_{7j} &= -\iota \frac{\omega}{V_j} \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0}, & a_{78} &= -\iota \frac{\omega}{V_0} \sin \theta_0, & a_{79} &= \iota \frac{\omega}{V_2} \sqrt{1 - \frac{V_2^2}{V_0^2} \sin^2 \theta_0}, \\
a_{8i} &= \iota \frac{\omega}{V_i} \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0}, & a_{8j} &= \iota \frac{\omega}{V_0} \sin \theta_0, & a_{88} &= \iota \frac{\omega}{V_1} \sqrt{1 - \frac{V_1^2}{V_0^2} \sin^2 \theta_0}, & a_{89} &= -\iota \frac{\omega}{V_0} \sin \theta_0, \\
a_{9i} &= \iota d_i \frac{\omega}{V_i} \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0}, & a_{9j} &= a_{98} = a_{99} = 0 & (i = 1, 2, 3, 4 \text{ and } j = 5, 6, 7)
\end{aligned} \tag{35}$$

where

$$\begin{aligned}
d_1 &= \frac{\rho c_1^2}{\nu T_0}, & d_2 &= \frac{\lambda}{\nu T_0}, & d_3 &= \frac{\lambda_0}{\nu T_0}, & d_4 &= \frac{\mu}{\nu T_0}, & d_5 &= \frac{K}{\nu T_0}, & d_6 &= \frac{\gamma}{L^2 \nu T_0}, & d_7 &= \frac{b_0}{L^2 \nu T_0}, \\
d_8 &= \frac{\alpha_0}{L^2 \nu T_0}, & d_9 &= \frac{b_0}{L^2 \nu T_0}, & d_{10} &= \frac{k_4}{L^3 c_1 \nu T_0}, & d_{11} &= \frac{k_5 + k_6}{L^3 c_1 \nu T_0}, & d_{12} &= \frac{k_5}{L^3 c_1 \nu T_0}, & d_{13} &= \frac{k_6}{L^3 c_1 \nu T_0}, \\
\bar{d}_1 &= \frac{\lambda^e + 2\mu^e}{\nu T_0}, & \bar{d}_2 &= \frac{\lambda^e}{\nu T_0}, & \bar{d}_3 &= \frac{\mu^e}{\nu T_0}
\end{aligned}$$

For incident LD-wave:

$$\begin{aligned}
A^* &= S_{01}, S_{02} = S_{03} = S_{04} = S_{05} = S_{06} = S_{07} = 0, & Y_1 &= -a_{11}, & Y_2 &= a_{21}, & Y_3 &= -a_{31}, & Y_4 &= a_{41}, \\
Y_5 &= -a_{51}, & Y_6 &= a_{61}, & Y_7 &= -a_{71}, & Y_8 &= a_{81}, & Y_9 &= a_{91}
\end{aligned}$$

For incident T-wave:

$$\begin{aligned}
A^* &= S_{02}, S_{01} = S_{03} = S_{04} = S_{05} = S_{06} = S_{07} = 0, & Y_1 &= -a_{12}, & Y_2 &= a_{22}, & Y_3 &= -a_{32}, & Y_4 &= a_{42}, \\
Y_5 &= -a_{52}, & Y_6 &= a_{62}, & Y_7 &= -a_{72}, & Y_8 &= a_{82}, & Y_9 &= a_{92}
\end{aligned}$$

For incident LM-wave:

$$\begin{aligned}
A^* &= S_{03}, S_{01} = S_{02} = S_{04} = T_{05} = T_{06} = T_{07} = 0, & Y_1 &= -a_{13}, & Y_2 &= a_{23}, & Y_3 &= -a_{33}, & Y_4 &= a_{43}, \\
Y_5 &= -a_{53}, & Y_6 &= a_{63}, & Y_7 &= -a_{73}, & Y_8 &= a_{83}, & Y_9 &= a_{93}
\end{aligned}$$

For incident LT-wave:

$$\begin{aligned}
A^* &= S_{04}, S_{01} = S_{02} = S_{03} = S_{05} = S_{06} = S_{07} = 0, & Y_1 &= -a_{14}, & Y_2 &= a_{24}, & Y_3 &= -a_{34}, & Y_4 &= a_{44}, \\
Y_5 &= -a_{54}, & Y_6 &= a_{64}, & Y_7 &= -a_{74}, & Y_8 &= a_{84}, & Y_9 &= a_{94}
\end{aligned}$$

For incident CD-I wave:



$$A^* = S_{05}, \quad S_{01} = S_{02} = S_{03} = S_{04} = S_{06} = S_{07} = 0, \quad Y_1 = a_{15}, \quad Y_2 = -a_{25}, \quad Y_3 = a_{35}, \quad Y_4 = -a_{45}, \\ Y_5 = a_{55}, \quad Y_6 = -a_{65}, \quad Y_7 = a_{75}, \quad Y_8 = -a_{85}, \quad Y_9 = a_{95} = 0$$

