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Wave Propagation in Fibre-Reinforced Transversely Isotropic Thermoelastic Media with Initial Stress at the Boundary Surface

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ABSTRACT

The reflection and transmission of thermoelastic plane waves at an imperfect boundary of two dissimilar fibre-reinforced transversely isotropic thermoelastic solid half-spaces under hydrostatic initial stress has been investigated. The appropriate boundary conditions are applied at the interface to obtain the reflection and transmission coefficients of various reflected and transmitted waves with incidence of quasi-longitudinal (qP), quasi-thermal (qT) & quasi- transverse (qSV) waves respectively at an imperfect boundary and deduced for normal stiffness, transverse stiffness, thermal contact conductance and welded boundaries.The reflection and transmission coefficients are functions of frequency, initial stress and angle of incidence. There amplitude ratios are computed numerically and depicted graphically for a specific model to show the effect of initial stress. Some special cases are also deduced from the present investigation.

Keywords : Fibre-reinforced; Hydrostatic initial stress; Reflection; Transmission; Thermoelasticity.

1 INTRODUCTION

FIBRE-REINFORCED are widely used in engineering structures, due to their superiority over the structural materials in applications requiring high strength and stiffness in lightweight components. Consequently, characterization of their mechanical behavior is of particular importance for structural design using these materials. Fibres are assumed an inherent material property, rather than some form of inclusion in models as Spencer [1]. In the case of an elastic solid reinforced by a series of parallel fibres it is usual to assume transverse isotropy.

Lord and Shulman [2] introduced a theory of generalized thermoelasticity with one relaxation time for an isotropic body. The theory was extended for anisotropic body by Dhaliwal and Sherief [4]. In this theory, a modified law of heat conduction including both the heat flux and its time derivatives replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both coupled and uncoupled theories of thermoelasticity. Erdem [5] derived heat conduction equation for a composite rigid material containing an arbitrary distribution of fibres. Recently, Kumar [6] discussed the wave motion in an anisotropic fibre-reinforced thermoelastic solid.

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Many authors have studied the wave propagation in isotropic thermoelasticity. For example, Deresiewicz [7] studied the effects of boundaries on the waves in a thermoelastic solid and reflection of plane waves from a plane boundary. Sinha and Sinha [8] and Sinha and Elsibai [9] discussed the reflection of thermoelastic waves at a solid half-space in context of the Lord and Shulman [2] and Green and Lindsay [3] theories.

Sinha and Elsibai [10] studied the reflection of thermoelastic waves at the interface of two semi-infinite media being in welded contact. Singh [11] and Abd-Alla et al. [12] discussed some problems concerning reflection of the generalized magneto-thermo-viscoelastic plane waves from a stress-free surface. Singh [13] discussed the reflection of SV waves from the free surface of an elastic solid with generalized thermoelastic diffusion. Song et al. [14] studied the wave propagation at interface between two half-spaces of micropolar viscoelastic media. Singh and Khurana [15] studied reflection and transmission of P and SV waves at the interface between two monoclinic elastic half-spaces. Kumar and Singh [16] discussed the reflection and transmission at an imperfectly bounded interface between two orthotropic, generalized thermoelastic half-spaces.

The study of wave propagation in a generalized thermoelastic media with additional parameters like prestress, porosity, viscosity, microstructure, temperature and other parameters provide vital information about existence of new or modified waves. The Earth is assumed to be under high initial stresses. Such information may be useful for experimental seismologists in correcting earthquake estimation. It is therefore of much interest to study the influence of these stresses on the propagation of stress waves. Biot [17] showed the acoustic propagation under initial stresses which was fundamentally different from that under stress-free state. He has obtained the velocities of longitudinal and transversal waves along the co-ordinate axis only. Some problems of reflection and transmission phenomena of plane waves in unbounded medium under initial stresses were investigated by Chattopadhyay et al. [18], Sidhu and Singh [19], Dey et al. [20] and Selim [21].

Montanaro [22] investigated the isotropic linear thermoelasticity with hydrostatic initial stress. Singh et al. [23], Singh [24] and Othman and Song [25] used the theory given by Montanaro [22] to study the reflection of thermoelastic waves from a free surface under hydrostatic initial stress, in context of different theories of the generalized thermoelasticity. [26] Abd-Alla and Alsheikh showed the effect of the initial stresses on the reflection and transmission on plane quasi vertical transverse waves in piezoelectric materials. Chattopadhyay [27] investigated reflection and transmission of quasi P and SV waves at the interface of fibre-reinforced media. Recently, Singh and Zorammuana [29] studied the reflection of plane waves at a plane free fibre-reinforced thermoelastic half-space.

In the present paper, the governing equations of fibre-reinforced transversely isotropic thermoelastic solid medium are formulated to study the problem of reflection and transmission at the boundary surface. The boundary conditions at the interface are formulated and the expressions of reflection and transmission coefficients are obtained and computed for a particular model. Numerical results are shown graphically to show the effect of initial stresses on the reflection and transmission coefficients of various reflected and transmitted waves.

2 BASIC EQUATIONS

The basic equations in a homogeneous thermally conducting fibre-reinforced medium with an initial hydrostatic stress without body forces and heat sources are given by Lord and Shulman [2], Othman & Abbas [28] as:

$$\sigma_{ij,j} - P\omega_{ij,j} = \rho \ddot{u}_i \,, i,j = 1,2,3 \tag{1}$$

and heat conduction equation is given by

$$k_{ij} T_{,ij} = T_o \beta_{ij} \dot{u}_{i,j} + \rho C_e \dot{T} \,,\ i,j = 1,2,3 \tag{2}$$

The constitutive equations for thermally conducting transversely isotropic, fibre-reinforced linearly elastic medium [4, 5] are

$$\begin{aligned}\sigma_{ij} = {} & \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) \\ & + \beta(a_k a_m e_{km} a_i a_j) - \beta_{ij} T \,,\quad i,j,k,m = 1,2,3\end{aligned} \tag{3}$$

where

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \omega_{ij} = \frac{1}{2}\left(u_{j,i} - u_{i,j}\right), i, j = 1,2,3 \tag{4}$$

and ρ is the mass density , σ_{ij} are components of stress tensor, u_i are displacement components, e_{ij} are components of infinitesimal strain, T the temperature change of a material particle, T_o the reference uniform temperature of the body, k_{ij} are coefficients of thermal conductivity, β_{ij} are thermal elastic coupling tensor, C_e the specific heat at constant strain, δ_{ij} is the kronecker delta, P is component of the initial stress, The comma in subscript notation is used for spatial derivatives and superimposed dot represents time differentiation. a_j are components of a, all referred to Cartesian coordinate. The vector a may be a function of position. We choose a so that its components are (1, 0, 0).

