Magnetic Stability of Functionally Graded Soft Ferromagnetic Porous Rectangular Plate

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ABSTRACT

This study presents critical buckling of functionally graded soft ferromagnetic porous (FGFP) rectangular plates, under magnetic field with simply supported boundary condition. Equilibrium and stability equations of a porous rectangular plate in transverse magnetic field are derived. The geometrical nonlinearities are considered in the Love-Kirchhoff hypothesis sense. The formulations are compared to those of homogeneous isotropic plates were given in the literature. In this paper the effect of pore pressure on critical magnetic field of plate and the effect of important parameters of poroelastic material on buckling capacity are investigated. Also the compressibility of fluid and porosity on the buckling strength are studied.

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Keywords : Buckling analysis; Rectangular plate; Functionally graded plate; Porous material; Magnetic field.

1 INTRODUCTION

T ECHNOLOGY development in field of making materials with functional properties for engineering usages including aerospace and medical industries etc., has led lots of researchers to study the behavior of these materials with engineering structure like beam, plate and shell. The researchers have conducted vast studies in order to investigate the behavior of these materials under mechanical forces and thermal field but rarely any researches have been done about the effect of magnetic field on behavior of these materials. Moon and Pao [1] studied the behavior of homogeneous rectangular ferromagnetic plates under the magnetic field. They found that when the intensity of magnetic field is increased up to a certain degree, the plate becomes instable and buckles. After that, other researches tried to formulate this behavior of the plates for example, Zhou and Zheng [2], expressed the energy of rectangular magnetic plate with a variation formulation. They compared their own results with the laboratory results which were obtained by Moon and Pao. By using this variation formula, they obtained the governing equations and behavior of the plate under magnetic field.

Wentao et al. [3], studied the stability of a thin homogeneous rectangular plate with simple supports under a magnetic field. They obtained the critical forces using variation method which was produced by the magnetic field. Their results were reasonably close to the results obtained by Moon and Pao. Using Zhou and Zheng's theory, Zhao et al. [4] studied the bending of homogeneous rectangular plate under magnetic field. By changing the angle of the magnetic field, deformation of the plate was shown. It was also demonstrated that the plate had the highest stability under transverse magnetic field by using FEM (Finite Element Method). Zang et al. [5], examined a rectangular plate made of the materials with ferromagnetic properties. They studied the bending and buckling of the plate with

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the simple support using Zhou and Zheng theory. By using a numerical method, they obtained the critical field for a rectangular plate and they compared it with the laboratory results of Moon and Pao. In their article, they showed the effect of applied magnetics field's angle on the plate. Zhao et al. [6], studied the buckling and post-buckling of a rectangular plate made of ferromagnetic materials. They examined the plate's behavior under magnetic field by using the classical theory. Studying the pre-buckling and post buckling of the plate, they concluded that by applying the transverse magnetic field, the plate buckles and becomes unstable without bending. Zheng and Wang [7] studied the behavior of a plate with ferromagnetic and non-linear magnetic properties; they compared the obtained results with the results from the linear ferromagnetic properties. Wang et al. [8], presented a variational model for expressing the behavior of rectangular plates under thermal and magnetic fields. They studied the thermal and magnetic instability of the rectangular plate with a simple support. The magnetic properties of the plate were assumed to be fixed in their study. They managed to obtain the stability limits of the plate under the magnetic and thermal fields by formulating the stability equations of the rectangular plate and solving the equations. Wang et al. [9], investigated stability of the rectangular plate with simple supports under the transverse magnetic field, applying Zhou and Zheng's energy method. Furthermore, they presented the effects of heat on the magnetic force and field. Considering Van Carmen's theory, they demonstrated the mechanical behavior of the plate and their results were also evaluated by using FEM. Magnetic properties of the plate have been considered as a function of temperature in their article. Zhou et al. [10], examined a rectangular plate with simple supports under magnetic and thermal fields, applying buckling and post-buckling energy method. They established the bending and buckling of the plate under angles of applied magnetic field, with use of a numerical method. They also showed that the plate buckles without transverse deformation while it is under the magnetic field. Zheng and Wang [11], formulated the behavior of a shell made of ferromagnetic materials, considering the energy method. Using FEM, they showed the behavior of the shell under the magnetic field. Er-gang et al. [12], studied the buckling of a rectangular plate under the magnetic field and they achieved the effect of plate's symmetrical and asymmetrical deformations on the critical load. The dynamic stability of ferromagnetic plate under transverse magnetic field and harmonic external force were studied by Wanga and Lee [13]. The effect of Lorentz's dynamic forces and the forces originated from the simultaneous stagnation magnetic field on dynamic stability of the rectangular plate with simple supports have been investigated. To express the plate's deformation, the classical theory has been applied by Wanga and Lee [13]. Dai et al. [14], studied a FGM cylinder with magnetic properties under a uniform magnetic field. Using Lorentz's forces, they obtained the stresses in the cylinder due to the magnetic field. Bhangale and Ganesan [15], obtained the behavior of a sandwich plate with a layer which was made of the materials with functional properties using FEM. Xing [16], obtained the natural frequencies of the ferromagnetic beam with a circular cross-section, utilizing Zhou and Zheng's energy method. They presented a dynamic model in order to calculate the beam's behavior under the transverse magnetic field. Raikher et al. [17], studied deformation of a circular membrane under transverse magnetic field which was made of a ferromagnetic material. The critical magnetic field for the circular membrane with ferromagnetic properties was also obtained. Kankanala and Triantafyllidis [18], examined the buckling of the ferromagnetic rectangular block under a uniform transverse magnetic field. Jin et al. [19], presented a model of the magnetic field and the magnetic forces which were caused by the magnetic field. They compared the model and the forces with Zhou and Zheng's model.

