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Nonlinear Vibration Analysis of the Beam Carrying a Moving Mass Using Modified Homotopy

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ABSTRACT

In the present study, the analysis of nonlinear vibration for a simply-supported flexible beam with a constant velocity carrying a moving mass is studied. The amplitude of vibration assumed high and its deformation rate is assumed slow. Due to the high amplitude of vibrations, stretching is created in mid-plane, resulting in, the nonlinear strain-displacement relations is obtained, Thus, Nonlinear terms governing the vibrations equation is revealed. Modified homotopy equation is employed for solving the motion equations. The results shown that this method has high accuracy. In the following, analytical expressions for nonlinear natural frequencies of the beams have been achieved. Parametric studies indicated that, due to increasing of the velocity concentrated mass, the nonlinear vibration frequency is reduced. On the other hand, whatever the mass moves into the middle of beam, beam frequency decreases.

Keywords: Non-linear vibration; Modified homotopy; Concentrated mass.

1 INTRODUCTION

BEAMS are one of the most important engineering members, that are used extensively in a various types of structures. These structures include the micro and nono scale-Beams, such as very small vibrating; to macro scale-beams such as aeroplane wing, flexible satellite, and the large span bridges. Whereas, the beams are used extensively in civil engineering, mechanical engineering, and industrial; to achieve an appropriate design of the beam-shaped structures; awareness of beam vibration in transvers mode and understand how to obtain its natural frequencies is very important. When the vibration amplitude is much more, the phenomenon of fatigue on the beam's material and the overall failure of beam structural can happen frequently; In which case, the mentioned effects on natural frequency can be much more evident [1]. In such cases, the existance of nonlinear effects in equations governing the vibration of beam motion, is more important. The sources that created the nonlinear terms in the equation, can be geometry, inertia or material. Nonlinear geometry may be created in result of mid-plan stretching and big curvature of beam. The nonlinear relationship between stress and strain of the beam material is the cause of material nonlinearity. Also, concentrated mass and asymmetrically distributed caused the nonlinear created inertia [2]. Euler–Bernoulli theory based on the assumption, that during of beam deformation, the plans of beam's cross-section remain continuously like a plan and also perpendicular to the main axis, as the predeformation [3]. In this theory, in fact, the effects of shear deformation and transverse-vertical strain is neglected [4]. Pirbodaghi and co-workers employed the analytical homotopy method and investigated a nonlinear behavior of a beam under an

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axial load. Hence, an appropriate term for expression nonlinear frequency is obtained [5]. The same theory was used by sedighi and co-workers. They employed analysis of damping of non-linear vibrations and achieved appropriate results [6]. Foda employed multiple time scale method in investigating of nonlinear vibration for simply supported beam. For derivation of its equation, the effects of shear deformation and rotary inertia was considered [7]. Rafiee using of a particular condition, in order modeling the equations of a carbon nanotube; where, effect of the mid-plan stretching has been considered. Afterwards, is employed from the aforementioned method in investigation of frequency response, in both cases of primary resonance and secondary resonance [8]. Ramazani and co-workers examined the vibrations of a simply supported beam. They concluded that for achieving an accurate investigation at the right time in the study of microbial-nano based on electro-mechanical, effects of shear deformation and rotary inertia must be considered, [9]. Most of studies were performed in conjunction with the investigation of nonlinear vibrations. Nonlinear cubic function term was considered more often; and effects of high degree terms is ignored, then analytical solution will be considered [10-12]. Sarma and co-workers discussed various formulation of Finite Element for large amplitude free vibration of beams; and presented analytical formulation based on Rayleigh-Ritz method [13]. The motion of a cantilever beam that is under a uniform loading, investigated by Parnell and Cobble based on Euler–Bernoulli model [14]. On the other hand, Chin and Nayfeh, examined a hinged-clamped beam under initial excitation [15].