For incident CD-II wave:

$$A^* = S_{06}, \quad S_{01} = S_{02} = S_{03} = S_{04} = S_{05} = S_{07} = 0, \quad Y_1 = -a_{14}, \quad Y_2 = a_{24}, \quad Y_3 = -a_{34}, \quad Y_4 = a_{44}, \\ Y_5 = a_{56}, \quad Y_6 = -a_{66}, \quad Y_7 = a_{76}, \quad Y_8 = -a_{86}, \quad Y_9 = a_{96} = 0$$

For incident CD-III wave:

$$A^* = S_{06}, \quad S_{01} = S_{02} = S_{03} = S_{04} = S_{05} = S_{06} = 0, \quad Y_1 = -a_{14}, \quad Y_2 = a_{24}, \quad Y_3 = -a_{34}, \quad Y_4 = a_{44}, \\ Y_5 = a_{57}, \quad Y_6 = -a_{67}, \quad Y_7 = a_{77}, \quad Y_8 = -a_{87}, \quad Y_9 = a_{97} = 0$$

and

$$Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{S_2}{A^*}, \quad Z_3 = \frac{S_3}{A^*}, \quad Z_4 = \frac{S_4}{A^*}, \quad Z_5 = \frac{S_5}{A^*}, \quad Z_6 = \frac{S_6}{A^*}, \quad Z_7 = \frac{S_7}{A^*}, \quad Z_8 = \frac{\bar{S}_1}{A^*}, \quad Z_9 = \frac{\bar{T}_1}{A^*}$$

where  $Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7$  are the amplitude ratios of reflected LD-wave, T-wave, LM-wave, LT-wave and coupled CD-I, CD-II, CD-III waves in medium  $M_1$  and  $Z_8, Z_9$  are the amplitude ratios of transmitted P-wave and SV-wave in medium  $M_2$ .

## 5 PARTICULAR CASES

(a) If we neglect micropolarity effect in medium  $M_1$  i.e  $K=0$ , then we obtain amplitude ratios at an interface of microstretch thermoelastic half-space with microtemperatures without microrotational effect and elastic solid half space as:

$$\sum_{j=1}^8 a_{ij} Z_j = Y_i; \quad (i=1, 2, 3, 4, 5, 6, 7, 8) \quad (36)$$

where the values of  $a_{ij}$  are given as:

$$a_{1i} = -\left( d_1 \left( 1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0 \right) + d_2 \frac{V_i^2}{V_0^2} \sin^2 \theta_0 \right) \frac{\omega^2}{V_i^2} - l_i + d_3 g_i, \quad a_{1j} = (d_2 - d_1) \frac{\omega^2}{V_j V_0} \sin \theta_0 \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0}, \\ a_{17} = \left[ \left[ \bar{d}_1 \left( 1 - \frac{\bar{V}_1^2}{V_0^2} \sin^2 \theta_0 \right) + \bar{d}_2 \frac{\bar{V}_1^2}{V_0^2} \sin^2 \theta_0 \right] \frac{\omega^2}{V_1^2} \right], \quad a_{18} = (\bar{d}_2 - \bar{d}_1) \frac{\omega^2}{V_2 V_0} \sin \theta_0 \sqrt{1 - \frac{\bar{V}_2^2}{V_0^2} \sin^2 \theta_0}, \\ a_{2i} = -(2d_4 + d_5) \frac{\omega^2}{V_i V_0} \sin \theta_0 \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0}, \quad a_{2j} = d_4 \frac{\omega^2}{V_j^2} \left( 1 - 2 \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right) + d_5 \frac{\omega^2}{V_j^2} \left( 1 - \frac{V_j^2}{V_0^2} \sin^2 \theta_0 \right), \\ a_{27} = -2\bar{d}_3 \frac{\omega^2}{V_1 V_0} \sin \theta_0 \sqrt{1 - \frac{\bar{V}_1^2}{V_0^2} \sin^2 \theta_0}, \quad a_{28} = -\left[ \bar{d}_3 \frac{\omega^2}{V_2^2} \left( 1 - 2 \frac{\bar{V}_2^2}{V_0^2} \sin^2 \theta_0 \right) \right], \\ a_{3i} = \iota d_8 \frac{\omega}{V_i} \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2 \theta_0} g_i, \quad a_{3j} = -\iota d_9 \frac{\omega}{V_0} \sin \theta_0 n_j, \quad a_{37} = a_{38} = 0,$$