    The porous materials are composed of two elements: the body is consisted of the solid phase and the other element is either liquid or gas. This kind of materials are frequently found in nature, such as wood, stone, and layers of dust. For many years, porous material structures, such as beams, plates, and shells, have been widely discussed in structural design problems. The problem of deflection and buckling of the poro plates has been studied by many authors. For example, the buckling of a fluid-saturated porous slab under axial compression was considered by Biot [20]. He investigated the pore compressibility effect on critical buckling load and expressed that the critical load is proportional to the pore compressibility. He showed the lower and upper critical value for a rectangular plate, more over he provided identical behavior which was derived by analogy for a thermoelastic slab with a critical range between isothermal and adiabatic buckling. Jabbari et al. [21, 22] studied the buckling of a porous circular plate based on classical plate theory. They investigated the effect of porosity and pore fluid properties on the critical buckling load. The effect of a piezoelectric plate, which was added to the circular plate, on the stability of the circular plate was studied. Buckling of porous beams with varying properties was described by Magnucki and Stasiewicz [23]. They used shear deformation theory for solving the critical load. In this work, the effect of porosity on the strength was investigated as well as the buckling load of the beam. Magnucki et al. [24] investigated bending and buckling of a rectangular plate made of foam material. He obtained some findings about the poro/nonlinear symmetric distribution plate. Buckling of a circular porous plate with varying properties and simply supported boundary conditions was described by Magnucka-Blandzi [25]. Javaheri and Eslami [26] reported mechanical buckling of rectangular functionally graded plates (FGM) based on the classical plate theory (CPT). Jabbari et al.

[27-28] investigated on thermal buckling analysis of porous circular plate and investigated on the effect of different parameters on the thermal buckling load. Khorshidvand et al. [29] investigated on the effect of sensor actuator patches on critical buckling load for porous circular plate. Magnucki et al. [30] investigated on theoretical and experimental study of a sandwich circular plate under pure bending.

The present paper deals with critical buckling of FGFP rectangular plates, under magnetic field with simply supported boundary condition. It was assumed that properties of FGFP materials were changed through thickness according to power law functions and its behavior followed poroelastic relationship. Furthermore, its pores were saturated with fluid and there was no electric field, charge distribution or conduction current. General equilibrium and stability equations were derived by applying energy method and calculus of variations based on the classical plate theory. Then, closed form solutions for the rectangular plates under transverse magnetic field are obtained.

2 DERIVATION OF THE GOVERNING EQUATIONS

We consider a thin rectangular porous plate Fig. 1, length a, width b and thickness h under traverse magnetic field. The system coordinate (x, y, z) is established on the middle of this plate. The material properties are assumed to vary through the thickness according to the following power law distribution (see Fig. (1)).The functional relationship between G and z for porous plate is assumed as three different tips [21]

A - poro/nonlinear nonsymmetric distribution with shear modulus [21]

$$G(z)=G_0\left[1-e_1\cos\left(\left(\frac{\pi}{2h}\right)\left(z+\frac{h}{2}\right)\right)\right] \tag{1}$$

B - poro/nonlinear symmetric distribution with shear modulus [21]

$$G(z)=G_0\left[1-e_1\cos\left(\frac{\pi z}{h}\right)\right] \tag{2}$$

C - poro/monotonous distribution with shear modulus [21]

$$G(z)=G_0\left[1-e_1\right] \tag{3}$$

where e_1 is the coefficient of plate porosity $0<e_1<1$, G_1 and G_0 are the shear modulus at $z=-h/2$ and $z=h/2$, respectively. G_0 is shear modulus for perfect plate, ($G_0 \geq G_1$). The relationship between the modulus of elasticity for $j=0$ and 1 is $E_j=2G_j(1+v)$.

