

Response of Two Temperatures on Wave Propagation in Micropolar Thermoelastic Materials with One Relaxation Time Bordered with Layers or Half Spaces of Inviscid Liquid

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ABSTRACT

The present study is concerned with the propagation of Lamb waves in a homogeneous isotropic thermoelastic micropolar solid with two temperatures bordered with layers or half spaces of inviscid liquid subjected to stress free boundary conditions. The generalized theory of thermoelasticity developed by Lord and Shulman has been used to investigate the problem. The secular equations for symmetric and skew-symmetric leaky and nonleaky Lamb wave modes of propagation are derived. The phase velocity and attenuation coefficient are computed numerically and depicted graphically. The amplitudes of stress, microrotation vector and temperature distribution for the symmetric and skew-symmetric wave modes are computed analytically and presented graphically. Results of some earlier workers have been deduced as particular cases.

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1 INTRODUCTION

ERINGEN [1] developed the theory of micropolar elasticity which has aroused much interest in recent years because of its possible utility in investigating the deformation properties of solids for which the classical theory is inadequate.

Lord and Shulman [2] considered a wave type heat equation by postulating a new law of heat conduction (the Maxwell Cattaneo equation) to replace the classical Fourier law. The heat equation of this theory is of wave type, therefore it automatically ensures finite speeds of propagation for heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motions and constitutive relations remain the same as those for the coupled and uncoupled theories.

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effect. The comprehensive review on the subject was given by Eringen [3, 4] and Nowacki [5]. Touchert et al [6] also derived the basic equations of the linear theory of micropolar coupled thermoelasticity. The generalized thermoelasticity was presented by Dost and Taborrok [7] by using Green and Lindsay theory. Chandrasekharaiah [8] developed a heat flux dependent micropolar thermoelasticity. Boschi and Iesan [9] extended

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the generalized theory of micropolar thermoelasticity that permits the transmission of heat as thermal waves at finite speed.

Thermoelasticity with two temperatures is one of the non-classical theories of thermoelasticity of elastic solids. The thermal dependence is the main difference of this theory with respect to the classical one. Chen et al [10,11] have formulated a theory of heat conduction in deformable bodies, which depend on two distinct temperatures, the conductive temperature Φ and thermodynamic temperature T . For time independent situations, the difference between these two temperatures is proportional to the heat supply. For time dependent problems and for wave propagation problem in particular, the two temperatures are in general different, regardless of the presence of heat supply. Warren and Chen [12] investigated the wave propagation in the two temperature theory of thermoelasticity.

For non-destructive evaluation of solid structures, the study of the interaction of elastic waves with fluid loaded solids has been recognized as a viable means. The reflected acoustic field from a fluid solid interface has great information, which reveals details of many characteristics of solids.

Theoretical and experimental verifications of these phenomenon have been conducted for a wide variety of solids extending from the simple isotropic semi-space to the much more complicated systems of multilayered anisotropic media. Nayfeh [13] has presented a detailed review of the available literature on this subject. The influence of viscous fluid loading on the propagation of leaky Rayleigh wave in the presence of heat conduction effects was studied by Qi [14]. Subsequently, Wu and Zhu [15] suggested an alternative approach to the treatment of Qi [14]. They presented solutions for the dispersion relations of leaky Rayleigh waves, when heat conduction is neglected. The same method was adopted by Zhu and Wu [16] for Lamb waves in submerged and fluid coated plates.

Nayfeh and Nagy [17] derived the exact characteristic equations for leaky waves propagating along the interfaces of several systems involving isotropic elastic solids loaded with viscous fluids, including semi- spaces and finite thickness fluid layers. The technique adopted by Nayfeh and Nagy [17] removed certain inconsistencies that unnecessarily reduce the accuracy and range of validity of the Zhu and Wu [16] results.

Youssef [18] presented a new theory of generalized thermoelasticity by taking into account the theory of heat conduction in deformable bodies, which depends on distinct conductive and thermodynamics temperatures. He also established a uniqueness theorem for the equation of two temperature generalized linear thermoelasticity for a homogeneous and isotropic body. Various authors studied the problems of thermoelastic medium with two temperatures notable among them are Puri and Jordan [19], Youssef and Al-Lehaibi [20], Youssef and Al-Harby [21], Magana and Quintanilla [22], Mukhopadhyay and Kumar [23], Roushan and Santwana [24], Kaushal et al [25,26].

Various authors investigated the problem of wave propagation in thermoelastic plates e.g. Nowacki and Nowacki [27], Kumar and Gogna [28], Tomar [29,30], Kumar and Pratap [31,32,33,34,35,36], Sharma *et.al.* [37], Sharma and Kumar [38].

In this paper, we study the propagation of wave in an infinite homogeneous micropolar thermoelastic plate with two temperatures bordered with layers or half-space of inviscid liquid. The secular equations for different conditions of solutions have been deduced from the present one. The phase velocity and attenuation coefficient are computed numerically and depicted graphically. The amplitudes of stress, microrotation vector and temperature distribution for the symmetric and skew-symmetric wave modes are computed analytically and presented graphically for LS-theory.

2 BASIC EQUATIONS

Following Eringen [1] and Ezzat and Awad [39], the field equations in an isotropic, homogeneous, micropolar elastic medium in the context of generalized theory of thermoelasticity with two temperatures, without body forces, body couples and heat sources, are given by

$$(\lambda + 2\mu + K)\nabla(\nabla\vec{u}) - (\mu + K)\nabla \times (\nabla \times \vec{u}) + K(\nabla \times \vec{\phi}) - \nu \nabla(1 - a\nabla^2)\Phi = \rho \frac{\partial^2 \vec{u}}{\partial t^2}, \quad (1)$$

$$(\alpha + \beta + \gamma)\nabla(\nabla\vec{\phi}) - \gamma \nabla \times (\nabla \times \vec{\phi}) + K \nabla \times \vec{u} - 2K\vec{\phi} = \rho j \frac{\partial^2 \vec{\phi}}{\partial t^2}, \quad (2)$$

$$K^* \nabla^2 \Phi = \rho c^* \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (1 - a \nabla^2) \Phi + \nu T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\nabla \vec{u}), \quad (3)$$