In the present study, after deriving the equations governing the vibrational motion of simply supported beam, in which a concentrated mass moving on it, with employing of combination the homotopy method and traditional perturbation, the response time of beam has been obtained. In the derived dynamic equation, a geometric type of nonlinearity is considered which is due to the stretching effect of the mid-plane of the beam. Afterwards, in order to accurately assess and examine the precision of calculation, analytical solution with the numerical solution by Runge - Kutta Method Fourth Order is compared. The results show that in this case, the respond of analytical method has a good correspondence with a numerical method. In continue, an analytical expression for the nonlinear natural frequency is obtained. By this expression, parametrical study will be investigated. The results show that, increasing of concentrated mass velocity caused the beam nonlinear frequency decrease.

2 FORMULATION OF PROBLEM

The partial differential equation governing the nonlinear vibration in high amplitude of simply supported beam, which a moving mass with constant velocity is moving on, is as follows [16]

$$m\frac{\partial^2 w(\mathrm{x,t})}{\partial t^2}+EI\frac{\partial^4 w(\mathrm{x,t})}{\partial x^4}-\left\{\frac{EA}{2L}\int_0^L\left[\frac{\partial w(\mathrm{x,t})}{\partial x}\right]^2 dx-\frac{EA}{8L}\int_0^L\left[\frac{\partial w(\mathrm{x,t})}{\partial x}\right]^2 dx\right\}\frac{\partial^2 w(\mathrm{x,t})}{\partial x^2}=-M\ddot{w}(\mathrm{x,t})\delta(x-s) \tag{1}$$

The nonlinear terms in aforementioned equation is created due to high amplitude and mid-plane stretching in beam. The right term in Eq. (1) represents the concentrated mass affect in case of moving on beam.

In Eq. (1), w(x,t) is the amount of transvers deformation of beam in time of t and coordinate x of left corner of beam. Also S=vt is displacement of concentrated mass and v is velocity. In this investigation, x=S is intended. If it is assumed that deformation is large but performed slowly, the right of Eq. (1) can be calculated as [16]

$$\ddot{w}(x,t)=M\left(\frac{\partial^2 w(x,t)}{\partial t^2}+\dot{x}^2\frac{\partial^2 w(x,t)}{\partial x^2}\right)\delta(x-s) \tag{2}$$

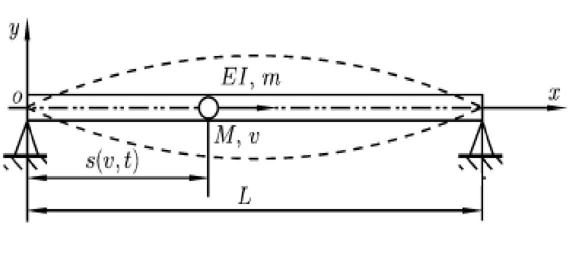

Fig. 1
Simply supported beam carrying a concentrated mass in motion.

In fact, the first term of right Eq. (2) is the lateral acceleration of the center of moving-concentrated mass. Substituting of Eq. (2) in Eq. (1) and performing the dimensional processes with define some new variables, Eq. (1) can be written as:

$$\frac{\partial^2\eta}{\partial\tau^2}+\frac{\partial^4\eta}{\partial\xi^4}-\left\{\frac{1}{2}\beta\int_0^1\left(\frac{\partial\eta}{\partial\xi}\right)^2 d\xi-\frac{1}{8}\beta\int_0^1\left(\frac{\partial\eta}{\partial\xi}\right)^4 d\xi\right\}\frac{\partial^2\eta}{\partial\xi^2}=-\alpha\left(u^2\frac{\partial^2\eta}{\partial\xi^2}+\frac{\partial^2\eta}{\partial\tau^2}\right)\delta\left(\xi-\xi_0\right) \quad (3)$$

which new variables are the following as:

$$\eta=\frac{w}{L},\ \xi=\frac{x}{L},\ \xi_0=\frac{s}{L},\ \alpha=\frac{M}{mL},\ \beta=\frac{AL^2}{I},\ u=\sqrt{\frac{m}{EI}}vL,\ \tau=\left(\frac{EI}{m}\right)^{\frac{1}{2}}\frac{t}{L^2} \quad (4)$$

