Journal of Solid Mechanics Vol. 8, No. 1 (2016) pp. 104-115

A New Numerical Procedure for Determination of Effective Elastic Constants in Unidirectional Composite Plates

S. Daryazadeh [1], L. Lvov Gennadiy [1], M. Tajdari [2,*]

[1]*National Technical University , Kharkov Polytechnic Institute, Ukraine, Kharkov*
[2]*Department of Mechanical Engineering, Islamic Azad University, Arak Branch, Arak, Iran*

Received 7 November 2015; accepted 4 January 2016

ABSTRACT

In this paper a composite plate with similar unidirectional fibers is considered. Assuming orthotropic structure, theory of elasticity is used for investigating the stress concentration. Also, complex variable functions are utilized for solving the plane stress problems. Then the effective characteristics of this plate are studied numerically by using ANSYS software. In this research a volume element of fibers in square array is considered. In order to investigate the numerical finite element modeling, the modeling of a quarter unit cell is considered. For determining the elasticity coefficients, stress analysis is performed for considered volume with noting to boundary conditions. Effective elasticity and mechanical properties of composite which polymer epoxy is considered as its matrix, are determined theoretically and also by the proposed method in this paper with finite element method. Finally, the variations of mechanical properties with respect to fiber-volume fraction are studied.

Keywords : Composite plate ; Unidirectional fibers ; Effective elastic constants ; Orthotropic plate.

1 INTRODUCTION

COMPOSITE materials which consist of two or more constituent materials are commonly used in advanced structural applications, e.g. in the marine and aerospace industry. This is because of appropriate mechanical properties such as high specific strength and stiffness, low density and high resistance to corrosion. However, the limited understanding of the composite material behavior requires more research. This is further complicated by the fact that these materials behavior is dependent on lay-up, loading direction, specimen size and environmental effects such as temperature and moisture.

Research on determination of effective elastic constants for anisotropic materials is very important in composite structures.

Unidirectional fiber reinforced resin matrix composites are used in some structural applications, due to their various reasons especially to their excellent mechanical behavior in terms of their specific stiffness in the direction of the fibers. The prediction of the mechanical properties of unidirectional composites has been the main objective of many researchers. The well-known models that have been proposed and used to evaluate the properties of unidirectional composites are Voigt [1] and Reuss [2] models. The Voigt model is also known as the rule of mixture model or the iso-strain model, while the Reuss model is also known as the inverse of mixture model or the iso-strain model. Semi empirical models have emerged to correct the rule of mixture model where correcting factors are

*Corresponding author.
E-mail address: m.tajdari@srbiau.ac.ir (M. Tajdari).

introduced. Under this category, it is noticed three important models: the modified rule of mixture, the Halpin-Tsai model [3] and Chamis model [4]. The Halpin-Tsai model emerged as a semi-empirical model that tends to correct the transverse Young's modulus and longitudinal shear modulus. The Chamis micromechanical model is the most used and trusted model which give a formulation for all five independent elastic properties. Hashin and Rosen [5] initially proposed a composite cylinder assemblage model to evaluate the elastic properties of unidirectional composites. Moreover, Christensen [6] proposed a generalized self-consistent model in order to better evaluate the transversal shear modulus. Also the Mori-Tanaka model [7] is a famous model which is widely used for modeling different kinds of composite materials. This is an inclusion model where fibers are simulated by inclusions embedded in a homogeneous medium. The self-consistent model has been proposed by Hill [8] and Budianski [9] to predict the elastic properties of composite materials reinforced by isotropic spherical particulates. Later the model was presented and used to predict the elastic properties of short fibers composites [10]. Recently, a new micromechanical model has been proposed by Huang et al. [11, 12]. The model is developed to predict the stiffness and the strength of unidirectional composites.

In this paper a composite plate with unidirectional fibers is considered. Assuming orthotropic structure and using ANSYS software, effective characteristics of this plate are studied. Numerical studies are performed for some stress states in a representative cell for determination the effective elastic properties of unidirectional reinforced composite.

2 COMPUTATIONAL PROCEDURE

2.1 Definition

This study considers a composite plate with unidirectional fibers, as shown in Fig. 1. As it is shown, unidirectional fibers are parallel to "z" direction.

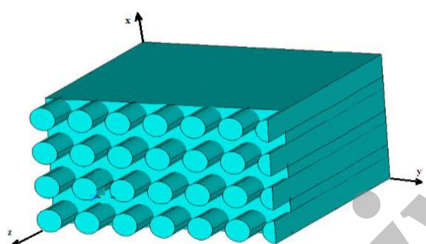


Fig.1
Schematic and coordinate of composite plate with unidirectional fibers.