3 FORMULATION OF THE PROBLEM

We consider fibre-reinforced transversely isotropic thermoelastic media M_1 & M_2 with different elastic and thermal properties. Rectangular Cartesian coordinate system is taken as $Ox_1x_2x_3$,O is the origin at the interface of two media M_1 & M_2 and x_2 is pointing vertically downward in the medium. All quantities with superscript '*m*' correspond to medium M_2.

The displacement components for medium M_1 are taken as:

$$u = (u_1, u_2, 0) \tag{5}$$

Eqs .(1) and (2) with the help of Eqs .(3), (4) and (5), take the form

$$C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + \left(C_{12} + C_0 - \frac{P}{2}\right)\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + \left(C_0 + \frac{P}{2}\right)\frac{\partial^2 u_1}{\partial x_2^2} - \beta_{11}\frac{\partial T}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{6}$$

$$C_{22}\frac{\partial^2 u_2}{\partial x_2^2} + \left(C_{12} + C_0 - \frac{P}{2}\right)\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \left(C_0 + \frac{P}{2}\right)\frac{\partial^2 u_2}{\partial x_1^2} - \beta_{22}\frac{\partial T}{\partial x_2} = \rho\frac{\partial^2 u_2}{\partial t^2}, \tag{7}$$

$$\frac{\partial^2 T}{\partial x_1^2} + \bar{k}\frac{\partial^2 T}{\partial x_2^2} - \frac{\rho C_e}{k_{11}}\left(\frac{\partial T}{\partial t} + \tau_o\frac{\partial^2 T}{\partial t^2}\right) = \varepsilon\left[\left(\frac{\partial^2 u_1}{\partial x_1 \partial t} + \tau_o\frac{\partial^3 u_1}{\partial x_1 \partial t^2}\right) + \bar{\beta}\left(\frac{\partial^2 u_2}{\partial x_2 \partial t} + \tau_o\frac{\partial^3 u_2}{\partial x_2 \partial t^2}\right)\right] \tag{8}$$

where

$$\begin{aligned} &\beta_{11} = \left(C_{11} + C_{12}\right)\alpha_{11} + C_{12}\alpha_{22}, \beta_{22} = \left(C_{12} + C_{23} - C_{55}\right)\alpha_{11} + C_{22}\alpha_{22}, C_{12} = \lambda + \alpha, \\ &C_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta, C_{22} = C_{33} = \lambda + 2\mu_T, C_{44} = C_{66} = 2\mu_L, C_{55} = 2\mu_T, \\ &C_{23} = C_{33} - C_{55}, C_o = \frac{C_{44}}{2}, \varepsilon = \frac{T_o\beta_{11}}{k_{11}}, \bar{k} = \frac{k_{22}}{k_{11}}, \bar{\beta} = \frac{\beta_{22}}{\beta_{11}}, \end{aligned} \tag{9}$$

and $\lambda, \alpha, \beta, \mu_L, \mu_T$ are material constants, α_{11}, α_{22} are components of linear thermal expansion, τ_o is thermal relaxation time.

To facilitate the solution, the following dimensionless quantities are introduced

$$\left(x_1^{'}, x_2^{'}\right) = \frac{\omega^*}{v_1}\left(x_1, x_2\right), \left(u_1^{'}, u_2^{'}\right) = \frac{\rho v_1 \omega^*}{\beta_{11} T_o}\left(u_1, u_2\right), t_{ij}^{'} = \frac{t_{ij}}{\beta_{11} T_o}, T^{'} = \frac{T}{T_o}, t^{'} = \omega^* t, \tau_o^{'} = \omega^* \tau_o \tag{10}$$

where $\omega^* = \dfrac{C_e C_{11}}{k_{11}}, v_1^2 = \dfrac{C_{11}}{\rho}$

4 SOLUTION OF THE PROBLEM

We assume the solutions of the form

$$\left(u_1, u_2, T\right) = \left(Ad_1, Ad_2, \xi B\right)\exp\left\{i\xi\left(x_1 p_1 + x_2 p_2 - vt\right)\right\} \tag{11}$$

where $p\left(p_1, p_2, 0\right)$ denote the unit propagation vector, $d\left(d_1, d_2, 0\right)$ is the unit displacement vector, $v = \omega/\xi$ is the non-dimensional phase velocity, *A,B* are the arbitrary constants, ω is the frequency and ξ is the wave number of the plane waves propagating in $x_1 x_2$ plane. Substituting the values of u_1, u_2 and *T* from the Eq. (11) in Eqs .(6), (7) and (8), we obtain

$$\left[\left(v^2 - p_1^2\right) - \frac{1}{C_{11}}\left(C_o + \frac{P}{2}\right)p_2^2\right]Ad_1 - \left[\frac{1}{C_{11}}\left(C_{12} + C_o - \frac{P}{2}\right)p_1 p_2 + \frac{1}{C_{11}}\left(C_o + \frac{P}{2}\right)p_2^2\right]Ad_2 + \left[ip_1\right]B = 0 \tag{12}$$

$$\left[\frac{1}{C_{11}}\left(C_{12} + C_o - \frac{P}{2}\right)p_1 p_2\right]Ad_1 + \left[\frac{C_{22}}{C_{11}}p_2^2 + \frac{1}{C_{11}}\left(C_o + \frac{P}{2}\right)p_1^2 - v^2\right]Ad_2 + \left[i\,\bar{\beta}p_2\right]B = 0 \tag{13}$$

$$\left[\varepsilon_1 v p_1 (1 - i\,\tau_o\omega)\right]Ad_1 + \left[\varepsilon_1 v p_2\left(\bar{\beta} - i\,\tau_o\omega\right)\right]Ad_2 + \left[\xi\left(p_1^2 + \bar{k}p_2^2\right) - iv\left(1 - i\,\tau_o\omega\right)\right]B = 0 \tag{14}$$