To transform Eq. (4) into an usual partial differential equation, Galerkin method is employed. For this purpose, consider the answer of equation as follows

$$\eta\left(\xi,\tau\right)=q\left(t\right)\sin\left(\pi\xi\right) \quad (5)$$

In fact, the product of the first mode shape is assumed the simply supported beam and time dependent. In continue, employing of Galerkin method, Eq. (3) will be written as follows

$$\ddot{q}\left(t\right)+\omega_0^2\left(u,\xi_0\right)q\left(t\right)+f\left(u,\xi_0\right)q\left(t\right)^3-\gamma\left(u,\xi_0\right)q\left(t\right)^5=0 \quad (6)$$

which in Eq. (6), coefficients is achieved as follows

$$\begin{aligned}
\omega_0^2(u,\xi_0)&=\frac{\left[\pi^2-2u^2\alpha\sin^2(\pi\xi_0)\right]\pi^2}{1+2\alpha\sin^2(\pi\xi_0)}\\
f(\xi_0)&=\frac{\beta\pi^4}{4\left[1+2\alpha\sin^2(\pi\xi_0)\right]}\\
\gamma(\xi_0)&=\frac{3\beta\pi^6}{64\left[1+2\alpha\sin^2(\pi\xi_0)\right]}
\end{aligned} \quad (7)$$

In all of the mentioned equations, ξ_0, u are the location of the instantaneous concentrate of mass and velocity, respectively. In continue be consider of solution of Eq. (6) by modified homotopy method.

3 VIBRATIONAL ANALYSIS

In order to achieve beam's response time and it's parametrical study, homotopy perturbation method is employed. Hence, it is assumed that beam included free vibration; as a result, the vibration equation of beam in boundary conditions can be expressed as:

$$\begin{aligned}
&\ddot{q}\left(t\right)+\omega_0^2\left(u,\xi_0\right)q\left(t\right)+f\left(u,\xi_0\right)q\left(t\right)^3+\gamma\left(u,\xi_0\right)q\left(t\right)^5=0\\
&\begin{cases} q\left(0\right)=a_0\\ \dot{q}\left(0\right)=0\end{cases}
\end{aligned} \quad (8)$$

To continue solution of this problem by homotopy perturbation method, the shape of homotopy with employing of homotopy parameter (p) for Eq. (8) formulated as:

$$\left(1-P\right)\left(\ddot{q}\left(t\right)+\omega_0^2\left(u,\xi_0\right)q\left(t\right)\right)+p\left(\ddot{q}\left(t\right)+\omega_0^2\left(u,\xi_0\right)q\left(t\right)+f\left(u,\xi_0\right)q\left(t\right)^3+\gamma\left(u,\xi_0\right)q\left(t\right)^5\right)=0 \quad (9)$$

with facilitated the Eq. (9)

$$\ddot{q}(t)+\omega_0^2(u,\xi_0)q(t)+p\left(f(u,\xi_0)q(t)^3+\gamma(u,\xi_0)q(t)^5\right)=0 \tag{10}$$

Eq. (10) is such that, if homotopy parameter equals to zero it becomes a linear equation, and if equal to 1 the equation is the same governing problem [17,18]. Now for solution of Eq. (10) by the mentioned method, answer such a power series of homotopy parameter is considered

$$q(t)=q_0(t)+Pq_1(t)+P^2q_2(t)+........ \tag{11}$$

On the other hand, in nonlinear equation has been proven that nonlinear natural frequency isn't constant system and is functional of applied parameter and initial condition of problem [2]. For this reason, the linear frequency of governing equation expanded and can be formulated as a power series

$$\omega_0^2=\omega^2(u,\xi_0)+P\omega_{11}(u,\xi_0)+P^2\omega_{22}(u,\xi_0) \tag{12}$$