and the constitutive relations are

$$t_{ij} = \lambda u_{r,r} \delta_{ij} + \mu (u_{i,j} + u_{j,i}) + K (u_{j,i} - \varepsilon_{ijr} \phi_r) - \nu T \delta_{ij}, \quad (4)$$

$$m_{ij} = \alpha \phi_{r,r} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i}, \quad i, j, r = 1, 2, 3 \quad (5)$$

where ∇^2 is the Laplacian operator, λ and μ are Lamé's constants. K , α , β and γ are micropolar constants. t_{ij} are the components of the stress tensor and m_{ij} are the components of couple stress tensor. \vec{u} and $\vec{\phi}$ are the displacement and microrotation vectors, ρ is the density, \hat{j} is the microinertia, K^* is the thermal conductivity, c^* is the specific heat at constant strain, T is the thermodynamic temperature, Φ is the conductive temperature, T_0 is the reference temperature, $\nu = (3\lambda + 2\mu + K)\alpha_T$, where α_T is the coefficient of linear thermal expansion, δ_{ij} is the Kronecker delta, ε_{ijr} is the alternating symbol. T and Φ are connected by the relation $T = (1 - a \nabla^2) \Phi$.

Following Achenbach [40], the field equations can be expressed in terms of velocity potential for inviscid fluid as:

$$p_L = -\rho_L \dot{\phi}_L, \quad (6)$$

$$\left(\phi_{L,ii} - \frac{1}{c_L^2} \ddot{\phi}_L \right) = 0, \quad (7)$$

$$u_L = \nabla \phi_L,$$

where $c_L^2 = \frac{\lambda_L}{\rho_L}$ is the velocity of acoustic fluid, λ_L is the bulk modulus, ρ_L is the density of the fluid, p_L is the acoustic pressure in the fluid, ϕ_L is the velocity potential of the fluid, u_L is the velocity vector, ∇ is gradient operator, ∇^2 is the Laplacian operator.

3 FORMULATION OF THE PROBLEM

We consider an infinite homogeneous isotropic, thermally conducting micropolar thermoelastic plate of thickness $2d$ initially undisturbed and at uniform temperature T_0 . The plate is bordered with infinitely large homogeneous inviscid liquid half spaces or layers of thickness h on both sides as illustrated in Figs. 1(a) and 1(b). We take origin of the co-ordinate system (x_1, x_2, x_3) on the middle surface of the plate and x_1 -axis is taken normal to the solid plate.

For two dimensional problem, we take

$$\vec{u} = (u_1(x_1, x_3), 0, u_3(x_1, x_3)), \quad \vec{\phi} = (0, \phi_2(x_1, x_3), 0) \quad (8)$$

For inviscid fluid, we take

$$u_L = (u^L(x_1, x_3), 0, w^L(x_1, x_3))$$

For convenience, the following non-dimensional quantities are introduced

$$\begin{aligned}
 x_1' &= \frac{\omega^* x_1}{c_1}, & x_3' &= \frac{\omega^* x_3}{c_1}, & u_1' &= \frac{\rho \omega^* c_1}{\nu T_0} u_1, & u_3' &= \frac{\rho \omega^* c_1}{\nu T_0} u_3, \\
 \phi_2' &= \frac{\rho c_1^2}{\nu T_0} \phi_2, & t' &= \omega^* t, & T' &= \frac{T}{T_0}, & \Phi' &= \frac{\Phi}{T_0}, \\
 t_{ij}' &= \frac{1}{\nu T_0} t_{ij}, & m_{ij}' &= \frac{\omega^*}{c_1 \nu T_0} m_{ij}, & \tau_0' &= \omega^* \tau_0, \\
 u^{L'} &= \frac{\rho c_1}{\nu T_0} u^L, & w^{L'} &= \frac{\rho c_1}{\nu T_0} w^L, & \phi_L' &= \frac{\rho \omega^*}{\nu T_0} \phi_L, & c_L^2 &= \frac{\lambda_L}{\rho_L}, & p_L' &= \frac{1}{\nu T_0} p_L, \\
 h' &= \frac{c_1 h}{\omega^*}, & d' &= \frac{\omega^* d}{c_1}, & a' &= \frac{\omega^{*2}}{c_1^2} a
 \end{aligned}
 \tag{9}$$

where $\omega^* = \frac{\rho c^* c_1^2}{K^*}$, $c_1^2 = \frac{\lambda + 2\mu + K}{\rho}$, ω^* is the characteristic frequency of the medium, c_L is the velocity of sound in the liquid, ρ_L is the density of the liquid and λ_L is the bulk modulus.

The expressions relating the displacement components u_1 and u_3 to the potential functions ϕ, ψ in dimensionless form after suppressing the primes are taken as:

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}, \tag{10}$$

In the liquid layers, we have

$$u_i^L = \frac{\partial \phi_{L_i}}{\partial x_1}, \quad w_i^L = \frac{\partial \phi_{L_i}}{\partial x_3}, \tag{11}$$

where ϕ_{L_i} are the scalar velocity potential components for the top liquid layer ($i = 1$) and for the bottom liquid layer ($i = 2$). u_i^L and w_i^L ($i = 1, 2$) are the x_1 and x_3 components of the particle velocity for the top liquid layer and the bottom liquid layer respectively.

Making use of Eqs. (8)-(11) in Eqs. (1)-(7) and after suppressing the primes, we obtain

$$\nabla^2 \phi - (1 - a \nabla^2) \Phi - \frac{\partial^2 \phi}{\partial t^2} = 0, \tag{12}$$

$$\nabla^2 \psi + a_1 \phi_2 - a_2 \frac{\partial^2 \psi}{\partial t^2} = 0, \tag{13}$$

$$\nabla^2 \phi_2 - a_3 \nabla^2 \psi - a_4 \phi_2 - a_5 \frac{\partial^2 \phi_2}{\partial t^2} = 0, \tag{14}$$

$$\nabla^2 \Phi = a_6 \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} (1 - a \nabla^2) \Phi + a_7 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \nabla^2 \phi, \tag{15}$$

$$\nabla^2 \phi_{L_i} - \frac{1}{\delta_L^2} \frac{\partial^2 \phi_{L_i}}{\partial t^2} = 0, \quad i = 1, 2 \tag{16}$$

where

$$a_1 = \frac{K}{\mu + K}, \quad a_2 = \frac{\rho c_1^2}{\mu + K}, \quad a_3 = \frac{Kc_1^2}{\gamma \omega^{*2}}, \quad a_4 = 2a_3, \quad a_5 = \frac{\rho \hat{J} c_1^2}{\gamma},$$

$$a_6 = \frac{\rho c^* c_1^2}{K^* \omega^*}, \quad a_7 = \frac{v^2 \Gamma_0}{\rho K^* \omega^*}, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, \quad \delta_L^2 = \frac{c_L^2}{c_1^2}$$