$$\begin{aligned}
a_{4i} &= -\left(d_{10} + d_{11}\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}\right)\frac{\omega^2}{V_i^2}f_i, & a_{4j} &= -d_{11}\frac{\omega^2}{V_jV_0}m_j\sin\theta_0\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0}, \\
a_{47} &= a_{48} = 0, & a_{5i} &= -(d_{12} + d_{13})f_i\frac{\omega^2}{V_iV_0}\sin\theta_0\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}, \\
a_{5j} &= (d_{13}(1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0) - d_{12}\frac{V_j^2}{V_0^2}\sin^2\theta_0)\frac{\omega^2}{V_j^2}, & a_{57} &= a_{58} = 0, \\
a_{6i} &= \iota\frac{\omega}{V_0}\sin\theta_0, & a_{6j} &= -\iota\frac{\omega}{V_j}\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0}, & a_{67} &= -\iota\frac{\omega}{V_0}\sin\theta_0, & a_{68} &= \iota\frac{\omega}{V_2}\sqrt{1 - \frac{V_2^2}{V_0^2}\sin^2\theta_0}, \\
a_{7i} &= \iota\frac{\omega}{V_i}\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}, & a_{7j} &= \iota\frac{\omega}{V_0}\sin\theta_0, & a_{77} &= \iota\frac{\omega}{V_1}\sqrt{1 - \frac{V_1^2}{V_0^2}\sin^2\theta_0}, \\
a_{78} &= -\iota\frac{\omega}{V_0}\sin\theta_0, & a_{8i} &= \iota\frac{\omega}{V_i}\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}, & a_{8j} &= a_{87} = a_{88} = 0 \\
& (i = 1, 2, 3, 4 \text{ and } j = 5, 6)
\end{aligned} \tag{37}$$

## 6 NUMERICAL RESULTS AND DISCUSSION

The following values of relevant parameters are taken for numerical computations. Following Eringen [32], the values of micropolar constants are taken as:

$$\begin{aligned}
\lambda &= 9.4 \times 10^{10} \text{ Nm}^{-2}, & \mu &= 4.0 \times 10^{10} \text{ Nm}^{-2}, & K &= 1.0 \times 10^{10} \text{ Nm}^{-2}, & \gamma &= 7.779 \times 10^{-8} \text{ N}, \\
j &= 2 \times 10^{-20} \text{ m}^2, & \rho &= 1.74 \times 10^3 \text{ Kg m}^{-3}.
\end{aligned}$$

and thermal parameters are taken from Dhaliwal and Singh [33]:

$$c^* = 1.04 \times 10^3 \text{ Nm Kg}^{-1} \text{ K}^{-1}, \quad T_0 = 0.298 \text{ K}, \quad K^* = 1.7 \times 10^2 \text{ N sec}^{-1} \text{ K}^{-1}.$$

Microstretch parameters are taken as:

$$\begin{aligned}
j_0 &= 0.19 \times 10^{-17} \text{ m}^2, & b &= 0.15 \times 10^{-9} \text{ N}, & \lambda_0 &= 0.21 \times 10^{11} \text{ Nm}^{-2}, & \lambda_1 &= 0.007 \times 10^{12} \text{ Nm}^{-2}, \\
\alpha_0 &= 0.00008 \times 10^{-5} \text{ N}
\end{aligned}$$

and microtemperatures parameters are taken as:

$$\begin{aligned}
k_1 &= 0.0035 \text{ N s}^{-1}, & k_2 &= 0.045 \text{ N s}^{-1}, & k_3 &= 0.055 \text{ N K}^{-1} \text{ s}^{-1}, & k_4 &= 0.065 \text{ N s}^{-1} \text{ m}^2, & k_5 &= 0.076 \text{ N s}^{-1} \text{ m}^2, \\
k_6 &= 0.096 \text{ N s}^{-1} \text{ m}^2, & \mu_1 &= 0.0085 \text{ N}, & \mu_2 &= 0.0095 \text{ N}
\end{aligned}$$

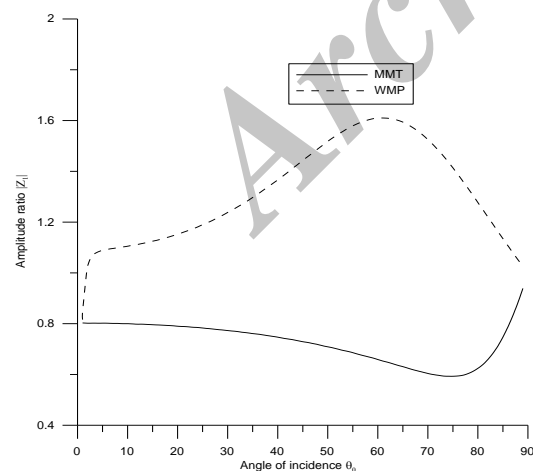
The values of amplitude ratios have been computed at different angles of incidence. In Figs.1-18, MMT corresponds to microstretch thermoelastic solid with microtemperatures and WMP corresponds to microstretch thermoelastic solid with microtemperatures without microrotational effect.

