

A New Mathematical Model for Permeability of Composites

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ABSTRACT

Study of permeability of Fibrous Composites is important in several natural and industrial processes in mechanical engineering. In this study, a comprehensive mathematical model is presented for calculation of normal permeability of ordered elliptical fibrous media. An innovative scale-analysis technique is employed for determining the normal permeability of elliptical fibrous media. In this technique, the permeability is related to the porosity, elliptical fiber diameters, and tortuosity of the medium. In other word, the normal permeability of the circular fibrous structures, which presented in the literature, is extended to the general case of elliptical fibrous media. The composite material is represented by a "unit cell" which is assumed to be repeated throughout the media. A closed-form relation is obtained for non-dimensional permeability using scale analysis approach. Due to lack of experimental data for permeability of fibrous porous media (composite media), with elliptical cross section, a numerical analysis is also employed. The governing equations are solved numerically in the unit cells using finite volume method. The results obtained by numerical solution are compared with those presented by scale analysis method. The presented relation for normal permeability can suitably cover the case of fibrous media with circular cross section. The results are also compared with those presented in the literature for the case of cylindrical fibers. The developed compact relationships are successfully verified through comparison with the present experimental results and the data reported by others. © 2013 IAU, Arak Branch. All rights reserved.

Keywords: Mathematical model; Permeability; Elliptical; Composite; Scale analysis

1 INTRODUCTION

ANALYSIS of fibrous composites (as a porous medium) is important in several natural and industrial processes including: thermal insulations, physiological systems, filtration and separation of particles, composite fabrication, heat exchangers, and fuel cells [1-3]. As such, investigate in porous media have been the focus of numerous studies since 19th century. Researchers have employed various theoretical and experimental techniques to solve the problem. Comprehensive reviews of the pertinent literature can be found in Refs. [1,4,5]. According to experimental data, permeability can be achieved by using Darcy's equation [4], which defined that a linear relationship exists between the volume-averaged superficial fluid velocity, U_D , and the pressure gradient:

$$-\nabla P = \frac{\mu}{K} U_D \quad (1)$$

where μ is the fluid viscosity and K is the permeability of the medium. Darcy's equation holds when flow is in creeping regime, however Darcy's relationship is empirical, convenient, and widely accepted. To use Darcy's

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equation should know the permeability of the medium. Permeability, which can be interpreted as the flow conductance of the solid matrix, related to several geometrical factors including: porosity, particles shape, pore distribution and particles arrangement.

Scale analysis, experimental and numerical approach and analytical method are used to investigate on the permeability of fibrous media. Fibrous materials can be divided into 1, 2, and 3 directional media. In one-directional (1D) structures the axes of fibers are parallel to each other. In two directional (2D) fibrous matrices the fibers axes are located on planes parallel to each other with random positions and orientations on these planes. The axes of fibers in three-directional (3D) are randomly positioned and oriented in space [6].

Experimental works [7, 8] on the permeability of fibrous media dates back to 1940s and theoretical analyses [9-12] in 1950s. Sparrow and Loeffler [9] and Hasimoto [10] used series solutions for estimate the permeability of ordered arrangement of cylinders. Kuwabara [11], solving stream function and vorticity transport equations and also employing limited boundary layer approach, predicted the permeability of flow normal to randomly arranged fibers for materials with high porosities. Studies of permeability of 2D and 3D materials are not as frequent as 1D arrangement. Happel and Brenner [12] analytically solved the Stokes equation for parallel and normal flow to a single cylinder with free surface model. They assumed that the flow resistance of a random 3D fibrous structure is equal to two third of the normal flow resistances of 1D array of cylinders plus one third of the parallel. Later, Sangani and Acrivos [13], performed analytical and numerical studies on viscous permeability of square and staggered arrays of cylinders, when their axes were perpendicular to the flow direction. Their analytical models were agreement with the lower and higher limits of porosity. Drummond and Tahir [14] solved Stokes equations for normal and parallel flow towards different ordered structures. They used a distributed singularities method to find the flow-field in square, triangular, hexagonal and rectangular arrays. They [14] compared their results with numerical values of Sangani and Acrivos [13] for normal flow, Their model for normal permeability was very close to the analytical model of Sangani and Acrivos [13]. Sangani and Yao [15] extended the studies of [13] to random media and reported numerical results for the permeability of random 1D fibers towards normal and parallel flows.