2.1 Basic equations

The linear poroelasticity theory of Biot has two features [20]:
   1. An increase of pore pressure induces a dilation of pore.
   2. Compression of the pore causes a rise of pore pressure.
   The stress-strain law for the poroelasticity is given by

$$\sigma_{ij}=2G\varepsilon_{ij} + \lambda\varepsilon\delta_{ij} - p\alpha\delta_{ij} \tag{4}$$

$$\Delta\varepsilon_{kk} =\frac{\Delta\sigma_{kk}}{3k} + \frac{\alpha\Delta p}{k} \tag{5}$$

$$\Delta\varepsilon_{kk} = \frac{\Delta\sigma_{ij}}{2G} \quad , \quad i \neq j \tag{6}$$

where

$$p = M\left(\zeta - \alpha\varepsilon_{kk}\right) \tag{7}$$

$$M = \frac{2G\left(v_u - v\right)}{\alpha^2\left(1 - 2v_u\right)\left(1 - 2v\right)} \tag{8}$$

$$v_u = \frac{v + \alpha B_s\left(1 - 2v\right)/3}{1 - \alpha B_s\left(1 - 2v\right)/3} \tag{9}$$

Here, p is pore fluid pressure, M is Biot's modulus, λ is Lame's parameters, v is drained Poisson's ratio, which is assumed to be constant across the plate thickness, v_u is undrained Poisson's ratio $v < v_u < 0.5$, α is the Biot coefficient of effective stress $0 < \alpha < 1$. The Biot coefficient ($\alpha = 1 - \frac{G(z)}{G_0}$) indicates the effect of porosity on the solid constituents of poroelastic plate and it shows the effect of generated stresses in the pores on the poroelastic material in undrained condition, B_s is the Skempton coefficient, the pore fluid properties is introduced by the Skempton coefficient, ζ is variation of fluid volume content, and ε_{kk} is the volumetric strain. The two dimensional stress-strain law for plane-stress condition in the Cartesian coordinates for the undrained condition ($\zeta = 0$ and $K = K_u$) is given by

$$\sigma_{xx} = A_1\varepsilon_{xx} + B_1\varepsilon_{yy} \qquad \sigma_{yy} = A_1\varepsilon_{yy} + B_1\varepsilon_{xx} \tag{10}$$

$$\sigma_{xy} = G(z)\gamma_{xy} \qquad p = M\left(-\alpha\varepsilon\right) \tag{11}$$

K_u is undrained bulk modulus. Where the constants A_1, B_1 in terms of the constants C_1, C_2 are

$$
\begin{aligned}
C_1 &= 2\left[1 + \frac{v_u}{1 - 2v_u} + \frac{v_u - v}{\left(1 - 2v_u\right)\left(1 - 2v\right)}\right]G(z) \qquad C_2 = C_1 - 2G(z) \\
A_1 &= \left(\frac{2}{1 - v_u^2}\right)\left[1 + v_u + \frac{\left(v_u - v\right)\left(1 + v_u\right)}{1 - 2v_u}\left(1 - \frac{C_2}{C_1}\right)\right]G(z) \\
B_1 &= \left(\frac{2}{1 - v_u^2}\right)\left[\left(1 + v_u\right)v_u + \frac{\left(v_u - v\right)\left(1 + v_u\right)}{1 - 2v_u}\left(1 - \frac{C_2}{C_1}\right)\right]G(z)
\end{aligned}
\tag{12}
$$

According to the Love-Kirchhoff hypothesis, based on the classical plate theory, the strain components at distance z from the middle plane are given by Javaheri and Eslami [26]

$$\varepsilon_{xx} = \overline{\varepsilon_{xx}} + zk_{xx} \qquad \varepsilon_{yy} = \overline{\varepsilon_{yy}} + zk_{yy} \qquad \gamma_{xy} = \overline{\gamma_{xy}} + 2zk_{xy} \tag{13}$$

where the z-axis is assumed positive outward. Here, $(\overline{\varepsilon_{xx}}, \overline{\varepsilon_{yy}}, \overline{\gamma_{xy}})$ are the engineering strain components in the median surface and (k_{xx}, k_{yy}, k_{xy}) are the curvatures which can be expressed in terms of the displacement

components. The relations between the middle plane strains, the curvatures, and the displacement components according to the Sanders assumption are [26]

$$\overline{\varepsilon_{xx}} = u_{,x} + \frac{1}{2}\left(w_{,x}\right)^2 \quad \overline{\varepsilon_{yy}} = v_{,x} + \frac{1}{2}\left(w_{,y}\right)^2 \quad \overline{\gamma_{xy}} = u_{,x} + v_{,y} + w_{,x}w_{,y} \quad k_{xx} = -w_{,xx} \quad k_{yy} = -w_{,yy} \tag{14}$$

where (u, v, w) represent the corresponding components of the displacement of a point on middle surface of plate.