2.2 Elasticity effective parameters in orthotropic composite plates

Theory of elasticity can be used for investigating the stress concentration of composite plates with unidirectional fibers. The generalized Hook's law relating strains to stresses can be written as follows:

$$\langle \varepsilon_{ij} \rangle = [A]\langle \sigma_{kl} \rangle \qquad (i, j = x, y, z) \tag{1}$$

where $[A]$ is the stiffness matrix and $\langle \varepsilon_{ij} \rangle$ and $\langle \sigma_{ij} \rangle$ are the strain and stress components, respectively. Proof of the form of the stress-strain relations for the various cases of material property symmetry is given by Tsai [8]. For example, if there are two orthogonal planes of material property symmetry for a material, the stress-strain relations in coordinates aligned with principal material directions are as follows and are said to define an orthotropic material.

$$\begin{bmatrix} \langle \varepsilon_{xx} \rangle \\ \langle \varepsilon_{yy} \rangle \\ \langle \varepsilon_{zz} \rangle \\ \langle \varepsilon_{xy} \rangle \\ \langle \varepsilon_{yz} \rangle \\ \langle \varepsilon_{xz} \rangle \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix} \times \begin{bmatrix} \langle \sigma_{xx} \rangle \\ \langle \sigma_{yy} \rangle \\ \langle \sigma_{zz} \rangle \\ \langle \sigma_{xy} \rangle \\ \langle \sigma_{yz} \rangle \\ \langle \sigma_{xz} \rangle \end{bmatrix} \tag{2}$$

where:

$$a_{11}=\frac{1}{\langle E_x\rangle}\ ,\ a_{22}=\frac{1}{\langle E_y\rangle}\ ,\ a_{33}=\frac{1}{\langle E_z\rangle}\ ,\ a_{12}=-\frac{\langle\nu_{yx}\rangle}{\langle E_y\rangle}=-\frac{\langle\nu_{xy}\rangle}{\langle E_x\rangle}\ ,\ a_{13}=-\frac{\langle\nu_{zx}\rangle}{\langle E_z\rangle}=-\frac{\langle\nu_{xz}\rangle}{\langle E_x\rangle}\ ,$$
$$a_{23}=-\frac{\langle\nu_{zy}\rangle}{\langle E_z\rangle}=-\frac{\langle\nu_{yz}\rangle}{\langle E_y\rangle}\ ,\ a_{44}=\frac{1}{\langle G_{xy}\rangle}\ ,\ a_{55}=\frac{1}{\langle G_{yz}\rangle}\ ,\ a_{66}=\frac{1}{\langle G_{xz}\rangle} \tag{3}$$

$\langle E_x\rangle,\langle E_y\rangle,\langle E_z\rangle$ - Young's moduli in the *x , y* and *z*-directions.

$\langle\upsilon_{xy}\rangle,\langle\upsilon_{yz}\rangle,\langle\upsilon_{zx}\rangle$ - Poisson's ratio in the *xy* plane, *yz* plane and *zx* plane.

$\langle G_{xy}\rangle,\langle G_{yz}\rangle,\langle G_{zx}\rangle$ - Shear modulus in the *xy* plane, *yz* plane and *zx* plane.

3 THE EFFECT OF MATRIX AND'S PARAMETERS ON THE ELASTICITY COEFFICIENT OF ORTHOTROPIC PLATES

Complex variable functions are used for solving the plane stress problems [13]. In this procedure, the elasticity coefficients of composite structures are dependent to the material properties of matrix and fibers and also to the situation of the fibers in the matrix. These coefficients are as follows:

$$\langle G_{xy}\rangle=G_m\frac{1-\xi+(1-\xi)G_m/G_b}{1-\xi+(1+\xi)G_m/G_b}\ ,\ \langle G_{yz}\rangle=G_m\frac{(1-\xi)\chi+(1+\chi\xi)G_m/G_b}{\chi+\xi+(1-\xi)G_m/G_b}\ ,\ \langle\upsilon_{xy}\rangle=\upsilon_m-\frac{(\chi_m+1)(\upsilon_m-\upsilon_b)\xi}{2-\xi+\chi_m\xi+(1-\xi)(\chi_b-1)G_m/G_b}$$
$$\langle E_x\rangle=\xi E_b+(1-\xi)E_m+\frac{8G_m\xi(1-\xi)(\upsilon_b-\upsilon_m)}{2-\xi+\chi_m\xi+(1-\xi)(\chi_b-1)G_m/G_b},$$
$$\frac{1}{\langle E_y\rangle}=\frac{(\upsilon_{xy})^2}{E_x}+\frac{1}{8G_m}\left[\begin{array}{l}\dfrac{2(1-\xi)(\chi_m-1)+(\chi_b-1)(\chi_m-1+2\xi)G_m/G_b}{2-\xi+\chi_m\xi+(1-\xi)(\chi_b-1)G_m/G_b}+\\ 2\dfrac{\chi_m(1-\xi)+(1+\xi\chi_m)G_m/G_b}{\chi_m+\xi+(1-\xi)G_m/G_b}\end{array}\right] \tag{4}$$
$$\langle\upsilon_{yx}\rangle=\langle\upsilon_{xy}\rangle\frac{E_y}{E_x}\ ,\ \langle\upsilon_{zy}\rangle=\langle\upsilon_{yz}\rangle\frac{E_z}{E_y}\ ,\ \langle\upsilon_{xy}\rangle\langle\upsilon_{yz}\rangle\langle\upsilon_{zx}\rangle=\langle\upsilon_{yx}\rangle\langle\upsilon_{zy}\rangle\langle\upsilon_{xz}\rangle$$