Van der Westhuizen and Du Plessis [16] numerically determined the permeability of cylinders to normal flow and proposed a correlation for the normal permeability of 1D fibers. Using numerical simulations, Sahraoui and Kaviani [17] determined the permeability of cylinders to normal flow included inertial effects. They proposed a correlation which was accurate in limited range of porosity, i.e., $0.4 < \varepsilon < 0.7$. Hellou et al. [18] theoretically predicted the permeability of general triangular arrangement. They also proposed a correlation for determination of permeability of periodic triangular arrangements. Sobera and Kleijn [19] recently analytically and numerically studied the permeability of random 1D and 2D fibrous media. Their model's comparison with numerical results showed that their model was accurate in highly porous materials. Their analytical model was a modification of the scale analysis proposed by Clauge et al. [3].

Several investigations were tried to quantify the permeability of random fibrous media, i.e., real materials. A number of researchers related the permeability of random media to the values of parallel and normal permeability of 1D fiber. The model proposed by Jackson and James [20] was based on numerical approach. However, limitations of this model were shown in the literature [1]. Tomadakis and Sotirchos [21] defined a model for anisotropic permeability through 1D, 2D, and 3D random fibrous. In some cases, their model has considerable errors; see Ref. [1] for more details. Avellaneda and Torquato [22] by using the similarity between permeability and electrical conduction in porous media, proposed an upper bound for the permeability of fibrous media. Using experimental data, Tomadakis and Robertson [1] stated that the upper and lower bounds for fibrous media with random orientation of fibers were normal and parallel permeability of 1D arrangements, where showed that this bound was violated by several data points available in the literature [22].

Several experimental studies have been existed for determination of the permeability of fibrous media [23-29]. Comprehensive reviews of these experimental works are available in [1, 5, 20].

Our literature review can be summarized:

- Less attention has been paid for determination of permeability of non-cylindrical fibers.
- Most of the theoretical models found in the literature are not general and fail to predict the permeability over the entire range of porosity.
- Scale analysis method didn't used frequently.

Tamayol and Bahrami [30] studied permeability of touching and non-touching ordered fibrous media towards normal and parallel flow. Analytical models were developed using the concept of "unit cell" and introducing an "integral technique". Assuming a parabolic velocity profile within the unit cells and integrating the continuity and momentum equations, compact analytical relationships were reported for pressure drop and permeability of considered patterns. Tamayol et al. [31] analyzed the effects of mechanical compression and PTFE content on the through-plane gas permeability of gas diffusion layers of PEM fuel cells both experimentally and theoretically. The

experimental data showed a reverse relationship between the through-plane permeability and both PTFE content and mechanical compression.

Nabovati et al. [32] studied the effect of porosity heterogeneity on the bulk hydrodynamic properties (permeability and tortuosity) of simulated gas diffusion layers. Using the results of pore-level simulations, the effect of porosity distribution was characterized on the predicted in- and cross-plane permeability and tortuosity. Nazari et al. [33] investigated the normal flow permeability of an ordered fibrous structure, with square cross section analytically. The presented method can predict the permeability of fibrous media, especially at high porosity.

Due to lack of experimental data for permeability of fibrous porous media with elliptical cross section we were not able to verify our model, for this reason we used the numerical method and also compared the obtained results with cylindrical experimental data for $D=d$.

The objectives of the present work are to:

- Use the numerical results for verifying the normal flow model where a lack of experimental data exists in the literature.
- Develop an analytical approach that is applicable to 1D elliptical fibrous structure. The analytical solution captures the trends observed in experimental data.
- Investigate the effect of relevant geometrical parameters.