2.2 Equilibrium and stability equations

The total potential energy V is the sum of the strain energy U_s and the potential energy of the magnetic energy U_m.

$$V = U_s + U_m \tag{15}$$

$$U_s = \frac{1}{2}\int \sigma_{ij} d\varepsilon_{ij}^E dV \tag{16}$$

where ε_{ij}^E is elastic strain. Elastic strain energy for porous materials is comprised of elastic strain energy for solid body and fluid in pores. Substituting Eq. (7) into Eq. (5) in the undrained condition and use of $(K_u = \frac{\alpha M}{B_s})$, volumetric strain is obtained as:

$$\varepsilon_{ij} = \frac{\left(1 - \alpha B_s\right)}{3K_u}\sigma_{ii} \quad , \quad i = x, y, z \tag{17}$$

Here, $\left(1 - \alpha B_s\right)$ is relation between drained bulk modulus and undrained bulk modulus, αB_s is coupling between pore fluid effects and macroscopic deformation. Substituting $\left(1 - \alpha B_s\right)$ in the Eq. (17) gives

$$\varepsilon_{ij}^E = \frac{\varepsilon_{ii}}{\left(1 - \alpha B_s\right)} \quad , \quad i = x, y, z \tag{18}$$

where σ_{kk} and ε_{kk}^E are volumetric stress and elastic strain, respectively. Substituting Eqs. (10) and (18) into Eq. (16) gives strain energy for the undrained condition

$$U_s = \frac{1}{2}\int_x\int_y\int_z\left(\frac{A_1}{1 - \alpha B_s}\left(\varepsilon_{xx}^2 + \varepsilon_{yy}^2\right) + \frac{2B_1}{\left(1 - \alpha B_s\right)}\varepsilon_{xx}\varepsilon_{yy} + G(z)\gamma_{xy}^2\right)dxdydz \tag{19}$$

The magnetic energy functional for the system as follows [2]

$$U_m = \frac{1}{2}\int_{V^+(w)}\mu_r\mu_0\left(\nabla\phi^+\right)^2 dxdydz + \frac{1}{2}\int_{V^-(w)}\mu_0\left(\nabla\phi^-\right)^2 dxdydz + \frac{1}{2}\int_{s_0}n.B\,\phi^- dxdy \tag{20}$$

In which ϕ is the magnetic scalar potential which satisfies $-\nabla\phi = H$, where $V^+(w)$ and $V^-(w)$, respectively, represent inside and outside regions of the deformed ferromagnetic plate with displacement vector denoted by w, ∇ is a 3D gradient operator, n is a unit vector outward normal to the surface s of the ferromagnetic plate, s_0 denotes a closed surface which surrounds and is far away from the ferromagnetic medium, B is magnetic induction vectors (relationship between magnetic induction vectors and magnetic field vectors for linear magnetic materials is,

$B^+ = \mu_0\mu_r H^+$), μ_0 and μ_r (constant for the entire plate) denote the magnetic permeability of vacuum and the relative permeability of the ferromagnetic plate, respectively.

Integrating Eqs. (19) and (20) with respect to z from $z = \dfrac{h}{2}$ to $z = -\dfrac{h}{2}$ and substituting into Eq. (15), it becomes

$$
\begin{aligned}
V &= \frac{1}{2}\int_x\int_y\left(A_2\left[\left(\overline{\varepsilon_{xx}}\right)^2+\left(\overline{\varepsilon_{yy}}\right)^2\right]+2B_2\,\overline{\varepsilon_{xx}}\,\overline{\varepsilon_{yy}}+C_2\left(\overline{\gamma_{xy}}\right)^2\right)dxdydz\\
&+\frac{1}{2}\int_x\int_y\left(2A_3\left[k_{xx}\,\overline{\varepsilon_{xx}}+k_{yy}\,\overline{\varepsilon_{yy}}\right]+2B_3\left[k_{yy}\,\overline{\varepsilon_{xx}}+k_{xx}\,\overline{\varepsilon_{yy}}\right]+4C_3 k_{xy}\,\overline{\gamma_{xy}}\right)dxdy\\
&+\frac{1}{2}\int_x\int_y\left(A_4[(k_{xx})^2+(k_{yy})^2]+2B_4[k_{xx}k_{yy}]+4C_4(k_{xy})^2\right)dxdy\\
&+\int_x\int_y q_m w\,dxdy+\frac{1}{2}\int_x\int_y D^+dxdy+\frac{1}{2}\int_x\int_y D^-dxdy
\end{aligned}
\tag{21}
$$

In which

$$
A_2=\int_{-h/2}^{h/2}\frac{A_1}{1-\alpha B_s}dz\,,\quad A_3=\int_{-h/2}^{h/2}\frac{A_1}{1-\alpha B_s}z\,dz\,,\quad A_4=\int_{-h/2}^{h/2}\frac{A_1}{1-\alpha B_s}z^2dz
$$

$$
B_2=\int_{-h/2}^{h/2}\frac{B_1}{1-\alpha B_s}dz\,,\quad B_3=\int_{-h/2}^{h/2}\frac{B_1}{1-\alpha B_s}z\,dz\,,\quad B_4=\int_{-h/2}^{h/2}\frac{B_1}{1-\alpha B_s}z^2dz
\tag{22}
$$

$$
C_2=\int_{-h/2}^{h/2}G(z)dz\,,\quad C_3=\int_{-h/2}^{h/2}G(z)z\,dz\,,\quad C_4=\int_{-h/2}^{h/2}G(z)z^2dz
$$

$$
D^+=\int_{-h/2}^{h/2}\mu_r\mu_0(\nabla\phi^+)^2dz \qquad\qquad D^-=\int_{-h/2}^{h/2}\mu_0(\nabla\phi^-)^2dz
\tag{23}
$$

q_m is an equivalent magnetic force, which exerted on the plate is given by Zhou and Zheng [2]

$$
q_m=-\int_{-h/2}^{h/2}\frac{\mu_r\mu_0\chi}{2}\frac{d}{dz}(\nabla\phi^+)^2dz
\tag{24}
$$

χ is the susceptibility of the ferromagnetic medium ($\chi=\mu_r-1$).