In Eq. (12), ω_{11} and ω_{22} is higher approximations for frequency and nonlinear natural frequency of system is ω. During the solution process, at each step one of the approximations being calculated and in the end, the answer is obtained from the sum of all approximations. With tending of homotopy parameter towards 1 in Eqs. (11) and (12), the analytical-approximation solution for response time of system and nonlinear natural frequency of system is achieved. In continue, the Eqs. (11) and (12) are inserted in original equation, which can be written as Eq. (13)

$$\begin{aligned}&\left(\ddot{q}_0(t)+P\ddot{q}_1(t)+P^2\ddot{q}_2(t)\right)+\left(\omega^2+P\omega_{11}+P^2\omega_{22}\right)\left(q_0(t)+Pq_1(t)+P^2q_2(t)\right)+\\&f\left(q_0(t)+Pq_1(t)+P^2q_2(t)\right)^3-\gamma\left(q_0(t)+Pq_1(t)+P^2q_2(t)\right)^5=0\end{aligned} \tag{13}$$

Eq. (13) being adjusted by powers of homotopy parameter and is transformed into ordinary differential equations

$$\begin{aligned}&O(P^0)\\&\ddot{q}_0(t)+\omega^2q_0=0\end{aligned} \tag{14}$$

where the initial condition is

$$q_0(0)=a_0 \quad , \quad \dot{q}_0(0)=0$$

And another ordinary differential equation is

$$\begin{aligned}&O(P):\\&\ddot{q}_1(t)+\omega^2q_1=-\left[f(u,\xi_0)q_0(t)^3-\gamma(u,\xi_0)q_0(t)+\omega_{11}q_0(t)\right]\end{aligned} \tag{15}$$

with initial conditions as follows

$$q_1(0)=0 \quad , \quad \dot{q}_1(0)=0$$

To achieve a more accurate approximation, also the other ordinary differential equation is obtained as aforementioned investigation. As the approximate number of levels increases, also the accuracy increases and in the interval time that the results are consistent with the numerical results, are also increased [18,19]. It is seen that the next steps of approximation solution depends on previous steps solution. For this reason, at first the solution of differential equation is considered, that its solution with applying initial condition, can be obtained as Eq. (16)

$$q_0(t)=a_0\cos(\omega t) \tag{16}$$

Substituting Eq. (16) into Eq. (15)

$$\ddot{q}_1(t)+\omega^2 q_1=-\left[\omega_{11}a_0\cos(\omega t)+f(u,\xi_0)\left(a_0\cos(\omega t)\right)^3-\gamma(u,\xi_0)\left(a_0\cos(\omega t)\right)^5\right] \tag{17}$$

which simplify the right of Eq. (17) and prevent of the creation long time terms such as tcos(ωt) in the final solution, which is sum of all approximated times; should coefficient of cos(ωt) term at the left of solution be zero, then it can be achieved

$$\omega_{11}=-\frac{3}{4}f(u,\xi_0)a_0^2+\frac{5}{8}\gamma(u,\xi_0)a_0^4 \tag{18}$$

With substituting of Eq. (18) in Eq. (12) ,the first approximation of nonlinear frequency of system is achieved. On the other hand, with solution of differential Eq. (17) and applying its initial condition

$$q_1(t)=A_1\cos(\omega t)+(A_2+A_3)\cos(3\omega t)+A_4\cos(5\omega t) \tag{19}$$

which introduced coefficient in Eq. (19) can be considered

$$\begin{gathered}A_1=A_2+A_3+A_4\\ A_2=\frac{1}{32}\frac{a_0^3 f(u,\xi_0)}{\omega^2}\qquad A_3=\frac{5}{128}\frac{\gamma(u,\xi_0)a_0^5}{\omega^2}\qquad A_4=-\frac{1}{384}\frac{\gamma(u,\xi_0)a_0^5}{\omega^2}\end{gathered} \tag{20}$$

In this way, the higher approximation of solution can be obtained. Also in the other equations , as it has been described, the creation long time terms should be prevented of. Because in this case, the higher approximation of nonlinear frequency can be achieved.