In this study, the normal permeability of ordered elliptical fibrous media is studied both numerically and theoretically. A scale analysis technique is employed for determining the normal permeability of elliptical fibrous media. In this technique, the permeability is related to the porosity, elliptical fiber diameters, and tortuosity of the medium. In other word, the normal permeability of the circular fibrous structures, which presented in the literature, is extended to the general case of elliptical fibrous media. Clauge et al. [34] applied this method to fibrous media and this was modified to improve its accuracy. The developed solutions are successfully compared with analytical and numerical results for a wide range of geometries.

2 MODELING APPROACH

2.1 Numerical model development

In the present work, the 1D fibrous media are studied. Following the approach used in several applications, (such as spherical packed beds [35] and flow in fibrous porous media [36]), a unit cell is considered for analyzing the media. The unit cell (or basic cell) is defined the smallest volume which can represent the characteristics of the whole microstructure. Porous media are assumed to be periodic and the unit cells repeat throughout the material. Because of the large length of fibers, variations in the z -direction can be neglected for the normal flow case, although the velocity distribution in porous material in fact is 3D.

Fig.1 shows how unit cells are produced for simulating normal flow. The unit cells are selected as the space between adjacent elliptical cylinders as shown in Fig.1. The square arrangements of elliptical cylinders are considered in this study. Porosity for this arrangement can be determined from:

$$\varepsilon = 1 - \frac{\pi Dd}{4S^2} \quad (2)$$

And the solid volume fraction defined,

$$\varphi = 1 - \varepsilon \quad (3)$$

where D , d and S are the big diameter of ellipse, small fiber diameter and the distance between two adjacent fibers, respectively.

Darcy's equation Eq. (1) holds when flow is in creeping regime. The inertial effects are negligible in this regime. Therefore, the Stokes and continuity equations govern the flow-field. The flow assumed to be steady state, incompressible and porous media is completely saturated. The flow Reynolds number should be kept sufficiently low to ensure negligible effects of inertial terms. Therefore, to guarantee that creeping flow exists, the inlet velocity is set low enough, which the Reynolds number based on the fibers diameter, d , is below 0.05 for all cases.

In this paper, the results of the cross flow permeability are reported in the case of fully developed flow. It means that the volume averaged velocity does not change in the consecutive unit cells, in the fully developed region.

Therefore, suitable boundary conditions should be considered for the numerical analysis of fluid flow through a unit cell. Inlet and outlet boundaries of a fully developed cell should be considered as periodic boundaries. The pressure gradient, i.e. $\Delta P/S$, is the same for the unit cells that are located in the fully developed section.

Another way to simulate the fluid flow through the unit cell is consideration of a set of 7-10 unit cells in series. The selected series geometries have to be arranged so that the fully developed condition is reached. The velocity profiles are compared at the entrance to each unit cell. In other word, the unit cells which are located far from the inlet can be considered as fully developed cells. The inlet velocity of the media is assumed to be uniform and velocity inlet boundary condition is applied. The normal gradient of properties along the outlet is zero and the values of all properties at the outlet are interpolated from the computational domain. The symmetry boundary condition is applied on the side borders of the considered unit cells; this means that normal velocity and gradient of parallel component of the velocity on the side borders are zero. The Finite volume method is used for solving the governing equations and SIMPLE algorithm is selected for pressure-velocity coupling. Second order upwind scheme is also employed to discrete the governing equations. More descriptions about the numerical method will be added in Section 3.1.

2.2 Scale analysis model development

According to experimental observations, Darcy’s law [4] defined that a linear relationship exists between the volume-averaged superficial fluid velocity, U_D , and the pressure gradient:

$$-\frac{dP}{dx} = \frac{\mu}{K} U_D \tag{4}$$

where, μ is the fluid viscosity and K is the permeability of the medium. Darcy’s relationship is empirical and widely accepted; this equation holds when flow is in creeping regime [37]. However, for use Darcy’s equation, one should know the permeability. The permeability can be calculated with pore-scale analysis through the flow in the solid matrix. The pore-scale velocity is governed by Stokes equation (in the creeping regime):

$$\nabla \cdot \vec{V} = 0 \tag{5}$$

$$\mu \nabla^2 \vec{V} = -\nabla P \tag{6}$$

In the scale analysis approach, in governing equations, the scale or the range of variation of the parameters is substituted. Moreover, the derivatives in the governing equations are approximated with differences [38]. Half of the minimum opening between two adjacent ellipses, δ_{min} , is selected as the characteristic length scale over which rapid changes of the velocity occurs, following [6, 19, 34]. This selection is due to the fact that most of pressure drop (directly related to the permeability) occurs at the entrance/exit regions of the unit cell.