We consider the propagation of plane waves in the x_1x_3 -plane with a wavefront parallel to the x_2 -axis, therefore, $\phi, \psi, \phi_2, \Phi, \phi_{L_1}$ and ϕ_{L_2} are independent of x_2 -coordinates. We assume the solutions of Eqs. (12)-(16) of the form

$$(\phi, \psi, \phi_2, \Phi, \phi_{L_1}, \phi_{L_2}) = [f_1(x_3), f_2(x_3), f_3(x_3), f_4(x_3), f_5(x_3), f_6(x_3)] e^{i\xi(x_1 - ct)}, \quad (17)$$

where $c = \frac{\omega}{\xi}$ is the non-dimensional phase velocity, ω is the frequency and ξ is the wave number.

Using Eq. (17) in Eqs. (12)-(16), we get

$$(\nabla^{*2} + \xi^2 c^2) f_1(x_3) = [1 - a \nabla^{*2}] f_4(x_3), \quad (18)$$

$$(\nabla^{*2} + a_6(i\xi c + \tau_0 \xi^2 c^2)(1 - a \nabla^{*2})) f_4(x_3) = -a_7(i\xi c + \tau_0 \xi^2 c^2) \nabla^{*2} f_1(x_3), \quad (19)$$

$$(\nabla^{*2} + a_2 \xi^2 c^2) f_2(x_3) = -a f_3(x_3), \quad (20)$$

$$(\nabla^{*2} + a_5 \xi^2 c^2 - a_4) f_3(x_3) = a_3 \nabla^{*2} f_2(x_3), \quad (21)$$

$$(d^2 / dx_3^2 - \gamma_L^2) f_k(x_3) = 0, \quad (22)$$

Eliminating $f_4(x_3)$ from Eqs. (18) and (19) and eliminating $f_3(x_3)$ from Eqs. (20) and (21), we obtain

$$(\nabla^{*4} + A \nabla^{*2} + B) f_1(x_3) = 0, \quad (23)$$

$$(\nabla^{*4} + C \nabla^{*2} + D) f_2(x_3) = 0, \quad (24)$$

where $\nabla^{*2} = d^2 / dx_3^2 - \xi^2$ and A, B, C and D are given by

$$A = \frac{-aa_6 \tau_0' + \frac{1}{\xi^2 c^2} + \frac{a_6 \tau_0'}{\xi^2 c^2} + \frac{a_7 i}{\xi^3 c^3}}{\frac{-aa_6 \tau_0'}{\xi^2 c^2} + \frac{1}{\xi^4 c^4} - \frac{aa_7}{\xi^3 c^3} i}, \quad B = \frac{a_6 \tau_0'}{\frac{-aa_6 \tau_0'}{\xi^2 c^2} + \frac{1}{\xi^4 c^4} - \frac{aa_7}{\xi^3 c^3} i},$$

$$C = \xi^2 c^2 \left((a_5 + a_2) + \frac{a_1 a_3 - a_4}{\xi^2 c^2} \right), \quad D = a_2 \left(a_5 - \frac{a_4}{\xi^2 c^2} \right) \xi^4 c^4, \quad \tau_0' = \tau_0 + \frac{i}{\xi c}, \quad \gamma_L^2 = \xi^2 \left(1 - \frac{c^2}{\delta_L^2} \right)$$

The roots of Eqs. (23) and (24) are given as $n_{1,2}^2 = \frac{1}{2} [-A \pm \sqrt{A^2 - 4B}]$ and $n_{3,4}^2 = \frac{1}{2} [-C \pm \sqrt{C^2 - 4D}]$. The appropriate potentials $\phi, \Phi, \psi, \phi_2, \phi_{L_1}$ and ϕ_{L_2} will be obtained as:

$$\phi = (A_1 \cos n_1 x_3 + A_2 \cos n_2 x_3 + B_1 \sin n_1 x_3 + B_2 \sin n_2 x_3) e^{i\xi(x_1 - ct)}, \quad (25)$$

$$\Phi = (h_1 A_1 \cos n_1 x_3 + h_2 A_2 \cos n_2 x_3 + h_1 B_1 \sin n_1 x_3 + h_2 B_2 \sin n_2 x_3) e^{i\xi(x_1 - ct)}, \tag{26}$$

$$\psi = (A_3 \cos n_3 x_3 + A_4 \cos n_4 x_3 + B_3 \sin n_3 x_3 + B_4 \sin n_4 x_3) e^{i\xi(x_1 - ct)}, \tag{27}$$

$$\phi_2 = (h_3 A_3 \cos n_3 x_3 + h_4 A_4 \cos n_4 x_3 + h_3 B_3 \sin n_3 x_3 + h_4 B_4 \sin n_4 x_3) e^{i\xi(x_1 - ct)}, \tag{28}$$

$$\phi_{L_1} = (E_5 e^{\gamma_L x_3} + F_5 e^{-\gamma_L x_3}) e^{i\xi(x_1 - ct)}, \tag{29}$$

$$\phi_{L_2} = (E_6 e^{\gamma_L x_3} + F_6 e^{-\gamma_L x_3}) e^{i\xi(x_1 - ct)}, \tag{30}$$

where

$$h_i = \frac{(r_1^2 - n_i^2)}{(1 + a\xi^2 + an_i^2)}, \quad h_j = \frac{-(r_2^2 - n_j^2)}{a_1}, \quad i = 1, 2 \quad j = 3, 4$$

$$n_i^2 = \xi^2 (c^2 \alpha_i^2 - 1), \quad i = 1, 2, 3, 4, \quad r_1^2 = \xi^2 (c^2 - 1), \quad r_2^2 = \xi^2 (c^2 a_2 - 1),$$

$$\alpha_i^2 = \frac{k_0 \pm \sqrt{k_0^2 - 4a_6 \tau_0 k_1}}{2k_1},$$

$$\alpha_j^2 = \frac{((a_5 + a_2) + \frac{a_1 a_3 - a_4}{\xi^2 c^2}) \pm \sqrt{((a_5 + a_2) + \frac{a_1 a_3 - a_4}{\xi^2 c^2})^2 - 4(a_5 - \frac{a_4}{\xi^2 c^2}) a_2}}{2}, \quad i = 1, 2, \quad j = 3, 4$$

where

$$k_0 = -aa_6 \tau_0 + \frac{1}{\xi^2 c^2} + [\frac{a_6 \tau_0}{\xi^2 c^2} + \frac{a_7}{\xi^2 c^2} \tau_0], \quad k_1 = \frac{-aa_6 \tau_0}{\xi^2 c^2} + \frac{1}{\xi^4 c^4} - aa_7 \tau_0 \frac{1}{\xi^2 c^2}, \quad k_1' = \xi^2 c^2 k_1$$