Applying the Euler equations for total functional of q_m in Eq. (21), we obtain

$$
N_{x,x}+N_{xy,y}=0 \qquad\qquad N_{y,y}+N_{xy,x}=0 \qquad\qquad M_{x,xx}+M_{y,yy}+2M_{xy,xy}+q_m=0
\tag{25}
$$

$$
\nabla^2\phi^-=0 \qquad\qquad \nabla^2\phi^+=0
\tag{26}
$$

where M_x, M_y are bending moments, M_{xy} is twist moment, N_x, N_y are mid-plane internal force, N_{xy} is shearing force, which can be expressed as:

$$
\left\{\begin{matrix}N_{xx}\\N_{yy}\\N_{xy}\end{matrix}\right\}=\int_{-h/2}^{+h/2}\left\{\begin{matrix}\dfrac{\sigma_{xx}}{(1-\alpha B_s)}\\[2mm]\dfrac{\sigma_{yy}}{(1-\alpha B_s)}\\[2mm]\sigma_{xy}\end{matrix}\right\}dz
\tag{27}
$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{+h/2} \begin{Bmatrix} \dfrac{\sigma_{xx}}{(1-\alpha B_s)} \\ \dfrac{\sigma_{yy}}{(1-\alpha B_s)} \\ \sigma_{xy} \end{Bmatrix} z\,dz \tag{28}$$

The stability equations of the rectangular plate are derived using the adjacent equilibrium criterion [27].

We assumed u_0, v_0 and w_0 as the displacement components of the equilibrium state and u_1, v_1 and w_1 as the virtual displacements corresponding to a neighboring state. The displacement components of neighboring state are

$$u = u_0 + u_1 \qquad v = v_0 + v_1 \qquad w = w_0 + w_1 \tag{29}$$

The magnetic field is divided into two parts [1]:

1. The magneto-statics solution for the undeformed plate
2. The magneto-statics solution for the deformed plate.

ϕ_0^+ and ϕ_0^- are supposed to be the magnetic scalar potential before the deformation of the plate and ϕ_1^- and ϕ_1^+ as the magnetic scalar potential due to a small deformation of the plate, that is

$$\phi^+ = \phi_0^+ + \phi_1^+ \qquad\qquad \phi^- = \phi_0^- + \phi_1^- \tag{30}$$

The terms in the resulting equations with superscript 0 satisfy the equilibrium condition and therefore drop out of the equations. Also, the non-linear terms with superscript 1 are ignored because they are small compared to the linear terms. Deformation of the plate prior to buckling for arbitrary boundary conditions and subjected to transverse magnetic field is obtained by solving Eqs. (25) and (26) with the nonlinear term is set equal to zero.

According to the adjacent equilibrium criterion in the neighboring state of equilibrium, the stability equations are found. Similar to Eqs. (29), the stress and moment resultants are found to be the sum of those related to the equilibrium and neighboring states as:

$$\begin{aligned} N_x &= N_{x0} + N_{x1} &,& \qquad M_x = M_{x0} + M_{x1} \\ N_y &= N_{y0} + N_{y1} &,& \qquad M_y = M_{y0} + M_{y1} \\ N_{xy} &= N_{xy0} + N_{xy1} &,& \qquad M_{xy} = M_{xy0} + M_{xy1} \end{aligned} \tag{31}$$

Considering all of these mentioned above, and substituting relations (29), (30) and (31) into Eqs. (25) and (26) collecting the second order terms, the stability equations are obtained as:

$$N_{x1,x} + N_{xy1,y} = 0 \qquad N_{y1,y} + N_{xy1,x} = 0 \qquad M_{x1,xx} + M_{y1,yy} + 2M_{xy1,xy} + q_m = 0 \tag{32}$$

$$\nabla^2 \phi_1^- = 0 \qquad\qquad \nabla^2 \phi_1^+ = 0 \tag{33}$$

2.3 Magnetic buckling analysis

Consider a rectangular plate, subjected to a uniform transverse magnetic field B_0. Thus, for this case of discussion of stability equations is satisfied and the first to third of Eqs. (32), based on the displacement components yield

$$A_2 u_{1,xx} - A_3 w_{1,xxx} + B_2 v_{1,xy} - B_3 w_{1,xyy} + C_2(v_{1,xy} + u_{1,yy}) - 2C_3 w_{1,xyy} = 0$$