Sobera and Klein [19] proposed to use the average velocity in the section with minimum frontal area as the characteristic velocity scale. However, this assumption is only accurate for highly porous structures, $\epsilon > 0.8$, and over predicts the pressure drop in low porosities.

In addition, Carman [6] argued that a fluid particle to path through a sample of size L should travel in a tortuous path of length L_e . Therefore, it is expected that the resulting velocity scale is inversely related to L_e / L (when a constant pressure difference applies). This ratio is called tortuosity factor, τ . Thus, the pore-level velocity scale becomes:

$$V \delta_{min} = \frac{U_D S}{\tau} \Rightarrow V = \frac{U_D}{\tau} \frac{S}{\delta_{min}} \tag{7}$$

where $\beta = \delta_{min}/S$ is the ratio of the minimum to the total frontal areas in the unit cell. Substituting from Eq. (7) for velocity scale and using δ_{min} as the length scale, permeability can be calculated as:

$$-\frac{dP}{dx} = \frac{\mu}{K} U_D \Rightarrow \text{using Eq.(6)}: \frac{\mu}{K} U_D \approx \mu \frac{V}{\delta_{\min}^2} \Rightarrow \text{using Eq.(7)}: \frac{\mu}{K} U_D \approx \mu \frac{\tau \beta}{\delta_{\min}^2} \Rightarrow K = C \beta \delta_{\min}^2 \tau \quad (8)$$

where C is a constant that should be determined through comparison with experimental/Numerical data. Therefore, one needs to know the ratio between tortuosity factor, τ , and minimum to total frontal area, β , to calculate the permeability.

The ratio of the average distance, L_e , that a particle should travel to cover a direct distance of L called tortuosity factor. Because its importance in mass and thermal diffusion, several empirical and theoretical relationships have been proposed for tortuosity calculation in the literature; good reviews can be found elsewhere [39-41].

Any relationship proposed for tortuosity should satisfy the following conditions [40]:

$$\begin{aligned} \tau > 1 &\longrightarrow \lim_{\varepsilon \rightarrow 1} \tau = 1 \\ \lim_{\varepsilon \rightarrow 0} \tau &\rightarrow \infty \end{aligned} \quad (9)$$

The Archie's law [41] is one of the most popular empirical models for determination of tortuosity. This law satisfies all the conditions presented in Eq. (9).

$$\tau = \left(\frac{1}{\varepsilon}\right)^p = \left(\frac{1}{1-\phi}\right)^p \quad (10)$$

where ε is the porosity and p is a constant. Boudreau [40] Showed that $p = 0.5$ provides a good estimate for tortuosity in packed beds. Due to similarity of flow in packed beds and flow in 1D and 2D fibers, p is assumed to be equal to 0.5.

For 3D structures, p is assumed to be equal to 0.3, as presented by Tomadakis and Robertson [1]. They showed that 3D fibrous structures are less tortuous in comparison with 1D and 2D matrices.

3 RESULTS AND DISCUSSION

Eq. (8) defined the permeability of fibrous media related to the minimum opening between adjacent fibers, δ_{\min} , ratio between minimum to total frontal area, β , and tortuosity factor, τ . The tortuosity factor can be calculated from Eq. (10).

In the following sections, C will be calculated using geometrical properties and numerical analysis. The permeability is then related to the solid volume fraction for square arrangement defined. Numerical results are obtained from 2D analyses for normal flows. In the following subsections, our focus will be on the pressure drop and permeability for the normal flow through presented fibrous structure.