where $\varepsilon_1 = \dfrac{T_o\beta_{11}^2}{\rho\omega^* k_{11}}$, the system of Eqs. (12)-(14) has a non-trivial solution if the determinant of the coefficients $\left[d_1, d_2, B\right]^T$ vanishes, which yields to the following polynomial characteristic equation

$$v^6 + d_{21}v^4 + d_{31}v^2 + d_{41} = 0 \tag{15}$$

where

$$\begin{aligned}
&\Delta_1 = \left(-\xi A^2\tau_0\right),\ \Delta_2 = \left[A\xi\tau\left(-H + F\right) + A^2 R + A^2\xi\varepsilon_1\left(\bar{\beta}p_2^2\tau_b - p_1^2\tau\right)\right] \\
&\Delta_3 = \left[AR\left(H + F\right) + \varepsilon_1 A\xi p_2^2\tau_b\left(-F\bar{\beta} + p_1^2 EA\right) + \xi\tau\left(FH + p_1^2 H\varepsilon_1 A - Gp_1 p_2 A\left(E + \bar{\beta}\varepsilon_1\right)\right)\right] \\
&\Delta_4 = \left[-FHR + p_1 p_2 GEAR\right],\ d_{21} = \frac{\Delta_2}{\Delta_1},\ d_{31} = \frac{\Delta_3}{\Delta_1},\ d_{41} = \frac{\Delta_4}{\Delta_1}
\end{aligned} \tag{16}$$

and,

$$\begin{aligned}
&D = \frac{1}{C_{11}}\left(C_o + \frac{P}{2}\right),\ E = \frac{1}{C_{11}}\left(C_{12} + C_o - \frac{P}{2}\right),\ F = A\left(p_1^2 + Dp_2^2\right),\ G = \left(-Ep_1 p_2 + Dp_2^2\right)A, \\
&H = -A\left(\frac{C_{22}}{C_{11}}p_2^2 + Dp_1^2\right),\ R = \xi\left(p_1^2 + \bar{k}p_2^2\right),\ \tau = \tau_o + \frac{i}{\omega},\ \tau_b = \tau_o + \frac{i\bar{\beta}}{\omega}
\end{aligned} \tag{17}$$

The characteristic Eq. (15) is cubic in v^2 and hence possesses three roots $\left(v_p^2, p=1,2,3\right)$. Therefore, there exist three types of quasi-waves in transversely isotropic thermoelastic half-space namely quasi-longitudinal waves (qP), quasi-thermal waves (qT) and quasi-transverse waves (qSV).

5 REFLECTION AND TRANSMISSION

5.1 Incident qP waves

Consider homogeneous fibre-reinforced transversely isotropic thermoelastic half-spaces occupying the regions $x_2>0$ (lower medium M_1) and $x_2<0$ (upper medium M_2). We consider the incidence of qP wave passing through medium M_1, at interface $x_2=0$, resulting to this incident wave, we get three reflected wave (i) qP , (ii) qT and (iii) qSV in lower medium M_1 and three transmitted wave (i) qP , (ii) qT and (iii) qSV in upper medium M_2 respectively. We label these waves (i) incident as (n=0), three reflected waves as (n=1, 2, 3) and three transmitted waves as (n=4, 5, 6) respectively. The complete geometry showing the angle of incidence, angles of reflection and angles of transmission are shown in Fig.1.

$$
\begin{aligned}
&u_1=\sum_{j=0}^{3}A_j d_1^j \exp(i\eta_j),\ u_2=\sum_{j=0}^{3}A_j d_2^j \exp(i\eta_j),\ T=\sum_{j=0}^{3}\xi_j B_j \exp(i\eta_j) \qquad \left[x_2>0\right]\\
&u_1^m=\sum_{j=4}^{6}A_j d_1^j \exp(i\eta_n),\ u_2^m=\sum_{j=4}^{6}A_j d_2^j \exp(i\eta_n),\ T^m=\sum_{j=4}^{6}\xi_j B_j \exp(i\eta_j) \qquad \left[x_2<0\right]
\end{aligned} \tag{18}
$$

where

$$
\eta_j=\xi_j\left(x_1p_1^j+x_2p_2^j-v_jt\right), \qquad \left(j=0,1,2,3,4,5,6\right) \tag{19}
$$

The expression for displacements and temperature field for the medium M_1 and M_2 are

$$
\left(u_1,u_2,T\right)=\sum_{j=0}^{3}\left(d_1^j,d_2^j,\xi_jF_jd_1^j\right)A_j\exp(i\eta_j),
$$

and

$$
\left(u_1^m,u_2^m,T^m\right)=\sum_{j=4}^{6}\left(d_1^j,d_2^j,\xi_jF_jd_1^j\right)A_j\exp(i\eta_j), \tag{20}
$$

where

$$
\begin{aligned}
&F_j=\frac{F_{1j}}{F_{2j}},\ F_{1j}=\left[\left(-E_jp_1^jp_2^j\right)\left(\varepsilon v_jp_2^j\left(\bar{\beta}-i\tau_0\xi_jv_j\right)\right)+\left(\varepsilon v_jp_1^j\left(1-i\tau_0\xi_jv_j\right)\right)\left(\frac{C_{22}}{C_{11}}\left(p_2^j\right)^2+D_j\left(p_1^j\right)^2-\left(v_j\right)^2\right)\right]\\
&F_{2j}=\left[-\left(\frac{C_{22}}{C_{11}}\left(p_2^j\right)^2+D_j\left(p_1^j\right)^2-\left(v_j\right)^2\right)\left(R_j-iv_j-\tau_0\xi_j\left(v_j\right)^2\right)+\left(\varepsilon v_jp_2^j\left(\bar{\beta}-i\tau_0\xi_jv_j\right)\right)\left(i\bar{\beta}p_2^j\right)\right]
\end{aligned}
$$

For incident qP waves

$$
p_1^{(0)}=\sin\theta_0,\ p_2^{(0)}=-\cos\theta_0, d_1^{(0)}=\cos\theta_0, d_2^{(0)}=\sin\theta_0, v_0=v_{p1} \tag{21}
$$

For reflected qP waves

$$p_1^{(1)} = \sin\theta_1, p_2^{(1)} = \cos\theta_1, d_1^{(1)} = \sin\theta_1, d_2^{(1)} = \cos\theta_1, v_1 = v_{p1} \tag{22}$$