$$A_2 v_{1,yy} - A_3 w_{1,yyy} + B_2 u_{1,xy} - B_3 w_{1,xxy} + C_2(v_{1,xx} + u_{1,xy}) - 2C_3 w_{1,xxy} = 0$$

$$A_3 u_{1,xxx} - A_4 w_{1,xxxx} + B_3 v_{1,xxy} - 2B_4 w_{1,xxyy} + A_3 v_{1,yyy} - A_4 w_{1,yyyy}$$

$$+ B_3 u_{1,xyy} + 2C_3(u_{1,xyy} + v_{1,xxy}) - 4C_4 w_{,xxyy} + q_m = 0$$

$$\text{(34)}$$

For a rectangular plate which is simply-supported at both edges:

$$w_1(x,0) = w_1(x,b) = w_1(0,y) = w_1(a,y) = 0$$

$$v_1(x,0) = v_1(x,b) = u_1(0,y) = u_1(a,y) = 0 \tag{35}$$

$$M_{y1}(x,0) = M_{y1}(x,0) = M_{x1}(0,y) = M_{x1}(a,y) = 0$$

where

$$M_{x1} = A_3(u_{1,x} - w_{1,xx}) + B_3(v_{1,y} - w_{1,yy})$$

$$M_{y1} = B_3(u_{1,x} - w_{1,xx}) + A_3(v_{1,y} - w_{1,yy}) \tag{36}$$

The functions for displacements that satisfy the governing equations and boundary conditions are

$$u_1 = U_1 \cos(\beta x) \sin(\gamma y)$$

$$v_1 = V_1 \sin(\beta x) \cos(\gamma y) \tag{37}$$

$$w_1 = W_1 \sin(\beta x) \sin(\gamma y)$$

where

$$\beta = \frac{m\pi}{a} \quad , \quad \gamma = \frac{n\pi}{b} \quad , \quad k = \sqrt{\beta^2 + \gamma^2} \quad , \quad n,m = 1,2,3,...$$

U_1, V_1 and W_1 are constant coefficients. The following boundary conditions for the magnetostatic potential from the continuity of the corresponding field vector components are obtained:

$$\phi^+ = \phi^- \quad , \quad \mu_r \phi^+_{,n} = \phi^-_{,n} \quad , \quad on \quad \pm h/2$$

$$-\mu_0 \nabla \phi^- = B_0 \quad , \quad on \quad s_0 \tag{38}$$

The functions for magnetic scalar potential that satisfy the magnetic governing equations and boundary conditions are

$$\phi_0^+ = \frac{B_0}{\mu_r \mu_0} z + c \qquad\qquad \phi_0^- = \frac{B_0}{\mu_0} z + c$$

$$\phi_1^+ = \frac{B_0 \chi}{\mu_r \mu_0} \left[\frac{\cosh(kz)}{\mu_r \sinh(kh/2) + \cos(kh/2)} \right] w_1 \qquad \phi_1^- = -\frac{B_0 \chi}{\mu_0} \left[\frac{\sinh(kz) \exp k(h/2 - z)}{\mu_r \sinh(kh/2) + \cos(kh/2)} \right] w_1 \tag{39}$$

where *c* is constant. Substitution of Eqs. (30) and (39) into Eq. (24) yields

$$q_m = q_1 B_0^2 w_1 \tag{40}$$

where

$$q_1 = \frac{2\chi^2}{\mu_r \mu_0}\left[\frac{k\,\sinh(kh/2)}{\mu_r\,\sinh(kh/2) + \cos(kh/2)}\right] \tag{41}$$

Substituting Eqs. (37) into the stability equations and using the kinematic and constitutive relations, yields a system of three homogeneous equations for U_1, V_1 and W_1.

$$[K_{ij}]\begin{Bmatrix}U_1 \\ V_1 \\ W_1\end{Bmatrix} = 0 \tag{42}$$

In which K_{ij} is a symmetric matrix with the components as follows:

$$\begin{aligned}
K_{11} &= A_2\gamma^2 + C_2\beta^2 \\
K_{12} &= \gamma\beta(B_2 + C_2) \\
K_{13} &= A_3\gamma^3 + \beta^2\gamma(B_3 + 2C_3) \\
K_{22} &= A_2\beta^2 + C_2\gamma^2 \\
K_{23} &= A_3\beta^3 + \beta\gamma^2(B_3 + 2C_3) \\
K_{33} &= A_4(\beta^4 + \gamma^4) + \beta^2\gamma^2(2B_4 + 4C_4) - q_1 B_0^2
\end{aligned} \tag{43}$$

For a nontrivial solution of Eq. (42), the coefficients of functions must be set to zero setting $|K_{ij}| = 0$, the value of the B_0 is found as:

$$B_0^2 = \frac{1}{q_1}\left[A_4(\beta^4 + \gamma^4) + \beta^2\gamma^2(2B_4 + 4C_4) - \frac{P_1}{P_2}\right] \tag{44}$$

where

$$\begin{aligned}
P_1 &= 2K_{12}K_{13}K_{23} - K_{11}K_{23}^2 - K_{22}K_{13}^2 \\
P_2 &= K_{12}^2 - K_{11}K_{22}
\end{aligned} \tag{45}$$