### 6.1 Incident LD-Wave

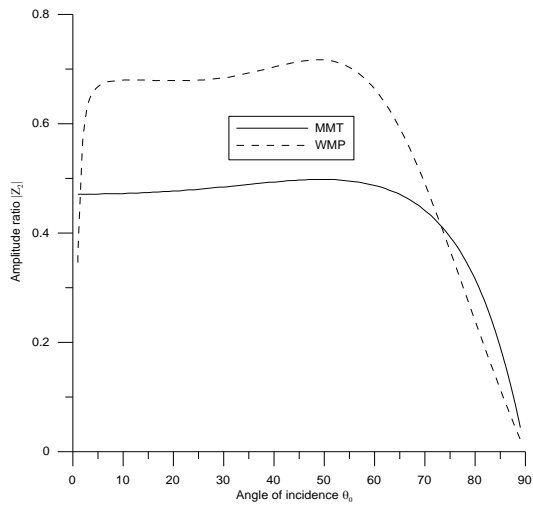
Variations of amplitude ratios  $|Z_i|$ ;  $1 \leq i \leq 9$  with the angle of incidence  $\theta_0$ , for incident LD-wave are shown in Figs. 2 through 10. Fig. 2 shows that the values of  $|Z_1|$  for MMT decrease monotonically in the range  $0^\circ < \theta_0 < 77^\circ$  and then increase, whereas the values for WMP increase monotonically in the range  $0^\circ < \theta_0 < 64^\circ$  and then decrease in the further range. It is noticed that the values for WMP are greater than the values for MMT in the whole range.

It is evident from Fig. 3 that the values of amplitude ratio  $|Z_2|$  for MMT first increase and then decrease near the grazing incidence. The values for WMP follow oscillatory pattern in the whole range. The values for WMP in comparison with MMT are greater in the whole range, except near the normal incidence and grazing incidence. Fig. 4 shows that the values for  $|Z_3|$  for MMT decrease in the whole range. The values for WMP attain maximum value in the intermediate range. The values for MMT are greater than the values for WMP in the whole range, except the range  $39^\circ < \theta_0 < 76^\circ$ , where the behavior is reversed. Fig. 5 depicts that the values of amplitude ratio  $|Z_4|$  for MMT decrease from normal incidence to grazing incidence. The values for WMP are greater than the values for MMT in the range  $20^\circ < \theta_0 < 80^\circ$ . The values of amplitude ratio for MMT and WMP are magnified by multiplying by  $10^6$ .

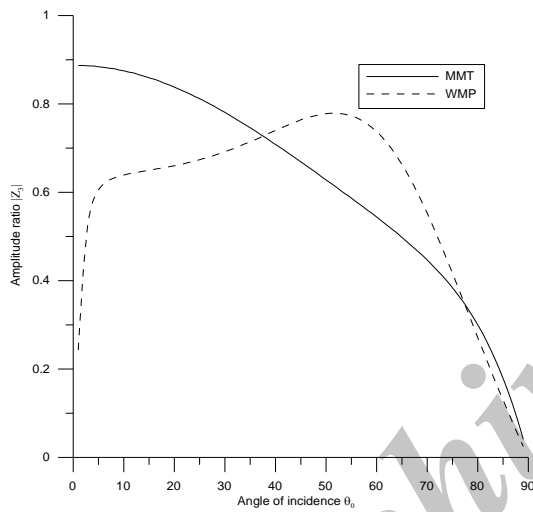
Fig. 6 shows that the values of  $|Z_5|$  for WMP attain maximum value at the normal incidence. The values of amplitude ratio for MMT and WMP are magnified by multiplying by 10. The values for MMT increase in the range  $0^\circ < \theta_0 < 61^\circ$  and then decrease. Fig. 7 shows that the values of  $|Z_6|$  for MMT increase monotonically in the interval  $0^\circ < \theta_0 < 61^\circ$  and then decrease as  $\theta_0$  increases further, while the values for WMP decrease monotonically in the whole range. It is noticed from Fig. 8 that the values of  $|Z_7|$  for MMT increase to attain maximum value at  $\theta_0 = 55^\circ$  and then decrease with further increase in  $\theta_0$ . The values for MMT are magnified by multiplying by  $10^2$ . It is noticed from Fig. 9 that values of  $|Z_8|$  for MMT decrease with increase in  $\theta_0$ . The values for WMP oscillate and attain maximum value at normal incidence. Fig. 10 depicts that the values of amplitude ratio  $|Z_9|$  for MMT increase in the interval  $0^\circ < \theta_0 < 65^\circ$  and decrease for  $\theta_0 \geq 65^\circ$ . The values for MMT are greater in comparison to the values for WMP in the whole range. The values of amplitude ratio for WMP are magnified by multiplying by  $10^2$ .