For reflected qT waves

$$p_1^{(2)} = \sin\theta_2, p_2^{(2)} = \cos\theta_2, d_1^{(2)} = -\cos\theta_2,\ d_2^{(2)} = \sin\theta_2, v_2 = v_{p2} \tag{23}$$

For reflected qSV waves

$$p_1^{(3)} = \sin\theta_3, p_2^{(3)} = \cos\theta_3, d_1^{(3)} = -\cos\theta_3, d_2^{(3)} = \sin\theta_3, v_3 = v_{p3} \tag{24}$$

For transmitted qP waves

$$p_1^{(4)} = \sin\theta_4, p_2^{(4)} = -\cos\theta_4, d_1^{(4)} = \sin\theta_4, d_2^{(4)} = -\cos\theta_4, v_4 = v_{n1} \tag{25}$$

For transmitted qT waves

$$p_1^{(5)} = \sin\theta_5, p_2^{(5)} = -\cos\theta_5, d_1^{(5)} = \cos\theta_5, d_2^{(5)} = \sin\theta_5, v_5 = v_{n2} \tag{26}$$

For transmitted qSV waves

$$p_1^{(6)} = \sin\theta_6, p_2^{(6)} = -\cos\theta_6, d_1^{(6)} = \cos\theta_6, d_2^{(6)} = \sin\theta_6, v_6 = v_{n3} \tag{27}$$

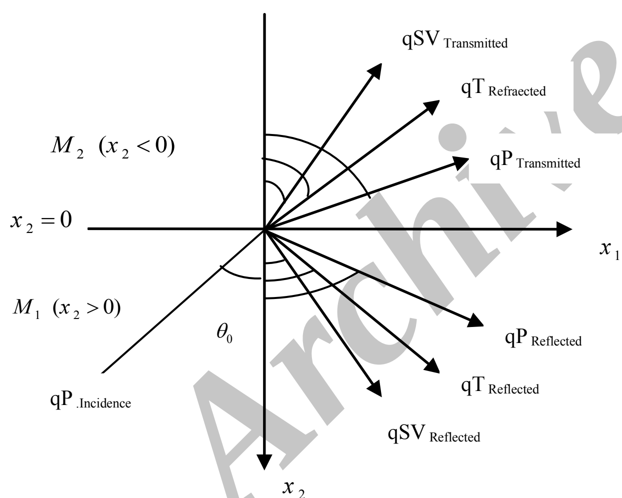

Fig.1
Geometry of the Problem.

5.1 Incident qt & qsv waves

This case is similar to the earlier case 4. In this case, n=0 to be considered for incident qT & incident qSV waves respectively. In the Eq. (21), v_0 is to be replaced by v_{p2} to get incident qT and v_{p3} to get incident qSV waves respectively. All the calculations are similar to incident qP waves.

6 BOUNDARY CONDITIONS

The appropriate boundary conditions at imperfect boundary surface $x_2 = 0$ are given by:

$$t_{22}^{m} = K_n\left(u_2 - u_2^{m}\right) \tag{28}$$

$$t_{21}^{m} = K_t\left(u_1 - u_1^{m}\right) \tag{29}$$

$$K^{*}\left(\frac{\partial T^{m}}{\partial x_2}\right) = K_\theta\left(T - T^{m}\right) \tag{30}$$

$$t_{22} = t_{22}^{m} \tag{31}$$

$$t_{21} = t_{21}^{m} \tag{32}$$

$$K^{*}\left(\frac{\partial T}{\partial x_2}\right) = \left(K^{*}\right)^{m}\left(\frac{\partial T^{m}}{\partial x_2}\right) \tag{33}$$

where the component of stresses are given by $t_{22} = C_{12}u_{1,1} + C_{22}u_{2,2} - \beta_{22}T,\;\; t_{21} = C_0\left(u_{1,2} + u_{2,1}\right)$ and $K_n, K_t \,\&\, K_\theta$ are normal force stiffness, transverse force stiffness of dimension N/m^3 and thermal contact conductance with dimension $W/m^2K\,\text{sec}$. respectively. $K^{*} = C_e\left(\lambda + 2\mu\right)/4$ and $\left(K^{*}\right)^{m} = C_e^{m}\left(\lambda^{m} + 2\mu^{m}\right)/4$ are the material characteristic constant.

The boundary conditions given by (28)-(33) must be satisfied for all values of x_1 , so we have

$$\eta_0\left(x_1,0,t\right) = \eta_1\left(x_1,0,t\right) = \eta_2\left(x_1,0,t\right) = \eta_3\left(x_1,0,t\right) = \eta_4\left(x_1,0,t\right) = \eta_5\left(x_1,0,t\right) = \eta_6\left(x_1,0,t\right) \tag{34}$$

Then from (19) and (34), we have

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} = \frac{\sin\theta_4}{v_4} = \frac{\sin\theta_5}{v_5} = \frac{\sin\theta_6}{v_6} = \frac{1}{v}$$

which corresponds to the Snell's law in this case,

Substituting the value of $u_1, u_2, T, u_1^{m}, u_2^{m} \,\&\, T^{m}$ from Eq. (20) in (28)-(33) and with the aid of (5) & (10), after simplification we obtain

$$\sum_{j=1}^{6} a_{ij}Z_j = -a_{i0}\,, \qquad \left(i = 1,2...6\right), \tag{35}$$

where

$$a_{1j} = \frac{-d_2^{j}K_n\beta_{11}T_0}{C_{11}}\,, \qquad (j = 0,1,2,3)$$

$$a_{1j} = \left[p_1^{j}d_1^{j} + \left(\frac{C_{22}}{C_{11}}\right)p_2^{j}d_2^{j} - \left(\frac{\beta_{22}C_{11}}{\beta_{11}C_{12}}\right)\xi_jF_jd_1^{j} + \left(\frac{\beta_{11}T_O}{C_{11}}\right)K_nd_2^{j}\right], \qquad (j = 4,5,6)$$

$$a_{2j} = \frac{-d_1^{j}K_t\beta_{11}T_0}{C_0}\,, \qquad (j = 0,1,2,3)$$

$$a_{2j} = \left[p_2^{j}d_1^{j} + p_1^{j}d_2^{j} + \left(\frac{\beta_{11}T_O}{C_0}\right)K_td_1^{j}\right], \qquad (j = 4,5,6)$$