4 BOUNDARY CONDITIONS

The boundary conditions at the solid-liquid interfaces $x_3 = \pm d$ are given by:

(i) The magnitude of the normal component of the stress tensor $(t_{33})_s$ of the plate should be equal to the pressure of the liquid p_L .

$$(t_{33})_s = -p_L \tag{31}$$

(ii) The tangential component of the stress tensor should be zero.

$$(t_{31})_s = 0 \tag{32}$$

(iii) The tangential component of the couple stress tensor should be zero.

$$(m_{32})_s = 0 \tag{33}$$

(iv) The normal velocity component of the solid should be equal to that of the liquid.

$$(\dot{u}_3)_s = (w^L) \tag{34}$$

(v) The thermal boundary conditions is given by

$$\frac{\partial \Phi}{\partial x_3} + H \Phi = 0, \quad (35)$$

where H is the surface heat transfer coefficient. Here $H \rightarrow 0$ corresponds to thermal insulated boundaries and $H \rightarrow \infty$ refers to isothermal one.

4.1 Leaky lamb waves

The complete solutions for solid media of finite thickness $2d$ sandwiched between two liquid half spaces is given by Eqs. (25)-(28) and

$$\phi_{L_1} = E_5 e^{\gamma_L(x_3+d)} e^{i\xi(x_1-ct)}, \quad -\infty < x_3 < -d \quad (36)$$

$$\phi_{L_2} = F_6 e^{-\gamma_L(x_3-d)} e^{i\xi(x_1-ct)}, \quad d < x_3 < \infty \quad (37)$$

4.2 Nonleaky lamb waves

The corresponding solutions for a solid media of finite thickness $2d$ sandwiched between two finite liquid layers of thickness h is given by Eqs. (25)-(28) and

$$\phi_{L_1} = E_5 \sinh \gamma_L [x_3 + (d+h)] e^{i\xi(x_1-ct)}, \quad -(d+h) < x_3 < -d \quad (38)$$

$$\phi_{L_2} = F_6 \sinh \gamma_L [x_3 - (d+h)] e^{i\xi(x_1-ct)}, \quad d < x_3 < (d+h) \quad (39)$$

Nonleaky and leaky lamb waves are distinguished by selecting the functions ϕ_{L_1} and ϕ_{L_2} in such a way that the acoustical pressure is zero at $x_3 = \mp(d+h)$. This shows that ϕ_{L_1} and ϕ_{L_2} are solutions of standing wave and travelling wave for nonleaky lamb waves and leaky lamb waves respectively.

5 DERIVATION OF THE DISPERSION EQUATIONS

We apply the already shown formal solutions in this section to study the specific situations with inviscid fluid.

5.1 Leaky lamb waves

We consider an isotropic thermoelastic micropolar plate with two temperatures completely immersed in a inviscid liquid as shown in Fig. 1(a). The thickness of the plate is $2d$ and thus the lower and upper portions of the fluid extend from $x_3 = d$ to ∞ and $x_3 = -d$ to $-\infty$ respectively. In this case, the partial waves are in both the plate and the fluid. The appropriate formal solutions for the plate and fluids are those given by Eqs. (25)-(28), (36) and (37). By applying the boundary conditions (31)-(35) at $x_3 = \pm d$ and subsequently requiring nontrivial values of the partial wave amplitudes E_k and F_k , ($k=1, 2, 3, 4$); E_5, F_6 and $\gamma_L \neq 0$, we arrive at the characteristic dispersion equations as:

$$\left[\frac{T_1}{T_3} \right]^{\pm 1} - \left[\frac{T_2}{T_3} \right]^{\pm 1} P_1 - \left[\frac{T_1}{T_4} \right]^{\pm 1} P_2 + \left[\frac{T_2}{T_3} \right]^{\pm 1} P_1 P_2 + \frac{P_3 S}{n_4 G} \left[\frac{P_4 n_3 h_3}{h_4 n_2 [T_4]^{\pm}} - \frac{P_5}{[T_3]^{\pm}} \right] = \frac{-P_3 Q m_3 n_3 (h_4 - h_3)}{h_4} \quad (40)$$

where

$$P_1 = \frac{n_1 h_1 m_2}{n_2 h_2 m_1}, \quad P_2 = \frac{n_3 h_3 m_6}{n_4 h_4 m_5}, \quad P_3 = \frac{(h_1 - h_2) n_1}{m_1 m_5 h_2}, \quad P_4 = (\xi^2 c m_3 + R m_6), \quad P_5 = (\xi^2 c m_3 + R m_5)$$

For stress free thermally insulated boundaries ($H \rightarrow 0$) of the plate and

$$\left[\frac{T_1}{T_3} \right]^{\pm 1} - \left[\frac{T_2}{T_3} \right]^{\pm 1} P_6 + \left[\frac{T_4}{T_3} \right]^{\pm 1} \frac{P_7 h_3 m_6}{n_4 Q} + \left(1 - \frac{P_4 n_3 h_3}{P_5 n_4 h_4} \left[\frac{T_4}{T_3} \right]^{\pm 1} \right) P_8 T_3 \left(\left[\frac{T_1}{T_3} \right]^{\pm 1} - \left[\frac{T_2}{T_3} \right]^{\pm 1} P_6 \right) = \frac{P_7 h_4 m_5}{n_3} \quad (41)$$

where

$$P_6 = \frac{n_2 h_1}{n_1 h_2}, \quad P_7 = \frac{(m_1 h_2 - m_2 h_1)}{n_1 h_2 m_3 (h_4 - h_3)}, \quad P_8 = \frac{P_5 h_4 S}{n_3 m_3 Q G (h_3 - h_4)}$$

For stress free isothermal boundaries ($H \rightarrow \infty$) of the plate.