where,

$$\chi=3-4\upsilon \tag{5}$$

In this study a composite plate with unidirectional fibers parallel to "z" direction is considered. So, for orthotropic plates:

$$\langle E_x\rangle=\langle E_y\rangle\ ,\ \langle G_{xz}\rangle=\langle G_{yz}\rangle\ ,\ \langle\upsilon_{xy}\rangle=\langle\upsilon_{yx}\rangle\ ,\ \langle\upsilon_{yz}\rangle=\langle\upsilon_{xz}\rangle\ ,\ \langle\upsilon_{zx}\rangle=\langle\upsilon_{zy}\rangle$$

In the above equations, $\langle E_x\rangle,\langle E_y\rangle,\langle E_z\rangle,\langle G_{xy}\rangle,\langle G_{yz}\rangle,\langle G_{zx}\rangle,\langle\upsilon_{xy}\rangle,\langle\upsilon_{yz}\rangle$ and $\langle\upsilon_{zx}\rangle$ are the mean composite material modulus and E_m,G_m,υ_m and E_b,G_b,υ_b are the matrix and fiber's coefficients, respectively.

The strength of a fiber-reinforcement composite material must be determined in terms of the strengths of the fibers and the matrix and also their relative volumes (relative to the total volume of the composite material). So, for calculating these coefficients, it is necessary to define the fiber-volume fraction which is dependent to the fiber-reinforcement geometries and the distance of the fibers.

Considering the fiber-reinforcement geometry in square arrangement illustrated in Fig. 2 and two vectors as w_1 and $w_2 = w_1 b e^{i\alpha}$, fiber-volume fraction for a composite plate with unidirectional fibers is:

$$\xi = \frac{\pi . a^2}{w_1^2 b . \sin \alpha} \tag{6}$$

where "a" is the radius of the fibers.

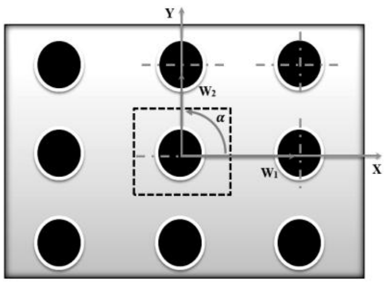


Fig.2
Schematic diagram of square fiber arrangements.

Using Eq. (6), fiber-volume fraction it can be calculated for different fiber arrangement structures.

In this study, composites with unidirectional similar fibers and constant radius are investigated as orthotropic materials.

These materials with volume "V", stress and strain are described as follows:

$$\langle \sigma_{ij} \rangle = \int_V \sigma_{ij} dV \quad and \quad \langle \varepsilon_{ij} \rangle = \int_V \varepsilon_{ij} dV \tag{7}$$

In Cartesian coordinates, Hook's law is as follows:

$$\begin{aligned} &\langle \sigma_x \rangle = b_{11} \langle \varepsilon_x \rangle + b_{12} \langle \varepsilon_y \rangle + b_{13} \langle \varepsilon_z \rangle, \\ &\langle \sigma_y \rangle = b_{21} \langle \varepsilon_x \rangle + b_{22} \langle \varepsilon_y \rangle + b_{23} \langle \varepsilon_z \rangle, \\ &\langle \sigma_z \rangle = b_{31} \langle \varepsilon_x \rangle + b_{32} \langle \varepsilon_y \rangle + b_{33} \langle \varepsilon_z \rangle, \\ &\langle \tau_{xy} \rangle = b_{44} \langle \gamma_{xy} \rangle, \langle \tau_{yz} \rangle = b_{55} \langle \gamma_{yz} \rangle, \langle \tau_{zx} \rangle = b_{66} \langle \gamma_{zx} \rangle \end{aligned} \tag{8}$$

where b_{ij} is the coefficient of elasticity of composite material. Matrix of coefficient of elasticity is symmetric so, $b_{ij} = b_{ji}$.