$$a_{3j} = -K_\theta\left(\xi_j F_j d_1^j\right), \qquad (j = 0,1,2,3)$$
$$a_{3j} = \left(p_2^j + K_\theta\right)\xi_j F_j d_1^j, \qquad (j = 4,5,6)$$
$$a_{4j} = \left[d_1^j p_1^j + \left(\frac{C_{22}}{C_{11}}\right)d_2^j p_2^j - \left(\frac{\beta_{22}C_{11}}{\beta_{11}C_{12}}\right)\xi_j F_j d_1^j\right], \qquad (j = 0,1,2,3)$$
$$a_{4j} = -\left[d_1^j p_1^j + \left(\frac{C_{22}}{C_{11}}\right)d_2^j p_2^j - \left(\frac{\beta_{22}C_{11}}{\beta_{11}C_{12}}\right)\xi_j F_j d_1^j\right], \qquad (j = 4,5,6)$$
$$a_{5j} = \left(p_2^j d_1^j + p_1^j d_2^j\right), \qquad (j = 0,1,2,3)$$
$$a_{5j} = -\left(p_2^j d_1^j + p_1^j d_2^j\right), \qquad (j = 4,5,6)$$
$$a_{6j} = \xi_j F_j d_1^j p_2^j, \qquad (j = 0,1,2,3)$$
$$a_{6j} = -\left(\frac{K_2^*}{K_1^*}\right)\xi_j F_j d_1^j p_2^j, \qquad (j = 4,5,6)$$

and $Z_j = \dfrac{A_j}{A_0}, \qquad (j = 1,2,3,4,5,6)$

Here, Z_1, Z_2 & Z_3 are real-values of reflection coefficients (or amplitude ratio) of reflected qP, qT & qSV waves respectively and Z_4, Z_5 & Z_6 are reflection coefficients (or amplitude ratio) of transmitted qP, qT & qSV waves respectively.

7 PARTICULAR CASES

7.1 Normal force stiffness

In this case ($K_n \neq 0,\ K_t \to \infty,\ K_\theta \to \infty$), we have a boundary with normal stiffness and obtain a system of six non-homogeneous equations as given by (35) with the changed values of a_{ij} as:

$$a_{2j} = \frac{a_{2j}}{k_t}, \ (j = 0,1,2,3) \ ; a_{2j} = \left(\frac{\beta_{11}T_O}{C_0}\right)d_1^j, (j = 4,5,6)$$
$$a_{3j} = \frac{a_{3j}}{K_\theta}, (j = 0,1,2,3) \ ; a_{3j} = \xi_j F_j d_1^j, (j = 4,5,6)$$

7.2 Transverse force stiffness

In this case ($K_n \to \infty,\ K_t \neq 0,\ K_\theta \to \infty$), the imperfect boundary reduces to the transverse stiffness and we obtain a system of six non-homogeneous equations as given by (35) and modified values of a_{ij} are

$$a_{1j} = \frac{a_{1j}}{k_n}, \ (j = 0,1,2,3) \ ; a_{1j} = \left(\frac{\beta_{11}T_O}{C_0}\right)d_2^j, (j = 4,5,6)$$
$$a_{3j} = \frac{a_{3j}}{K_\theta}, (j = 0,1,2,3) \ ; a_{3j} = \xi_j F_j d_1^j, (j = 4,5,6)$$

7.3 Thermal contact conductance

In this case ($K_n \to \infty,\ K_t \to \infty,\ K_\theta \neq 0$), the imperfect boundary reduces to a thermally conducting imperfect surface, getting system of six non-homogeneous equations given by (35) with the changed values of a_{ij} as:

$$a_{1j} = \frac{a_{1j}}{k_n},\ (j=0,1,2,3)\ ;\ a_{1j} = \left(\frac{\beta_{11}T_O}{C_0}\right) d_2^j\ ,(j=4,5,6)$$
$$a_{2j} = \frac{a_{2j}}{k_t},\ (j=0,1,2,3)\ ;\ a_{2j} = \left(\frac{\beta_{11}T_O}{C_0}\right) d_1^j\ ,(j=4,5,6)$$

7.4 Welded contact

In this case ($K_n \to \infty,\ K_t \to \infty,\ K_\theta \to \infty$), a system of six non-homogeneous equations given by (35) with the modified values of a_{ij} as:

$$a_{1j} = \frac{a_{1j}}{k_n},\ (j=0,1,2,3)\ ;\ a_{1j} = \left(\frac{\beta_{11}T_O}{C_0}\right) d_2^j\ ,(j=4,5,6)$$
$$a_{2j} = \frac{a_{2j}}{k_t},\ (j=0,1,2,3)\ ;\ a_{2j} = \left(\frac{\beta_{11}T_O}{C_0}\right) d_1^j\ ,(j=4,5,6)$$
$$a_{3j} = \frac{a_{3j}}{K_\theta}\ ,(j=0,1,2,3)\ ;\ a_{3j} = \xi_j F_j d_1^j\ ,(j=4,5,6)$$

8 NUMERICAL RESULTS AND DISCUSSION

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results. For computation, we take the following values of the relevant parameters for fibre-reinforced transversely isotropic generalized thermoelastic solid:

For medium M_1 as:

$$\lambda = 7.76\times10^{10}\,N/m^2,\ \mu = 3.86\times10^{10}\,N/m^2,\ \rho = 8954Kg/m^3,\ \mu_T = 2.46\times10^{10}\,N/m^2, P=1$$
$$\mu_L = 5.66\times10^{10}\,N/m^2,\ \alpha = -1.28\times10^{10}\,N/m^2,\ \beta = 220.90\times10^{10}\,N/m^2,\ T_o = 293K$$
$$K_{11} = 0.690\times10^2\,Jm^{-1}\deg^{-1}s^{-1}, K_{22} = 0.701\times10^2\,Jm^{-1}\deg^{-1}s^{-1},\ \omega = 2s^{-1}, \tau_o = 0.05\,s$$
$$\alpha_1 = 0.017\times10^{-4}\deg^{-1}, \alpha_2 = 0.015\times10^{-4}\deg^{-1}, C_e = 0.3831\times10^3\,JKg^{-1}\deg^{-1}$$