By substitution ($m = 1, n = 1$), the critical magnetic field for magneto-poroelastic plate buckling obtained. Introducing dimensionless form for B_{cr} as $B_{cr}^* = \left(\frac{B_0}{\sqrt{G_0\mu_0}}\right)$. The value of the B_{cr}^* is found as:

$$B_{cr}^* = \frac{1}{\sqrt{q_1 G_0 \mu_0}}\left[A_4(\beta^4 + \gamma^4) + \beta^2\gamma^2(2B_4 + 4C_4) - \frac{P_1}{P_2}\right]^{0.5} \tag{46}$$

3 RESULTS AND DISCUSSION

In this paper, investigating the stability of FGFP rectangular plates with simply supported boundary condition under magnetic field contributed to achieving the effects of poroelastic plate properties on critical magnetic field (B_{cr}^*) such as pores compressibility, pores distribution, plate thickness, shear modulus and plate magnetism characteristics. The plate shear modulus varies with pore distribution across thickness direction in symmetric and non-symmetric conditions (Fig. 1). Shear modulus decreases by increasing the porosity therefor the plate would be more unstable under applied magnetic field. Fig. 2 shows that increasing the poroelastic plate porosity decreased the

critical magnetic field. Also, this figure shows the maximum stability for homogenous isotropic condition. By substituting $(e_1 = 0)$ into Eqs. (8) and (43) the critical magnetic field for homogenous isotropic plate is obtained as:

$$B_{cr}^* = \left[\frac{k^3 h^3 (1+v)(\mu_r \sinh(kh/2) + \cosh(kh/2))}{12\mu_r \sinh(kh/2)(1-v^2)}\right]^{0.5} \tag{47}$$

This result is similar to Wang's (2003) result for homogenous isotropic plate.

The Biot coefficient indicates the effects of porosity and pores distribution. In considered plate, pores varies in thickness direction and pores distribution influenced on resistance of plate against the force of magnetic field. Fig. 3 shows the effect of pores distribution on critical magnetic field (B_{cr}^*). Porous plate with symmetric pores has higher strength than other cases. The poro/monotonous distribution plate has lowest resistance against the applied transverse magnetic field. The effect of pores distribution on plate resistance against the magnetic field, increases with thickness increasing. Furthermore the homogenous/isotropic plate has highest resistance than other cases. In porous materials, Skempton coefficient introduces the saturated fluid compressibility. If the compressibility of the fluid is high, the Skempton coefficient would be zero $(B_s \to 0)$ and when the fluid is incompressible, the Skempton coefficient would be one $(B_s \to 0)$. Hence, with respect to the fluid compressibility the Skempton coefficient changes between two values 0 and 1, $(0 < B_s < 1)$. In Fig. 4, the effect of the Skempton coefficient on stability of poroelastic plate is shown. As seen in this figure, by decreasing the Skempton coefficient the stability of plate decreases. Also, by increasing the Skempton coefficient, the stability of plate increases and behavior of the plate bears resemblance to that of homogeneous isotropic plate. It can be seen in this figure that plate becomes unstable with lower applied magnetic field, when magnetic permeability (μ_r) rises. This figure shows that changes in mechanical properties plate are more influential than changes in magnetic properties plate on critical point. In this article, the effect of porosity on changes in magnetic properties plate is disregarded because existing of cavity on ferromagnetic plate causes slight change on amount of (μ_r) and by regarding those changes in magnetic properties plate has negligible influence on critical field, the effect of changes in magnetic properties plate can be disregarded and only pursue its effect on mechanical properties.
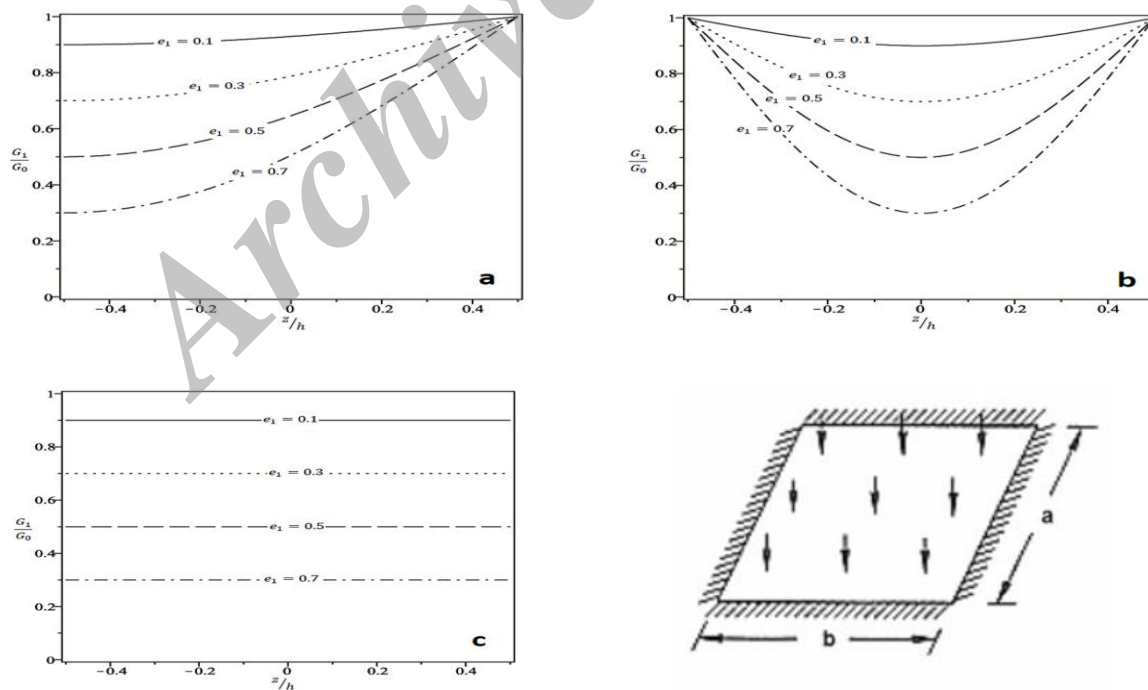