3.1 Numerical results

Permeability can be calculated from Darcy's relationship:

$$K = \frac{\mu S}{\Delta P} U_D \quad (11)$$

where μ is the viscosity, ΔP is the pressure drop in the unit cell, S is the unit cell length and U_D is the volume averaged velocity. Once the pressure drop is known, permeability can be evaluated from Eq. (11). In this equation, the pressure drops are the values obtained from the analysis of fully developed unit cells (refer to Section 2.1). As presented in Section 2.1, a set of 7-10 unit cells (in series) is considered as computational domain. The selected series geometries have to be arranged so that the fully developed condition is reached. The velocity profiles are compared at the entrance to each unit cell. In other word, the unit cells which are located far from the inlet can be considered as fully developed cells. The inlet velocity of the media is assumed to be uniform and velocity inlet

boundary condition is applied. The outflow boundary condition is applied in the computational domain outlet, i.e., normal gradient of properties along the outlet is zero. Different numerical grids are employed to check the grid independency of the results. The number of grids (in a periodic unit cell) is presented in the Table 1. , for each porosity.

Using continuity equation, U_D (volume averaged velocity in a cell) can be calculated from the following relationship; which the inlet velocity is set equally for all cases.

$$U_D = u_{inlet} \frac{S-d}{S} \tag{12}$$

The predicted values for the non-dimensional permeability, i.e. $K^*=K/(dD)$, volume averaged velocity in the cell and the pressure loss across the unit cell (in the case of normal flow) are reported in Table 1. , considering different configurations used in the numerical analysis. The inlet velocity is set low enough to guarantee that creeping flow exists. The Reynolds number based on the fibers small diameter, d , is below 0.05 for all cases. For specific values of the big and small diameters, D and d , the porosity of the unit cell is a function of the distance between two adjacent fibers, S .

3.2 Tortuosity method

For the three different ordered 1D unit cells shown in Fig.1, it can be seen that $\beta = (S - d) / S$ and $\delta_{min} = (S - d)$. Therefore, Eq. (8) can be rewritten as,

$$K = C \frac{(S-d)^3}{S\sqrt{1-\phi}} \tag{13}$$

The values of normal flow permeability versus porosity were reported in Table 1. using numerical finite volume approach. By comparing the numerical results (for all porosities) with the presented model, i.e. Eq. (13), one would calculate the constant parameter, $C=0.133$. Therefore, the dimensionless permeability of the ordered structures is,

$$K^* = 0.133 \frac{(S-d)^3}{dDS\sqrt{1-\phi}} \tag{14}$$

Considering Eqs. (2 - 3), after some mathematical manipulations, one can write,

$$K^* = 0.133 \frac{\left[\frac{\pi}{4\phi} - 3\sqrt{\frac{\pi}{4\alpha\phi}} + \frac{3}{\alpha} - \sqrt{\left(\frac{1}{\alpha}\right)^3 \frac{4\phi}{\pi}} \right]}{\sqrt{1-\phi}} \tag{15}$$

where α is the ratio of big diameter to the small diameter of ellipse, $\alpha=D/d$.

In Eq. (15), a constant coefficient is selected for parameter C , i.e. equal to 0.133, in order to make coincidence between tortuosity model, Eq. (13), and the numerical results (see Table 1.). The best coincidence between the tortuosity model and the numerical data occurs when the 3rd order regression method is used in the simulation. In other word, a 3rd order polynomial (i.e. $C=a_0+a_1\varepsilon+ a_2\varepsilon^2+ a_3\varepsilon^3$) is employed in the regression; and the error function is minimized with respect to the unknown parameters to obtain the a_i coefficients. The error function is defined based on the least-square procedure. The coefficient C should be selected such a way that the error between tortuosity model and the obtained numerical data is minimized. After some manipulations, the following relation can be obtained,

$$K^* = C(\varphi) \frac{\left[\frac{\pi}{4\varphi} - 3\sqrt{\frac{\pi}{4\alpha\varphi}} + \frac{3}{\alpha} - \sqrt{\left(\frac{1}{\alpha}\right)^3 \frac{4\varphi}{\pi}} \right]}{\sqrt{1-\varphi}} \tag{16a}$$

$$C = 2.71\varepsilon^3 - 5.72\varepsilon^2 + 3.965\varepsilon - 0.78, \text{ and } \varepsilon = 1 - \varphi \tag{16b}$$

In this case, the results of tortuosity model have good agreement with numerical data in the entire range of porosity. The non-dimensional permeability of the normal flow is shown in Fig. 2, for different values of $\alpha=D/d$. As shown in this figure, the non-dimensional permeability has a strong dependency to the diameter ratio especially at small porosities.