4 RESULTS AND DISCUSSION

In this section, results of the analysis are investigated for the nonlinear free vibration beam with a moving mass on it. Also, to evaluate the accuracy of the results in homotopy perturbation analytical method, a numerical example is solved. In this regard, the results are compared with Runge - Kutta method fourth order, that is one of the most accurate numerical methods. As can be seen in Fig.2, data from analytical method are entirely consistent with numerical results and represents a very high accuracy and a very high convergence rate of this method. In Figs. 3 and 4, the effect of masses ratio and also the velocity of moving-concentrated mass on response time is investigated, respectively. As indicated in the following figures, increasing the value of these parameters causes the period increased and other means, lead to reduce the vibration frequency.

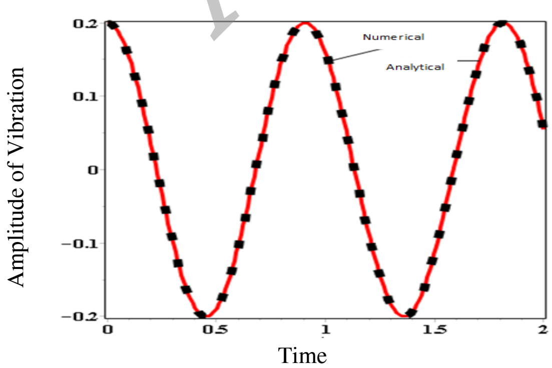

Fig. 2
Comparing of numerical method and analytical method.

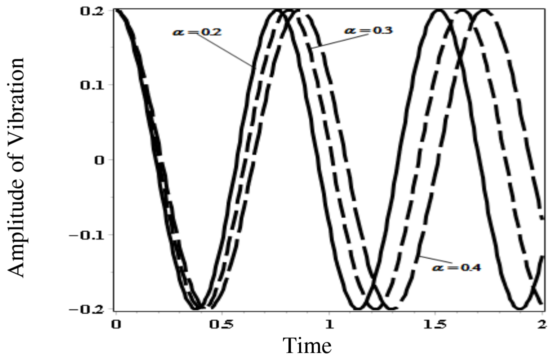

Fig. 3
Effect of masses ratio on response-time system.

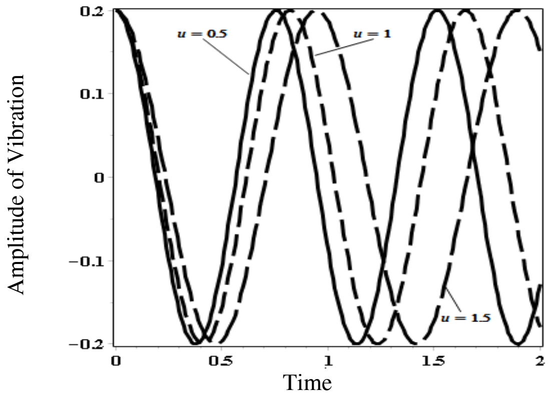

Fig. 4
Effect of masses velocity on response-time system.

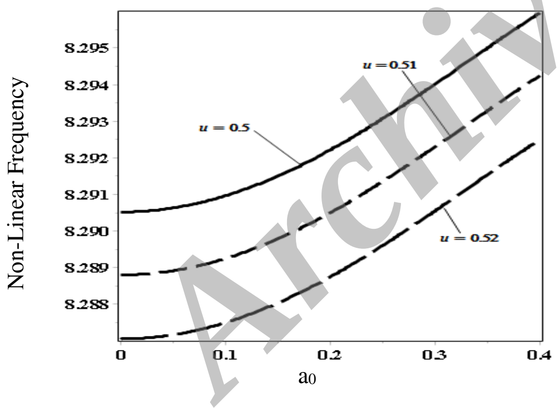

Fig. 5
Effect of the initial condition of amplitude on the nonlinear frequency.