Since the composite has the same elasticity properties in "x" and "y" directions:

$$b_{11} = b_{22} \quad , \quad b_{13} = b_{23} \quad , \quad b_{55} = b_{66} \tag{9}$$

4 DETERMINATION OF COEFFICIENTS OF ELASTICITY BY NUMERICAL ANALYSIS

4.1 Finite element modeling

The numerical finite element modeling is widely used in predicting the mechanical properties of composites. In this paper for numerical analysis, a plane element of fibers in square array is considered which symmetric exists on all of its lines. In order to investigate the numerical finite element modeling, a quarter unit cell for a square array is considered in plane "xy" using ANSYS software as shown in Fig. 3.

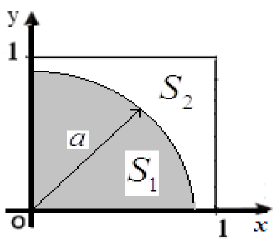


Fig.3
Quarter square array element in plane "xy".

Numerical analysis of volume element causes to study about stress-strain and stress concentration. The infinite flat plate problem with a circular hole is evaluated using ANSYS while applying uniform normal pressure on the hole border. PLANE182 element was selected. This is a triangular element with four nodes and two degree of freedom in each node. The 4-node element is defined by four nodes having two degrees of freedom at each node; translations in the nodal x and y directions. This element can be used either as a plate element (in plane strain and plane stress) or as an axisymmetric problems. This element is used in flat problems and the models with symmetrical stress mode. One of the important characteristics of this element is that the orthotropic material axle is consistent with the direction of the coordinate system of the element.

A regular two-dimensional arrangement of fiber in a matrix was adequate to describe the overall behavior of the composite, and was modeled as a regular uniform arrangement, as illustrated in Fig. 2. This model assumed that the fiber was a perfect cylinder of radius 0.8, in a square (1 × 1) of the matrix. It is assumed that the geometry, material and loading of the unit cell are symmetrical with respect to x-y coordinate system as demonstrated in Fig. 3. For composite with fibers in constant radius as *0<a<1,* fiber-volume fraction can be calculated by Eq. (6). So, considering $\alpha = 90^\circ, a = 0.8, b = 1, w_1 = 2$ fiber-volume fraction is obtained as $\xi = 0.504$. Therefore, 50.4% volume fraction fibers were inserted into the square matrix uniformly as illustrated in Fig. 4.

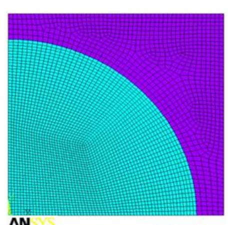


Fig.4
Plane element in mesh formed.

In this analysis polymer epoxy and glass are considered as matrix and fiber, respectively. Mechanical properties of epoxy matrix and glass fibers are given in Table 1. [4].

Table 1
Mechanical properties of the matrix and reinforcement of the composite plate.

Material	E (MPa)	G (MPa)	ν
Epoxide matrix	4200	1500	0.4
glass fibers	74800	31000	0.2

For determining the elasticity coefficients b_{ij}, stress analysis is performed for considered volume with noting to boundary conditions. In the present procedure, normal strains are applied to two directions and shear strains are applied to two planes as follows.

4.2 The first numerical testing

The first numerical testing is unidirectional tension in "z" direction. In this condition, tensor of average values for strains is as follows:

$$\langle \varepsilon_x \rangle = 0, \langle \varepsilon_y \rangle = 0, \langle \varepsilon_z \rangle = 10^{-3}, \langle \gamma_{xy} \rangle = 0, \langle \gamma_{yz} \rangle = 0, \langle \gamma_{xz} \rangle = 0. \quad (10)$$

Boundary conditions for this structural analysis are as follows:

In plane xy: $u_z = 10^{-3}$ and $\tau_{xy} = \tau_{xz} = 0$

where u_z is displacement in "z" direction.

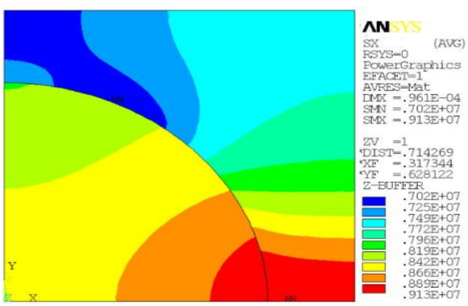
Stress components are determined as:

$$\langle \sigma_x \rangle = \int_0^1 \sigma_x (x = 1, y)dy \quad , \quad \langle \sigma_y \rangle = \int_0^1 \sigma_y (x, y = 1)dx \quad , \quad \langle \sigma_z \rangle = \int_0^1 \int_0^1 \sigma_z dxdy \tag{11}$$