5.2 Nonleaky lamb waves

We consider an isotropic thermoelastic micropolar plate with two temperatures bordered with layers of inviscid liquid on both sides as shown in Fig. 1(b).

The appropriate formal solutions for the plate and fluids are given by Eqs. (25)-(28), (38) and (39). By applying the boundary conditions (31)-(35) at $x_3 = \pm d$ and subsequently requiring nontrivial values of the partial wave amplitudes E_k and F_k , ($k=1, 2, 3, 4$); E_5, F_6 and $\gamma_L \neq 0$, we arrive at the characteristic dispersion equations as:

$$\left[\frac{T_1}{T_3} \right]^{\pm 1} - \left[\frac{T_2}{T_3} \right]^{\pm 1} P_1 - \left[\frac{T_4}{T_4} \right]^{\pm 1} P_2 + \left[\frac{T_2}{T_3} \right]^{\pm 1} P_1 P_2 + \frac{P_3 S}{n_4 G} T_5 \left[\frac{P_4 n_3 h_3}{h_4 n_2 [T_4]^{\pm}} - \frac{P_5}{[T_3]^{\pm}} \right] = - \frac{P_3 Q m_3 n_3 (h_4 - h_3)}{h_4} \quad (42)$$

For stress free thermally insulated boundaries ($H \rightarrow 0$) of the plate.

$$\left[\frac{T_1}{T_3} \right]^{\pm 1} - \left[\frac{T_2}{T_3} \right]^{\pm 1} P_6 + \left[\frac{T_4}{T_3} \right]^{\pm 1} \frac{P_7 h_3 m_6}{n_4 Q} + \left(1 - \frac{P_4 n_3 h_3}{P_5 n_4 h_4} \left[\frac{T_4}{T_3} \right]^{\pm 1} \right) P_8 T_3 T_5 \left(\left[\frac{T_1}{T_3} \right]^{\pm 1} - \left[\frac{T_2}{T_3} \right]^{\pm 1} P_6 \right) = \frac{P_7 h_4 m_5}{n_3} \quad (43)$$

For stress free isothermal boundaries ($H \rightarrow \infty$) of the plate.

where

$$\begin{aligned} m_i &= [d_1 l_i + d_2 n_i^2 + b_i h_i], \\ m_3 &= (2d_4 + d_5) i \xi, \quad m_k = (d_4 + d_5) n_j^2 - d_4 \xi^2 - d_5 h_j, \quad i = 1, 2, \quad j = 3, 4, \quad k = 5, 6 \\ S &= \frac{\rho_L}{\rho} i \xi c, \quad R = i \xi c, \quad Q = i \xi d_2, \quad l_i = \xi^2 + n_i^2, \quad G = \gamma_L, \quad b_i = (1 + a \xi^2 + a n_i^2), \\ s_i &= \sin n_i d, \quad s_j = \sin n_j d, \quad c_i = \cos n_i d, \quad c_j = \cos n_j d, \quad T_5 = \tanh \gamma_L h, \\ d_1 &= \frac{\lambda}{\rho c_1^2}, \quad d_2 = \frac{(2\mu + K)}{\rho c_1^2}, \quad d_4 = \frac{\mu}{\rho c_1^2}, \quad d_5 = \frac{K}{\rho c_1^2} \\ T_i &= \tan n_i d, \quad i = 1, 2, 3, 4 \end{aligned}$$

Here the superscript +1 refers to skew-symmetric and -1 refers to symmetric modes.

Eqs. (40) and (43) are the general dispersion relations involving wave number and phase velocity of various modes of propagation in a micropolar thermoelastic plate bordered with layers of inviscid liquid or half spaces on both sides.

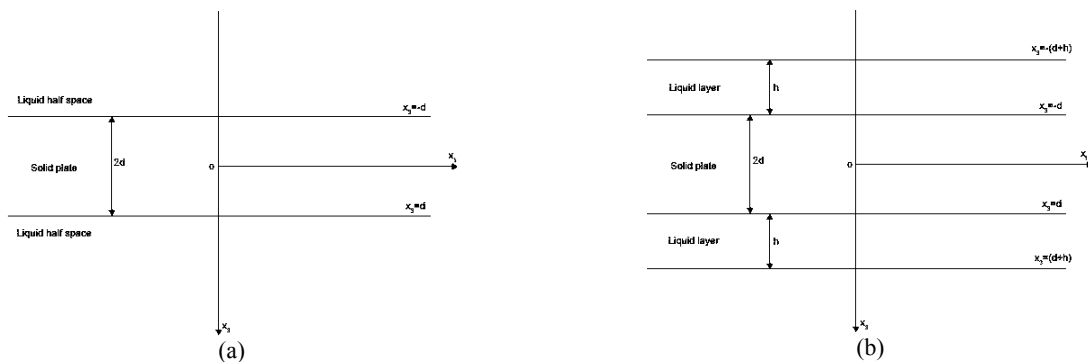


Fig. 1

a) Geometry of leaky Lamb waves. (b) Geometry of nonleaky Lamb waves.

6 SPECIAL CASES

If the liquid layers or half spaces on both sides are removed, then we are left with the problem of wave propagation in micropolar thermoelastic solid with two temperatures. For this, we take $\rho_L = 0$ in Eqs. (40) and (42), the secular equations for stress free thermally insulated boundaries ($H \rightarrow 0$) for the said case reduce to

$$(T_1 T_3)^{\pm} N_1 + (T_1 T_4)^{\pm} N_2 + (T_2 T_3)^{\pm} N_3 + (T_2 T_4)^{\pm} N_4 + (T_3 T_4)^{\pm} N_5 = 0$$

$$N_1 = h_2 h_3 n_2 n_3 m_1 m_6 G, \quad N_2 = -h_2 h_4 n_2 n_4 m_1 m_5 G, \quad N_3 = -h_1 h_3 n_1 n_3 m_2 m_6 G,$$

$$N_4 = h_1 h_4 n_1 n_4 m_2 m_5 G, \quad N_5 = n_1 n_2 n_3 n_4 m_3 QG (h_4 - h_3)(h_2 - h_1)$$

Subcase (i): In this case, if $a = 0$ then we obtain the secular equations in micropolar generalized thermoelastic plate.