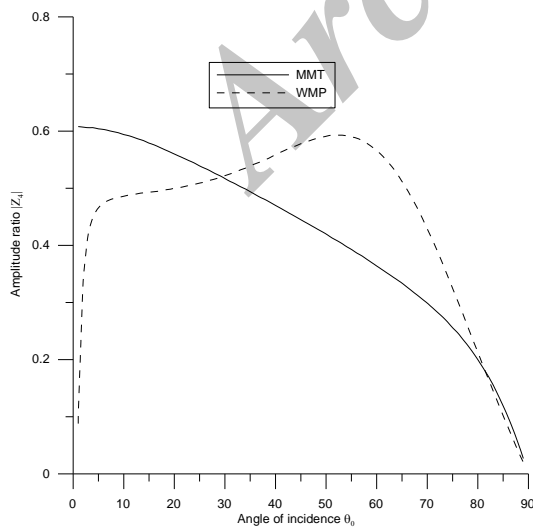
**Fig. 2**  
Variations of amplitude ratios  $|Z_1|$  with the angle of incidence for LD-Wave.



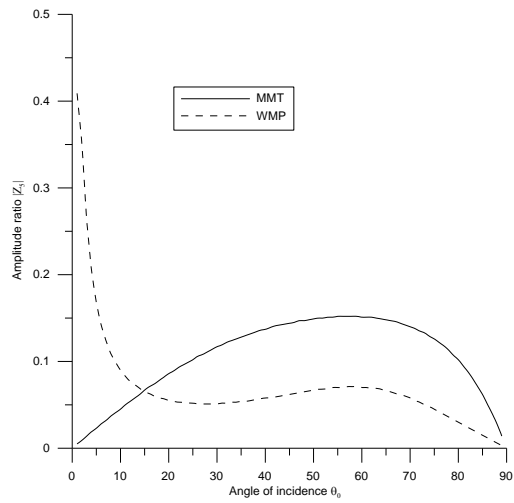
**Fig. 3**  
Variations of amplitude ratios  $|Z_2|$  with the angle of incidence for LD-Wave.



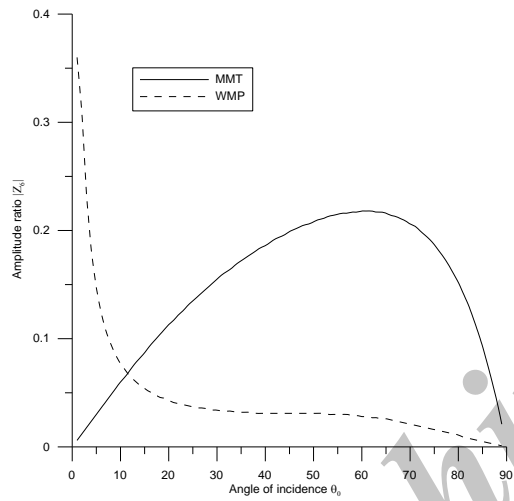
**Fig. 4**  
Variations of amplitude ratios  $|Z_3|$  with the angle of incidence for LD-Wave.



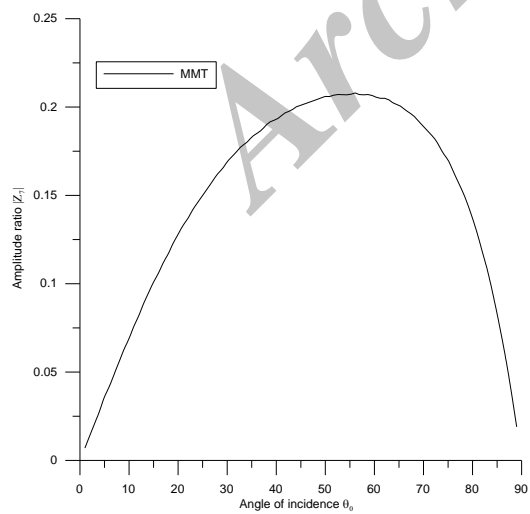
**Fig. 5**  
Variations of amplitude ratios  $|Z_4|$  with the angle of incidence for LD-Wave.