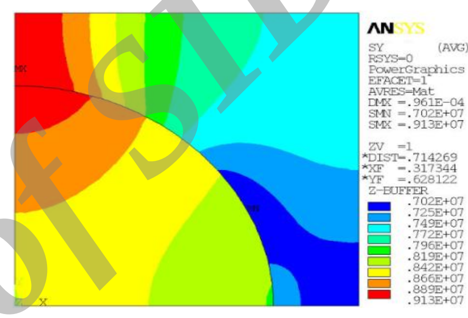
Therefore, according to Eqs. (8), by the first numerical testing three coefficients of elasticity can be determined as follows:

$$b_{13} = \frac{\langle \sigma_x \rangle}{\langle \varepsilon_z \rangle} \quad , \quad b_{23} = \frac{\langle \sigma_y \rangle}{\langle \varepsilon_z \rangle} \quad , \quad b_{33} = \frac{\langle \sigma_z \rangle}{\langle \varepsilon_z \rangle} \tag{12}$$

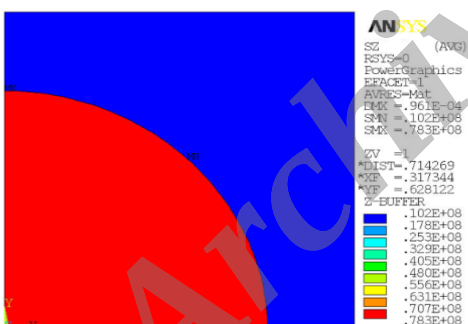
In Fig. 5 the result of the first condition for $\xi = 0.504$ is shown.



(a)



(b)



(c)

Fig.5
Stress distribution of normal stresses: a) σ_x , b) σ_y and c) σ_z .

4.3 The second numerical testing

Unidirectional tension in "x" direction is considered as the second numerical testing. In this condition, tensor of average values for strains is as follows:

$$\langle \varepsilon_x \rangle = 10^{-3}, \langle \varepsilon_y \rangle = 0, \langle \varepsilon_z \rangle = 0, \langle \gamma_{xy} \rangle = 0, \langle \gamma_{yz} \rangle = 0, \langle \gamma_{xz} \rangle = 0 \tag{13}$$

Boundary conditions for this structural analysis are as follows:

On the line x=1: $u_x = 10^{-3}$ and $\tau_{xy} = \tau_{zx} = 0$.

where u_x is displacement in "x" direction. In this situation, there are symmetric conditions on other lines and the following equations are obtained:

$$\langle \sigma_x \rangle = \int_0^1 \sigma_x (x = 1, y) dy \ , \ \langle \sigma_y \rangle = \int_0^1 \sigma_y (x, y = 1) dx \tag{14}$$

Therefore, according to Eqs. (8), by the second numerical testing two coefficients of elasticity can be determined as follows:

$$b_{11} = \frac{\langle \sigma_x \rangle}{\langle \varepsilon_x \rangle}, b_{21} = \frac{\langle \sigma_y \rangle}{\langle \varepsilon_x \rangle} \tag{15}$$

4.4 The third numerical testing

The third numerical testing is shearing in "xy" plane. In this condition, tensor of average values for strains is as follows:

$$\langle \varepsilon_x \rangle = 0, \langle \varepsilon_y \rangle = 0, \langle \varepsilon_z \rangle = 0, \langle \gamma_{xy} \rangle = 10^{-3}, \langle \gamma_{yz} \rangle = 0, \langle \gamma_{xz} \rangle = 0 \tag{16}$$

Boundary conditions for this structural analysis are as follows:
On the lines x=0 , x=1: $\sigma_x = 0$ and $u_y = 0$.
On the line y=1: $\sigma_y = 0$, $u_x = 10^{-3}$ and the of line y=0: $\sigma_y = 0$ and $u_x = 0$.
where u_y is displacement in "y" direction.
The shear stress component is as follows:

$$\langle \tau_{xy} \rangle = \int_0^1 \tau_{xy} (x = 1, y) dy \tag{17}$$

Therefore, according to Eqs. (8), by the third numerical testing another coefficient of elasticity can be determined as follows:

$$b_{44} = \frac{\langle \tau_{xy} \rangle}{\langle \gamma_{xy} \rangle} \tag{18}$$

4.5 The fourth numerical testing

Shearing in "yz" plane is considered as the fourth numerical testing. To carry out numerical analysis on longitudinal shear using the capabilities of ANSYS software system, the construction of a 3D model of cell is required. But it is possible to restrict the 2-D analysis in the 2D dimensional, if we use a mathematical analogy between the problems of longitudinal shear (as a special case anti- plane shear deformation) and the problems of steady-state temperature distribution in a plane domain.