As shown in Fig. 5, the effect of initial condition of excitation amplitude on the nonlinear natural frequency is investigated. As observed in this figure, the nonlinear natural frequency can be increased within the specified range; this figure is also confirmed Fig. 4. In Figs. 6 and 7, effect of the nonlinear-terms coeffecients in the equation of three and five degree on nonlinear natural frequency are shown. As seen in these figures, the effects of both nonlinears are opposed to each other; the increasing of nonlinear cubic equation caused the increase frequency; on the other hand, with increasing of nonlinear equation of five degree, the frequency decreased. As shown in the Fig. 8, the effect of concentrate mass location on response time period is investigated. As can be observed, when the mass is closer to the center, caused the motion period to be increased.

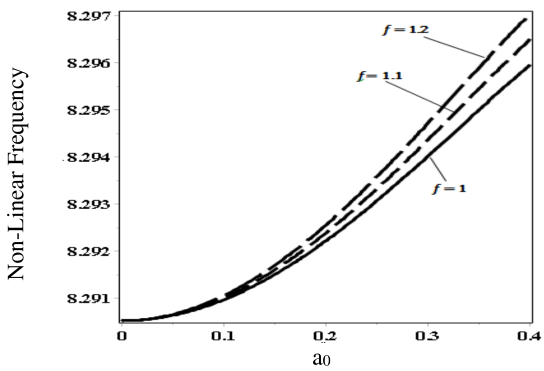

Fig. 6
Effect of the nonlinear-terms coefficient in the equation of three.

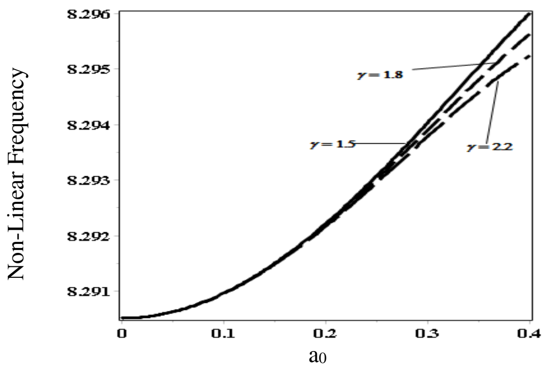

Fig. 7
Effect of the nonlinear-terms coefficient in the equation of five.

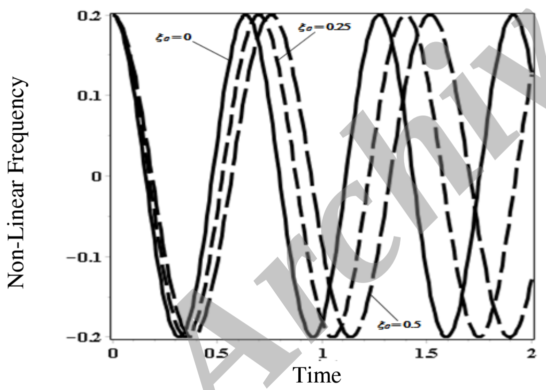

Fig. 8
Effect of concentrate mass instantaneous location.

5 CONCLUSIONS

In the present study, the analysis of nonlinear vibration for a simply-supported beam that with a constant velocity carrying a concentrated mass, has been studied. To solve the problem, modified homotopy method which consists of homotopy method and traditional perturbation, with a very high accuracy, was employing. Parametric studies showed that increasing the velocity of concentrated mass caused the decrease of nonlinear natural frequency. On the other hand, for cases that mass are closer to the center, the results were the same.

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