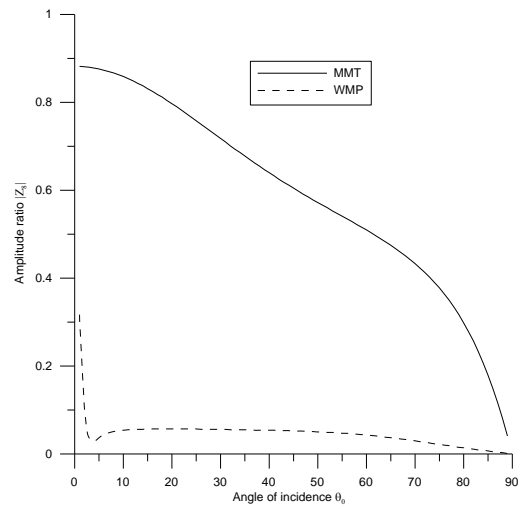
**Fig. 6**  
Variations of amplitude ratios  $|Z_5|$  with the angle of incidence for LD-Wave.



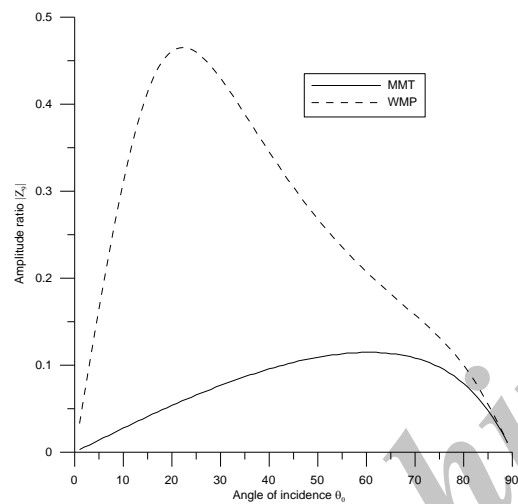
**Fig. 7**  
Variations of amplitude ratios  $|Z_6|$  with the angle of incidence for LD-Wave.



**Fig. 8**  
Variations of amplitude ratios  $|Z_7|$  with the angle of incidence for LD-Wave.



**Fig. 9**  
Variations of amplitude ratios  $|Z_8|$  with the angle of incidence for LD-Wave.



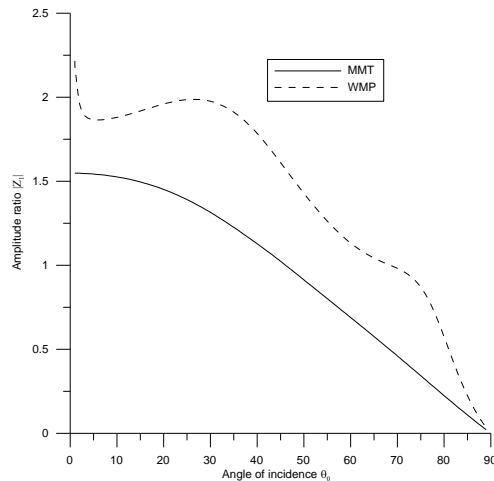
**Fig. 10**  
Variations of amplitude ratios  $|Z_9|$  with the angle of incidence for LD-Wave.

## 6.2 Incident LM-Wave

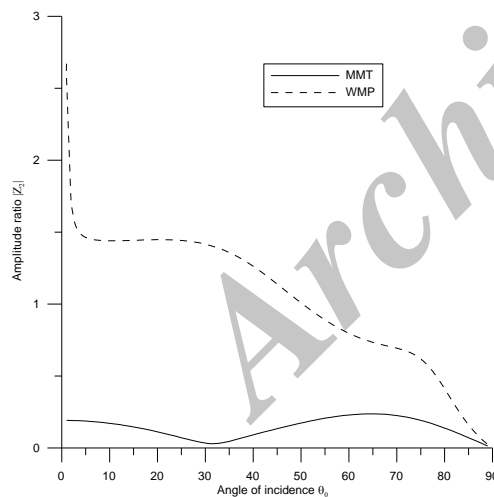
Variations of amplitude ratios  $|Z_i|$ ;  $1 \leq i \leq 9$ , with the angle of incidence  $\theta_0$ , for incident LM-wave are shown in Figs. 11 through 19. Figs. 11 show that the values of  $|Z_1|$  for MMT decrease and WMP oscillate in the whole range. The values for WMP remain more than the values for MMT in the whole range. Fig. 12 depicts that the values of  $|Z_2|$  for WMP are greater than the values for MMT in the whole range. It is evident from Fig. 13 that the values of  $|Z_3|$  for WMP attain maximum value in the range  $65^\circ < \theta_0 < 75^\circ$ . The values for WMP in comparison to the values for MMT are more in the whole range.