The solution of anti- plan problem for the case of longitudinal shear is defined as the transverse displacement $w(x,y)$ in the direction of reinforcement. Displacement function must satisfy the harmonic equation in domain S in the sectional of volume $S = S_1 \cup S_2$ (as shown in Fig. 3).The harmonic equation has the same form in domains S_1 and S_2 :

$$\nabla^2 w = 0 \tag{19}$$

The shear stress components can be expressed in terms of the displacement $w(x,y)$ with taking into account the different shear modulus for the fibers and matrix as follows:

$$\tau_{xz}^{1} = G_1 \frac{\partial w}{\partial x}, \tau_{yz}^{1} = G_1 \frac{\partial w}{\partial y}, \tau_{xz}^{2} = G_2 \frac{\partial w}{\partial x}, \tau_{yz}^{2} = G_2 \frac{\partial w}{\partial y} \tag{20}$$

At the external boundaries of the domain S under specified conditions: $\tau_{xz}\, l + \tau_{xy}\, m = p_{zn}$, where l and m are components of the unit normal to the plane, and p_{zn} is the external longitudinal load.

On the other hand the stationary heat conduction problem is defined as the harmonic equation for the temperature field $T(x,y)$:

$$\nabla^2 T = 0 \tag{21}$$

The heat fluxes in different areas are determined by the following formulas:

$$\left\langle Q_x^1 \right\rangle = -K_1 \left\langle \frac{\partial T}{\partial x} \right\rangle, Q_y^1 = K_1 \frac{\partial T}{\partial y}, Q_x^2 = K_2 \frac{\partial T}{\partial x}, Q_y^2 = K_2 \frac{\partial T}{\partial y} \tag{22}$$

where K_1 and K_2 are the coefficients of thermal conductivity for respective areas.

At the external boundaries of the domain S under specified conditions: $Q_x .l + Q_y .m = Q_n$, where Q_n is external heat flux defined on the boundary.

As it can be seen, the boundary value problem of longitudinal shear (19, 20), are the same as the boundary-value problem of heat conduction (21, 22). So solving the stationary the heat conduction problem for determination of Q_x, Q_y , can help us for determination of shear stress components in the longitudinal shear problem. It means:

$$Q_x = \tau_{xz}, Q_y = \tau_{yz} \tag{23}$$

Therefore, according to Eqs. (8), by the fourth numerical testing another coefficient of elasticity can be determined as follows:

$$b_{55} = \frac{\left\langle \tau_{yz} \right\rangle}{\left\langle \gamma_{yz} \right\rangle} \tag{24}$$

The results of this simulation at temperature (T = 10−3°C) in nodes on line y=1 are presented in Fig. 6. These results correspond to the displacement w at longitudinal shear.

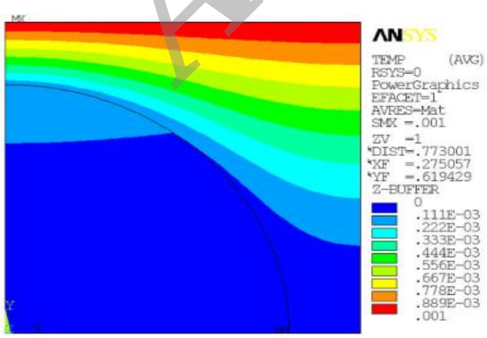
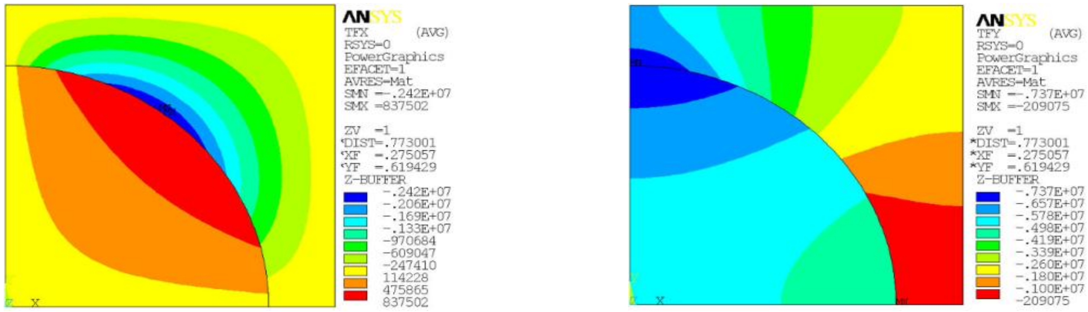


Fig.6
The distribution of temperature at temperature ($T = 10^{-3}$°C) on line y=1.

Fig. 7 shows the heat fluxes in nodes at temperature (T = 10−3°C) on line y=1. These results correspond to the stress τ_{xz} and τ_{yz} .



(a) (b)

Fig.7
The heat flux: a) Q_x , b) Q_y in nodes at temperature ($T = 10^{-3}$°C) on line y=1 and $\xi = 0.504$.