7 AMPLITUDES OF DILATATION, MICROROTATION AND TEMPERATURE DISTRIBUTION

In this section the amplitudes of dilatation, microrotation and temperature distribution for symmetric and skew-symmetric modes of waves have been computed for micropolar thermoelastic plate. Using Eqs. (18)-(25) and (28)-(35), we obtain

$$(e)_{sy} = [-M_1 \cos n_1 x_3 + \frac{M_2 L s_1}{s_2} \cos n_2 x_3] A_1 e^{i\xi(x_1 - ct)}, \quad (44)$$

$$(e)_{asy} = [-M_1 \sin n_1 x_3 + \frac{M_2 L c_1}{c_2} \sin n_2 x_3] B_1 e^{i\xi(x_1 - ct)}, \quad (45)$$

$$(\phi_2)_{sy} = [h_3 \sin n_3 x_1 - \frac{h_3 n_3 c_3}{n_4 c_4} \sin n_4 x_3] B_3 e^{i\xi(x_1 - ct)}, \quad (46)$$

$$(\phi_2)_{asy} = [h_3 \cos n_3 x_3 - \frac{h_3 n_3 s_3}{n_4 s_4} \cos n_4 x_3] A_3 e^{i\xi(x_1 - ct)}, \quad (47)$$

$$(T)_{sy} = [N h_1 \cos n_1 x_3 - \frac{h_1 n_1 s_1}{n_2 s_2} N' \cos n_2 x_3] A_1 e^{i\xi(x_1 - ct)}, \quad (48)$$

$$(T)_{asy} = [Nh_1 \sin n_1 x_3 - \frac{h_1 n_1 c_1}{n_2 c_2} N' \sin n_2 x_3] B_1 e^{i\xi(x_1 - ct)}, \tag{49}$$

where

$$L = \frac{h_1 n_1}{h_2 n_2}, \quad N = (1 + a\xi^2 + an_1^2), \quad N' = (1 + a\xi^2 + an_2^2), \quad M_1 = (k^2 + n_1^2), \quad M_2 = (k^2 + n_2^2)$$

8 NUMERICAL RESULTS AND DISCUSSION

To illustrate theoretical results obtained in the preceding sections and to compare these in the context of Lord and Shuman theory of thermoelasticity, we now present some numerical results. The material chosen for this purpose is Magnesium crystal (micropolar elastic solid), the physical data for which is given below:

(i) Micropolar parameters

$$\lambda = 9.4 \times 10^{10} \text{ Nm}^{-2}, \quad \mu = 4.0 \times 10^{10} \text{ Nm}^{-2}, \quad \gamma = 0.779 \times 10^{-9} \text{ N},$$

$$\kappa = 1.0 \times 10^{10} \text{ Nm}^{-2}, \quad \rho = 1.74 \times 10^3 \text{ kgm}^{-3}, \quad j = 2.0 \times 10^{-20} \text{ m}^2;$$

(ii) Thermal parameters

$$\nu = 0.268 \times 10^7 \text{ Nm}^{-2} \text{ K}^{-1}, \quad c^* = 1.04 \times 10^3 \text{ NmKg}^{-1} \text{ K}^{-1}, \quad T_0 = 0.298 \text{ K}, \quad K^* = 1.7 \times 10^2 \text{ Nsec}^{-1} \text{ K}^{-1},$$

$$a = 0.5 \text{ m}^2, \quad \tau_0 = 0.813 \times 10^{-12} \text{ sec}, \quad \omega = 1, \quad d = 1.0 \text{ m}$$

For numerical calculations, water is taken as liquid and the speed of sound in water is given by $c_L = 1.5 \times 10^3 \text{ m / sec}$.

In general, wave number and phase velocity of the waves are complex quantities, therefore, the waves are attenuated in space. If we write

$$C^{-1} = V^{-1} + i \omega^{-1} Q, \tag{50}$$

Then $\xi = R + iQ$, where $R = \frac{\omega}{V}$ and Q are real numbers. This shows that V is the propagation speed and Q is the attenuation coefficient of waves. Using Eq. (50) in secular Eqs. (40) and (42), the value of propagation speed V and attenuation coefficient Q for different modes of propagation can be obtained.

In Figs. 2 to 9, LLS and LNLS refer to leaky and nonleaky symmetric waves in micropolar thermoelastic solid with two temperatures, LLSK and LNLSK refer to leaky and nonleaky skew-symmetric waves in micropolar thermoelastic solid with two temperatures, LALS and LANLS refer to leaky and nonleaky symmetric waves in micropolar thermoelastic solid, LALS and LANLS refer to leaky and nonleaky symmetric waves in micropolar thermoelastic solid. In Figs. 10 to 15, GT represents the amplitude for micropolar thermoelastic solid with two temperatures and TS represents the amplitude for micropolar thermoelastic solid.

8.1 Phase velocity

It is evident from Figs. 2 and 4 that for symmetric leaky lamb wave modes of propagation, it is noticed that phase velocity for lowest symmetric mode for LALS remain more than the values for LLS for wave number $\xi d = 2, 5, 10$ and in the remaining region, the behavior is reversed. For symmetric non-leaky lamb wave modes of propagation, the phase velocity for LALS and LLS coincide for wave number $5 \leq \xi d \leq 10$. There is slight difference in the phase velocity for LLS and LALS for (n=1) symmetric leaky lamb wave mode of propagation, the phase velocities for LLS remain more than the velocities for LALS for wave number $4 \leq \xi d \leq 10$ and for (n=1) symmetric nonleaky lamb wave mode of propagation, the velocities for LANLS remain more than the velocities for LNLS for wave

number $3 \leq \xi d \leq 7$. It is noticed that for $(n=2)$ symmetric nonleaky lamb wave modes of propagation, the phase velocity for LNLS remain more than in case of LANLS for wave number $\xi d = 1, 7, 8, 9$ and in the remaining range, the behavior is reversed. For symmetric leaky lamb wave mode of propagation $(n=2)$, the phase velocities for LLS remain more than the velocities for LALS for wave number $3 \leq \xi d \leq 6, \xi d = 8$.