For medium M_2 as:

$$\lambda^m = 9.4\times10^{10}\,N/m^2,\ \mu^m = 4.0\times10^{10}\,N/m^2, \rho^m = 1740Kg/m^3, K_{11}^m = 1.7\times10^2\,Jm^{-1}\deg^{-1}s^{-1},$$
$$K_{22}^m = 1.8\times10^2\,Jm^{-1}\deg^{-1}s^{-1}, \mu_T^m = 2.36\times10^{10}\,N/m^2, \mu_L^m = 5.56\times10^{10}\,N/m^2, T_o^m = 296K$$
$$\alpha^m = -1.24\times10^{10}\,N/m^2, \beta^m = 215.90\times10^{10}\,N/m^2, C_e^m = 1.04\times10^3\,JKg^{-1}\deg^{-1}, \tau_o^m = 0.05\,s,$$
$$\alpha_1^m = 0.025\times10^{-4}\deg^{-1}, \alpha_2^m = 0.027\times10^{-4}\deg^{-1},$$

with non-dimensional interface parameters as $K_n = 1.8,\ K_t = 2.0,\ \&\ K_\theta = 1.2$ .Using the above parameters for two different medium in contact, the system of Eqs. (35) are solved with the help of MATLAB PROGRAM. The absolute values of amplitude ratios (or reflection and transmission coefficients) of reflected and transmitted qP, qT and qSV waves are computed numerically with the range $0^0 \le \theta \le 60^0$ of angle of incidence of qP, qT and qSV waves. The variations of these amplitude ratios are shown graphically in Figs. 2 - 19. The solid curves with square symbol in these figures correspond to the amplitude ratios in fibre reinforced transversely isotropic thermoelastic material with initial stress(FTTIIS), solid line with round symbol corresponds to fibre reinforced transversely isotropic thermoelastic material without initial stress(FTTIWIS) and solid line with triangular symbol represents the fibre reinforced isotropic thermoelastic material with initial stress(FTISIS).

8.1 Incident qP-wave

It is noticed that the amplitude ratio $|z_1|$ of reflected qP wave first increase sharply to peak value at an angle $\theta = 5^0$ for the values of FTTIIS, FTTIWIS & FTISIS, then decrease sharply for the range $6^0 \le \theta \le 12^0$ and attain its minimum value at $\theta = 12^0$. Fig.3 indicates the variations of amplitude ratio $|z_2|$ of reflected qT-wave which shows that $|z_2|$ has certain maxima, particularly at $\theta = 5^0$ and at $\theta = 55^0$ for FTTIIS and FTTIWIS respectively. Behavior of FTTIWIS is just opposite to other two cases within the range $20^0 \le \theta \le 60^0$.The amplitude ratio $|z_3|$ shows similar behavior with $|z_1|$, but difference in their magnitude value. Moreover, small variations are noted between the values for FTTIIS and FTISIS.

The amplitude ratio $|z_4|$ of the transmitted P-wave w.r.t. the angle of incidence is shown in Fig.5 which indicates that $|z_4|$ attains maximum value at $\theta = 6^0$ and minimum value at $\theta = 11^0$ for the case FTTIWIS. On the other hand, the magnitude of values for FTTIIS is almost zero within the whole range. Fig.6 shows the variations of amplitude ratio $|z_5|$ of transmitted qT-wave which indicates that magnitude of $|z_5|$ for FTTIWIS is more as compared to FTTIIS and FTISIS. A sudden increment in the values of $|z_5|$ at the points $\theta = 5^0$, $\theta = 30^0$ and $\theta = 55^0$ are noted for the FTTIWIS, at these values of θ, magnitude of $|z_5|$ increases sharply to peak values and decreases smoothly towards minima at $\theta = 12^0, \theta = 25^0$ and $\theta = 38^0$ respectively. In Fig.7, the behavior of the curve FTISIS is almost similar to the curve FTTIWIS in Fig.6.

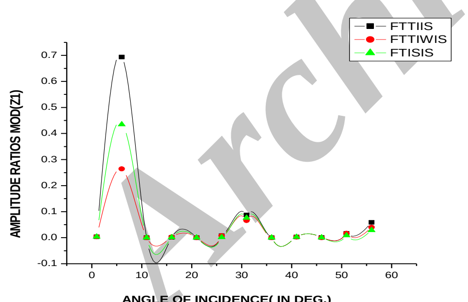

Fig.2
Reflection coefficient of qP waves due to incidence of qP wave.

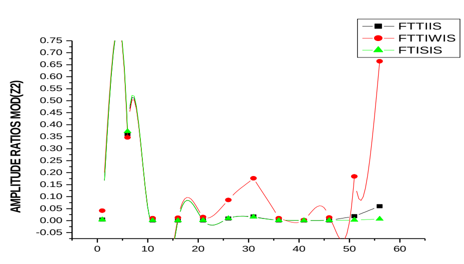

Fig.3
Reflection coefficient of qT waves due to incidence of qP wave.

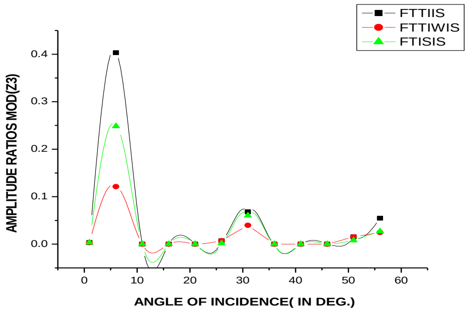

Fig.4
Reflection coefficient of qS waves due to incidence of qP wave.

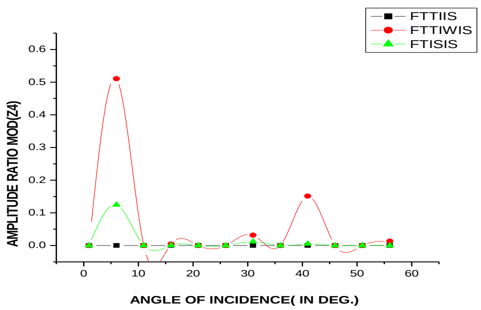

Fig.5
Refrection coefficient of qP waves due to incidence of qP wave.

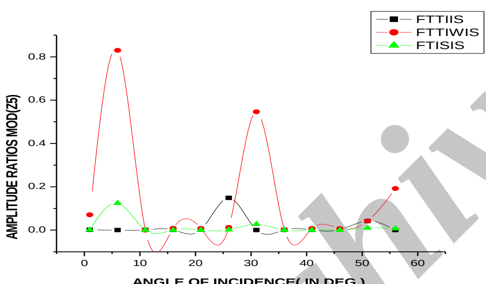

Fig.6
Refrection coefficient of qT waves due to incidence of qP wave.