In this way, elasticity coefficients such as poison ratio, modulus of elasticity and shear modulus of composite plate can be obtained.

5 NUMERICAL ANALYSIS RESULTS

Effective elasticity properties for $\xi = 0.504$ is determined by theory of complex functions and numerical procedure proposed in this research. Table 2 . shows theoretical and numerical effective elastic constants. Numerical values are calculated by ANSYS.

Table 2
Theory and numerical results of effective elasticity properties for $\xi = 0.504$

Elasticity properties	symbol	Numerical Method (by ANSYS)	Theoretical Method (by Vanin Formula)
Modulus of elasticity (MPa)	$\langle E_x \rangle$	15868	18050
	$\langle E_y \rangle$	15868	18050
	$\langle E_z \rangle$	48875	43840
Modulus of shear (MPa)	$\langle G_{xy} \rangle$	4100	4080
	$\langle G_{xz} \rangle$	4960	4900
	$\langle G_{yz} \rangle$	4960	4900
Poisson's ratio	$\langle \upsilon_{xy} \rangle$	0.29	0.28
	$\langle \upsilon_{yz} \rangle$	0.13	0.118
	$\langle \upsilon_{zx} \rangle$	0.32	0.33

6 RESULTS AND DISCUSSION

In this section, variation of $E_1 = \langle E_x \rangle / E_m$, $E_2 = \langle E_y \rangle / E_m$ and $G = \langle G_{xy} \rangle / G_m$ versus different values of ξ are obtained for unidirectional glass fibers in a square pattern. Mechanical properties of composite are determined theoretically (by Vanin formula) and also by the proposed numerical method in this paper (by ANSYS).

The variation of E_1 versus different values of ξ for unidirectional glass fibers in a square pattern is depicted in Fig. 8. $\langle E_x \rangle$ is modulus of elasticity of composite in fibers direction and E_m is modulus of elasticity of matrix. In this figure, the curve 1 is obtained from theoretical formulation and the curve 2 is obtained by the method of this paper. As it can be seen, the behaviors of curves are linear. Fig. 8 shows that for small value of ξ the value of E_1 is near to 1. Also for the maximum value of ξ ($\xi = 0.504$), the value of E_1 is near to the modulus of elasticity of fibers E_b / E_m, as it is predicted.

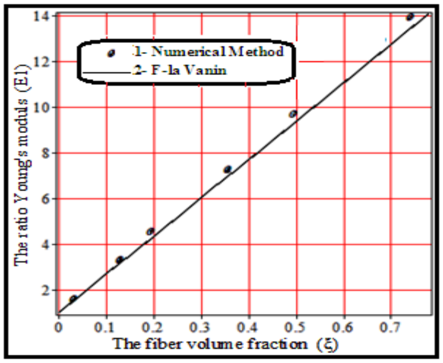


Fig.8
The variation of E_1 versus different values of ξ for unidirectional glass fibers in a square pattern.

Fig. 9 shows the variation of E_2 versus different values of ξ for unidirectional glass fibers in a square pattern. $\langle E_y \rangle$ is modulus of elasticity of composite in perpendicular direction of fibers and E_m is modulus of elasticity of matrix. In this figure, the curve 1 is obtained from theoretical formulation and the curve 2 is obtained by the method of this paper. As it can be seen, the behaviors of curves are nonlinear. In Fig. 9, we see that for small value of ξ, the value of E_2 is near to 1. Also for the maximum value of ξ ($\xi = 0.504$), E_2 is near to a value that is smaller than the modulus of elasticity of fibers E_b / E_m, because these results are obtained in perpendicular direction of fibers.

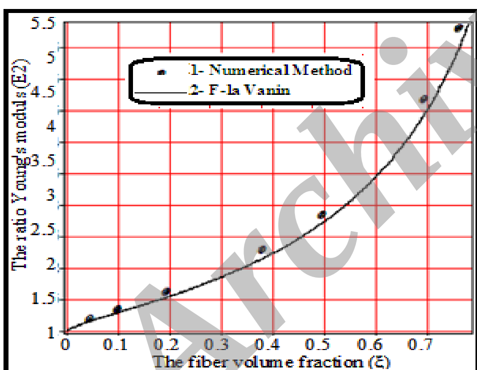


Fig.9
The variation of E_2 versus different values of ξ for unidirectional glass fibers in a square pattern.

The variation of G versus different values of ξ for unidirectional glass fibers in a square pattern is shown in Fig. 10. $\langle G_{xy} \rangle$ is shear modulus of composite in xy plane and G_m is the shear modulus of matrix. In this figure, the curve 1 is obtained from theoretical formulation and the curve 2 is obtained by the method of this paper. As it can be seen, the behaviors of curves are nonlinear. Fig. 10 shows in small value of ξ, the value of G is near to 1. Also for the maximum value of ξ ($\xi = 0.504$), G is near to a value that is smaller than the shear modulus of fibers G_b / G_m. This result is coincident to curves obtained by Vanin.