It is noticed from Fig. 3 that the phase velocities for lowest skew-symmetric leaky and nonleaky lamb wave mode of propagation coincide. For $(n=1)$ skew-symmetric leaky lamb wave mode of propagation the phase velocities for LLSK and LALS are similar. Fig. 5 depicts that for $(n=1)$ skew-symmetric nonleaky lamb wave mode of propagation, there is slight difference in the velocities in the region $\xi d \leq 2$ and in further region the velocities coincide. It is observed that for $(n=2)$ mode, the phase velocities for LLSK are greater than the values for LALS for $\xi d = 2, 3$ and for further increase in wave number, the phase velocities coincide. For $(n=2)$ skew-symmetric mode for nonleaky lamb waves, the phase velocities for LANLSK coincide with the values for LNLSK.

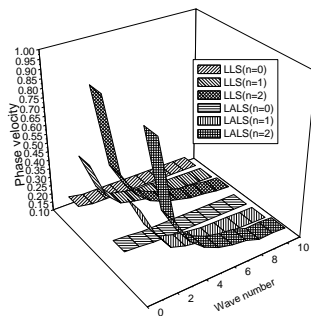


Fig .2

Variation of phase velocity for symmetric leaky lamb waves.

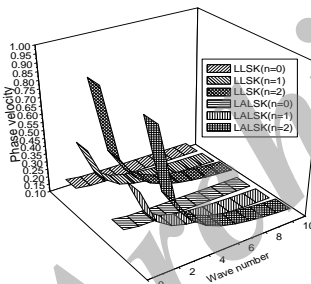


Fig .3

Variation of phase velocity for skew-symmetric leaky lamb waves.

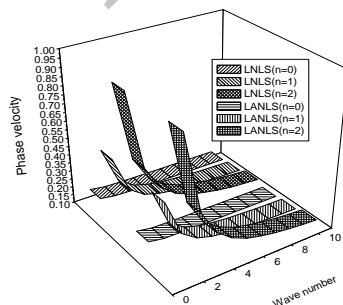


Fig .4

Variation of phase velocity for symmetric nonleaky lamb waves.

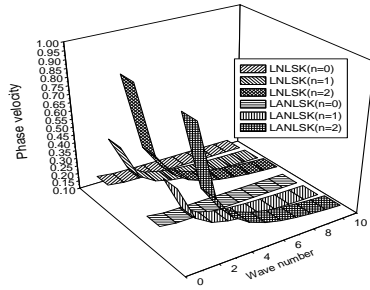


Fig .5
Variation of phase velocity for skew-symmetric nonleaky lamb waves.

8.2 Attenuation coefficients

Fig. 6 depicts that for symmetric leaky Lamb wave mode ($n=0$), the magnitude of attenuation coefficient for LLS remain more than the value of attenuation coefficient for LALS in the region $\xi d = 1, 4, 6 \leq \xi d \leq 10$. For ($n=1$) symmetric mode the values for LLS remain more than the values for LALS in the whole region, except for $1 \leq \xi d \leq 3, \xi d = 5$ and in the remaining region, the behavior is reversed. It is observed that for ($n=2$) symmetric mode the phase velocities for LLS remain more than the values for LALS in the whole region.

Fig. 7 shows that the magnitude of attenuation for ($n=0$) mode for LLSK attain maximum value 0.000021 at $\xi d = 1$ and LLSK remain more than the values for LALS in the whole region except the region $1 \leq \xi d \leq 3$. For ($n=1$) skew symmetric mode, the values for LLSK remain slightly more than the values for ALSK for wave number $2 \leq \xi d \leq 6$ and $8 \leq \xi d \leq 10$. It is noticed that for ($n=2$) mode, the magnitude of attenuation coefficient for LALS remain more than in case of LLSK in the whole region, except at $\xi d = 1$, where the values are similar.

It is evident from Fig. 8 that for symmetric nonleaky lamb wave mode ($n=0$), the attenuation coefficient for LNLS attain maximum value at $\xi d = 2$ and LANLS at $\xi d = 3$. It is noticed that for ($n=1$), the magnitude of attenuation coefficient for LNLS and LANLS attain maximum value at $\xi d = 3$. For ($n=2$) mode, the values for LANLS remain more than the values for LNLS in the region $1 \leq \xi d \leq 4, \xi d = 9, \xi d = 10$.

It is noticed from Fig. 9 that for ($n=0$) skew symmetric nonleaky lamb wave mode of propagation, the magnitude of attenuation coefficient for LANLSK remain more than the values for LNLSK in the whole region except $\xi d = 7$. It is observed that for ($n=1$), NLSK attains maximum value 0.00164 at $\xi d = 2$. For ($n=2$) mode LNLSK and LANLSK attains maximum value 0.0007076 at $\xi d = 3$.

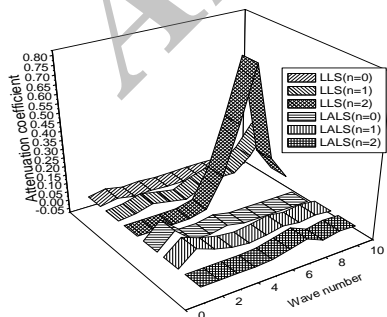


Fig .6
Variation of attenuation coefficient for symmetric leaky lamb waves.

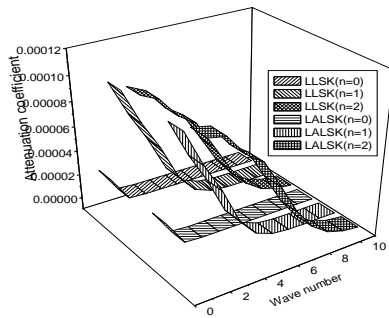


Fig .7
Variation of attenuation coefficient for skew-symmetric leaky lamb waves.

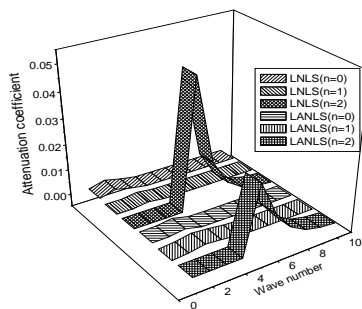


Fig .8
Variation of attenuation coefficient for symmetric nonleaky lamb waves.