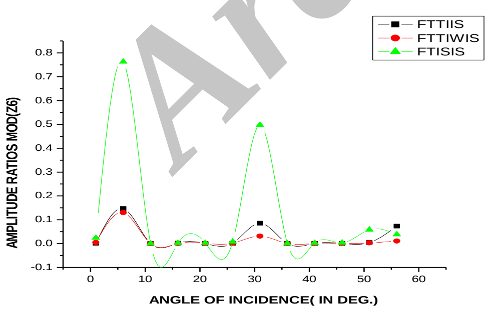

Fig.7
Refrection coefficient of qS waves due to incidence of qP wave.

8.2 Incident qT-wave

The variations of amplitude ratios of various reflected and transmitted waves when qT-wave is incident on the interface are shown in Figs.8-13. All the three curves show similar behavior of amplitude ratio $|z_1|$.The values of amplitude ratio $|z_1|$ first strictly increase within the range $0^0 \leq \theta \leq 5^0$ and then show a sudden fall within the range

$5^0 \le \theta \le 10^0$, which oscillates within rest of the range. Fig.9 indicates that amplitude ratio $|z_2|$ for FTTIIS and FTISIS have small variation in their magnitude as compared to FTTIWIS. Near the end of the range at angle $\theta = 60^0$, the behavior of all three cases are almost different and FTTIWIS shows a great variation in the values $|z_2|$ as compared to the presence of initial stress. The amplitude ratio $|z_3|$ shows similar behavior for all three cases as the amplitude ratio $|z_1|$ shows in Fig.8, but the values are different in magnitude within the whole range of angle of incidence.

Fig.11 indicates the amplitude ratio $|z_4|$ of transmitted qP-wave due to incidence of qT-wave. The effect of initial stress is more in isotropic case as compared to transversely isotropic case within the range $0^0 \le \theta \le 10^0$. For the amplitude ratio $|z_5|$ of transmitted qT-wave, the observed FTISIS reveals great impact as compared to FTTIIS within the whole range, which indicates that magnitude of $|z_5|$ in isotropic case is much more then transversely isotropic case. The amplitude ratio $|z_6|$ indicates that variation in the magnitude of $|z_6|$ for isotropic case is much more than transversely isotropic case. The behavior of all three curves is similar, but more variations in their magnitude of $|z_6|$ can be observed within the range $0^0 \le \theta \le 45^0$ in Fig.13.

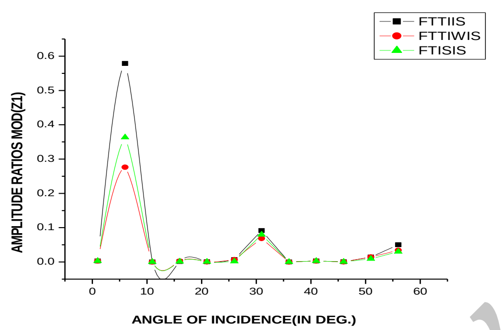

Fig.8
Reflection coefficient of qP waves due to incidence of qT wave.

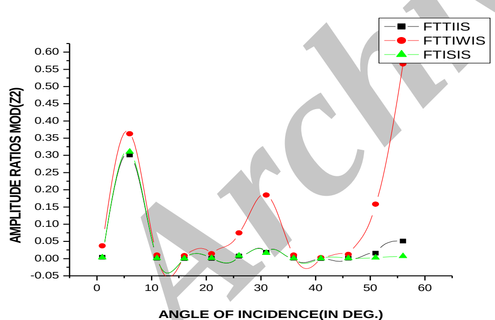

Fig.9
Reflection coefficient of qT waves due to incidence of qT wave.

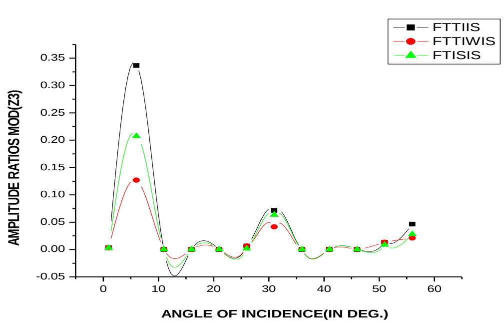

Fig.10
Reflection coefficient of qS waves due to incidence of qT wave.

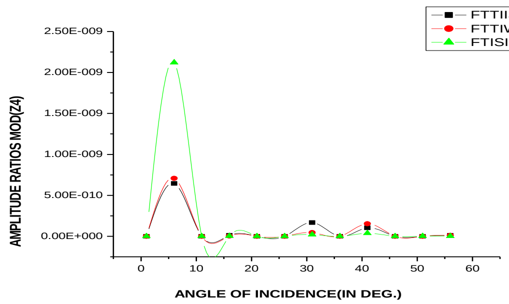

Fig.11
Refraction coefficient of qP waves due to incidence of qT wave.

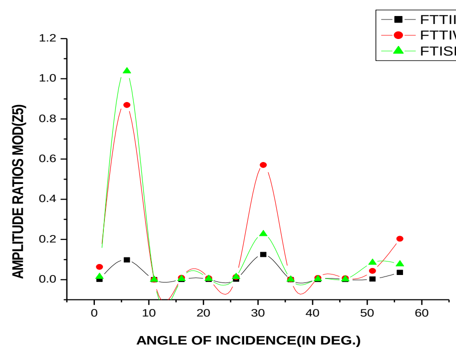

Fig.12
Refraction coefficient of qT waves due to incidence of qT wave.

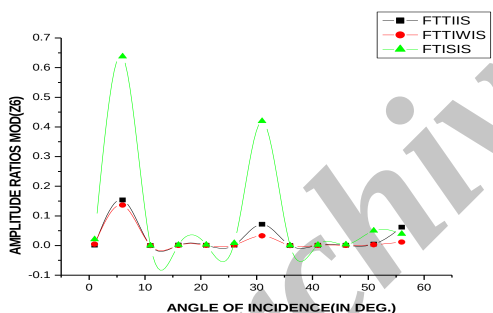

Fig.13
Refraction coefficient of qS waves due to incidence of qT wave.