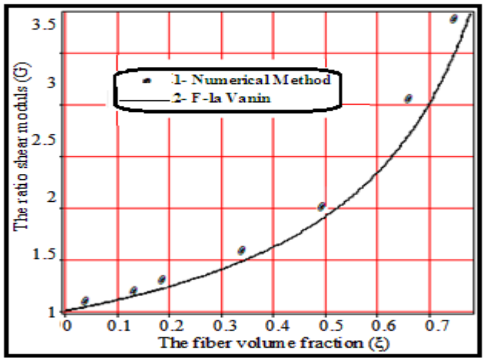


Fig.10
The variation of G versus different values of ξ for unidirectional glass fibers in a square pattern.

7 CONCLUSIONS

In this research assuming orthotropic structure for a composite plate with unidirectional fibers, the effective elasticity and mechanical properties are determined theoretically and also by finite element method. A volume element of fibers in square array is considered which plane symmetric exists on all of its planes. In order to investigate the numerical finite element modeling, the modeling of a quarter unit cell is considered. For determining the elasticity coefficients, stress analysis is performed for considered volume with noting to boundary conditions. In the present procedure, normal strains are applied to three directions and shear strains are applied to three planes. So, the effective elasticity and mechanical properties of composite which polymer epoxy is considered as its matrix are determined theoretically and also by the proposed method in this paper.

The variations of mechanical properties with respect to fiber-volume fraction ξ are studied and the following results are obtained:

1. In direction of fibers, the behaviors of ratio E_1 due to ξ are linear. The results show that for small value of ξ, the value of E_1 is near to 1. Also for the maximum value of ξ, the value of E_1 is near to the modulus of elasticity of fibers E_b/E_m, as it is predicted.
2. In perpendicular direction of fibers, the behaviors of ratio E_2 due to ξ are nonlinear. The results show that for small value of ξ, the value of E_2 is near to 1. Also for the maximum value of ξ, the value of E_2 is near to a value that is smaller than the modulus of elasticity of fibers E_b/E_m, because these results are obtained in perpendicular direction of fibers.
3. The behaviors of ratio G are nonlinear. The results show that in small value of ξ, the value of G is near to 1. Also for the maximum value of ξ, the value of G is near to a value that is smaller than the shear modulus of fibers G_b/G_m. This result is coincident to curves obtained by Vanin.

REFERENCES

[1] Voigt W., 1889, Uber die beziehung zwischen den beiden elastizitatskonstanten isotroper korper, *Wiedemann's Annalen* **38** : 573-587.

[2] Reuss A., 1929, Berechnung der fliessgrense von mischkristallen auf grund der plastizitatsbedingun fur einkristalle, *Zeitschrift Angewandte Mathematik und Mechanik* **9** : 49-58.

[3] Halpin J.C., Kardos J.L., 1976, The halpin-tsai equations: a review, *Polymer Engineering and Science* **16**(5): 344-352.

[4] Chamis C.C., 1989, Mechanics of composite materials: past, present and future, *The Journal of Composites Technology and Research* **11**: 3-14.

[5] Hashin Z., Rosen B.W., 1964, The elastic moduli of fiber reinforced materials, *Journal of Applied Mechanics* **31**: 223-232.

[6] Christensen R.M., 1990, A critical evaluation for a class of micromechanical models, *Journal of Mechanics and Physics of Solids* **38**(3): 379- 404.

[7] Mori T., Tanaka K., 1973, Average stress in matrix and average elastic energy of materials with misfitting inclusions, *Acta Metallurgica* **21**: 571-574.

[8] Hill R., 1965, Theory of mechanical properties of fiber-strengthen materials-3 self-consistent model, *Journal of Mechanics and Physics of Solids* **13** :189-198.

[9] Bubiansky B., 1965, On the elastic modulli of some heterogeneous materials, *Journal of Mechanics and Physics of Solids* **13**: 223-227.

[10] Chou T.w., Nomura S., Taya M., 1980, A self- consistent approach to the elastic stiffness of short-fiber composites, *Journal of Composite Materials* **14**: 178-188.

[11] Huang Z.M., 2001, Simulation of the mechanical properties of fibrous composites by the bridging micromechanics model, *Composites: Part A* **32**: 143-172.

[12] Huang Z.M., 2001, Micromechanical prediction of ultimate strength of transversely isotropic fibrous composites, *International Journal of Solids and Structures* **38**: 4147-4172.

[13] Vanin G. A., 1985, *Micro-Mechanics of Composite Materials*, Nauka Dumka, Kiev.

[14] Carpeenosa D. M., 1985, *Composite Materials*, Nauka Dumka, Kiev.