Fig. 14 depicts that values of  $|Z_4|$  for MMT oscillate and attain maximum value near the normal incidence. The values of amplitude ratio for MMT are magnified by multiplying by  $10^6$  and WMP are magnified by multiplying by  $10^5$ . Fig. 15 shows that the values of  $|Z_5|$  for MMT increase in the range  $0^\circ < \theta_0 < 45^\circ$  and then decrease in further range. The values for WMP follow oscillatory pattern and attain peak value at  $\theta_0 = 0^\circ$ . The values of amplitude ratio for MMT and WMP are magnified by multiplying by 10. Fig. 16 shows that the values of  $|Z_6|$  for MMT increase in the interval  $0^\circ < \theta_0 < 55^\circ$  and then decrease in the subsequent range, while the values for WMP decrease sharply in

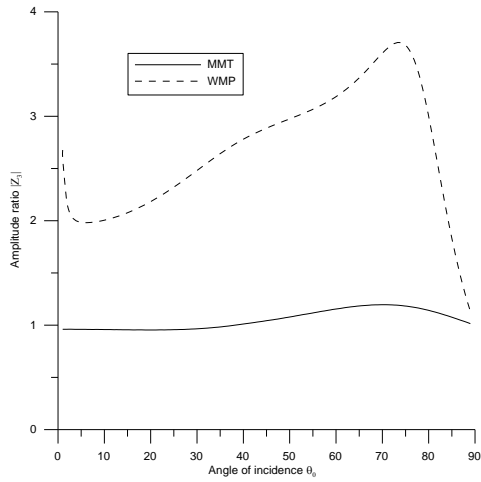
the whole range with slight oscillation in the finite region. Fig. 17 shows that the values of  $|Z_7|$  for MMT get increased to attain maximum value in the intermediate range and then decrease sharply in the further range. The values of amplitude ratio for WMP are magnified by multiplying by  $10^2$ . Fig. 18 shows that the values of  $|Z_8|$  for MMT decrease and WMP oscillate in the whole range and the values for MMT remain more as compared to the values for WMP in the whole range, except near the normal incidence where the behavior is reversed. Fig. 19 depicts that the values of amplitude ratio for MMT increase in the range  $0^\circ < \theta_0 < 56^\circ$  and then get decreased with increase in  $\theta_0$ . The values of amplitude ratio for WMP attain peak values in the range  $65^\circ < \theta_0 < 75^\circ$ . The values of amplitude ratio for MMT and WMP are magnified by multiplying by 10.



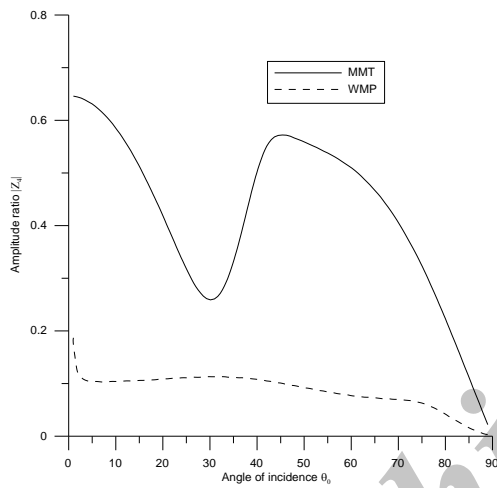
**Fig. 11**  
Variations of amplitude ratios  $|Z_1|$  with the angle of incidence for T-Wave.



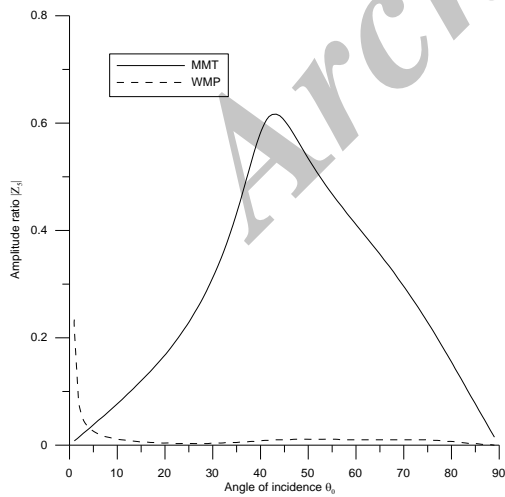
**Fig. 12**  
Variations of amplitude ratios  $|Z_2|$  with the angle of incidence for T-Wave.