Fig.1
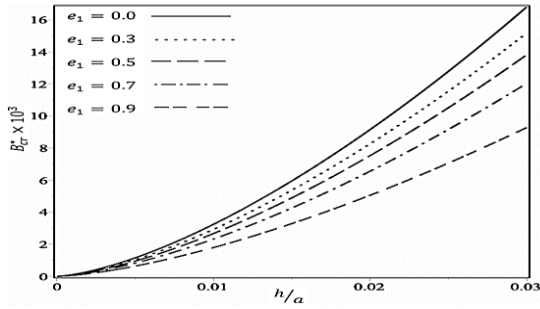The scheme of thin rectangular FG plate made of porous soft ferromagnetic.

Fig.2
$(B_{cr}^* \times 10^3)$ vs. thickness to length ratio of the plate, and for the cases of coefficient of plate porosity $[e_1 = 0.0, 0.3, 0.5, 0.7, 0.9]$ and $B = 0, \mu_r = 10^4$, with $\nu = 0.3$.
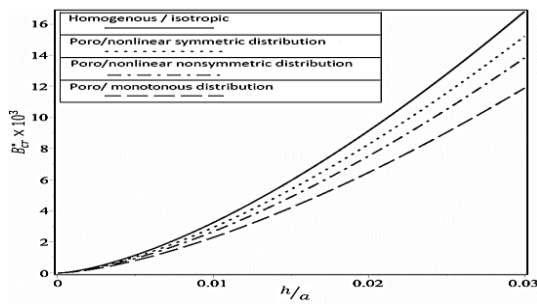

Fig.3
Critical magnetic field $(B_{cr}^* \times 10^3)$ vs. thickness to length ratio of the drained plate, and for the cases of poro/nonlinear nonsymmetric distribution, poro/nonlinear symmetric distribution and poro/monotonous distribution with $e_1 = 0.5, \mu_r = 10^4$, and homogenous/isotropic.
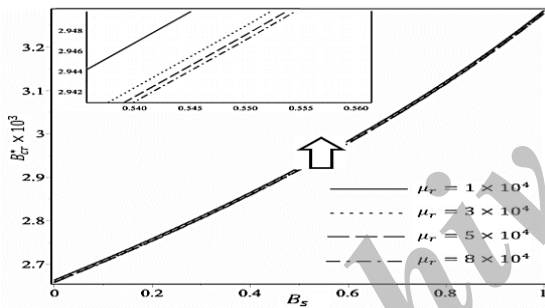

Fig.4
Critical magnetic field $(B_{cr}^* \times 10^3)$ vs. Skempton coefficient of the porous plate, and for the cases of magnetic permeability of plate $[\mu_r = 1 \times 10^4, 3 \times 10^4, 5 \times 10^4, 8 \times 10^4]$, with $\nu = 0.3$.

4 CONCLUSIONS

In the present article, the energy method is used for the buckling analysis of plate made of pore material in transverse magnetic field, where derivation is based on the classical plate theory with the assumption of power law composition for the constituent materials. The boundary conditions of plate is assumed to be simply supported.
It is concluded that:

1. The critical magnetic field (B_{cr}^*) decreased and the plate will be unstable by increasing the porosity.
2. The plate behavior tends to incline to homogeneous/isotropic behavior by reducing the porosity.
3. The critical magnetic field (B_{cr}^*) will be decreased by increasing the thickness.
4. Monotonous porosity is more unstable than the symmetric and nonsymmetric porosity. Location of the pores in porous materials is impressive on the strength of plate and critical magnetic field.
5. By increasing the compressibility of fluid within the pores the critical magnetic field will be reduced.

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