A comparison of the present results and several existing models is presented in Fig. 3, for the case of fluid flow through circular fibers, i.e $\alpha=1$. All of the models could capture trends of data in higher limits of porosity. The presented model predicts the trends of data accurately over the entire range of porosity.

Table 1

The numerical results of normal permeability, $D=1.4(\text{cm})$, $d=1(\text{cm})$ and $u_{\text{inlet}}=0.05(\text{m/s})$.

ε	No. of Grids in a Unit Cell	S (cm)	U_D (m/s)	ΔP (Pa)	K^*
0.51	1223	1.489	0.0166	18.7	0.00902347
0.72	1859	1.982	0.02477	7	0.04765776
0.79	2619	2.290	0.02817	4.79	0.091385
0.86	4292	2.804	0.03217	3.023	0.2024694
0.91	5921	3.496	0.0357	1.937	0.4372
0.96	6352	5.26	0.0405	0.93	1.554136
0.989	6784	10	0.045	0.34	8.98109

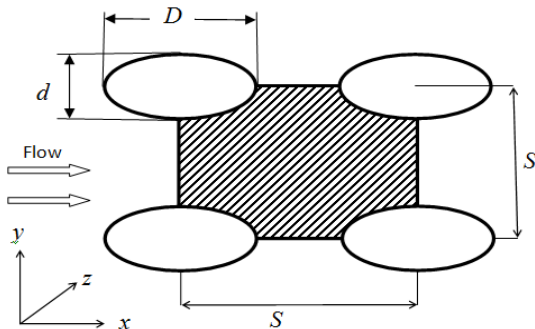


Fig. 1
Unit cell for a composite with elliptical fibers.

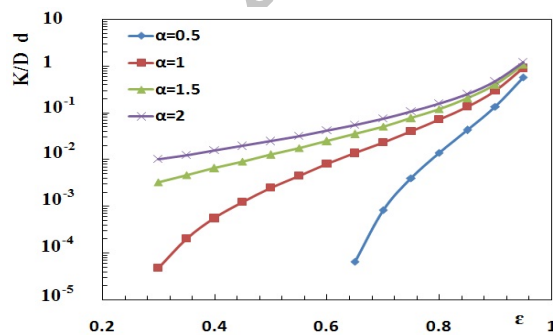


Fig. 2
The non-dimensional permeability of the normal flow for different values of α .

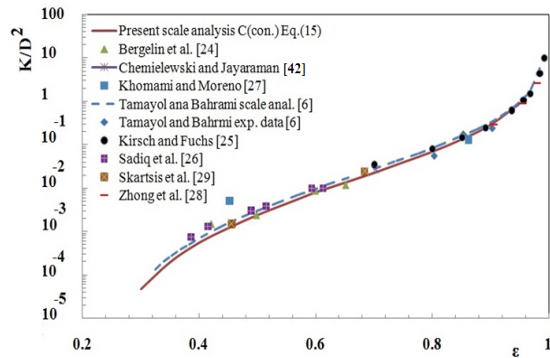


Fig. 3
Comparison of the scale analysis model, Eq. (15), with the existing results in the literature (for $\alpha=1$).

7 SUMMARY AND CONCLUSIONS

Scale analysis technique and numerical method were employed for analyzing pressure drop and permeability of an elliptical fibrous medium. The fibrous material was represented by a unit cell which was assumed to be repeated throughout the media. The Finite volume method was used for solving the governing equations to obtain the normal flow permeability. Moreover, a closed form relation was presented for the non-dimensional permeability (as a function of porosity) using the scale analysis technique. The presented relation will be applicable for wide range of porosities. The non-dimensional permeability has a strong dependency to the diameter ratio (of ellipse) especially at small porosities. The obtained results were compared with several existing models for the case of fluid flow through circular fibers, i.e $\alpha=1$. The presented model predicts the trends of data accurately over the entire range of porosity.

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