8.3 Incidence qSV-wave

In Fig.14, the variations of amplitude ratio $|z_1|$ shows an oscillating behavior attaining certain maxima & minima within the range $0^0 \le \theta \le 35^0$ and then the values of $|z_1|$ increase strictly with increasing the value of angle of incidence within the range $36^0 \le \theta \le 40^0$ but decrease monotonically from $41^0 \le \theta \le 48^0$. Fig. 15 indicates that the curve for $|z_2|$ shows similar behavior to the curves of Fig. 14, but the corresponding value of amplitude ratio $|z_2|$ are different in magnitude for all three cases. For the amplitude ratio $|z_3|$ of reflected qSV-wave, curves show that impact of initial stress is more within the range $0^0 \le \theta \le 10^0$ and $27^0 \le \theta \le 37^0$. Although a non-overlapping but oscillating behavior of curves is noticed due to relevant difference in the magnitude of $|z_3|$ in Fig.16.

It is noticed that within the range $0^0 \le \theta \le 35^0$, the presence or absence of initial stress in both isotropic and transversely isotropic cases doesn't put any impact for the amplitude ratio $|z_4|$ but the curve FTTIWIS increase strictly within the range $36^0 \le \theta \le 40^0$ and then fall sharply from $41^0 \le \theta \le 46^0$ by gaining its maxima at

$\theta = 40^0$ which then leads towards zero near the end of the range. The behavior of curves for Fig.18 and Fig. 19 are almost same for FTTIWIS AND FTTIIS, but the corresponding values of amplitude ratios are different in magnitude. In both figures, it is evaluated that within the whole range, the value of isotropic case in the presence of initial stress have more impact and curves show an oscillating behavior by attaining a number of maxima and minima in the corresponding range. The significant effect of initial stress can be noted for isotropic case at $\theta = 30^0$ which is a very good difference in the magnitude of amplitude ratio $|z_6|$.

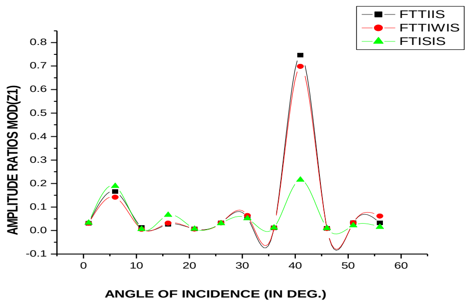

Fig.14
Reflection coefficient of qP waves due to incidence of qS wave.

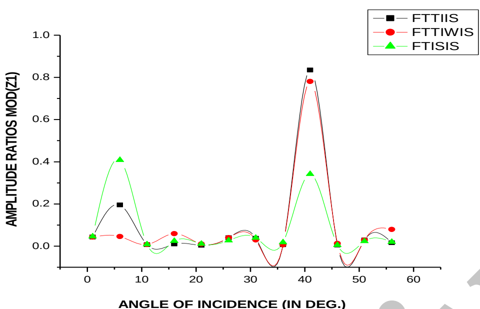

Fig.15
Reflection coefficient of qT waves due to incidence of qS wave.

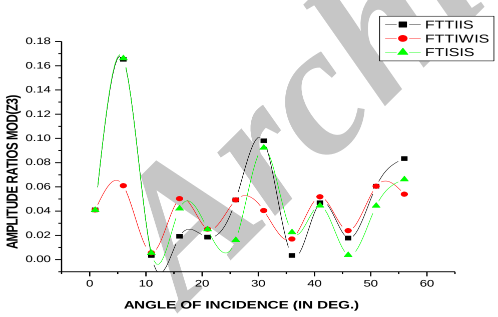

Fig.16
Reflection coefficient of qS waves due to incidence of qS wave.

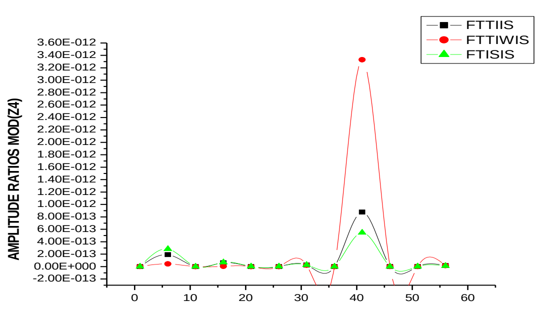

Fig.17
Refrection coefficient of qP waves due to incidence of qS wave.

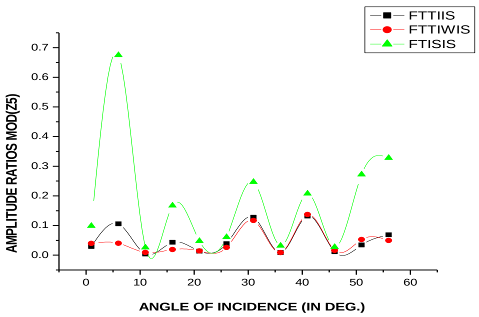

Fig.18
Refraction coefficient of qT waves due to incidence of qS wave.

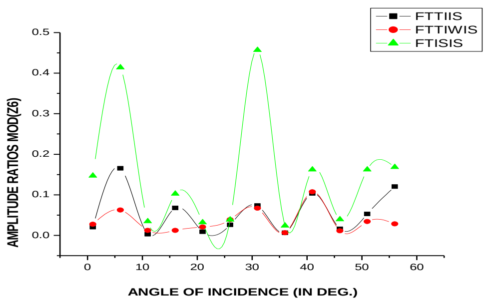

Fig.19
Refraction coefficient of qS waves due to incidence of qS wave.

9 CONCLUSIONS

The analytic behavior of amplitude ratio for various reflected and transmitted waves are obtained at the boundary surface between two different media. The expressions of reflection and transmission coefficients of various reflected and transmitted waves have been obtained for normal stiffness, transverse stiffness, thermally conducting and welded boundaries. An appreciable effect of initial stress and transversely isotropy is observed on amplitude ratio of various reflected and transmitted waves.It is observed from the above figures that the behavior of the amplitude ratios is oscillatory in nature and very much influenced with the effect of initial stress and anisotrophy near the beginning of the range i.e. $0^0 \le \theta \le 10^0$ with the incidence of qP and qT waves and near the end of the range with the incidence of qSV wave respectively. The model adopted in this paper is most realistic forms of the earth model and have the great importance for experimental seismologists.

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