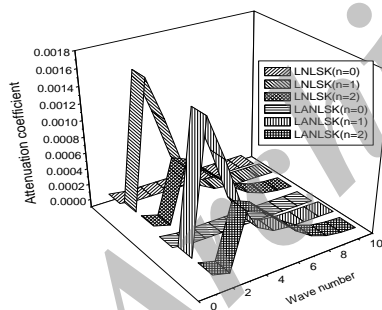


Fig .9
Variation of attenuation coefficient for skew-symmetric nonleaky lamb waves.

8.3 Amplitudes

In Figs. 10 to 15, $GT(a=0.5)$ represents the amplitude for micropolar thermoelastic solid with two temperatures and $TS(a=0)$ represents the amplitude for micropolar thermoelastic solid.

Figs. 10 to 11 shows the variations of symmetric and skew-symmetric amplitudes of dilatation for L-S theory for stress free thermally insulated boundary. The dilatation is minimum at the centre and maximum at the surfaces for symmetric and skew-symmetric modes. Also the dilatation for $GT(a=0)$ remain more than the dilatation for $TS(a=0.5)$ in the whole region.

It is evident from Figs. 12 to 13 that the amplitude of symmetric microrotation is minimum at the centre and the surfaces and attain maximum value in the region between centre and surface. The amplitude of skew-symmetric microrotation is maximum at the surfaces.

The amplitude of symmetric and skew-symmetric temperature is minimum at the centre and maximum at the surfaces as shown in Figs. 14 and 15. Also the amplitude of symmetric and skew-symmetric temperature for $GT(a=0.5)$ is greater than the amplitude for $TS(a=0)$.

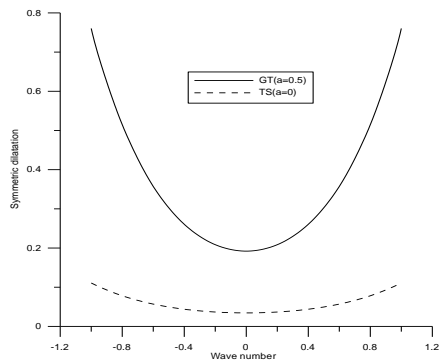


Fig .10
Amplitude of symmetric dilatation.

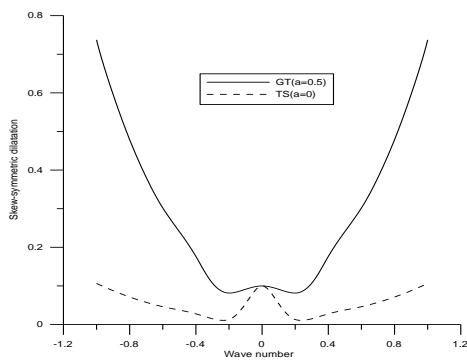


Fig .11
Amplitude of skew-symmetric dilatation.

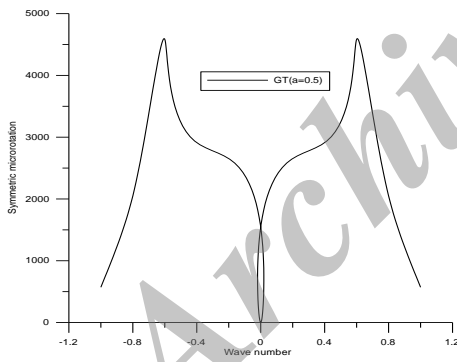


Fig .12
Amplitude of symmetric microrotation.

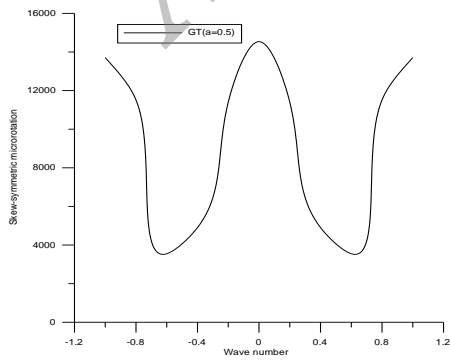


Fig .13
Amplitude of skew-symmetric microrotation.

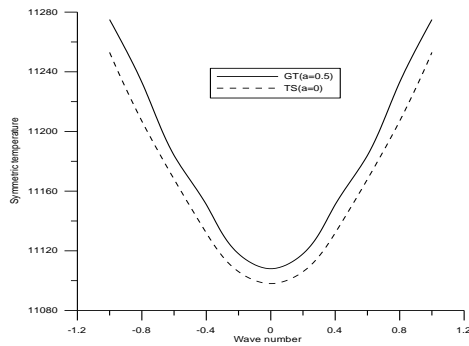


Fig .14
Amplitude of symmetric Temperature T .

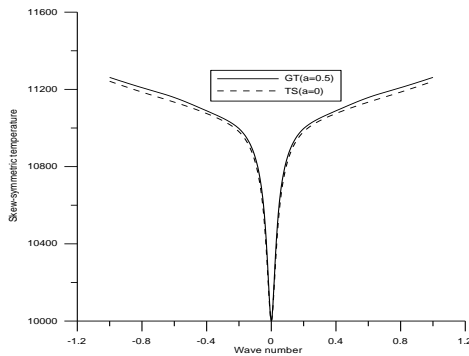


Fig .15
Amplitude of skew-symmetric Temperature T .

9 CONCLUSIONS

It is noticed that the variation of phase velocities of lowest symmetric and skew-symmetric mode for leaky and nonleaky Lamb waves shows slight variation in the intermediate range and then coincide with increase in wave number. Also the phase velocities for higher symmetric and skew-symmetric mode attain maximum value at vanishing wave number and as wave number increase the phase velocities decrease sharply. The values of attenuation coefficient for ($n=2$) symmetric leaky wave mode for LLS remain higher than in case of LALS. It is noticed that the values of attenuation coefficient for lowest symmetric and skew-symmetric mode for leaky and non leaky Lamb waves are very small as compared to the values for highest mode. The values of symmetric and skew-symmetric dilatation in case of $GT(a=0.5)$ are higher in comparison to $TS(a=0)$ and the values of symmetric and skew-symmetric temperature in case of $GT(a=0.5)$ are greater than in case of $TS(a=0)$.

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