**Fig. 13**  
Variations of amplitude ratios  $|Z_3|$  with the angle of incidence for T-Wave.

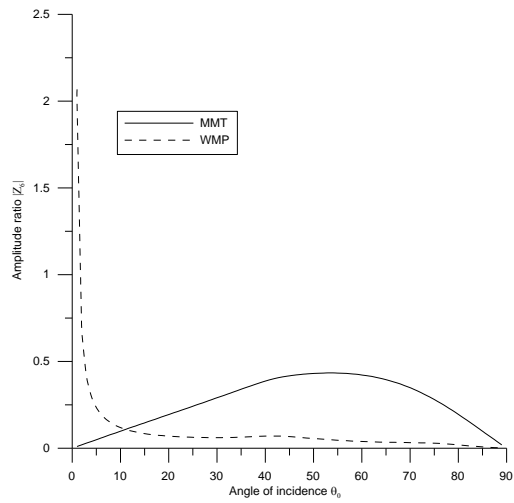


**Fig. 14**  
Variations of amplitude ratios  $|Z_4|$  with the angle of incidence for T-Wave.

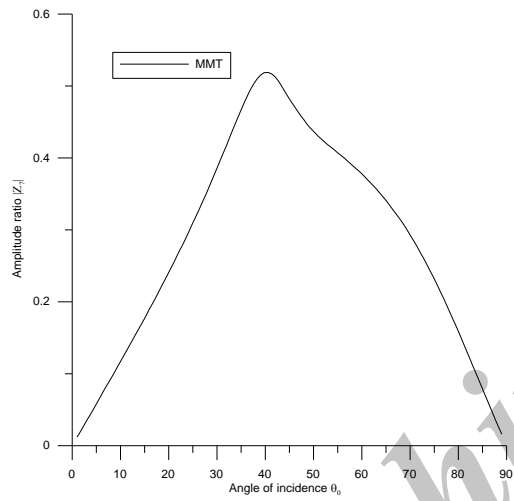


**Fig. 15**  
Variations of amplitude ratios  $|Z_5|$  with the angle of incidence for T-Wave.

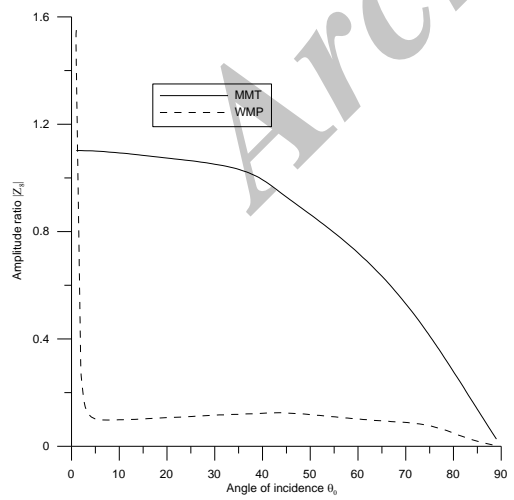




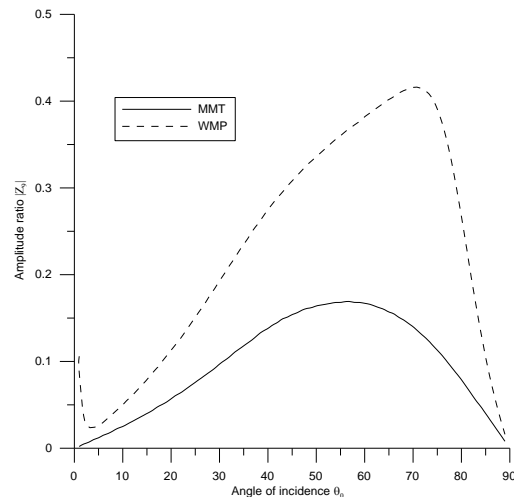
**Fig.16**  
Variations of amplitude ratios  $|Z_6|$  with the angle of incidence for T-Wave.



**Fig.17**  
Variations of amplitude ratios  $|Z_7|$  with the angle of incidence for T-Wave.



**Fig.18**  
Variations of amplitude ratios  $|Z_8|$  with the angle of incidence for T-Wave.



**Fig.19**  
Variations of amplitude ratios  $|Z_0|$  with the angle of incidence for T-Wave.

## 7 CONCLUSIONS

The reflection and transmission coefficients at an interface of microstretch thermoelastic solid half-space with microtemperatures and elastic solid half-space have been studied in the present paper. The amplitude ratios for an incidence of LD-wave and LM-wave have been obtained numerically and their variations with angle of incidence have been shown graphically. It is noticed that when LM-wave is incident, the values of reflected LD-wave, T-wave, LM-wave, LT-wave, transmitted SV-wave and when LD-wave is incident, the values of reflected LD-wave and transmitted SV-wave for WMP (without microrotational effect) are greater as compared to the values for MMT (with microrotational effect). The amplitude ratios for WMP are more oscillatory and attain peak value in the initial range.

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